A Novel Receive Algorithm for Clipped OFDM Signals

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***Abstract*— A major drawback of the *Orthogonal Frequency-Division Multiplex* (OFDM) principle is the high dynamic range of the transmit signal. The transmit *Power Amplifier* (PA) and other elements in the signal path have to be operated at high *Input Power Backoff* (IBO), which decreases the efficiency of the PA and leads to higher costs.**

This paper presents a novel receive algorithm for OFDM signals with limited dynamic range. Similar to existing estimators for clipped real processes, a closed-form expression of a Bayesian estimator for more practically relevant clipped complex Gaussian processes, observed over the *Additive White Gaussian Noise* (AWGN) channel, is derived. The estimator is based on the distribution of the real and imaginary parts of the complex baseband OFDM signal. It recovers the original undistorted process by minimizing the mean-square estimation error. It is shown that this estimator significantly improves the error rate of OFDM systems impaired by clipping at the transmitter.

***Index Terms*— OFDM, nonlinear distortion, Bayesian estimation**

I. Introduction

The time-domain signal in an OFDM data transmission system is the superposition of many carriers by means of an *Inverse Discrete Fourier Transform* (IDFT). This results in an approximately Gaussian distribution of the I- and Q-components of the complex baseband signal because of the central limit theorem [1]. Consequently, OFDM systems require transmit and receive signal processing blocks with a high dynamic range, which leads to costly RF components. The high *Peak-to-Average Power Ratio* (PAPR) of the OFDM transmit signal is especially problematic for the PA, as PAs with a large linear dynamic range are less efficient than PAs with a smaller linear dynamic range for a given supply voltage level [2].

There has been active research in recent years in the area of preprocessing of OFDM signals for PAPR reduction. Usually, some form of coding is employed to make cophasing of many OFDM subcarriers impossible. The work in [3] and [4] are examples for this approach. Clearly, the additional redundancy of PAPR reduction coding causes a reduction in user data rate for the OFDM system. Another approach not limited to OFDM systems is signal predistortion. The memoryless nonlinear distortion characteristic $f(x)$ of, for example, the PA, is assumed to be known. Then, the transmit signal can be predistorted with the inverse of $f(x)$, provided it exists. Of course, such an inversion works only as far as the available supply voltage goes, which is an important consideration since lower and lower supply voltage levels are used in real devices. An overview of predistortion techniques can be found in [2].

This work follows a third approach: If some nonlinear distortion at the transmitter is allowed, the requirements on the RF frontend can be relaxed. Then, improved receive algorithms are needed to recover the information from the noisy, distorted receive signal. In [5], a Bayesian estimator for real-valued clipped Gaussian random processes observed over the AWGN channel was derived. This paper extends this approach to complex-valued Gaussian processes, which is the more general case relevant for IEEE 802.11a or DVB-T signals, for example. This extension is done for the usual case where the real and imaginary parts of the time-domain OFDM signal have a normal distribution with zero mean and equal variance σ_x^2. Memoryless clipping affects the *magnitude* of this process, and thus clipping of the real and imaginary parts depends on each other. This is the main difference to the approach in [5], and will be explained in more detail in the next section.

II. System and Channel Model

Let $x(t)$ be a zero-mean, stationary complex normal random process with equal variance σ_x^2 per component. When such a process is distorted by a memoryless nonlinear device, the output process $z(t)$ can be analyzed by means of the Bussgang theorem [6]. It states that a zero-mean stationary Gaussian process distorted by a memoryless nonlinearity can be written as the sum of two uncorrelated stationary processes. One part is a scaled replica of the input process, the other part is the distortion noise $n_d(t)$. Therefore,

$$z(t) = \alpha x(t) + n_d(t)\,, \tag{1}$$

where α is a generally complex scaling factor depending on $f(x)$ and σ_x^2. Throughout this paper, the memoryless nonlinearity $f(x)$ is assumed to be a soft limiter acting on $r_x(t) = |x(t)|$. The magnitude $r_z(t) = |z(t)|$ is then given by

$$r_z(r_x) = \begin{cases} r_x & r_x \le A \\ A & r_x > A \end{cases}\,. \tag{2}$$

A is the clip level of the nonlinear device. The soft limiter nonlinearity was chosen for two reasons: First, it is a simple and general characteristic which is often used to model clipping effects. It allows the derivation of many important results in closed form. Secondly, the soft limiter models a situation where perfect predistortion is used to linearize a memoryless nonlinearity. The soft limiter is the residual distortion curve, since predistortion can invert a nonlinearity only as far as the supply voltage rail allows it.

After the nonlinearity, complex AWGN further corrupts $z(t)$ to yield

$$y(t) = z(t) + n(t)\,. \tag{3}$$

This system model is shown in Figure 1. The complex Gaussian random process $n(t)$ is assumed to be zero-mean. Its variance is $2\sigma_n^2$. The IBO for the soft limiter device is given by

$$\text{IBO} = \frac{A^2}{2\sigma_x^2}\,. \tag{4}$$

Since $f(x)$ is a real function, α will be also real. It can be computed via

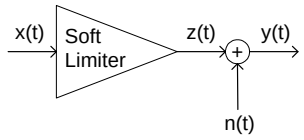

Fig. 1. System Model for Estimation Problem

$$\alpha = \frac{1}{2\sigma_x^2} \int_0^\infty r_x f(r_x) p_{r_x}(r_x) dr_x \tag{5}$$

as shown in [5], where

$$p_{r_x}(r_x) = \frac{r_x}{\sigma_x^2} e^{r_x^2/2\sigma_x^2}\,,\ 0 \le r_x < \infty \tag{6}$$

is the *Probability Density Function* (PDF) of $r_x(t)$. The result for α then reads

$$\alpha = \left(1 - e^{-\mathrm{IBO}}\right) + \sqrt{\pi \mathrm{IBO}}\, Q\left(\sqrt{2\mathrm{IBO}}\right), \tag{7}$$

where $Q(\cdot)$ is the Q-function (see [6]).

The system model for the estimation problem thus does not contain any signal processing specific to OFDM. The approach presented here just requires the input of the nonlinearity to be complex Gaussian. Consequently, the Bayesian estimator derived in the next section might be useful for other applications as well, for example multi-code CDMA signals. It is also important to note that, from an implementation point of view, this is an algorithm operating on complex baseband signals. Care must be taken to translate the RF clipping behaviour of a PA or another component to baseband. For memoryless devices, this can be done via the Chebychev transform described in [7] and [8]. Also, filtering effects in the RF part of the OFDM system must be taken into account, since the nonlinearity generates out-of-band signal power. On the other hand, the Bayesian estimator is potentially very useful in systems where the dynamic range of the baseband signal is deliberately limited at the transmitter to ensure proper PA operation. This approach also has the advantage of providing determined distortion characteristics, which are needed a-priori for optimum estimation.

III. Bayesian Estimation

Estimating $x(t)$ from the observation $y(t)$ is the estimation problem. These are two-dimensional signals, and it is important to choose a suitable set of coordinates for estimation. Cartesian coordinates are not well suited to the problem, because the soft limiter nonlinearity affects the magnitude of the signal which creates dependencies between the clipped I- and Q-components. Instead, polar coordinates are used; magnitude and phase are estimated separately.

The soft limiter does not affect the phase $\theta_x(t)$ of $x(t)$ at all, and thus the optimum estimate $\hat{\theta}_x(t)$ of $\theta_x(t)$ is just

$$\hat{\theta}_x(t) = \theta_y(t)\,, \tag{8}$$

where $\theta_y(t)$ is the phase of the received signal $y(t)$. Indeed, since the phase of the signal is only affected by the phase of $n(t)$, which is uniformly distributed on $[0, 2\pi)$, (8) is the best estimate for θ_x.

The only problem left is to estimate $r_x(t)$ from $r_y(t) = |y(t)|$. The optimum instantaneous estimator in a *Minimum Mean-Square Error* (MMSE) sense is expressed by the conditional expectation of $r_x(t)$ given $r_y(t)$. Generally, for each t,

$$\begin{aligned} \hat{r}_x &= E\{r_x|r_y\} = \int_{-\infty}^{\infty} r_x p_{r_x|r_y}(r_x, r_y) dr_x \\ &= \frac{1}{p_{r_y}(r_y)} \int_{-\infty}^{\infty} r_x p_{r_y|r_x}(r_x, r_y) p_{r_x}(r_x)\, dr_x\,, \end{aligned} \tag{9}$$

where $p_{r_y}(r_y)$ is the PDF of $r_y(t)$, and $p_{r_y|r_x}(r_x, r_y)$ is the conditional PDF of $r_y(t)$ given $r_x(t)$. $r_x(t)$, for any time t, is just a Rayleigh-distributed random variable with PDF

$$p_{r_x}(r_x) = \frac{r_x}{\sigma_x^2} e^{r_x^2/2\sigma_x^2}\,. \tag{10}$$

The PDF of $r_z(t) = |z(t)|$ is also needed. It can be written as

$$p_{r_z}(r_z) = p_{r_x}(r_z) u_A\left(r_z - \frac{A}{2}\right) + P_C \delta(r_z - A)\,, \tag{11}$$

where P_C is the probability that $r_x(t)$ exceeds the clip level A:

$$P_C = P\{r_x > A\} = \int_A^\infty \frac{r_x}{\sigma_x^2} e^{-r_x^2/2\sigma_x^2}\, dr_x = e^{-\mathrm{IBO}}\,, \tag{12}$$

and

$$u_A(x) = \begin{cases} 1 & -A/2 \le x \le A/2 \\ 0 & \text{elsewhere} \end{cases}\,. \tag{13}$$

The soft limiter therefore cuts off the tail of the Rayleigh distribution, and shifts all values $r_x > A$ to the impulse at $r_z = A$.

The determination of the effect of complex AWGN on the magnitude of the signal is not so straighforward. The noise can be written as

$$n(t) = r_n(t) e^{j\theta_n(t)}\,. \tag{14}$$

$r_n(t)$ is Rayleigh-distributed, while $\theta_n(t)$ is uniformly distributed on $[0, 2\pi)$. $z(t)$ and $n(t)$ add in the complex plane to yield $y(t)$; the angle θ_{zn} between these vectors is also uniformly distributed on the interval $[0, 2\pi)$. The value of $r_y(t)$ for each t can now be computed via

$$r_y = \sqrt{r_z^2 + r_n^2 + 2 r_z r_n \cos(\theta_{zn})}\,. \tag{15}$$

Of course, the PDF of this quantity is difficult to obtain, because the PDF of the product of the three random variables r_z, r_n, and θ_{zn} must be computed. The product $r_n \cos(\theta_{zn})$ is just a real Gaussian random variable, but this does not simplify the problem. To the best of the author's knowledge, there is no closed-form solution for this problem. To compute the Bayesian estimator, an approximation of r_y is computed instead: For $r_z > r_n$,

$$r_y \approx r_z + r_n \cos(\theta_{zn}) \,. \tag{16}$$

Thus, r_y is approximated as the sum of r_z and the *projection* of r_n onto r_z. This is a good approximation when r_n is smaller than r_z, which can also be verified by a Taylor series expansion. It can be expected, however, that the performance of the Bayesian estimator will degrade for lower *Signal-to-Noise Ratio* (SNR) because of that.

With (16), $p_{r_y}(r_y)$ can be computed as

$$p_{r_y}(r_y) = I_3(r_y) + I_4(r_y) \,, \tag{17}$$

where

$$I_3(r_y) = \frac{e^{-\mathrm{IBO}}}{\sqrt{2\pi\sigma_n^2}} e^{-(r_y-A)^2/2\sigma_n^2} \,, \tag{18}$$

and

$$\begin{aligned} I_4(r_y) &= \frac{e^{-c}}{4\sigma_x^2\sqrt{a^3 2\pi\sigma_n^2}} \Bigg\{ \sqrt{4a}\Big(1 - e^{bA-aA^2}\Big) \\ &+ \sqrt{\pi}\, b e^{b^2/4a} \\ &\cdot \left[\mathrm{erf}\left(\frac{2aA-b}{\sqrt{4a}}\right) + \mathrm{erf}\left(\frac{b}{\sqrt{4a}}\right) \right] \Bigg\} . \end{aligned} \tag{19}$$

a, b, and c are defined as

$$a = \frac{\sigma_x^2 + \sigma_n^2}{2\sigma_x^2\sigma_n^2}, \quad b = \frac{r_y}{\sigma_n^2}, \quad c = \frac{r_y^2}{2\sigma_n^2} \,, \tag{20}$$

and $\mathrm{erf}(\cdot)$ denotes the error function [6]. $I_3(r_y)$ and $I_4(r_y)$ are the result of convolving the PDFs of r_z and $r_n \cos(\theta_{zn})$.

To complete the Bayesian estimator, the integral in (9) must be solved. It is easy to see that

$$p_{r_y|r_x}(r_x, r_y) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \cdot \begin{cases} e^{-\frac{(r_y-r_x)^2}{2\sigma_n^2}} & r_x \le A \\ e^{-\frac{(r_y-A)^2}{2\sigma_n^2}} & r_x > A \end{cases} \,, \tag{21}$$

because $r_n \cos(\theta_{zn})$ is a Gaussian random variable with zero mean and variance σ_n^2. Again, the approximation (16) is used here. With this PDF, the integral in (9) comes out to be

$$\int_{-\infty}^{\infty} r_x p_{r_y|r_x}(r_x, r_y) p_{r_x}(r_x)\, dr_x = I_1(r_y) + I_2(r_y) \,, \tag{22}$$

where

$$\begin{aligned} I_1(r_y) &= \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(r_y-A)^2/2\sigma_n^2} \\ &\cdot \left[A\, e^{-\mathrm{IBO}} + \sqrt{\frac{\pi\sigma_x^2}{2}} \mathrm{erfc}\Big(\sqrt{\mathrm{IBO}}\Big) \right] , \end{aligned} \tag{23}$$

and

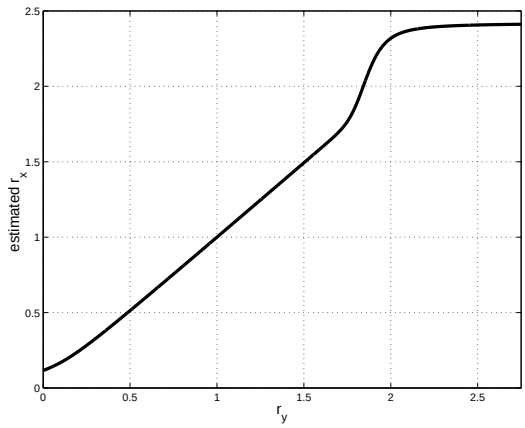

Fig. 2. Typical Bayesian estimator characteristic; Parameters: $2\sigma_x^2 = 2$, $A = 2$, $E_b/N_0 = 20dB$

$$\begin{aligned} I_2(r_y) &= \frac{e^{-aA^2-c}}{8\sigma_x^2\sqrt{a^5 2\pi\sigma_n^2}} \Bigg\{ \sqrt{4a}\Big[b e^{aA^2} - (2aA+b)\, e^{bA} \Big] \\ &+ \sqrt{\pi}(2a+b^2)\, e^{(aA^2+b^2)/4a} \\ &\cdot \left[\mathrm{erf}\left(\frac{2aA-b}{\sqrt{4a}}\right) + \mathrm{erf}\left(\frac{b}{\sqrt{4a}}\right) \right] \Bigg\} . \end{aligned} \tag{24}$$

$\mathrm{erfc}(\cdot)$ is the complementary error function ([6]).

Therefore, the Bayesian estimator in (9) takes on the following form:

$$\hat{r}_x = \frac{I_1(r_y) + I_2(r_y)}{I_3(r_y) + I_4(r_y)} \,. \tag{25}$$

It yields an estimate of the magnitude of x based on the observation of the magnitude of y for any time t.

IV. Properties of the Estimator

The Bayesian estimator is a memoryless transfer curve whose shape depends on the parameters σ_x^2, σ_n^2, and A. Figure 2 shows a typical curve for $2\sigma_x^2 = 2$ and $A = 2$. This corresponds to an IBO of 3 dB. The SNR with respect to the output power of the soft limiter was set to 20 dB. All values of E_b/N_0 in this paper are with respect to the output power of the soft limiter, because this is the power effectively used for transmitting data. Furthermore, this is the SNR measurable at the receiver. The output power $2\sigma_o^2$ of the soft limiter can be computed from

$$2\sigma_o^2 = 2(1 - e^{-\mathrm{IBO}})\sigma_x^2 \,. \tag{26}$$

It is observed that there is an almost linear region for medium to small r_y, but the curve does not cross the origin. This is clearly a result of the approximation (16); the condition $r_z > r_n$ is not fulfilled in this region. For small r_y, the estimator curve should obey $\hat{r}_x = r_y$. It can be expected that the performance of the estimator can be improved by forcing the curve to cross the origin.

For larger r_y in the range of A, the estimator inverts the soft limiter curve. The sharpness of the curve depends on the value of σ_n^2. For very large r_y, there seems to be a saturation, but

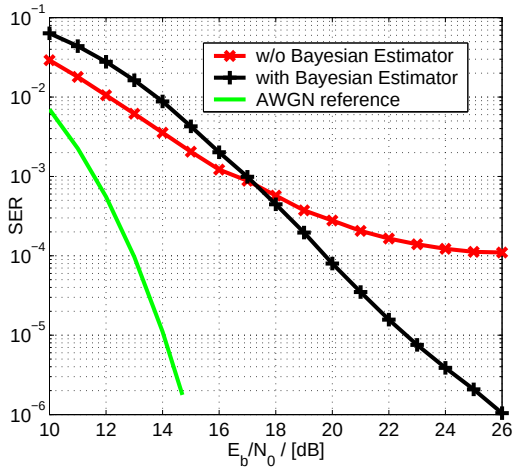

Fig. 3. Performance of Bayesian estimator: 16-QAM, 52 OFDM carriers, IBO $= 3dB$

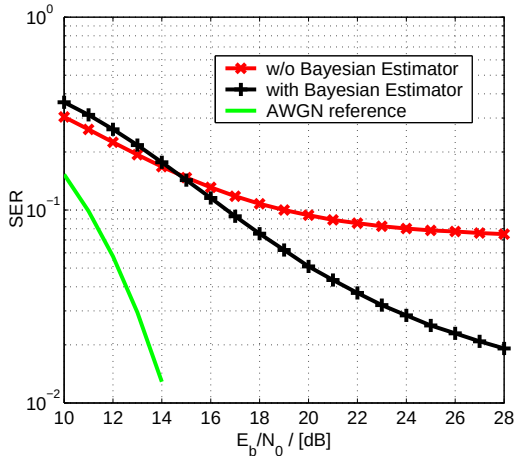

Fig. 4. Performance of Bayesian estimator: 64-QAM, 52 OFDM carriers, IBO $= 3dB$

the height of the saturation level is not entirely understood at this time. For small values of σ_n^2, the saturation level from simulation matches $E\{r_x|r_x > A\}$, which is intuitively expected. More general results for the saturation behaviour of the device for arbitrary levels of noise power are a subject of future research.

For practical applications, the transfer characteristic of the estimator would have to be stored in a lookup table, since the computation is involved and requires high numerical accuracy. Curve fitting is another possibility.

V. Application to OFDM

The Bayesian estimator can help to recover a clipped complex OFDM signal. Baseband simulations were conducted for typical OFDM system parameters. Figure 3 shows the *Symbol Error Rate* (SER) of a 16-QAM OFDM system with 52 carriers (similar to IEEE 802.11a parameters) for an IBO of 3 dB. This is a quite heavy distortion, since the dynamic range of the undistorted signal is in the range of 10 dB. The curve without Bayesian estimation is based on a linear estimator, that is, the scaling factor α is assumed to be known, but clipping noise is treated as AWGN. It can be seen that the estimator lowers the error floor considerably by incorporating knowledge on the distribution of $n_d(t)$. For lower SNR, performance is degraded because of the approximation in the derivation of the estimator.

64-QAM signals benefit from Bayesian estimation at higher SER regions. Figure 4 shows an improvement at high SNR. With appropriate error correction coding, this improvement makes transmission of a 64-QAM OFDM signal at an IBO of 3 dB possible. Simulation results for a coded DVB-T system will be reported elsewhere. Figure 5 shows the performance of 64-QAM signals and 6817 OFDM carriers (DVB-T, 8k mode) at IBO $= 3dB$. An improvement over the case with 52 carriers results from the fact that the assumption of a Gaussian transmit signal is more valid for higher numbers of OFDM carriers. Furthermore, the Bayesian estimator transfer curve was forced to cross the origin of the $r_y - \hat{r}_x$ plane, which reduces the performance loss for low SNR.

Finally, figure 6 shows the same scenario with an IBO of $6dB$. The last figure indicates that this receive algorithm is

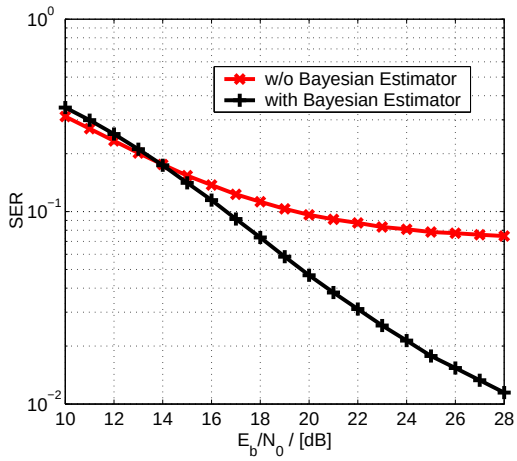

Fig. 5. Performance of improved Bayesian estimator: 64-QAM, 6817 OFDM carriers, IBO $= 3dB$

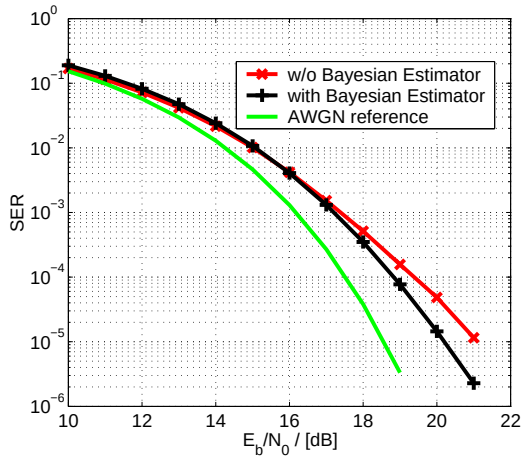

Fig. 6. Performance of improved Bayesian estimator: 64-QAM, 6817 OFDM carriers, IBO $= 6dB$

only beneficial when the clipping noise is the dominant noise term in the system. This observation was also made in [5]. For higher values of IBO, there is not much to be gained from Bayesian estimation.

VI. Conclusions

A novel receive algorithm for clipped OFDM signals was proposed. The key idea is to take the statistics of the clipping noise into account, rather than treating it as AWGN. A Bayesian estimator for clipped complex Gaussian signals was derived in closed form. Simulations showed the ability of this receive algorithm to improve OFDM system performance. Future research includes investigations of the properties of the estimator, especially the saturation behaviour. Furthermore, application of the estimator to OFDM corrupted by a fading channel and additive noise will be a subject of future work.

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