Technical Paper

On the preview control of limited bandwidth vehicle suspensions

C Pilbeam, BA, MSc and **R S Sharp,** BSc, MSc, FIMechE, MSEE
School of Mechanical Engineering, Cranfield Institute of Technology, Bedford

Using methods established in earlier work, calculations are carried out to determine the performance possibilities of a slow-active automobile suspension. The suspension consists of a limited bandwidth actuator in series with a passive spring, the combination being in parallel with a passive damper. Use of the correlation between the front and rear wheel inputs is made to determine near-optimal state feedback laws which improve on the uncorrelated (non-preview) laws found using standard linear quadratic regulator (LQR) theory. Various control laws are calculated for different vehicle speeds and the effects of placing the emphasis on either ride comfort or vehicle handling in the control law are studied. It is found that there is significant advantage to be gained in making use of the preview information available when the emphasis is placed on ride comfort in the cost function. Increasing the amount of preview (by reducing the vehicle speed) increases the amount of improvement made available by the use of preview information down to a speed of 10 m/s, below which there is nothing to be gained from having more preview. Due to the practical advantages associated with the use of limited bandwidth active control in comparison with full bandwidth systems and the small additional costs implied by adding to the signal processing required as specified, the results are considered important to the future development of active vehicle suspensions.

NOTATION

a	distance from c.g. to front axle
b	distance from c.g. to rear axle
c_1	front damper rate
c_2	rear damper rate
D	time delay between front and rear wheels
h_1	front tyre spring stiffness
h_2	rear tyre spring stiffness
I	principal pitch moment of inertia of sprung mass
J	cost function
k_1	front spring stiffness
k_2	rear spring stiffness
$\mathbf{K}$	gain matrix
L	vehicle wheelbase
m_1	front unsprung mass
m_2	rear unsprung mass
M	sprung mass
M_{aa}	constant defined in text
M_{ab}	constant defined in text
M_{bb}	constant defined in text
$q_1, \ldots, q_6$	weighting parameters
u_1	front control demand signal
u_2	rear control demand signal
u_1'	second-order filtered front control demand signal
u_2'	second-order filtered rear control demand signal
u_1''	fourth-order filtered front control demand signal
u_2''	fourth-order filtered rear control demand signal
$\boldsymbol{u}$	control demand vector
x_a	front wheel road input
x_b	rear wheel road input
$x_1, \ldots, x_{16}$	state variables
ζ	filter damping ratio
ρ_1, ρ_2	weighting parameters
ω	filter cut-off frequency

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1 INTRODUCTION

A great deal of work has been carried out in recent years investigating the benefits available from replacing conventional passive suspension components (springs and dampers) by active and/or adaptive elements.

Full bandwidth systems offer improvements in ride and handling but may suffer from other problems such as high power consumption and noise, vibration and harshness (NVH) problems (**1**). A good active system mainly gains over a conventional passive one by damping the body resonance much more effectively. It cannot do much better than the passive system higher up the frequency range. It has been shown (**2**) that slow-active systems can also offer significant improvements in the control of body resonances, with wheel-hop resonances being dealt with by a suitable choice of damper rate. Slow-active systems incorporate an actuator of limited bandwidth (around 5 Hz) in series with a passive spring, the combination being in parallel with a passive damper. This type of system offers advantages over full bandwidth active systems in the form of reduced power consumption and lower performance components (and hence lower capital cost) due to the lower frequency operation required, and they have been preferred for production (**3**, **4**). In fact, the series actuator and spring are realized commercially by gas springs and electro-hydraulic elements.

It has also been shown (**5**–**7**) that the use of preview information with a full bandwidth system can give an improvement in vehicle suspension system performance.

Preview can be of two types: (a) wheelbase preview, where knowledge of the front wheel states is used to improve performance at the rear of the vehicle, and (b) look-ahead preview, where a sensor of some description is used to measure the road profile ahead of the front wheels. Both types of preview can give substantial improvements in performance, depending to some extent on the control scheme adopted. Discrete optimal control theory was utilized in reference (**8**) to establish ultimate possibilities while the results of reference (**5**) showed that restriction to state feedback control only reduces performance capabilities a little (in the case of unlimited bandwidth systems). No corresponding information has yet been published for limited bandwidth systems.

This paper contains the results of an investigation into the performance potential of a limited bandwidth system with wheelbase preview. Control is restricted by assuming only state feedback and good control laws for the preview case are found using the Simplex method described in reference (**5**). The non-preview (uncorrelated input) case, calculated using linear quadratic regulator (LQR) theory, is used as a starting point for these calculations.

The amount of preview is determined by the vehicle speed and its wheelbase. The effect of changing the amount of preview by varying the vehicle speed is investigated, as is the effect of placing the emphasis in the control scheme on either ride comfort or vehicle handling.

Frequency response results are presented for a passive vehicle and for a slow-active system (with and without preview) for different vehicle speeds.

2 SYSTEM MODEL AND PERFORMANCE CALCULATIONS

The half-car model shown in Fig. 1 is the one used in the simulation calculations. The road input is an integrated delta-function (that is a unit step). A sub-optimal control scheme is to be found that makes use of wheelbase preview, that is it takes into account the fact that the rear wheel input is a time-delayed version of the front wheel input. The limited bandwidth nature of each actuator is represented in the model by including two similar second-order low-pass filters in series with a perfect actuator. Earlier work involved only one of these filters but results showed the actuator movement to be too vigorous in the wheel hop frequency range. The additional filters are effective in restricting the actuators to dealing with lower frequency body modes.

The equations for the low-pass second-order filters are of the form:

$$\ddot{y} + 2\zeta\omega\dot{y} + \omega^2 y = \omega^2 x \tag{1}$$

where

x = input signal

y = output signal

ω = cut-off frequency

ζ = damping ratio

Figure 2 shows a bode plot for the modelled actuator/filter combination showing the gain and phase characteristics of the system. State variables $x_a, x_b, x_1, \ldots, x_{16}$ are used, where

$$u_1' = x_{13} \tag{2}$$

$$u_2' = x_{15} \tag{3}$$

$$u_1'' = x_2 - x_3 \tag{4}$$

$$u_2'' = x_5 - x_6 \tag{5}$$

Equations (4) and (5) here correspond to the assumption that the actuator is ideal, and instantaneously produces any displacement that is demanded of it.

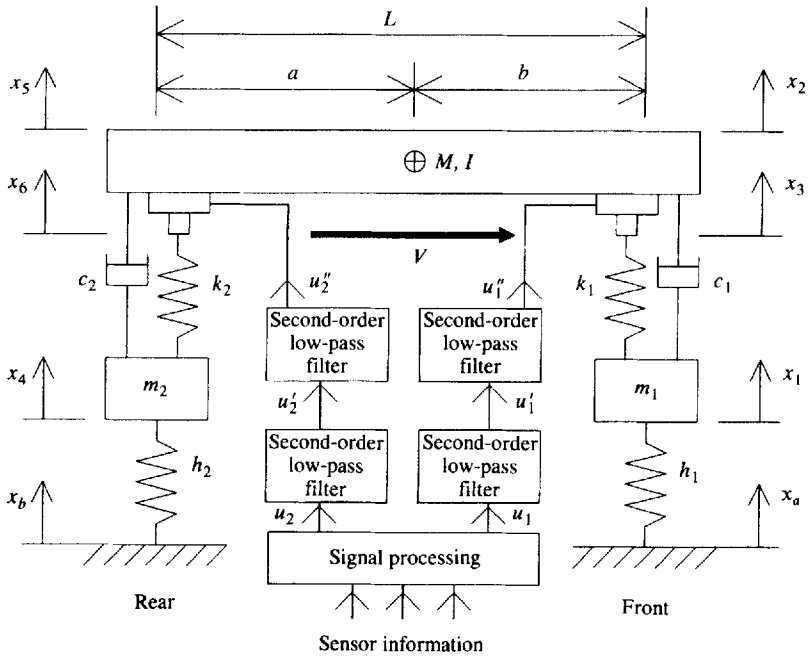

Fig. 1 Half-car model

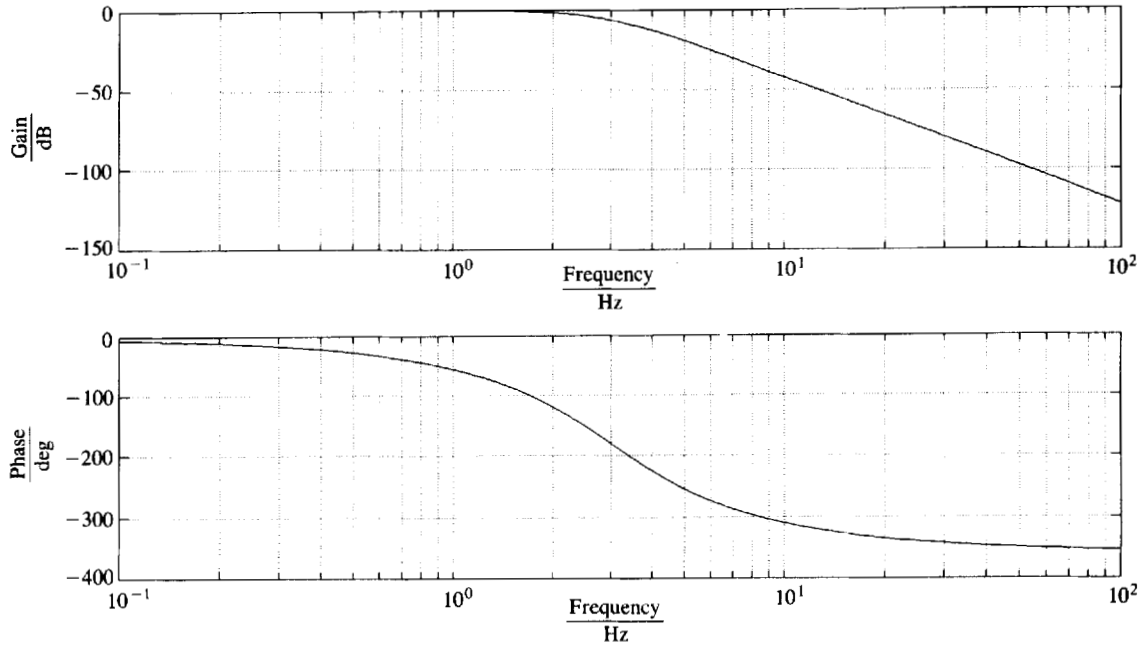

Fig. 2 Bode plot showing actuator/filter frequency response function

The equations of motion of the system in the state variables $x_a, x_b, x_1, \ldots, x_{16}$ are then

$$\dot{x}_a = \delta(t)$$

$$\dot{x}_b = \delta(t - D)$$

$$\dot{x}_1 = x_7$$

$$\dot{x}_2 = x_8$$

$$\dot{x}_3 = x_9$$

$$\dot{x}_4 = x_{10}$$

$$\dot{x}_5 = x_{11}$$

$$\dot{x}_6 = x_{12}$$

$$\dot{x}_7 = \frac{h_1}{m_1} x_a - \frac{h_1 + k_1}{m_1} x_1 + \frac{k_1}{m_1} x_3 - \frac{c_1}{m_1} x_7 + \frac{c_1}{m_1} x_8$$

$$\dot{x}_8 = \frac{k_1}{M_{aa}} x_1 - \frac{k_1}{M_{aa}} x_3 + \frac{k_2}{M_{ab}} x_4 - \frac{k_2}{M_{ab}} x_6 + \frac{c_1}{M_{aa}} x_7 - \frac{c_1}{M_{aa}} x_8 + \frac{c_2}{M_{ab}} x_{10} - \frac{c_2}{M_{ab}} x_{11}$$

$$\dot{x}_9 = \frac{k_1}{M_{aa}} x_1 + \omega^2 x_2 - \left(\frac{k_1}{M_{aa}} + \omega^2\right) x_3 + \frac{k_2}{M_{ab}} x_4 - \frac{k_2}{M_{ab}} x_6 + \frac{c_1}{M_{aa}} x_7 - \left(\frac{c_1}{M_{aa}} - 2\zeta\omega\right) x_8 - 2\zeta\omega x_9 + \frac{c_2}{M_{ab}} x_{10} - \frac{c_2}{M_{ab}} x_{11} - \omega^2 x_{13} \qquad (6)$$

$$\dot{x}_{10} = \frac{h_2}{m_2} x_b - \frac{h_2 + k_2}{m_2} x_4 + \frac{k_2}{m_2} x_6 - \frac{c_2}{m_2} x_{10} + \frac{c_2}{m_2} x_{11}$$

$$\dot{x}_{11} = \frac{k_1}{M_{ab}} x_1 - \frac{k_1}{M_{ab}} x_3 + \frac{k_2}{M_{bb}} x_4 - \frac{k_2}{M_{bb}} x_6 + \frac{c_1}{M_{ab}} x_7 - \frac{c_1}{M_{ab}} x_8 + \frac{c_2}{M_{bb}} x_{10} - \frac{c_2}{M_{bb}} x_{11}$$

$$\dot{x}_{12} = \frac{k_1}{M_{ab}} x_1 - \frac{k_1}{M_{ab}} x_3 + \frac{k_2}{M_{bb}} x_4 + \omega^2 x_5 - \left(\frac{k_2}{M_{bb}} + \omega^2\right) x_6 + \frac{c_1}{M_{ab}} x_7 - \frac{c_1}{M_{ab}} x_8 + \frac{c_2}{M_{bb}} x_{10} - \left(\frac{c_2}{M_{bb}} - 2\zeta\omega\right) x_{11} - 2\zeta\omega x_{12} - \omega^2 x_{15}$$

$$\dot{x}_{13} = x_{14}$$

$$\dot{x}_{14} = -\omega^2 x_{13} - 2\zeta\omega x_{14} + \omega^2 u_1$$

$$\dot{x}_{15} = x_{16}$$

$$\dot{x}_{16} = -\omega^2 x_{15} - 2\zeta\omega x_{16} + \omega^2 u_2$$

where

$$\frac{1}{M_{aa}} = \frac{1}{M} + \frac{a^2}{I}$$

$$\frac{1}{M_{ab}} = \frac{1}{M} - \frac{ab}{I}$$

$$\frac{1}{M_{bb}} = \frac{1}{M} + \frac{b^2}{I}$$

A variable transformation is applied to the system state equations in order to facilitate the optimal control computations (**7**). This transformation is also used in references (**8**) to (**10**).

The outputs u_1 and u_2 from the signal conditioning are linear combinations of the system states $x_1, \ldots, x_{16}$:

$$\boldsymbol{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -\mathbf{K}\boldsymbol{x}$$

where

$$\mathbf{K} = \begin{bmatrix} k_{1,1} & k_{1,2} & \cdots & k_{1,16} \\ k_{2,1} & k_{2,2} & \cdots & k_{2,16} \end{bmatrix}$$

The cost function used is of the form

$$J = \int_0^\infty \{q_1(x_1 - x_a)^2 + q_2(x_1 - x_2)^2 + q_3 \ddot{x}_2^2 + q_4(x_4 - x_b)^2 + q_5(x_4 - x_5)^2 + q_6 \ddot{x}_5^2 + \rho_1 u_1^2 + \rho_2 u_2^2\} \, dt \quad (7)$$

with ρ_1, ρ_2 fixed at unity; $q_1, \ldots, q_6$ can be varied to alter the relative emphasis placed on handling (q_1 and q_4), suspension working space (q_2 and q_5) and ride comfort (q_3 and q_6) in the control scheme. The cost function can be broken up into its constituent parts, consisting of four terms which correspond to the contribution to the total cost function due to dynamic tyre load variation (handling), suspension working space, body vertical acceleration (ride comfort) and the amount of actuator displacement used.

To find a state feedback control law that minimizes the cost function and takes into account the preview information available, a suitable starting point must be found. This is provided by solving the uncorrelated problem, which can be done easily since this is the standard LQR problem. The Simplex method of reference (**5**) is then used to vary the individual gains in the gain matrix **K**, a few at a time, to reduce the cost function as far as possible. Earlier work assuming unlimited bandwidth actuators (**5**) has shown that state feedback control generated in this fashion is capable of approaching optimal performance quite closely.

The vehicle parameters used for the calculations are taken from reference (**10**) and are

$M = 505.1$ kg
$I = 651.0$ kg m^2
$m_1 = 28.58$ kg
$m_2 = 54.43$ kg
$a = 1.098$ m
$b = 1.468$ m
$L = 2.566$ m
$k_1 = k_2 = 15\,000$ N/m
$c_1 = c_2 = 1000$ N s/m
$h_1 = h_2 = 155\,900$ N/m
$\zeta = 0.7071$
$\omega = 6\pi$ rad/s

The set of weighting parameters q_i was chosen initially such that the three terms corresponding to handling, suspension working space and ride comfort in the cost function each made equal contributions to the total cost function. This set of values was then used as a base set and the corresponding q values were increased one by one, by a factor of ten, to place the emphasis on the different performance criteria in the control scheme. The base set of q values used was

$q_1 = 340$

$q_2 = 80$

$q_3 = 0.0035$

The calculations were carried out for vehicle speeds of 10, 20 and 30 m/s.

Some frequency response functions were also calculated for passive and slow-active half-car models at different vehicle speeds. These give body bounce acceleration and pitch angular displacement gains, dynamic tyre load variation gains and suspension working space gains for a sinusoidal input at the wheels (the rear wheel input being a time-delayed version of the front wheel input).

3 RESULTS

Improvements in the performance index deriving from wheelbase preview are shown in Fig. 3. These results show that at all vehicle speeds there is a significant advantage to be gained in ride comfort by taking into account the preview information available for the slow-active model, but little to be gained in handling performance. The improvement is greater at lower vehicle speeds, since the lower the vehicle speed, the longer the preview time. This result could have been expected because of the limited bandwidth of the actuator. Body resonances which occur in saloon cars at frequencies around 1–1.5 Hz can be dealt with by an actuator with a bandwidth of about 5 Hz, but wheel-hop resonances, which typically occur at around 10 Hz, are out of the range of effective control of the actuator.

Several simulations were carried out at vehicle speeds between 30 and 5 m/s, corresponding to different amounts of preview. The improvements in the cost function from the uncorrelated (non-preview) case to the correlated (wheelbase preview) case increased as the speed was reduced down to around 10 m/s, but there was no further improvement available on reducing the speed below 10 m/s (0.26 s of preview). This shows that the point of diminishing returns lies at around a quarter of a second of preview, a figure close to those obtained for full bandwidth systems (**6**, **7**).

It was found that by further increasing the emphasis placed on ride comfort in the control scheme (putting $q_3 = q_6 = 0.1$), a further improvement in the cost function in going from the non-preview to the preview case was possible. At a speed of 10 m/s the cost function was reduced by 20.6 per cent and at a speed of 30 m/s the cost function was reduced by 15.8 per cent. The remaining results use these values of the q_i to investigate the performance of the slow-active system with wheelbase preview.

Figures 4 to 9 show the frequency response functions for the half-car with these parameter values, for a vehicle speed of 30 m/s. The input to the system is a constant amplitude sinusoidal velocity at the wheels. Results are given for a slow-active system, with and without preview, and for a passively suspended system with the standard parameter values. The trends shown in these results are substantially the same for different vehicle speeds. The graphs show that there is very little difference between the three cases at higher frequencies (around wheel-hop resonance). This could have been expected due to the limited bandwidth nature of the system.

The dynamic tyre load variation is little altered at the front of the vehicle, since the preview information is only able to exert a significant influence on the performance at the rear. Rear dynamic tyre load variation in the preview case is improved over the non-preview

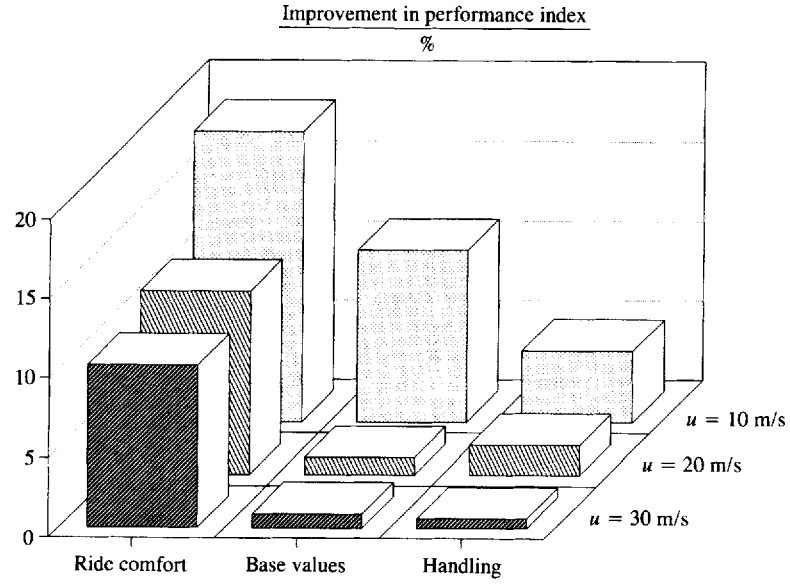

Fig. 3 Improvements in the cost function through wheelbase preview

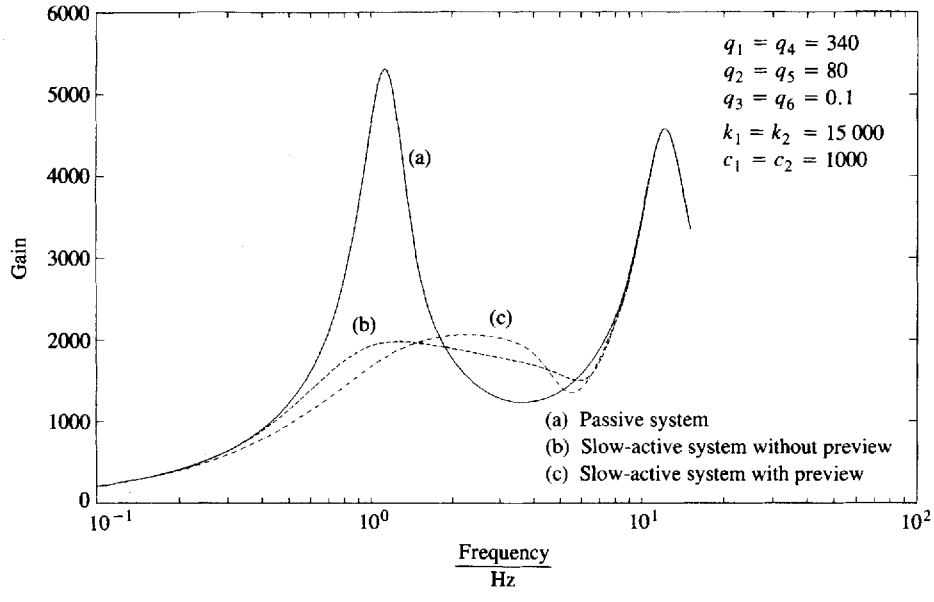

Fig. 4 Front dynamic tyre load variation, $u = 30$ m/s

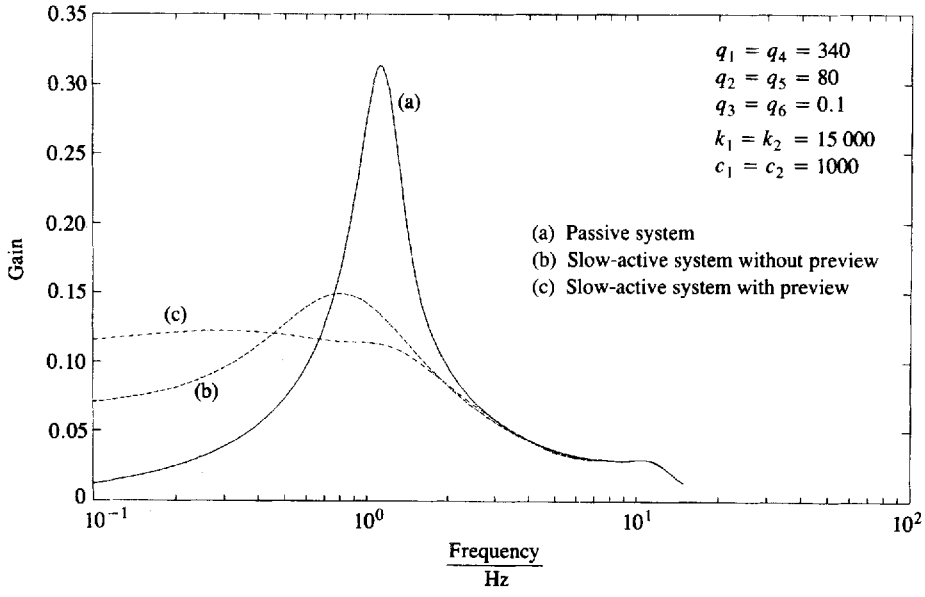

Fig. 5 Front suspension working space, $u = 30$ m/s

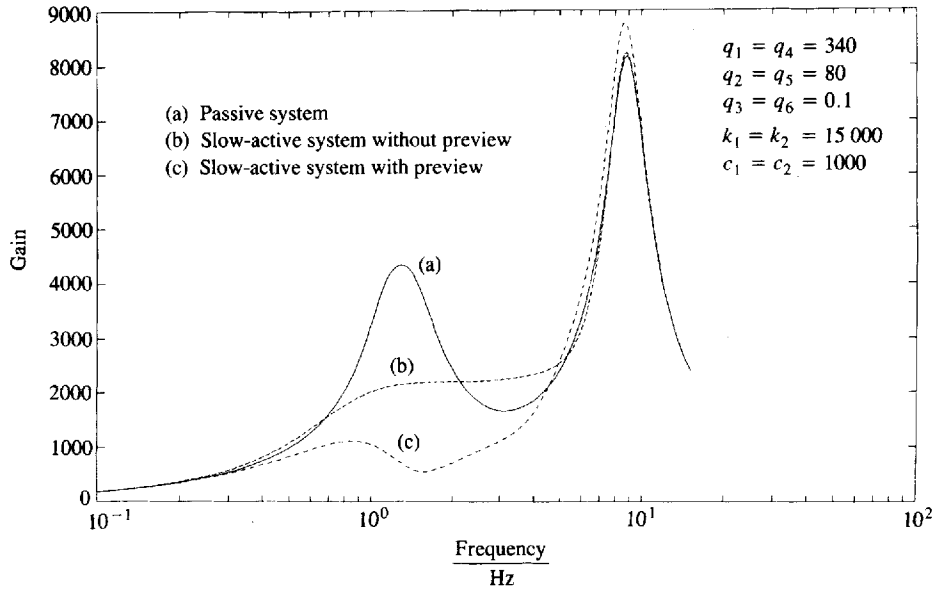

Fig. 6 Rear dynamic tyre load variation, $u = 30$ m/s

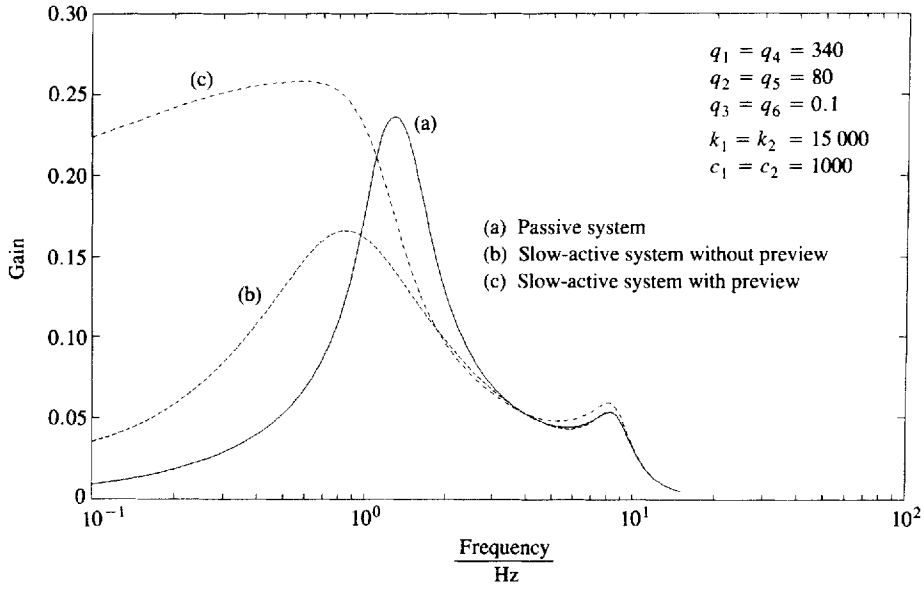

Fig. 7 Rear suspension working space, $u = 30$ m/s

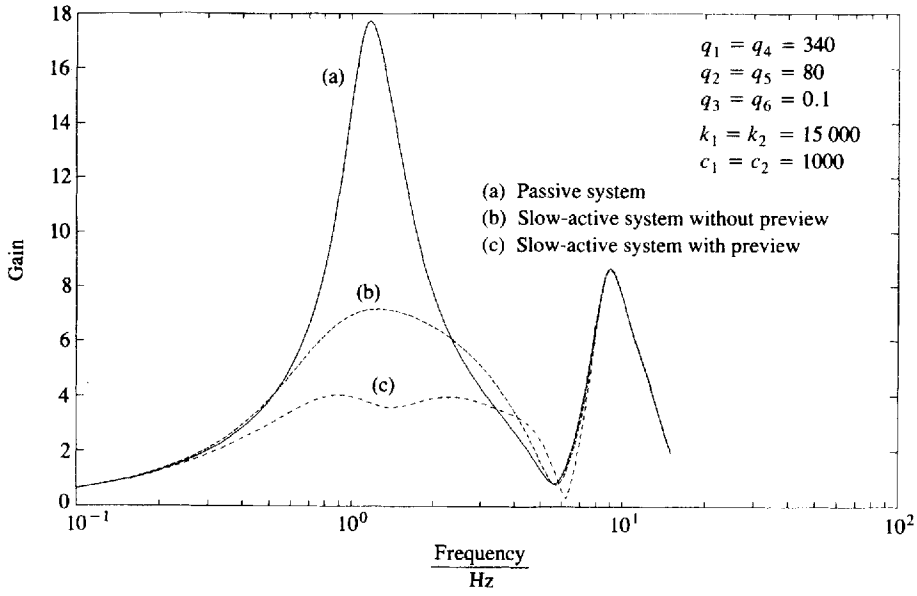

Fig. 8 Bounce acceleration, $u = 30$ m/s

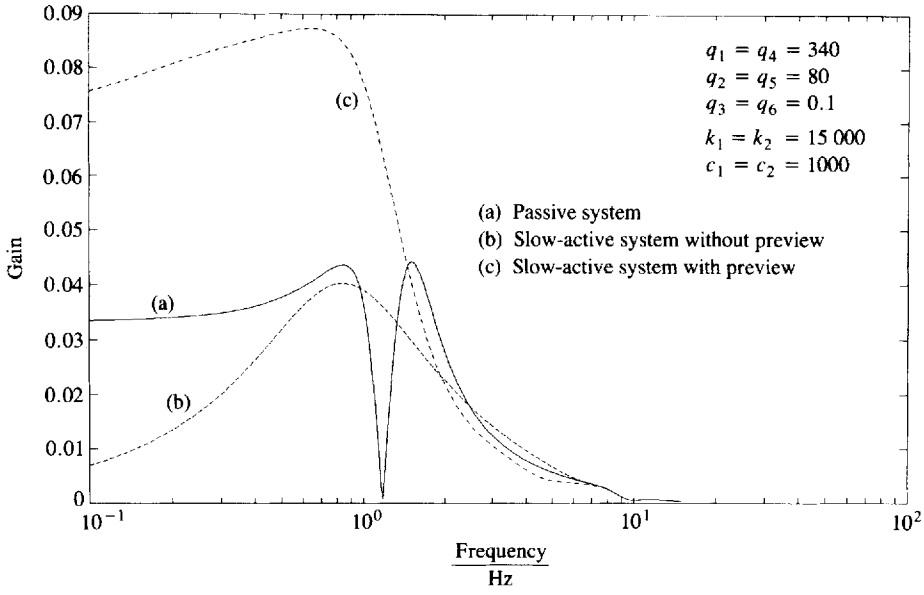

Fig. 9 Pitch angular displacement, $u = 30$ m/s

case up to about 5 Hz, while the rear suspension working space response deteriorates up to about 2 Hz. At lower vehicle speeds (around 10 m/s) the rear suspension working space is improved between about 0.5 and 3 Hz, but is still worse below 0.5 Hz.

Bounce acceleration is generally improved up to 5 or 6 Hz, but the improvement is less marked at lower vehicle speeds (below 15 m/s).

The pitch angle response is worse below about 2 Hz, although at lower speeds it is improved, except at very low frequencies below about 0.5 Hz. The fact that the improvements in the bounce and pitch angle responses are dependent on the vehicle speed is related to the fact that at different vehicle speeds the phase angle between the front and rear inputs varies. Hence if there is a whole number of wavelengths between the wheels the inputs are in phase and the bounce acceleration response will be worse than if the inputs are out of phase, in which case the pitch angle response will be large.

Figure 10 shows the time histories of the body vertical height over the front and rear axles for the non-preview and preview cases for a vehicle speed of 10 m/s. At both front and rear, the peak displacement is lower and the time to return to the equilibrium state shorter for the preview case, again with greater improvement at the rear.

Figures 11 and 12 show the time histories of the actuator displacements, front and rear, for vehicle speeds $u = 10$ m/s and $u = 30$ m/s, both for the preview and non-preview cases. At both speeds it can be seen that for the non-preview case the rear actuator movement is virtually a time-delayed version of that of the front actuator. In addition, the difference between the movements of the front actuator for the non-preview and preview cases is fairly small, as may be expected. The difference comes in the rear actuator movement. At the lower speed for the preview case the rear actuator executes large displacements before the disturbance reaches the rear wheels, which occurs at $t = 0.26$ s. At the

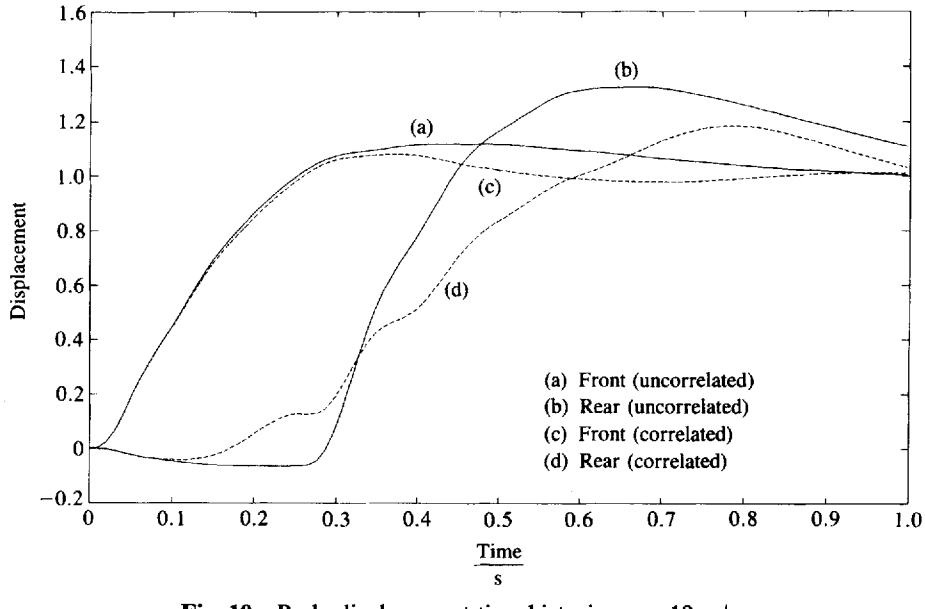

Fig. 10 Body displacement time histories, $u = 10$ m/s

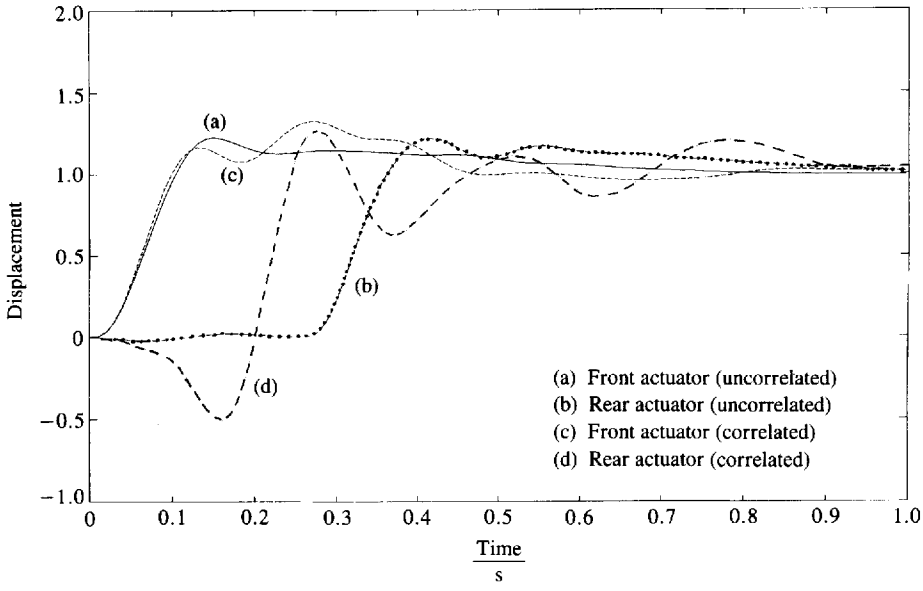

Fig. 11 Actuator displacement time histories, $u = 10$ m/s

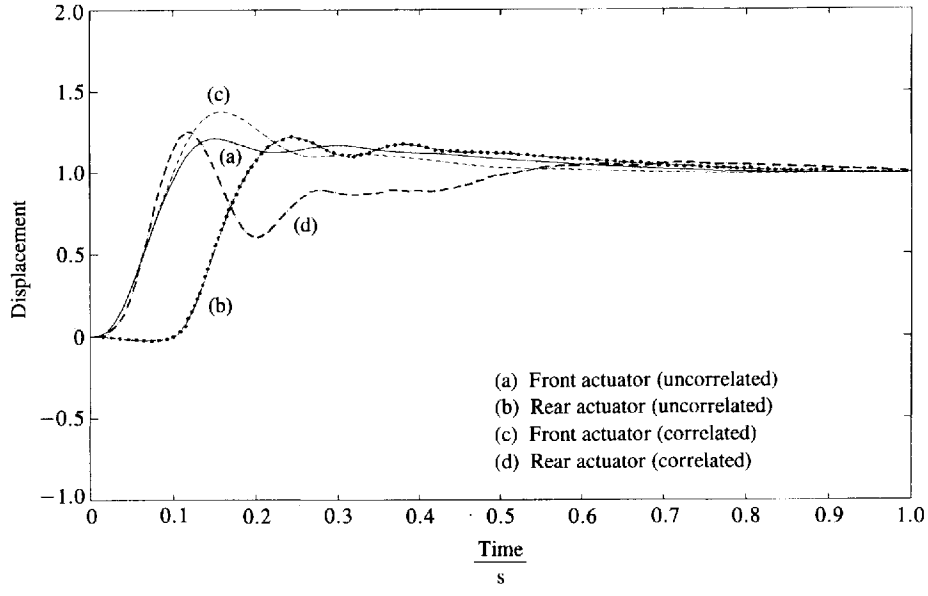

Fig. 12 Actuator displacement time histories, $u = 30$ m/s

higher speed the rear actuator again is in motion well before the step input reaches the rear axle, which occurs at $t = 0.09$ s, although the rear actuator does not perform such large displacements at the higher speed due to its limited speed of response.

4 CONCLUSIONS

The results included are of some practical significance, since slow-active suspensions are showing themselves to be commercially viable and any performance improvements available from the use of wheelbase preview can be realized at very low cost through additional signal processing. Some questions relating to performance improvements have been answered, while other questions have been raised.

Cost index values (Fig. 3) show significant potential performance gains, especially if ride comfort is accentuated in the weightings, and especially at lower vehicle speeds for which the wheelbase preview time is relatively long. Frequency response results show the improvements to be mainly from the rear suspension, with the front suspension acting similarly regardless of whether or not preview is accounted for in the (state feedback) control laws. Improvements occur in the region of the body mode resonant frequencies, with higher frequency behaviour little affected, as may be expected from the limited bandwidth of the actuators specified.

Step responses for low vehicle speeds show that the rear actuators push the wheels and body apart on approaching the step, pulling them back towards each other just before the wheels hit the step. This is the same pattern of behaviour as has been demonstrated theoretically for unlimited bandwidth suspension systems (**5**, **8**) and it confirms the viability of state feedback control laws generated by iterative improvement of LQR laws, which ignore the front/rear input correlations. For high vehicle speeds, there is insufficient preview time for this strategy to be advantageous and the rear actuators pull the wheels and body together as

soon as they receive information about the oncoming step from the front axle sensing system. This action apparently contributes to improving the state of the rear suspension when it reaches the step. Performance improvements from incorporation of the wheelbase preview information in the controls are obtained from more vigorous use of the rear actuators, and this implies some additional energy costs, which remain to be evaluated.

The state feedback control as derived has two unsatisfactory aspects from a practical point of view. These are shown in the frequency response results. The first concerns the behaviour of the active systems at frequencies well below the body resonant range, where they continue to work to maintain a fixed vehicle body elevation while the wheels follow the road profile. This is not viable in the real world of limited suspension working spaces and nor does it fulfil any useful purpose from the viewpoint of enhanced ride comfort because the body vertical accelerations implied at such low frequencies are very small. A simple solution appears to be obtainable by including high-pass filters in front of the actuators, although other more complex schemes have been developed for dealing with the problem (**11**). The simple solution proposed has not yet been developed and tested but further work will be done on the topic.

The second aspect is that the pitch response of the preview controlled vehicle is very strong at about 0.5–0.8 Hz and a real vehicle with this characteristic may be considered deficient subjectively. In formulating the optimization problem so far, the cost function has contained contributions from each end of the vehicle without reference to the relationship between the front and the rear. Thus there are no cost terms to discourage pitching of the vehicle body. It is a simple matter to alter this. It can be done by changing the form of the terms presently included to reflect differences between front and rear motions or by including some additional terms. Again, it is intended to pursue this matter in further work. Also, energy consumption computations will be done to quantify the costs associated with the performance gains available from the use of wheelbase preview.

ACKNOWLEDGEMENT

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