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Bounds on Maximum Rate-Per-Unit-Energy for Networks with Regenerative Relays

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***Abstract*— In this paper, the multi-hop relay network with transmissions over orthogonal AWGN channels is optimized in terms of maximum rate-per-unit-energy. For such a scenario, the optimal regenerative-forwarding strategy is still unknown. Furthermore, there is not a known one which always outperforms the others. The results for four different cases and discussion on the energy allocation and routing requirements are provided. First, as a comparative example, a lower bound is obtained by considering the traditional multi-hop relaying network where all communication are point-to-point reliable. The solution for this scenario is shown to have good distributed properties. Then a regenerative strategy, which is known to achieve the capacity for *allcast* communication is solved. In this case, the solution requires exhaustive computation. To maintain the point-to-point distributed properties a *unicast* strategy is investigated, with relays not decoding the source message but a bin index transmitted by the previous relay. Finally, an upper-bound on the above regenerative techniques is derived and analyzed.**

I. Introduction

Introducing relay capabilities in a network has a strong effect on the information flow that extends to all communication levels, from the achievable rate to the routing strategy. A fundamental understanding about the role of relays in a network is of paramount importance in the design of efficient network protocols for future wireless systems [1], [2].

In order to obtain insightful solutions the single source-destination network with multi-hop orthogonal relay transmissions is optimized to maximize the rate-per-unit-energy for different forwarding strategies. Joint solutions of problems traditionally belonging to different layers are provided: energy allocation (physical), relay selection and routing (network). In [3] the two basic relaying strategies were stated for the single relay network (hereafter referred to as *regenerative* and *non-regenerative*). This paper does not explore the possibility of *non-regenerative* strategies as *Amplify and Forward* (AF) or *Compress and Forward*. The main feature of these strategies is that hard decisions are only taken at destination node and the relays nodes just transmit information about the received signal, not about the message it carries. In the energy-efficient regime these strategies suffers from noise amplification at the relays [4]. In the case of *regenerative* strategies, on which the present work is instead focused, each relay makes a hard decision about a previously transmitted codeword before retransmitting. The optimal regenerative-forwarding strategy is still unknown. Furthermore, there is not a known one which always outperforms the others. First, as a comparative example, a lower-bound on the maximum rate-per-unit-energy is obtained by considering the traditional multi-hop relaying network where all communication are point-to-point reliable. For this strategy it is shown that the energy allocation only requires local channel state information (c.s.i) at the relays and the optimal route can be obtained in a distributed manner. Then, it is addressed, the regenerative strategy presented in [5] requiring all relays to fully decode the source message *allcast*, by properly combining all the received signals (*accumulative*). This strategy is known to achieve the *allcast* capacity, however, do not admits a solution with distributed properties, see also [6]. To maintain the point-to-point properties while obtaining the *accumulative* gain at destination. A *unicast* strategy with *non-accumulative* relays is investigated. Recently, for the *unicast* two-relay network a *binning* based technique proposed in [7], where the second relay is only required to decode a bin index of the source code-book is shown to outperform *allcast* schemes when the channel from the source to the second relay is rather poor. For our orthogonal transmission network, we extend this strategy to an arbitrary number of nodes. Finally, an upper-bound on the above regenerative techniques is derived and analyzed.

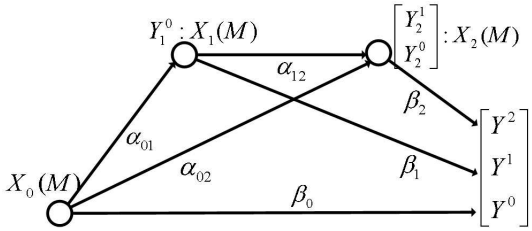

Fig. 1. Two-Relays *allcast* network

The remainder of the paper is organized as follows. In section II, the relay network model and the maximum rate-per-unit-energy problem metric are introduced. Subsequent sections III, IV, V and VI are each dedicated to solve each one of the regenerative relays scenarios previously introduced. Finally in section VII some simulation results and concluding remarks are provided.

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II. Network Model

Consider a $T-$relay network where a single source communicates to a single destination using multiple relays and orthogonal transmissions. In Fig. 1 the 2-relay case is depicted. Consider the transmission X_0 of a message M from a source terminal to a destination terminal which receives Y^j, $j \in (0,T)$ using up to T relay nodes, with input X_i and output Y_i^j, $i \in (1,T)$, and $j \in (1,i-1)$. The fraction of the channel for node i is denoted by τ_i, whereas its transmitted power by P_i. The channel gain from the relay terminal i to the destination is denoted by β_i, and the channel gain from the relay i to the relay j by $\alpha_{i,j}$. It is assumed, an AWGN channel, memoryless and time invariant during the transmission of a codeword. Node employs independent codebooks generated independent and identically distributed (i.i.d.) circularly symmetric, complex Gaussian. All channel gains are normalized such that the sampled noise at the receiver has unit variance and rates are computed in nats/s.

For the above described scenario we are interested in the energy efficient regime. This regime correspond to scenarios with low spectral efficiencies and low signal-to-noise ratios (SNRs). This is the case in wireless sensor networks. The main challenge is, therefore, maximizing the rate-per-unit-energy or reciprocally minimizing the energy-per-bit. Other works considering this performance metric for specific cooperative scenarios are [8], where the single relay case is extensively studied, and in [9] and [10] for the broadcast-relay channel.

Definition 1: Given a total network energy constraint $E \triangleq \sum E_i$ with the energy assigned to relay i defined as $E_i \triangleq \tau_i P_i$. The maximum rate $R(E)$ per unit energy E, is given by

$$\eta \triangleq \max_{E_i,\, \sum E_i = E} \frac{R(E)}{E}. \tag{1}$$

All the objective rate functions $R(E)$ considered in this work consist on a minimum over a set of positive, concave and strictly increasing functions in P_i for $P_i > 0$. For this type of rate functions, it was shown in [8] that η is achieved for $E_i \to 0$. In that case, the first-order approximation of the log function, i.e. $log(1+x) \approx x$, can be used while maintaining the validity of the results.

III. Point-to-Point Multi-hop

We start the discussion by analyzing a simple multi-hop relaying network where all communications are point-to-point reliable. Every network node employs one received signal to decode the source-transmitted message before re-encoding and transmitting. Given a the relay ordered set π (the route) with cardinality $|\pi| = T$. To simplify the notation, consider $\pi(i) = i$ and the source and destination, respectively, denoted by $i = 0$ and $i = |\pi| + 1$. The achievable rate is given by the minimum of the two-hop communications

$$R_0 \leq \max_{\tau_i, P_i} \min_{i \in (0,\pi)} \tau_i \log(1 + P_i \alpha_{i,i+1}). \tag{2}$$

In the energy efficient regime, using the first-order approximation of the log-function and $E_i = \tau_i P_i$, as discussed in section II. The maximum-rate-per-unit-energy problem, is defined as

$$\begin{aligned} R_0 = \quad & \max_{E_i, \pi} \min_{i \in (0,\pi)} C_i \\ \text{s.t} \quad & \sum_{i \in (0,\pi)} E_i = E \end{aligned} \tag{3}$$

with $C_i = E_i \alpha_{i,i+1}$, the solution is provided in the following Theorem.

Theorem 1: The maximum rate-per-unit-energy η_{PP} in a multi-hop point-to-point communication network is given by

$$\eta_{PP} = \frac{1}{\sum_{i \in (0,\pi^*)} \frac{1}{\alpha_{i,i+1}}}. \tag{4}$$

where π^* denotes the optimal relay ordered set.

The *energy allocation* achieving (4) is given by,

$$\begin{aligned} E_i &= \frac{\alpha_{i-1,i}}{\alpha_{i,i+1}} E_{i-1} \qquad i \in \pi^*, \\ E_0 &= \frac{E}{1 + \sum_{i \in \pi^*} \frac{\alpha_{0,1}}{\alpha_{i,i+1}}}. \end{aligned} \tag{5}$$

The *optimal relay order* set π^* can be found recursively and in a distributed manner as described in Algorithm 1.

Algorithm 1 Relay Selection and Ordering

Initialization:
Set $i = \mathcal{T}$, $\mathcal{N} = \mathcal{T}$ and $\mathcal{S} = \varnothing$.
Let every node to transmit directly to destination $\hat{\beta}_n^{\max} = \beta_n$, $\pi_n^{\max} = [\varnothing]$.
Centralized Entity:
1) Choose the node $n^* = \arg\max_{n \in \mathcal{N}} \hat{\beta}_n^{\max}$.
2) Move node n^* from $\mathcal{N}$ to node i in $\mathcal{S}$.
   Set $\hat{\beta}_i = \beta_{n^*}^{\max}$ and $\pi_i^* = \pi_{n^*}^{\max}$.
3) **if** i*s* is the Source **then**
   STOP.
   else
   Communicate $\hat{\beta}_i$ to *every node in* $\mathcal{N}$.
   end if

Each node in $\mathcal{N}$: Use $\hat{\beta}_i$ to compute $\hat{\beta}_n^{\max}$ and $\pi_n^{\max}$
1) $\frac{1}{\hat{\beta}_n^i} = \frac{1}{\alpha_{n,i}} + \frac{1}{\hat{\beta}_i}$.
2) **if** $\hat{\beta}_n^i > \hat{\beta}_n^{\max}$ **then**
   $\hat{\beta}_n^{\max} = \hat{\beta}_n^i$
   $\pi_n^{\max} = [i, \pi_i]$.
   end if
   $i = i - 1$.
3) Communicate $\hat{\beta}_n^{\max}$ to the *central entity*.

Previous to prove this result the notion of equivalent channel to destination is introduced and the Algorithm 1 is described. Denote the optimal route from the network node i to destination as π_i^*. To simplify notation consider $\pi_i^*(j) = i + j$.

Definition 2: The *equivalent channel to destination of node* i, $\hat{\beta}_i$ is equivalent to the achievable rate-per-unit-energy to destination of this node $\hat{\beta}_i \triangleq \frac{R_i}{E}$.

In the point-to-point network, $\hat{\beta}_i$ can be written as a function of $\hat{\beta}_{i+1}$, from the following relay $\pi_i^*(1) = i + 1$, as

$$\frac{1}{\hat{\beta}_i} = \frac{1}{\alpha_{i,i+1}} + \frac{1}{\hat{\beta}_{i+1}}. \tag{6}$$

Using this equality, for a given route π^* with $|\pi^*| > 1$, the $\eta_{PP} \triangleq \hat{\beta}_0$ in Theorem 1, can be obtained by computing (6) recursively from $i = T - 1$ to 0. Initially, set $\hat{\beta}_T = \beta_T$. The

algorithm 1 makes use of this recursive expression to obtain the optimal route, basically, it computes $\hat{\beta}_n$ and π_n^* of one node n in the network per iteration, until, after some iterations, the source is reached. Denote $\mathcal{T}$ as the set containing all nodes in the network, $\mathcal{S}$ as the set containing the nodes that have already compute $\hat{\beta}_n$ and π_n^* and $\mathcal{N}$ the remaining ones. The algorithm is divided in two main blocks. One block is computed at each network node $n \in \mathcal{N}$. The other block needs to be computed at a central entity. Define $\hat{\beta}_n^i$ as the equivalent channel of node n using node i as the following relay. At each iteration each node $n \in \mathcal{N}$ computes and stores the maximum equivalent channel to destination $\hat{\beta}_n^{\max}$ and the tentative route π_n^{max} using the best node i^* in $\mathcal{S}$ as its next relay and communicates the result to the central entity. The central entity decides that the nodes with maximum $\hat{\beta}_n^{\max}$ at that iteration $n^* = \arg\max_{n \in \mathcal{N}} \hat{\beta}_n^{\max}$ should follows the proposed route $\pi_{n^*}^* = \pi_{n^*}^{\max}$.

Proof: The problem in (3) is solved in two steps: First, the problem is solved assuming a fixed route π. Then, the optimality of the routing algorithm is justified using the iterative representation of the solution (6). Problem (3) is the maximization over the minimum of a set of linear functions $E_i \alpha_{i,i+1}$ with a total energy constraint. For a given ordered set π where all the relays inside the set accomplishes $E_i > 0$, the solution is obtained when all the linear functions coincide. Jointly with the total energy constraint, this forms a set of $T+1$ linear equations, whose solution provides η_{PP} and the energy allocation in Theorem 1. The *optimality of the routing algorithm* is based on the following lemma.

Lemma 1: The equivalent channel to destination $\hat{\beta}_n = \max_{i \in \mathcal{S}.} \hat{\beta}_n^i$ of any node n is always worst than the equivalent channel to destination $\hat{\beta}_i$ of any of its following relays $i \in \pi_n^*$.

Proof: This lemma can be easily proved from the recursive expression (6). Give relay $n \in \mathcal{N}$, the set of potential following relays $i \in \mathcal{S}$ that can improve their direct channel to destination β_n needs to satisfy $\hat{\beta}_n^i \geq \beta_n$. To be so, necessary conditions, although not sufficient, are that $\alpha_{n,i} \geq \hat{\beta}_n^i$ and, as claimed, $\hat{\beta}_i \geq \hat{\beta}_n^i \forall i$. ■

By choosing the large equivalent channel $\hat{\beta}_{n_1}^i$ among all nodes in $\mathcal{N}$ at each iteration, the algorithm ensures that no other node $n_2 \in \mathcal{N}$ could ever obtain an equivalent channel $\hat{\beta}_{i_2}$ such that $\hat{\beta}_{n_1}^{i_2} > \hat{\beta}_{n_1}^i$, since lemma 1 needs to be fulfilled. ■

Finally, some comments on the suitability for distributed implementation of the result in *Theorem* 1 are provided.

- The *energy allocation* at the relays only needs local channel state information (c.s.i), $(\alpha_{i-1,i}, \alpha_{i,i+1})$, while the source needs to know the equivalent channel to destination, $\hat{\beta}_0$, discovered by the routing algorithm.
- Network nodes only need to maintain a routing vector with two entries per destination, containing: the equivalent channel $\hat{\beta}_n$ and the following relay i in the route. The overall route is given by $\pi_n^* = [i, \pi_i^*]$.
  During the *route discovering*, the source does not need to wait until the algorithm ends. At each iteration, the source can select the optimal next relay i among its available $i \in S$. In addition, given a source-destination pair, the optimal routes of those nodes with better η_{PP} than that of the source, are also discovered. This would be useful in a wireless ad-hoc network.

IV. Allcast Multi-hop Relay Network

Two different *allcast regenerative* techniques for multiple-relay networks have been proposed, so far, in the literature: a strategy based on codebook partitioning *binning* in [1], and a different strategy, on which we are instead focused, generally achieving larger rates, based on results obtained in [11] for the multiple-access channel with generalized feedback in [5]. The following Theorem provides the achievable rate.

Theorem 2: Consider the T-relay network and orthogonal Gaussian noise channels, Fig. (1). Giving a transmission order π the *allcast* coding strategy in [5] achieves the rate below. To simplify notation consider $\pi(i) = i$, and $\alpha_{j,T+1} = \beta_j$.

$$R_0 \leq \max_{P_i \tau_i} \min_{i \in \{0,\pi\}} \sum_{j=0}^{i} \tau_j \log(1 + \alpha_{j,i+1} P_j), \tag{7}$$

The maximum rate-per-unit-energy problem, is defined by (3), but in this case

$$C_i = \sum_{j=0}^{i} E_j \alpha_{j,i+1}. \tag{8}$$

The solution, is provided by the following Theorem.

Theorem 3: Consider an order set of nodes π with $|\pi| = T$, and all the nodes inside $i \in \pi$ satisfying $\Gamma_j < \alpha_{01}, \forall j \leq i$. Then, the maximum rate-per-unit-energy can be computed as

$$\eta_{AC} = \max_{\pi} \eta_\pi, \tag{9}$$

$$\eta_\pi = \max_{1 \leq i \leq T} \frac{\alpha_{01}}{1 + \sum_{j=1}^{i-1} \Lambda_j + \frac{\alpha_{01} - \Gamma_{i-1}}{\beta_i}}, \tag{10}$$

with Λ_j, Γ_{i-1} obtained from

$$\Gamma_i = \Gamma_{i-1} + \Lambda_i \beta_i, \quad i \in (0, T-1) \tag{11}$$

$$\Lambda_i = \left(\frac{\alpha_{01} - \Upsilon_{i+1}}{\alpha_{i,i+1}} \right)^+, \quad i \in (1, T-1) \tag{12}$$

$$\Upsilon_i = \sum_{j=0}^{i-2} \Lambda_j \alpha_{j,i}, \quad i \in (2, T) \tag{13}$$

where $(x)^+ = \max(x, 0)$, $\Gamma_{-1} = 0$ and $\Lambda_0 = 1$.

The *energy allocation* achieving η_{AC} in (9) is given by

$$E_{i^*} = \frac{\alpha_{01} - \Gamma_{i^*-1}}{\beta_{i^*}} E_0 \tag{14}$$

$$E_i = \Lambda_i E_0, \forall i \in [1, i^*) \tag{15}$$

$$E_0 = \frac{\eta_\pi}{\alpha_{01}} E, \tag{16}$$

where i^* is the last relay in the route obtained as the argument of the maximum in (10).

Proof: Since the problem is, again, a maximization over the minimum of a set of linear functions in E_i, with a total energy constraint the solution is obtained when all the linear functions coincide. ■

Comments on result in Theorem 2:

- The *energy allocation* at the relays can not be done, only with local c.s.i. For a given route, the relay i needs to compute Λ_i. Assuming that $\alpha_{i,i+1}$ and α_{01} are known the relay still needs to compute Υ_{i+1}. This value depends on Λ_j, from all the *previous* relays but, also, on the local c.s.i of the immediately following relay $\alpha_{j,i+1}$, $j \in (0, i-1)$.
- The *routing algorithm* needs exhaustive search over all the ordered combinations of nodes. Furthermore, all nodes need to maintain the overall route to each destination. Note, that, for a given source with the optimal route π_0^* including node i at some point in the route $\pi_0^*(i) = i$. The optimal route of node i to destination π_i^* is different from the remaining route of the source $\pi_0^*(j > i) \neq \pi_i^*$.

V. Unicast Multi-hop Relay Network

The *one-way relay protocol* coding strategy presented in [7] for the two-relay network, here referred to as *successive binnig*, can be naturally extended to the multi-hop relaying network with $T-$relays. For this strategy, relays are not required to decode the source message, each relay has the ability to facilitate the transmission between the previous relay and the destination (not between the source and the next relay, as in [5]). The following Theorem provides the achievable rate.

Theorem 4: Consider the T-relay network and orthogonal Gaussian noise channels. Giving a transmission order π the *successive binning* coding strategy achieves the rate below. To simplify notation consider $\pi(i) = i$.

$$R_0 \leq \max_{P_i \tau_i} \min_{i \in \{0,\pi\}} (\tau_i \log(1+\alpha_{i,i+1}P_i) + \sum_{j=0}^{i-1} \tau_j \log(1+\beta_j P_j)). \tag{17}$$

Proof: It is a straightforward generalization of the result in [7] for two-relay networks particularized to the orthogonal Gaussian noise channel. Consider the scenario in Fig. 1 but, in this case, $\alpha_{0,2} = 0$ and $X_1(s_1|M)$, $X_2(s_2|s_1)$. Following the *binnnig* argument, given a transmission order $\pi(1 \dots T)$. Each relay employs a random partition of the previous relay codebook $\mathcal{C}_{i-1}$ in sets of size $|\mathcal{C}_{i-1}| / |\mathcal{C}_i| = 2^{n(R_{i-1}-R_i)}$ and transmits a bin index s_i at a rate R_i indexing the transmitted codeword s_{i-1}. Relays can only use the previous relay input to decode s_{i-1}, which is transmitted at rate $R_{i-1} \geq R_i$. The set of achievable rates is given by

$$\begin{array}{ll} R_i \leq \tau_i \log(1+\alpha_{i,i+1}P_i) & i = (0,T) \\ R_i \leq \tau_i \log(1+\beta_i P_i) + R_{i+1} & i = (0,T-1), \end{array} \tag{18}$$

with $R_{T+1} = 0$. The first inequality ensures that relay $i+1$ decodes the bin index transmitted by the previous relay i. The second inequality ensures successful *successive decoding* at destination by repeating the one-relay decoding scheme T times. Finally, eq. (17) is directly obtained by substituting R_{i+1} by its two possible values from $i = T$ to 0. ■

Remark 4.1: Although, the relay i only employs the channel from the immediately previous relay $\alpha_{i-1,i}$, this rate is not always worst than the one obtained in (7), where node i uses all previous transmissions. The reason is the source message decoding requirement at each relay.

The maximum rate-per-unit-energy problem, is defined by (3), with, in this case

$$C_i = \alpha_{i,i+1}E_i + \sum_{j=0}^{i-1} E_j \beta_j. \tag{19}$$

The solution, is provided by the following Theorem.

Theorem 5: Given a relay order π^*, with $|\pi^*| = T$ and considering the same scenario as in Theorem 4. The maximum rate per unit energy can be expressed as

$$\eta_{UC} = \hat{\beta}_0, \tag{20}$$

where $\hat{\beta}_0$ is obtained computing, from $i = T$ to 0

$$\hat{\beta}_i = \frac{\alpha_{i,i+1}\hat{\beta}_{i+1}}{\alpha_{i,i+1} + \hat{\beta}_{i+1} - \beta_i}. \tag{21}$$

The energy allocation achieving (21) is given by

$$E_0 = \frac{\hat{\beta}_0}{\alpha_{0,1}} E, \tag{22}$$

$$E_i = \frac{\hat{\beta}_i}{\alpha_{i,i+1}} \hat{E}_i = \frac{\alpha_{i-1,i} - \beta_{i-1}}{\alpha_{i,i+1}} E_{i-1}, \tag{23}$$

$$E_T = \frac{\alpha_{T-1,N} - \beta_{T-1}}{\beta_T} E_{T-1}, \tag{24}$$

with $i \in (0, T-1)$ and $\hat{E}_i$ defined as the remaining energy before relays i transmits

$$\hat{E}_i \triangleq E - \sum_{j<i} E_j. \tag{25}$$

The *optimal ordered set* π^* can be obtained using the same recursively algorithm 1 but, $\hat{\beta}_n^i$, at the relay block, computed as

$$\hat{\beta}_n^i = \frac{\alpha_{n,i}\hat{\beta}_i}{\alpha_{n,i} + \hat{\beta}_i - \beta_n}. \tag{26}$$

Proof: Using the recursive formulation in (18), particularized for the energy efficient regime $E \to 0$, the problem can be decomposed in T one-relay problems

$$\begin{array}{l} R_i = \min\left(\alpha_{i,i+1}E_i, \beta_i E_i + \hat{\beta}_{i+1}\hat{E}_{i+1}\right) \\ E_i + \hat{E}_{i+1} = \hat{E}_i, \end{array} \tag{27}$$

from $i = T-1$ to 0. The solution is given by $R_i = \hat{\beta}_i \hat{E}_i$, with $\hat{\beta}_i$ and $\hat{E}_i$ define in eq. (21) and eq. (25), respectively.

The proof for the optimal route strategy follows the guidelines of the point-to-point case. Lemma 1 is also fulfilled for the equivalent channel definition (26). In fact, in this case the conditions $\alpha_{n,i} \geq \hat{\beta}_n^i$ and $\hat{\beta}_i \geq \hat{\beta}_n^i \forall i$ are not only necessary but sufficient to satisfy $\hat{\beta}_n^i \geq \beta_n$. ■

Comments on the solution in Theorem 5:

- A more general scenario could be initially considered accomplishing the *non-accumulative* requirement, where each relay does not use the immediately previous transmission, but any of the previous decoded ones. That scenario would reduces to the one considered here in the energy-efficient regime. Note that, the $T-$relay network

is decomposed in T two-hop relay network. At each hop only the single best relay is required to transmit. Thus, no more than one relay use the same previous relay.

- The *energy allocation* requirements at relays admit two different interpretations: Using the first equality in (23), it is needed local c.s.i, $\hat{E}_i$ and $\hat{\beta}_i$ as forward c.s.i. Note that $\hat{E}_i$ can be computed using the received energy from previous relays $E_j\alpha_{j,i}$ and assuming local c.s.i. The second equality in (23), is very similar to the point-to-point solution but, it is also needed some backward c.s.i β_{i-1}. Finally, the source node needs to know, again, its achievable rate with the destination, $\hat{\beta}_0 E$.
- Concerning, the *route discovering* the same comments applicable to the point-to-point case are valid here.

VI. Upper-bound for Regenerative Techniques

One may think in hybrid schemes some relays *all-cast-* and others *uni-casat-based* relaying depending on what is easiest for a relay, to decode the whole message using all previous transmissions or just the party bits using only the transmission of only the previous relay. Here we provide an upper-bound on all this options to discuss whether it worth to employ complicated hybrid schemes. The following Theorem provides an explicit expression for the upper-bound

Theorem 6: Consider the T-relay network and orthogonal Gaussian noise channels. Giving a transmission order π the *allcast* coding strategy in [5] achieves the rate below.

$$R_0 \leq \max_{P_i\tau_i} \min_{i\in\{0,\pi\}} C_i, \tag{28}$$

$$\begin{aligned} C_i &= \tau_i \log(1+\alpha_{i,i+1}P_i) + \\ &\sum_{j=0}^{i-1} \tau_j \left(\log(1+\alpha_{j,i+1}P_j) + \log(1+\beta_j P_j)\right) \end{aligned}$$

with, in this case, $\alpha_{j,T+1} = 0$

Proof: The upper-bound is directly obtained by using the union of contribution in (7) and (17). ■

The intuitive interpretation of this upper-bound could be as follows. Similar to the *unicast* scenario, each relay is required to decode the *bin-index* transmitted by the previous relay s_{i-1}, which, indeed, has been created by indexing its own received bin s_{i-2}. To obtain the upper-bound consider that, some how, each relay can decode the bin index not only using the transmission of the immediately previous relay s_{i-1}, but also taking benefit of the energy received from transmissions containing the bin indexes on which s_{i-1} is based, s_j, $0 \leqslant j < i-1$.

The maximum rate-per-unit-energy problem, is defined by (3) with C_i given by

$$C_i = \sum_{j=0}^{i} \hat{\alpha}_{j,i+1} E_j, \tag{29}$$

with $\hat{\alpha}_{j,i+1} = \alpha_{j,i+1} + \beta_j$ when $j < i$ and $\hat{\alpha}_{j,i+1} = \alpha_{j,i+1}$ otherwise. Since, this is, exactly, the *allcast* scenario problem, Theorem (4), with $\hat{\alpha}_{j,i+1}$ instead of $\alpha_{j,i+1}$ provides the upper-bound for the maximum rate-per-unit-energy η_{UP_R}.

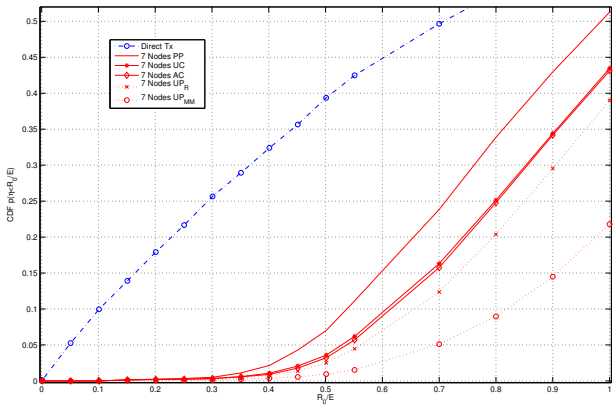

Fig. 2. CDF ($Pr(\eta < Ro/E)$) as a function of Ro/E

VII. Simulation Results and Conclusions

For simulations, it is considered a scenario with 7 nodes (up to 5 relay-hops) and channel gains assumed exponential random variables with mean $\mu = 1$. Fig. 2, compares the Cumulative Density Function (CDF) computed as ($Pr(\eta < Ro/E)$) for each of the strategies described. As a reference, the performance obtained by the direct transmission (Direct Tx) and the Max-flow Min-cut upper-bound (UP_{MM}), derived in [12], are also provided. The *unicast* strategy (UC) is shown to improve the point-to-point (PP) reliable network, with similar routing and energy allocation complexity. Interestingly, the (UC) curve yields extremely close to the *allcast* (AC) one for which *exhaustive* search and *accumulative* relays are needed and not very far away from the regenerative upper-bound (UP_R). However, the cut-set bound is far from being achievable with regenerative techniques.

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