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Space-Time-Frequency Transmit Diversity in Broadband Wireless OFDM Systems

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***Abstract*— We consider space-time -frequency transmit diversity with orthogonal designs in a broadband OFDM system. Based on capacity considerations for Gaussian as well as real world signal constellations, we show that the concatenation of an outer error control code, a bit interleaver and orthogonal spatial transmit diversity provides an attractive MIMO scheme which is simple and fairly robust to channel imperfections such as spatial correlation or keyhole effects. Particularly, we discuss orthogonal transmit diversity versus spatial multiplexing and space-time-frequency codes which have been discussed in literature recently. Furthermore, we propose a simple and pragmatic application of orthogonal designs for space-time-frequency transmit diversity and give design criteria which allow to use a simple combiner at the receiver also in time-varying and severly frequency-selective channels.**

I. Introduction

A broadband OFDM 4G candidate system offers different sources of diversity: Temporal diversity, frequency diversity and spatial diversity. The optimum solution to exploit all sources of diversity would be a joint design of a big space-time-frequency code which codes across subcarriers, multiple OFDM symbols and antennas. Such a space-time-frequency coding has been proposed e.g. in [1]. However, this approach has high complexity and low flexibility.

In this paper we deal with the question wether we loose significantly if simple space-time-frequency techniques are applied instead of theoretically optimum solutions. We have in mind that most wireless user equipment will remain small also in the future since it should be possible to carry it all the time. Therefore, in our considerations the number of antennas is limited to 2 or at most 4. Furthermore, we restrict ourselves to the realistic assumption that the transmitter has no channel state information which would allow beamforming or switching between different MIMO techniques.

One of the simplest MIMO techniques is transmit diversity from orthogonal designs [2], [3]. The main advantage is that the full spatial diversity is obtained using a simple combiner at the receiver. In OFDM, orthogonal designs can be applied as space-time or space-frequency block codes [4]. The principle will be reviewed in Section III after a description of the channel and system model in Section II.

Transmit diversity is the optimum solution in a MISO system with multiple transmit and a single receive antenna. However, in spatially uncorrelated MIMO systems with multiple transmit and receive antennas, spatial multiplexing, where independent data streams are transmitted simultaneously from multiple antennas, is theoretically optimum. Therefore, in Section IV-A we discuss the degradation in terms of capacity of transmit diversity in MIMO channels. We will show that the capacity loss is relatively small if realistic signal constellations are taken into consideration and that transmit diversity is fairly robust to channel imperfections such as spatial correlations or keyhole effects.

We can further ask if orthogonal designs are a good choice in a space-time-frequency coded system. In [4] it is claimed that orthogonal designs fail to exploit the available frequency diversity. Therefore, we discuss this statement in Section IV-B and show that virtually no capacity is lost in a MISO system if orthogonal designs are used as space-frequency block codes.

The next question which is treated in Section IV-C is wether it is necessary to design special codes for space-frequency coded modulation as proposed e.g. in [5]. We will show that similar as in single antenna systems, there is virtually no loss in terms of capacity for bit-interleaved space-time block coded modulation compared to space-time block coded modulation.

The aforementioned results justify to use a separated approach in OFDM systems, where an outer (convolutional) code in combination with a random interleaver takes care of time and frequency diversity and a simple orthogonal design provides spatial diversity.

Finally, in Section V we give a pragmatic mapping scheme and design criterion for the application of orthogonal designs in situations where the coherence time and/or the coherence bandwidth are smaller than the block size of an orthogonal design.

II. MIMO Channel and System Model

We consider OFDM over a frequency-selective MIMO channel with n_T transmit and n_R receive antennas and $D+1$ taps in each subchannel from a transmit to a receive antenna. In Figure 1 and Table I we set up three power delay profiles where $T_{s,SC}$ denotes the QAM symbol duration in a single carrier system. The symbol duration $T_{s,OFDM}$ on each subcarrier is increased by a factor of N_s compared to a single carrier system with the same bandwidth, i.e.

$$T_{s,OFDM} = N_s \cdot T_{s,SC}. \tag{1}$$

Model B is a preliminary channel model used for investigations on 4G mobile systems in [6].

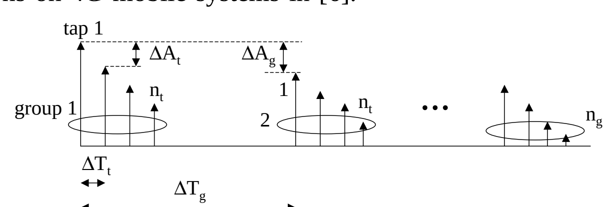

Fig. 1. Power delay profiles.

Model	n_g	n_t	ΔA_t	ΔA_g	ΔT_t	ΔT_g
A	1	2	3 dB		$60T_{s,SC}$	
B	1	24	0.5 dB		$8T_{s,SC}$	
C	2	6	1 dB	3	$4T_{s,SC}$	$150T_{s,SC}$

TABLE I

CHANNEL MODELS.

For high data rate transmission in 4G mobile systems, the number of subcarriers will be rather large, e.g. $N_s = 1024$ or 2048. Throughout the paper we set $T_{s,SC} = 7.4ns$ and consider a carrier frequency of $f_0 = 5GHz$.

III. TRANSMIT DIVERSITY USING ORTHOGONAL DESIGNS

A space-time block code [2], [3] is defined by a $P \times n_T$ orthogonal design $\mathbf{B}_{n_T}$ and provides a mapping rule for the M-QAM or M-PSK transmit symbols x_1 to x_K. For $n_T = 2$ and 4, orthogonal designs are given in [2], [7]:

$$\mathbf{B}_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \quad \mathbf{B_4} = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_3^* & -x_2^* & -x_1 \end{bmatrix}. \tag{2}$$

The entries in the same row of $\mathbf{B}_{n_T}$ are transmitted simultaneously from different antennas, the entries in the same column are transmitted from the same antenna in successive time slots. Under the assumption of constant channel coefficients during transmission of $\mathbf{B}_{n_T}$, the orthogonal structure allows to separate the symbols $x_1, ..., x_K$ and to achieve the maximum possible level of diversity by simple combining [3], [8]. Effectively, the MIMO channel is transformed to a SISO channel with a lower variance of the SNR [8].

If the channel varies during transmission of $\mathbf{B}_{n_T}$, the orthogonality is lost resulting in intersymbol interference. Of course, a maximum likelihood receiver could be applied to overcome this problem. However, since one of the key points of using orthogonal designs is the simple receiver, we wish to design the system such that the simple combiner can be applied.

In OFDM, orthogonal designs can be used as space-time or space-frequency block codes as described in the following sections.

A. Space-Time Block Codes in OFDM

In OFDM with a sufficient guard interval, each subcarrier provides a flat fading MIMO channel. Therefore, a space-time block code can be applied for each subcarrier. The detector assumes that the channel does not change during transmission of a space-time block code matrix, i.e. during P OFDM symbol durations $T_{s,OFDM}$. This is a more critical restriction in OFDM than in single carrier systems since the OFDM symbol duration is $N_s T_{s,SC}$, where $T_{s,SC}$ would be the symbol duration in a single carrier system. Therefore, the performance of space-time block codes will degrade in fast fading environments. This is particularly critical if more than $n_T = 2$ transmit antennas are applied. E.g. for $n_T = 3$ or 4, the channel needs to be constant over 4 or even 8 OFDM symbols depending on which orthogonal design is used [3], [7].

B. Space-Frequency Block Codes in OFDM

To avoid the problem of fast channel variations in time, the symbols of an orthogonal design can be transmitted on neighbouring subcarriers of the same OFDM symbol rather than on the same subcarrier of subsequent OFDM symbols [4]. This also reduces the transmission delay. However, the channel needs to be about constant over P neighbouring subcarriers. This is true in channels with low frequency-selectivity or can be accomplished by using a large number of subcarriers in order to make the subcarrier spacing very narrow. The performance of space-frequency block codes will degrade in heavily frequency-selective channels where the assumption of constant channel coefficients over a space-frequency block code matrix is not justified. Particularly, this is a problem for more than $n_T = 2$ transmit antennas, where $P \geq 4$ subcarriers are needed per space-frequency block code matrix.

IV. CAPACITY AND MUTUAL INFORMATION

In the following sections we motivate the application of transmit diversity from orthogonal designs by considerations based on capacity. The Shannon capacity assumes Gaussian transmit symbols and, therefore, refers to a huge symbol alphabet. We will also look at the mutual information for real world signal constellations which sometimes leads to modified conclusions.

The decisive quantity for the channel capacity is the number of effective MIMO dimensions. By a singular value decomposition, a flat fading MIMO channel with channel matrix $\mathbf{H}$ can be decomposed into rank $\mathbf{H}$ decoupled SISO channels, the tap gains of which are given by the singular values of $\mathbf{H}$ [9]. For spatially uncorrelated full rank channels, all rank $\mathbf{H}$ = min$\{n_T, n_R\}$ MIMO dimensions contribute to the capacity. In correlated channels, one singular value becomes dominant which reduces the number of effective MIMO dimensions and, consequently, the capacity.

In OFDM with a sufficiently long guard interval, each subcarrier s faces a flat fading MIMO channel with frequency domain channel matrix $\mathbf{H}_s$. Thus, the capacity of a MIMO OFDM system with N_s subcarriers is given by

$$C = E\{\frac{1}{N_s}\sum_{s=0}^{N_s-1} \log_2\{\det\{\mathbf{I}_{n_R} + \frac{E_s}{n_T N_0}\mathbf{H}_s\mathbf{H}_s^H\}\}\}, \tag{3}$$

where $E\{\}$ denotes expectation over the channel realizations.

A. Transmit Diversity versus Spatial Multiplexing

In spatial multiplexing, independent data streams are transmitted simultaneously from multiple antennas. This is the optimum strategy in terms of capacity if the channel offers n_T MIMO dimensions. On the other hand space-time block codes achieve transmit diversity by a transformation of the MIMO channel to a SISO channel with lower variance of the SNR [8]. Consequently, the resulting equivalent channel has always only one dimension. As long as the receiver is equipped with only one receive antenna, transmit diversity is the optimum strategy. However, space-time block codes cannot achieve the capacity of a multi-dimensional MIMO channel [8]. On the other hand, as we will show they offer a fairly robust solution in correlated and keyhole MIMO channels.

We will compare the capacities of the MIMO channel itself to the capacity of spatial multiplexing and a space-time block code for Gaussian transmit symbols as well as for QAM transmit symbols. In the latter case we compare at the same rate. If we e.g. consider a maximum transmission of 4 bit/channel use, we have spatial multiplexing with QPSK or BPSK in the case of $n_T = 2$ or 4 transmit antennas, respectively, and compare it to a space-time block code with 16-QAM.

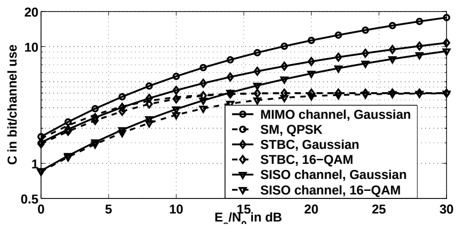

Fig. 2. Ergodic capacity for uncorrelated MIMO channel $n_T = n_R = 2$. Spatial multiplexing (SM) with QPSK, space-time block code (STBC) with 16-QAM.

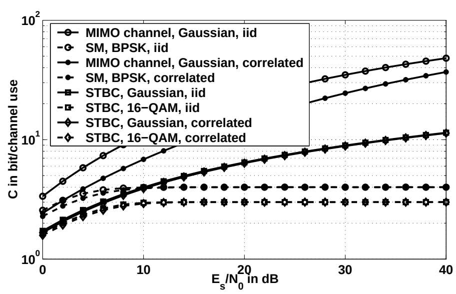

Fig. 3. Ergodic capacity for correlated MIMO channel $n_T = n_R = 4$. Spatial multiplexing (SM) with BPSK, space-time block code (STBC) with 16-QAM.

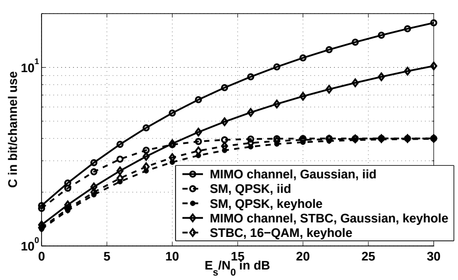

Fig. 4. Ergodic capacity for keyhole channel $n_T = n_R = 2$. Spatial multiplexing (SM) with QPSK, space-time block code (STBC) with 16-QAM.

The ergodic capacity for $n_T = n_R = 2$ is shown in Figure 2. Even though, with Gaussian transmit symbols we observe a significant capacity loss of the space-time block code compared to the MIMO channel capacity, the difference is small if realistic signal alphabets are taken into consideration. Figure 3 shows the ergodic capacity for a correlated channel with $n_T = n_R = 4$ transmit and receive antennas. The antenna spacing at both the base station and the user equipment is $\lambda/2$, where λ denotes the wavelength. The azimuth spread is uniform over 360^o at the mobile terminal and Laplacian distributed ($\sigma = 5^o$, angle of departure 20^o) at the base station. It can be observed that there is virtually no capacity loss compared to the uncorrelated channel for the space-time block code whereas spatial multiplexing suffers from spatial correlation. The degradation of the space-time block code compared to spatial multiplexing results from the rate loss of the orthogonal design $\mathbf{B}_4$ in (2). It might be reduced by other transmit diversity techniques such as quasi-orthogonal space-time block codes with rate 1 [10].

Another type of MIMO channel degradation might be a keyhole effect, i.e. all pathes have to travel through a narrow "keyhole" [11]. A keyhole channel has rank 1 even though the channel coefficients for all antennas are uncorrelated. From Figure 4 we conclude that transmit diversity is optimum in MIMO keyhole channels. The mutual information of spatial multiplexing is reduced due to the rank defficiency of the channel.

We conclude that even though transmit diversity is not optimum in uncorrelated full rank MIMO channels, it is a fairly robust solution in many practical applications.

B. Orthogonal Designs as Space-Frequency Block Codes

In [4] it was argued using pairwise error probabilities that the Alamouti scheme $\mathbf{B}_2$ in (2) applied as a space-frequency block code is unable to exploit the frequency diversity which is available in a frequency-selective channel and that therefore, orthogonal designs should not be used as space-frequency block codes. The first result reflects the fact that space-time block codes do not impose dependencies over all subcarriers. Moreover, orthogonal designs are designed to exploit only spatial diversity. The advantage of using orthogonal designs is that they enable simple diversity combining at the receiver given that the channel does not vary during transmission of a matrix. One might argue that the assumption of constant channel coefficients over an orthogonal design prevents to exploit available diversity. However, since space-frequency block codes assume high correlation of neighbouring subcarriers, we do not sacrifice diversity as no time or frequency diversity is available over the transmit symbol positions of one orthogonal design.

The relevant criterion for the application of orthogonal designs as space-frequency block codes is, if such a scheme causes a loss in capacity. In order to evaluate that, we compute the capacity of an orthogonal space-frequency block code for a frequency-selective channel, the frequency response of which is approximated as indicated in Figure 5 such that the channel is constant over P neighbouring subcarriers. Thereby, the available frequency diversity is reduced to the level which can be exploited in a system with orthogonal space-frequency block codes and simple combining.

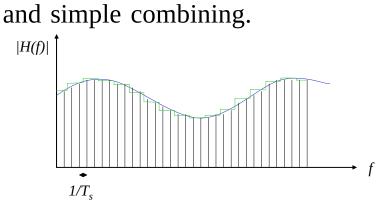

Fig. 5. Approximation of channel frequency response for orthogonal space-frequency block code.

The results in Figure 6 for channel B show, that in case of $n_T = 2$ transmit and $n_R = 1$ receive antenna, the capacity

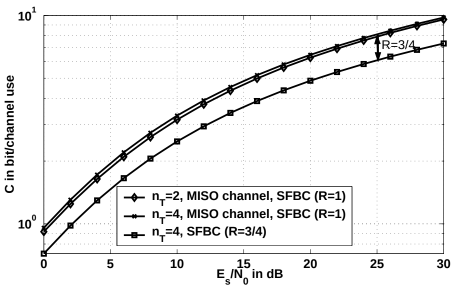

Fig. 6. Ergodic capacity for channel B and space-frequency block code (SFBC), $n_R = 1$, $N_s = 1024$.

is virtually the same for the true MISO channel, a space-time block coded system and a space frequency coded system. Therefore, we do not loose in terms of capacity by applying the Alamouti scheme as space-frequency block code. The available frequency and time diversity can be picked up by an outer FEC decoder. For $n_T = 4$, the degradation of the space-frequency block code with respect to the channel capacity is exactly the rate loss of $R = 3/4$. Therefore, even with $P = 4$, there is no loss in terms of capacity caused by the failure of space-time block codes to exploit frequency diversity. The capacity gap for $n_T = 4$ can be reduced using higher rate diversity schemes such as quasi-orthogonal space-time block codes [10].

C. Bit-Interleaved Space-Time-Frequency Coded Modulation

Recently, several proposals for space-frequency codes have been published, e.g. in [5]. However, the approach in [5] is essentially a concatenation of an outer error correcting code and an inner space-frequency code. Moreover, the examples given in that paper can exploit only a limited level of frequency diversity. We propose to use bit-interleaved coded modulation instead, where an orthogonal design provides spatial diversity and a standard (convolutional) code picks up time and frequency diversity. This approach is supported by the results in Figure 7 which demonstrate that bit-interleaved coded modulation in combination with a space-time or space-frequency block code provides virtually the same capacity as space-time respectively space-frequency coded modulation.

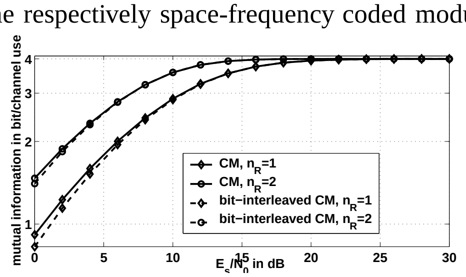

Fig. 7. Ergodic mutual information for space-time block code with coded modulation (CM) and bit-interleaved coded modulation (bit-interleaved CM), $n_T = 2$, 16-QAM with Gray mapping.

V. Space-Time-Frequency Block Codes in OFDM

So far, we have argued that transmit diversity with orthogonal designs might be a good choice in many applications. In this section we propose a simple pragmatic scheme for orthogonal space-time-frequency transmit diversity in broadband OFDM systems. It has already been mentioned that space-time block codes face problems in fast fading whereas space-frequency block codes suffer from frequency-selectivity. Therefore, we propose to distribute the elements of the or-

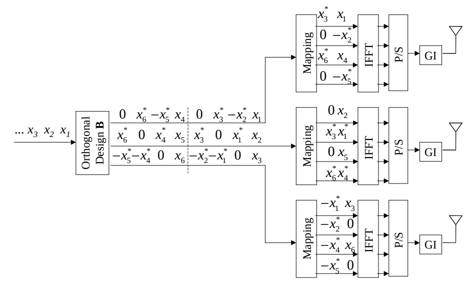

Fig. 8. Space-time-frequency block code in OFDM. $N_s = 4$, $n_T = 3$.

thogonal design both in time and frequency in order to relax the requirements for constant channel coefficients in both dimensions. This is particularly important for $n_T > 2$ transmit antennas where the channel coefficients have to be constant over $P \geq 4$ symbols. An example for $n_T = 3$ and $N_s = 4$ is shown in Figure 8 where the orthogonal design on the right hand side of (2) is used dropping the last column.

In order to design the mapping in time and frequency, we use measures for coherence time and coherence bandwidth. Space-time block codes require that the coherence time T_c of the channel meets

$$T_c > P \cdot T_{s,OFDM\_GI}, \tag{4}$$

where $T_{s,OFDM\_GI}$ is the OFDM symbol duration including the guard interval and P is the number of rows of the orthogonal design. In [12], the coherence time is defined as the time over which the time correlation function is above 0.5 and given by $T_c = 9/16\pi f_d$, where f_d is the maximum Doppler shift. Space-frequency block codes require that the coherence bandwidth B_c meets

$$B_c > P/T_{s,OFDM}, \tag{5}$$

where $1/T_{s,OFDM}$ is the subcarrier spacing. Figure 9 summarizes the coherence time $T_{c,1}$ normalized to $T_{s,OFDM\_GI}$ for different vehicular speed and different numbers of subcarriers. The horizontal lines in Figure 9 labelled with $P = 2$ and $P = 4$, respectively, mark the limits for the space-time codes (2). I.e. for $n_T = 2$ ($n_T = 4$) the normalized coherence time must be above the line marked with $P = 2$ ($P = 4$). The results indicate that given the orthogonal designs (2), for up to $n_T = 4$ transmit antennas problems may occur only for very high vehicular speed and more than $N_s = 2048$ subcarriers. Therefore, space-time block codes can be applied in 4G multi-carrier systems.

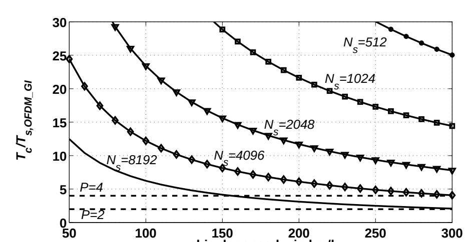

Fig. 9. $T_{c,1}/T_{s,OFDM\_GI}$ for vehicular speed v, $f_0 = 5GHz$, $T_{s,SC} = 7.4ns$.

We found that the definition

$$B_c = \frac{1}{15\sigma_\tau} \qquad (6)$$

is a more suitable criterion for the application of space-frequency block codes than other common rule of thumb definitions of the coherence bandwidth. In (6), σ_τ is the rms delay spread.

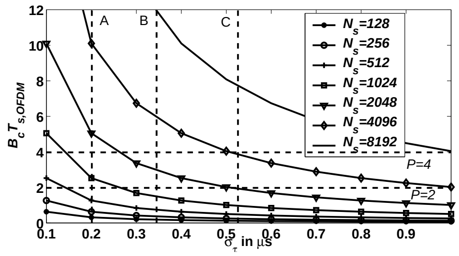

Fig. 10. $B_c T_{s,OFDM}$ for channels A,B,C according to Table I, $T_{s,SC} = 7.4ns$.

Figure 10 shows the normalized coherence bandwidth $B_c T_{s,OFDM}$ according to (6). The rms delay spread is marked for the channels A, B and C given in Table I. Obviously, the requirement of constant channel coefficients over an orthogonal design is much more critical for space-frequency block codes than for space-time block codes.

Figure 10 predicts that a space-frequency block code with $n_T = 2$ transmit antennas, i.e. $P = 2$ can be applied for $N_s \geq 1024$ subcarriers in channel A or with $N_s \geq 2048$ subcarriers in channel B. $n_T = 4$ transmit antennas, i.e. $P = 4$, can only be used with $N_s \geq 2048$ in channel A or $N_s \geq 4096$ in channel B.

VI. Simulation Results

In this section we present simulation results in order to demonstrate how the considerations of the previous sections transfer into BER performance. We use the system parameters and channel models of Section II. Unless stated otherwise the channel varies in time according to a Jakes spectrum with maximum Doppler shift f_d. In case of channel coding we use a rate 1/2 convolutional code with memory 4. All simulations are carried out in the frequency domain, i.e. effects of inter-carrier interference due to channel variations during transmission of one OFDM symbol are neglected. This is justified by the results depicted in Figure 9.

Figure 11 depicts the performance of a space-frequency block code in channel B for $N_s = 1024$, $n_T = 4$ and $n_R = 1$. For comparison we include also the performance of a space-time-frequency block code with a mapping as indicated in Figure 8 and of a space-time block code with and without outer FEC channel coding. According to Figure 10, a space-frequency block code cannot be used, which explains the error floor of the space-frequency block code However, a space-time-frequency block code seems to be a suitable alternative with lower delay compared to a space-time block code.

Figure 12 shows the BER performance in channel A for space-time and space-frequency block codes with $n_T = 2$ and $N_s = 1024$ or $N_S = 2048$, respectively. All schemes show nearly identical performance which supports the design criterion (5) with (6). In Figure 12 we also include results for

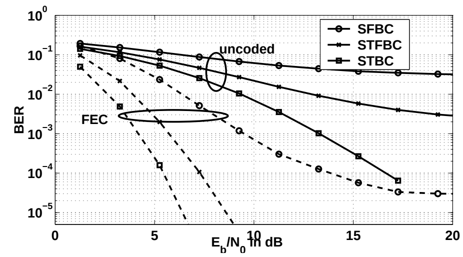

Fig. 11. BER of space-frequency block code (SFBC), space-time-frequency block code (STFBC) and space-time block code (STBC). Channel B, $f_d T_{s,OFDM\_GI} = 0.01$, $n_T = 4$, $n_R = 1$, $N_s = 1024$, QPSK modulation, 3000 info bits per block.

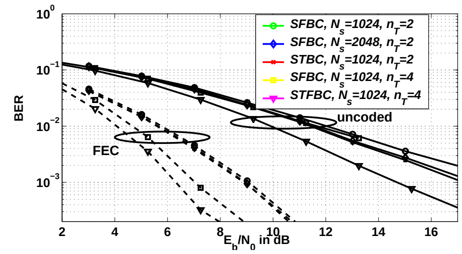

Fig. 12. BER of space-frequency block code (SFBC), space-time block code (STBC) and space-time-frequency block code (STFBC). Channel A, $f_d T_{s,OFDM\_GI} = 0.01$, $n_R = 1$, BPSK modulation, 2000 info bits per block.

a space-frequency and space-time-frequency block code with $n_T = 4$ and $N_s = 1024$.

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