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Effective Mass Number in Hadron-Nucleus Collisions at High Energies

Fujio TAKAGI

Department of Physics, Tohoku University, Sendai 980

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In some recent models for hadron-nucleus collisions at high energies, it is assumed that only A_e nucleons in the nucleus with mass number A interact as "a big hadron" with the incident hadron and the residual $A-A_e$ nucleons pass through as spectators. In this paper, two alternative formulae to calculate the distribution of the effective mass number A_e or its average $\langle A_e\rangle$ are derived. Distinction between A_e and ν, the number of hadron-nucleon inelastic collisions which the incident hadron experiences in passing the nucleus, is stressed. In fact, we obtain the result that $\langle A_e\rangle_{pA}\simeq(3.4\sim4.3)\langle\nu\rangle_{pA}$ for $A\gg1$ under a reasonable approximation. A confusion recently raised by considering that $A_e\equiv\nu$ is thereby settled.

§ 1. Introduction

It has recently been proposed by several authors that, in hadron (h)-nucleus (A) collisions at high energies, only A_e nucleons in the nucleus with mass number A interact as "a big hadron" with the incident hadron and the residual $A-A_e$ nucleons pass through as spectators.[1)~4)] The idea is illustrated in Figs. 1 (a) and (b), which show the situation just before and after the collision, respectively. The effective mass number A_e is the number of the constituent nucleons which are contained in a tube of radius ρ_h along the path of the incident hadron.

A theoretical estimate of the mean effective mass number $\langle A_e\rangle$ has been made by Berlad et al. by using the following formula for the probability distribution of $A_e (A_e=i$ in their notation):[2)]

$$P(A_e, A)=\frac{\displaystyle\int d^2\boldsymbol{b}\binom{A}{A_e}\left\{\frac{\sigma_h\rho_A(\boldsymbol{b})}{A}\right\}^{A_e}\left\{1-\frac{\sigma_h\rho_A(\boldsymbol{b})}{A}\right\}^{A-A_e}}{\displaystyle\int d^2\boldsymbol{b}\left[1-\left\{1-\frac{\sigma_h\rho_A(\boldsymbol{b})}{A}\right\}^{A}\right]}, \qquad (1\cdot1)$$

Fig. 1. (a) Hadron and nucleus just before they collide.
(b) Hadron, nucleus and the excited hadronic matter just after the collision.

where $\sigma_h \equiv \pi \rho_h^2$, $\boldsymbol{b}$ is the impact parameter vector, and $\rho_A(\boldsymbol{b})$ is the density distribution of the centers of mass of the nucleons in the nucleus A projected onto the transverse plane (the plane perpendicular to the incident momentum) and is normalized as

$$\int d^2\boldsymbol{b}\rho_A(\boldsymbol{b}) = A\,. \tag{1·2}$$

The point to be noted here is the fact that (1·1) turns out to be equivalent to the formula to calculate the probability distribution of another quantity ν, the number of hadron-nucleon inelastic collisions in the nucleus *if one identifies* σ_h *with the hadron-nucleon inelastic cross section* $\sigma_{\rm inel}^{hN}$. This is apparently the reason why several authors[4),5)] have thought that $A_{\rm e} \equiv \nu$. However, the following argument clearly shows that $A_{\rm e}$ introduced by Berlad et al.[2)] on the basis of the formula (1·1) is entirely different from ν. Berlad et al. obtained the general result:

$$R_{hA} \equiv \langle n_{\rm s}\rangle_{hA} / \langle n_{\rm ch}\rangle_{hp} = \langle A_{\rm e}^{1/4}\rangle_{hA}\,, \tag{1·3}$$

where $\langle n_{\rm s}\rangle_{hA}$ and $\langle n_{\rm ch}\rangle_{hp}$ are the mean shower particle multiplicity in h-A collisions and the mean charged hadron multiplicity in h-proton collisions at the same incident hadron energy, respectively. Applying (1·3) to the collisions of protons with emulsion nuclei, they obtained

$$R_{p\text{-emulsion}} = \langle A_{\rm e}^{1/4}\rangle_{p\text{-em}} \simeq 1.65 \tag{1·4}$$

in good agreement with experiments. From (1·4) one has

$$\langle A_{\rm e}^{1/4}\rangle_{p\text{-em}}^4 \simeq 7.4 \lesssim \langle A_{\rm e}\rangle_{p\text{-em}}\,. \tag{1·5}$$

On the other hand, $\langle \nu\rangle_{hA}$, the average of ν in h-A collisions is in general given by

$$\langle \nu\rangle_{hA} = A\sigma_{\rm inel}^{hN} / \sigma_{\rm inel}^{hA}\,, \tag{1·6}$$

where $\sigma_{\rm inel}^{hN}$ and $\sigma_{\rm inel}^{hA}$ are the h-nucleon inelastic cross section and h-A inelastic cross section, respectively. From (1·6) together with the data on $\sigma_{\rm inel}^{pp}$ and $\sigma_{\rm inel}^{pA}$,[6)] we have

$$\langle \nu\rangle_{p\text{-em}} \simeq 2.4\,. \tag{1·7}$$

By comparing (1·7) with (1·5), we conclude that $\langle A_{\rm e}\rangle$ in Berlad et al.'s model[2)] is about three times larger than $\langle \nu\rangle$ and hence $A_{\rm e} \not\equiv \nu$ in their model.*)

Phenomenological estimate of $\langle A_{\rm e}\rangle_{hA}$ has been given by the present author by analyzing $\langle n_{\rm s}\rangle_{hA}$[3),7)] and by Fredriksson by analyzing the production at large transverse momentum in h-A collisions[4)] with different conclusions that $\langle A_{\rm e}\rangle_{hA} > \langle \nu\rangle_{hA}$[3),7)] and $\langle A_{\rm e}\rangle_{hA} \simeq \langle \nu\rangle_{hA}$.[4)]

There is another way to estimate $\langle A_{\rm e}\rangle$. In certain experiments, one can measure $\langle N_p\rangle$, the mean number of the knocked-out proton per inelastic h-A collision.

*) This conclusion was confirmed by private communication from Dr. G. Eilam.

Then it may be reasonable to estimate $\langle A_e \rangle$ as[2)]

$$\langle A_e \rangle \simeq A \langle N_p \rangle / Z\,, \tag{1·8}$$

where Z is the atomic number of the nucleus A. Equation (1·8) with the experimental value of $\langle N_p \rangle$ yields

$$\langle A_e \rangle_{p\text{-em}} \simeq 19\,, \tag{1·9}$$

which is very large compared with $\langle \nu \rangle_{p\text{-em}} \simeq 2.4$.

We have a variety of possible values of $\langle A_e \rangle$ as discussed above. It is, therefore, important to establish an unambiguous method to calculate the A_e-distribution and/or $\langle A_e \rangle$. The purpose of this paper is to study various methods to calculate them with special emphasis on the distinction between A_e and ν. The general formula to calculate the probability distribution of ν is given in § 2. In § 3, the radius ρ_h of "the tube" is given in terms of the equivalent (transverse) radii of h and the nucleon. A formula to calculate the A_e-distribution is derived in § 4. An alternative formula to calculate $\langle A_e \rangle$ is given in § 5. Discussion and summary are given in § 6. The definition of the equivalent radius is given in the Appendix.

§ 2. General formula for ν-distribution

Since the distinction between ν and A_e is one of the main points of this paper, it is convenient to derive the general formula for the ν-distribution before entering into the study of the A_e-distribution or $\langle A_e \rangle$. The standard approximations of the Glauber theory[8)] are used in this section to derive the formula.

Consider a hadron h which is incident upon a nucleus A with the relative impact parameter $\boldsymbol{b}$. The origin of the two-dimensional impact parameter plane is set at the center of mass of the nucleus. The probability to find the center of mass of the k-th nucleon in the nucleus in an infinitesimally small region $d^2\boldsymbol{b}'$ around $\boldsymbol{b}'$ is given by $A^{-1}d^2\boldsymbol{b}'\rho_A(\boldsymbol{b}')$, which is really independent of k. Then the probability that the inelastic collision takes place between the hadron h incident with $\boldsymbol{b}$ and the k-th constituent nucleon in the region $d^2\boldsymbol{b}'$ around $\boldsymbol{b}'$ is given by $4A^{-1}d^2\boldsymbol{b}'\rho_A(\boldsymbol{b}')f_{hN}(\boldsymbol{b}-\boldsymbol{b}')$, where $f_{hN}(\boldsymbol{b})$ is the inelastic overlap function for h-nucleon elastic scattering, normalized as

$$4\int d^2\boldsymbol{b} f_{hN}(\boldsymbol{b}) = \sigma_{\text{inel}}^{hN}\,. \tag{2·1}$$

Hence the total probability that the inelastic collision occurs between h with the impact parameter $\boldsymbol{b}$ and the k-th nucleon in the nucleus turns out to be

$$p_{hA}(\boldsymbol{b}) = 4A^{-1}\int d^2\boldsymbol{b}'\rho_A(\boldsymbol{b}')f_{hN}(\boldsymbol{b}-\boldsymbol{b}')\,. \tag{2·2}$$

Now, it is easy to derive the probability that ν h-nucleon inelastic collisions take place per h-A inelastic collision. The result is

$$\bar{P}(\nu, A) = (\sigma_{\text{inel}}^{hA})^{-1} \int d^2\boldsymbol{b} \binom{A}{\nu} \{p_{hA}(\boldsymbol{b})\}^{\nu} \{1 - p_{hA}(\boldsymbol{b})\}^{A-\nu}, \tag{2·3}$$

where

$$\sigma_{\text{inel}}^{hA} = \int d^2\boldsymbol{b}[1 - \{1 - p_{hA}(\boldsymbol{b})\}^A]. \tag{2·4}$$

From (2·3), we obtain the well-known relation for $\langle\nu\rangle_{hA}$:

$$\begin{aligned}\langle\nu\rangle_{hA} &= \sum_{\nu=1}^{A} \nu\bar{P}(\nu, A) \\ &= A\sigma_{\text{inel}}^{hN}/\sigma_{\text{inel}}^{hA}, \end{aligned} \tag{2·5}$$

which is the same as (1·6).

§ 3. Radius of tube

The geometrical picture illustrated in Figs. 1(a) and (b) apparently suggests that the radius ρ_h of the tube is the most crucial factor in the determination of the A_e-distribution or $\langle A_e\rangle$. If the hadron h (the constituent nucleon N) is regarded as an extended object with radius R_h (R_N), there may be an appreciable interaction between h and N when the relative impact parameter is less than $R_h + R_N$. Then the reasonable assignment of ρ_h is

$$\rho_h = R_h + R_N, \tag{3·1}$$

and A_e is defined as the number of the constituent nucleons of which centers of mass are contained in the tube. The situation is illustrated in Fig. 2.

There are two possible choices for R_h. One is r_h, the usual equivalent radius defined in the three-dimensional space, and another is b_h, the equivalent transverse radius defined in the two-dimensional impact parameter plane. Their definitions are given in the Appendix for completeness. They are related to each other by the relation

$$b_h = 2r_h/\sqrt{5} \tag{3·2}$$

so that the two alternatives do not produce so large numerical difference. For example, we have

$$r_N \simeq 1.04 \text{ fm} \tag{3·3}$$

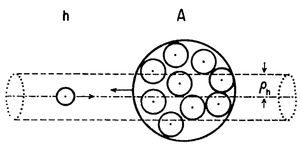

Fig. 2. Schematic illustration of the mean effective mass number in h-A collisions.

and

$$b_N \simeq 0.93\ \mathrm{fm} \tag{3·4}$$

by assuming that the hadronic matter distribution of the proton is proportional to the electric charge distribution. The base area $\sigma_N = \pi\rho_N^2 = 4\pi R_N^2$ of the tube in proton-nucleus collisions is now determined as

$$\sigma_N \simeq \binom{136}{109}\mathrm{mb} \quad \text{when} \quad R_N = \binom{r_N}{b_N}. \tag{3·5}$$

It should be noted here that σ_N is considerably larger than $\sigma_{\rm inel}^{NN}$ which is for instance about 32 mb in FNAL region.*)

§ 4. Formula for A_e-distribution

We derive a formula for the probability distribution of A_e by consideration which is analogous to that in § 2. The probability that the k-th nucleon in the nucleus lies within the tube when h is incident upon the nucleus at the impact parameter $\boldsymbol{b}$ is given by

$$\tilde{\rho}_A(\boldsymbol{b}, \rho_h)/A = A^{-1}\int_{|\boldsymbol{b}'|\leq\rho_h} d^2\boldsymbol{b}'\rho_A(\boldsymbol{b}+\boldsymbol{b}'), \tag{4·1}$$

where ρ_h is the radius of the tube. Since we are going to obtain the A_e-distribution for h-A inelastic collisions, we have to calculate the probability that h-A inelastic collision takes place, that is, at least one h-nucleon inelastic collision takes place when A_e nucleons are contained in the tube. Since it is difficult to derive the exact result for the probability and also we have already made an approximation that the tube has the definite radius ρ_h, we make another approximation (the grey disk approximation)

$$f_{hN}(\boldsymbol{b}) \simeq (\sigma_{\rm inel}^{hN}/4\pi b_{hN}^2)\,\theta(b_{hN}-|\boldsymbol{b}|), \tag{4·2}$$

where b_{hN} is the equivalent transverse radius of the inelastic overlap function for hN elastic scattering, i.e.,

$$b_{hN} = (2\langle b^2\rangle_{hN})^{1/2}, \tag{4·3}$$

with $\langle b^2\rangle_{hN}$ being the mean square impact parameter of $f_{hN}(\boldsymbol{b})$ which is to be inferred from experiments. For example, an analysis by Sakai and White gives $\langle b^2\rangle_{pp} \simeq (0.98\ \mathrm{fm})^2$ at $s=930\ \mathrm{GeV}^2$,[10] from which we have

$$b_{pN} \simeq 1.38\ \mathrm{fm}\,. \tag{4·4 **}$$

*) From the view-point of our big hadron picture for nucleus, there is no compelling reason to believe that $\sigma_h = \sigma_{\rm inel}^{hN}$ or πb_{hN}^2, where b_{hN} is the equivalent transverse radius of the inelastic overlap function for h-N elastic scattering. See § 4 also.

**) The Chou-Yang picture for proton-proton elastic scattering at high energies with Eqs. (A·4), (A·6) and (4·3) yields $b_{NN} \simeq 2\sqrt{2}r_N/\sqrt{5} = 1.32$ fm for $r_N = 1.04$ fm, being consistent with (4·4). Note that b_{NN} is by no means equal to $2b_N$ or $2r_N$.

Now the probability that A_e nucleons are contained in the tube when h collides with the nucleus at the impact parameter $\boldsymbol{b}$ is

$$\binom{A}{A_e}\left\{\frac{\tilde{\rho}_A(\boldsymbol{b},\rho_h)}{A}\right\}^{A_e}\left\{1-\frac{\tilde{\rho}_A(\boldsymbol{b},\rho_h)}{A}\right\}^{A-A_e},$$

while the probability that a certain nucleon in the tube with the radius ρ_h lies within the cocentric tube with the radius b_{hN} is given by $\tilde{\rho}_A(\boldsymbol{b},b_{hN})/\tilde{\rho}_A(\boldsymbol{b},\rho_h)$ and hence the probability for the h-A inelastic collision at the impact parameter $\boldsymbol{b}$ for a fixed A_e is

$$1-\left\{1-\frac{\sigma_{\text{inel}}^{hN}\tilde{\rho}_A(\boldsymbol{b},b_{hN})}{\pi b_{hN}^2\tilde{\rho}_A(\boldsymbol{b},\rho_h)}\right\}^{A_e}. \tag{4·5}$$

The probability distribution for A_e is thus given by

$$P(A_e,A)=(\sigma_{\text{inel}}^{hA})^{-1}\int d^2\boldsymbol{b}\binom{A}{A_e}\left[1-\left\{1-\frac{\sigma_{\text{inel}}^{hN}\tilde{\rho}_A(\boldsymbol{b},b_{hN})}{\pi b_{hN}^2\tilde{\rho}_A(\boldsymbol{b},\rho_h)}\right\}^{A_e}\right]$$
$$\times\left\{\frac{\tilde{\rho}_A(\boldsymbol{b},\rho_h)}{A}\right\}^{A_e}\left\{1-\frac{\tilde{\rho}_A(\boldsymbol{b},\rho_h)}{A}\right\}^{A-A_e}, \tag{4·6}$$

where

$$\sigma_{\text{inel}}^{hA}=\int d^2\boldsymbol{b}\left[1-\left\{1-\frac{\sigma_{\text{inel}}^{hN}\tilde{\rho}_A(\boldsymbol{b},b_{hN})}{\pi b_{hN}^2 A}\right\}^{A}\right]. \tag{4·7}$$

When the nucleus is so big that $\rho_A(\boldsymbol{b}+\boldsymbol{b}')$ for $|\boldsymbol{b}'|\leq\rho_h$ and b_{hN} is approximately given by $\rho_A(\boldsymbol{b})$, we have $\tilde{\rho}_A(\boldsymbol{b},x)\simeq\pi x^2\rho_A(\boldsymbol{b})$ for $x=\rho_h$ and b_{hN}, and hence

$$P(A_e,A)\simeq(\sigma_{\text{inel}}^{hA})^{-1}\int d^2\boldsymbol{b}\binom{A}{A_e}\left\{1-\left(1-\frac{\sigma_{\text{inel}}^{hN}}{\sigma_h}\right)^{A_e}\right\}$$
$$\times\left\{\frac{\sigma_h\rho_A(\boldsymbol{b})}{A}\right\}^{A_e}\left\{1-\frac{\sigma_h\rho_A(\boldsymbol{b})}{A}\right\}^{A-A_e}, \tag{4·8}$$

where

$$\sigma_h=\pi\rho_h{}^2 \text{ (the base area of the tube)}, \tag{4·9}$$

$$\sigma_{\text{inel}}^{hA}\simeq\int d^2\boldsymbol{b}\left[1-\left\{1-\frac{\sigma_{\text{inel}}^{hN}\rho_A(\boldsymbol{b})}{A}\right\}^{A}\right]. \tag{4·10}$$

Now, $\langle A_e\rangle$ is easily obtained from (4·8):

$$\langle A_e\rangle_{hA}\simeq(\sigma_{\text{inel}}^{hA})^{-1}\sigma_h\int d^2\boldsymbol{b}\rho_A(\boldsymbol{b})\left[1-\left(1-\frac{\sigma_{\text{inel}}^{hN}}{\sigma_h}\right)\left\{1-\frac{\sigma_{\text{inel}}^{hN}\rho_A(\boldsymbol{b})}{A}\right\}^{A-1}\right]. \tag{4·11}$$

Here, it is worthwhile to compare the above results (4·8)~(4·10) with the formula (1·1) which is used by Berlad et al.[2] It is obvious that both are identical with each other if $\sigma_h=\sigma_{\text{inel}}^{hN}$. As a matter of fact, σ_h is larger than $\sigma_{\text{inel}}^{hN}$, for example, $\sigma_p\simeq109\sim136$ mb (see (3·5)) while $\sigma_{\text{inel}}^{pp}\simeq32$ mb. Then, in general, a significant difference comes in through the weight factor $1-\{1-(\sigma_{\text{inel}}^{hN}/\sigma_h)\}^{A_e}$ on the r.h.s. of (4·8), which is missing in (1·1). This weight factor is necessary to calculate the correct probability distribution of A_e in h-A inelastic collisions. Since the weight factor is a monotonically increasing function of A_e when $0<\sigma_{\text{inel}}^{hN}/\sigma_h<1$, it increases the value of $\langle A_e\rangle$ compared with the case when there is no such factor as in (1·1).

For illustration, we make further approximations:

$$\left\{1-\frac{\sigma_{\text{inel}}^{hN}\rho_A(\boldsymbol{b})}{A}\right\}^{A-1}\simeq\exp\{-\sigma_{\text{inel}}^{hN}\rho_A(\boldsymbol{b})\}\,, \tag{4·12}$$

which should be valid when $A\gg1$, and

$$\hat{\rho}_A(\boldsymbol{r})=\frac{3}{4\pi r_0^3}\theta(R_A-|\boldsymbol{r}|)\,, \tag{4·13}$$

where $\hat{\rho}_A(\boldsymbol{r})$ is the density distribution of the center of mass of the nucleons in the nucleus A and $R_A=r_0A^{1/3}$ is the equivalent radius of the nucleus A. From (4·11)～(4·13), we obtain

$$\langle A_e\rangle_{hA}\simeq\langle\nu\rangle_{hA}\sigma_h/\sigma_{\text{inel}}^{hN}\quad\text{for}\quad A^{1/3}\gg1\,, \tag{4·14}$$

where $\langle\nu\rangle_{hA}$ is given by (2·5). For proton-nucleus collisions, we have

$$\langle A_e\rangle_{pA}\simeq(3.4\sim4.3)\times\langle\nu\rangle_{pA}\quad\text{for}\quad A^{1/3}\gg1\,. \tag{4·15}$$

§ 5. Alternative formula for $\langle A_e\rangle$

In this section, we derive an alternative formula for $\langle A_e\rangle$ without recourse to the probability distribution of A_e. Consider $\hat{\rho}_{mA}(\boldsymbol{r})$, the hadronic matter density of the nucleus A, and $\rho_{mA}(\boldsymbol{b})$, i.e., its projection onto the transverse plane:

$$\rho_{mA}(\boldsymbol{b})=\int_{-\infty}^{\infty}dz\hat{\rho}_{mA}(\sqrt{\boldsymbol{b}^2+z^2})\,, \tag{5·1}$$

where $\hat{\rho}_{mA}(\boldsymbol{r})$ is assumed to be spherically symmetric, i.e., $\hat{\rho}_{mA}(\boldsymbol{r})=\hat{\rho}_{mA}(|\boldsymbol{r}|)$. The normalization is

$$\int d^2\boldsymbol{b}\rho_{mA}(\boldsymbol{b})=A\,. \tag{5·2}$$

The hadronic matter contained in the tube of radius ρ_h is given by

$$\tilde{\rho}_{mA}(\boldsymbol{b},\rho_h)=\int_{|\boldsymbol{b}'|\le\rho_h}d^2\boldsymbol{b}'\rho_{mA}(\boldsymbol{b}+\boldsymbol{b}') \tag{5·3}$$

for h-A collision at the impact parameter $\boldsymbol{b}$. The probability that h-A inelastic collision takes place may be given by

$$1-\exp\{-\omega O_{hA}(\boldsymbol{b})\}\,, \tag{5·4}$$

where $O_{hA}(\boldsymbol{b})$ is the hadronic matter overlap:

$$O_{hA}(\boldsymbol{b}) = \int d^2\boldsymbol{b}_h \int d^2\boldsymbol{b}_A \delta^{(2)}(\boldsymbol{b}+\boldsymbol{b}_h-\boldsymbol{b}_A)\,\rho_{mh}(\boldsymbol{b}_h)\,\rho_{mA}(\boldsymbol{b}_A)\,, \tag{5·5}$$

and ω (which is independent of $\boldsymbol{b}$) is determined by the following relation:

$$\sigma_{\text{inel}}^{hA} = \int d^2\boldsymbol{b}[1-\exp\{-\omega O_{hA}(\boldsymbol{b})\}]. \tag{5·6}$$

Here, the idea of Chou-Yang model[10] has been used. The expectation value of the hadronic matter cut by the tube per h-A inelastic collision is then given by

$$(\rho_{\text{inel}}^{hA})^{-1}\int d^2\boldsymbol{b}\tilde{\rho}_{mA}(\boldsymbol{b},\rho_h)\,[1-\exp\{-\omega O_{hA}(\boldsymbol{b})\}].$$

It is reasonable to assume that $\langle A_e\rangle_{hA}$ is proportional to the above quantity. Then, by normalizing $\langle A_e\rangle_{hN}$ to unity, we have

$$\langle A_e\rangle_{hA} = \frac{\sigma_{\text{inel}}^{hN}\int d^2\boldsymbol{b}\tilde{\rho}_{mA}(\boldsymbol{b},\rho_h)\,[1-\exp\{-\omega O_{hA}(\boldsymbol{b})\}]}{\sigma_{\text{inel}}^{hA}\int d^2\boldsymbol{b}\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)\,[1-\exp\{-\omega O_{hN}(\boldsymbol{b})\}]}\,, \tag{5·7}$$

where $\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)=\tilde{\rho}_{mA}(\boldsymbol{b},\rho_h)|_{A=1}$.

For illustration we consider the case when

$$A\gg 1 \quad \text{so that} \quad R_A\gg\rho_h\,, \tag{5·8}$$

$$\rho_{mA}(\boldsymbol{r})\simeq\frac{3}{4\pi r_0^3}\theta(R_A-|\boldsymbol{r}|) \tag{5·9}$$

and

$$1-\exp\{-\omega O_{hA}(\boldsymbol{b})\}\simeq\frac{\sigma_{\text{inel}}^{hA}}{\pi b_{hA}^2}\theta(b_{hA}-|\boldsymbol{b}|)\,, \tag{5·10}$$

where b_{hA} is the equivalent radius of the inelastic overlap function for h-A elastic scattering and is expected to be between R_A and R_A+R_h. The integral on the numerator on the r.h.s. of (5·7) is now approximately given by $A\sigma_h\sigma_{\text{inel}}^{hA}/(\pi b_{hA}^2)$. On the other hand, $0\leq\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)\leq 1$ in general because $\lim_{\rho_h\to\infty}\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)=1$ and $\rho_{mN}(\boldsymbol{b})$ is definitely non-negative. In particular, $\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)\simeq 1$ for $0\leq|\boldsymbol{b}|\leq R_h$, $0<\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)<1$ for $R_h<|\boldsymbol{b}|<\rho_h+R_N=2R_N+R_h$, and $\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)\simeq 0$ for $2R_N+R_h<|\boldsymbol{b}|$, from which we have

$$\int d^2\boldsymbol{b}\tilde{\rho}_{mN}(\boldsymbol{b},\rho_h)\,[1-\exp\{-\omega O_{hN}(\boldsymbol{b})\}] \lesssim \int d^2\boldsymbol{b}[1-\exp\{-\omega O_{hN}(\boldsymbol{b})\}]=\sigma_{\text{inel}}^{hN}\,.$$

Combining these approximations with (5·7), we obtain

$$\langle A_e\rangle_{hA}\gtrsim\frac{A\sigma_h}{\pi b_{hA}^2}\,. \tag{5·11}$$

The black disk model is experimentally a fairly good approximation for collisions of the nucleon with heavy nuclei, which means that $\sigma_{\text{inel}}^{pA}/\pi b_{pA}^2 \simeq 1$. Hence we obtain

$$\langle A_{\text{e}}\rangle_{pA} \gtrsim \frac{A\sigma_p}{\sigma_{\text{inel}}^{pA}} = \langle\nu\rangle_{pA}\sigma_p/\sigma_{\text{inel}}^{pN}, \tag{5·12}$$

which is nearly the same as (4·14).

§ 6. Discussion and summary

We have derived two formulae to calculate the A_{e}-distribution and/or $\langle A_{\text{e}}\rangle$. In obtaining the first formula (4·6), the Glauber approximation has been used for convenience to determine the probability of the h-A inelastic collisions, (4·5). It provides a definite method to calculate the A_{e}-distribution. However, the concept of consecutive collisions involved in the Glauber approximation may rather conflict with the big hadron picture of the nucleus. Collisions of the usual hadron with the "big hadron" may not be described by the multiple scattering picture of the Glauber type. On the other hand, the second formula (5·7) has been obtained by using the idea of Chou-Yang model. Here, the nucleus, i.e., "the big hadron" is treated on the same footing as the usual hadron. The big hadron is different from the usual hadron only in the point that it has more hadronic matter and is bigger than the usual hadron. Therefore, the second method appears quite adequate for the big hadron picture. However, it is the defect of the second method that it does not provide the method to calculate the A_{e}-distribution. Anyway, the two different methods give numerically similar results (4·14) and (5·12) for the heavy nuclei.

Since $\langle\nu\rangle_{p\text{-emulsion}} \simeq 2.4$, we have

$$\langle A_{\text{e}}\rangle_{p\text{-emulsion}} \simeq 8 \sim 10 \tag{6·1}$$

by using (4·15). This figure is fairly smaller than 19, the value given in (1·9), which is obtained by using (1·8). There are at least three interpretations for this discrepancy: (i) The formula (1·8) is invalid; (ii) in the measurement of N_p (or N_{h}, the number of heavily ionizing prongs in emulsion experiments), not only the knocked-out protons but also the protons evaporated from the spectator fragment are counted; (iii) in emulsion experiments, a part of the heavily ionizing prongs may be due to slow pions. Unfortunately, we cannot choose any of these interpretations at present. Rather, all the three interpretations may be partially relevant.

It is crucial in getting $\langle A_{\text{e}}\rangle_{hA}$ which is much larger than $\langle\nu\rangle_{hA}$ that σ_h, the base area of the tube is much greater than $\sigma_{\text{inel}}^{hN}$. Berlad et al. have assumed that $\sigma_p = 2\pi r_0^2$ in (1·1).[2] Since $r_0 = (1.25 \pm 0.05)$ fm,[11] this means that $\sigma_p = (98 \pm 7)$ mb, which is quite large compared with $\sigma_{\text{inel}}^{pN}$. This is the reason why they have obtained so large $\langle A_{\text{e}}\rangle$ (see (1·5)) by using (1·1).

According to our formalism, the radius ρ_h of the tube is identified with the sum of the equivalent radii R_h and R_N (see (3·1)). Hence ρ_h is independent of the incident energy. However, if $\sigma_{\rm inel}^{hN}$ continues to grow with energy, ρ_h may have to be eventually replaced by an appropriate energy dependent radius $\rho_h(s)$ at such high energies where $\rho_h < b_{hN} = b_{hN}(s)$. A plausible choice for $\rho_h(s)$ is $\rho_h(s) = b_{hN}(s)$.

The results of the present paper are summarized as follows.

(i) Two alternative formulae have been derived to calculate the $A_{\rm e}$-distribution and/or $\langle A_{\rm e}\rangle$. Equations (4·6) and (5·7) are the main general results.

(ii) In particular, we have $\langle A_{\rm e}\rangle_{hA} \gtrsim \langle\nu\rangle_{hA}\sigma_h/\sigma_{\rm inel}^{hN}$ for $A\gg 1$, where σ_h is the base area of the tube with the radius $\rho_h = (\sigma_h/\pi)^{1/2} = R_h + R_N$ with R_h and R_N being the equivalent matter radii of the hadron h and the nucleon, respectively. Such a large value of $\langle A_{\rm e}\rangle_{hA}$ compared to $\langle\nu\rangle_{hA}$ (see (4·15)) is at least qualitatively consistent with our previous results[3),7)] of the analysis of the mean multiplicity as well as those obtained by Berlad et al.[2)] (Remember the discussion in § 1.)

(iii) The exact expression for the ν-distribution has been derived (Eq. (2·3)) within the framework of the Glauber approximation. It has been compared with the formula (4·6) for the $A_{\rm e}$-distribution, and the difference between ν and $A_{\rm e}$ has been thereby clarified.

Appendix

In terms of $\hat\rho_{{\rm m}h}(\boldsymbol{r})$, the spherically symmetric distribution function of the hadronic matter of h, the usual mean square radius $\langle r^2\rangle_h$ and the corresponding mean square transverse radius $\langle b^2\rangle_h$ are defined as

$$\langle r^2\rangle_h = \int d^3\boldsymbol{r}\, \boldsymbol{r}^2 \hat\rho_{{\rm m}h}(\boldsymbol{r}) \Big/ \int d^3\boldsymbol{r}\, \hat\rho_{{\rm m}h}(\boldsymbol{r}), \tag{A·1}$$

$$\langle b^2\rangle_h = \int d^2\boldsymbol{b}\, \boldsymbol{b}^2 \rho_{{\rm m}h}(\boldsymbol{b}) \Big/ \int d^2\boldsymbol{b}\, \rho_{{\rm m}h}(\boldsymbol{b}), \tag{A·2}$$

with

$$\rho_{{\rm m}h}(\boldsymbol{b}) = \int_{-\infty}^{\infty} dz\, \hat\rho_{{\rm m}h}(\sqrt{\boldsymbol{b}^2 + z^2}) \tag{A·3}$$

and $\hat\rho_{{\rm m}h}(\boldsymbol{r}) = \hat\rho_{{\rm m}h}(|\boldsymbol{r}|)$. The equivalent (transverse) radius r_h (b_h) is defined as the radius of the uniform distribution in the three-dimensional (two-dimensional impact parameter) space which gives the same mean square (transverse) radius $\langle r^2\rangle_h$ ($\langle b^2\rangle_h$). Then an elementary calculation leads to the following results:

$$r_h = (5\langle r^2\rangle_h/3)^{1/2}, \tag{A·4}$$

$$b_h = (2\langle b^2\rangle_h)^{1/2}, \tag{A·5}$$

$$\langle b^2\rangle_h = \tfrac{2}{3}\langle r^2\rangle_h, \tag{A·6}$$

from which we have

$$b_h = 2r_h/\sqrt{5}. \tag{A·7}$$

References

1) A. Z. Patashinskii, JETP Letters **19** (1974), 338.
2) G. Berlad, A. Dar and G. Eilam, Phys. Rev. **D13** (1976), 161.
3) F. Takagi, Nuovo Cim. Letters **14** (1975), 559.
4) S. Fredriksson, Nucl. Phys. **B111** (1976), 167.
5) A. Białas, W. Czyz and W. Czyz, Phys. Letters **58B** (1975), 325.
6) S. P. Denisov et al., Nucl. Phys. **B61** (1973), 62.
7) F. Takagi, Prog. Theor. Phys. **57** (1977), 939.
8) R. J. Glauber, *Lectures in Theoretical Physics* (Interscience Publ. Co., New York, 1959), p. 315.
9) N. Sakai and J. N. J. White, Nucl. Phys. **B59** (1973), 511.
10) T. T. Chou and C. N. Yang, Phys. Rev. **170** (1968), 1591.
11) A. P. Arya, *Fundamentals of Nuclear Physics,* (Allyn and Bacon, Inc., Boston, 1966).