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The Effects of Tire Nonlinearity on Vehicle Steering Stability at High Speed

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Abstract. The steering stability of a vehicle at high speed is the urgent key problem to be solved of automobile independent development. And it is also the premise and one of the necessary conditions of vehicle safety. Considering of the effects of tire nonlinearity, a 4-DOF dynamics model for a vehicle is established. The yaw rate responses, side slip angle, carriage roll angle and front wheel steering angle with different vehicle speeds are calculated. The calculated values are then compared with the values without considering of the effects of tire nonlinearity. The simulations results show that the vehicle responses can be reflected accurately by using nonlinear tire model. With the bigger vehicle speed, the effects of tire nonlinearity on vehicle high-speed steering stability become more obvious.

Introduction

With the development of vehicle technology, vehicle active safety is taken more and more attention. Vehicle handling stability is an important factor of vehicle active safety [1]. The steering characteristic and driving stability are important parts of the vehicle dynamic studies. In fact, there are many nonlinear factors affect vehicle performances. Especially the nonlinear characteristics of the tire have a significant impact on the vehicle steering characteristics and driving stability. When the vehicle is at high speed or before entering the instability state, tires are in the nonlinear working state. It is of no practical significance at the time for using linear tire model to analyze the vehicle handling stability [2,3].

The paper establishes a 4-DOF vehicle model including lateral movement, yaw movement, roll movement and steering movements on the base of considering the tires lateral force nonlinearity, and then analyzes the effects of the nonlinear characteristics of the tire on the vehicle steering stability at high speed.

Nonlinear Lateral Tire Force Model

Setting the vehicle a lateral acceleration of μg for a uniform circular motion the Eq.1 is established as[4]

$$a_y = \frac{v^2}{\rho} \tag{1}$$

where, ρ is the radius of circular motion, v is the vehicle speed, a_y is the vehicle lateral acceleration.

The vehicle centrifugal force is Ma_y. Front and rear wheel lateral forces on the ground can be expressed as

$$\begin{cases} F_{y1} = -Ma_y \dfrac{a}{L} \\ F_{y2} = -Ma_y \dfrac{b}{L} \end{cases} \tag{2}$$

Front and rear axle slip angles are

$$\begin{cases} \beta_1 = \dfrac{Ma}{k_1 L} a_y \\ \beta_2 = \dfrac{Mb}{k_2 L} a_y \end{cases} \tag{3}$$

where, M is the total vehicle mass, a , b are the distance from vehicle center-of-mass to front and rear axles, k_1 , k_2 are the front and rear axle tire comprehensive cornering stiffness, F_{y1} , F_{y2} are the front and rear wheel lateral forces on the ground, L is the wheel base, β_1 , β_2 are the front and rear axle slip angles.

Supposing that the lateral force is saturated and equal to $Mg\mu$. According to the Fiala tire cornering properties formula, lateral force of the tire can be approximately expressed with sideslip angle quadratic as [5,6]

$$\begin{cases} F_{y1}(\beta_1) = Ma_y \dfrac{b}{L} = k_1\beta_1 - \dfrac{a^2}{Mg\mu \dfrac{b}{L}} \beta_1^2 \\ F_{y2}(\beta_2) = Ma_y \dfrac{a}{L} = k_2\beta_2 - \dfrac{b^2}{Mg\mu \dfrac{a}{L}} \beta_2^2 \end{cases} \tag{4}$$

The lateral force of unit side slip angle while steering can be obtained through Eq. 4, and the equivalent cornering stiffness is also expressed as

$$\begin{cases} \dfrac{\partial F_{y1}(\beta_1)}{\partial \beta_1} = k_1 \sqrt{1 - \dfrac{a_y}{\mu g}} \\ \dfrac{\partial F_{y2}(\beta_2)}{\partial \beta_2} = k_2 \sqrt{1 - \dfrac{a_y}{\mu g}} \end{cases} \tag{5}$$

The nonlinear tire model is

$$\begin{cases} F_{y1} = K_1 \beta_1 \\ F_{y2} = K_2 \beta_2 \end{cases} \tag{6}$$

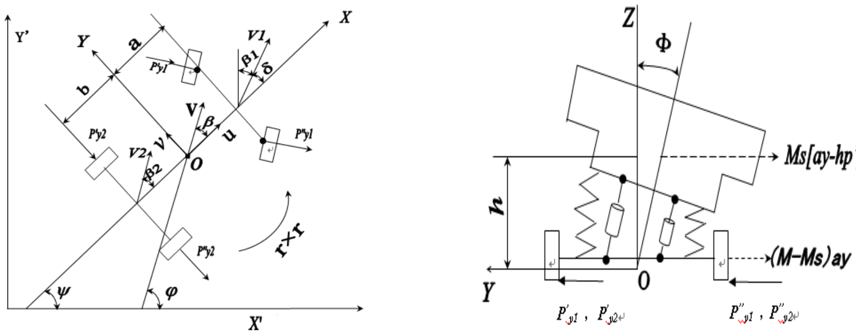
where,

$$K_1 = k_1\sqrt{1-\frac{a_y}{\mu g}} \tag{7}$$

$$K_2 = k_2\sqrt{1-\frac{a_y}{\mu g}} \tag{8}$$

Vehicle Dynamics Equation

To establish the input model of the vehicle steering wheel force, the following assumptions are made. (1) Keep the vehicle with a constant speed with a small corner and ignore the difference between inner and outer wheel steering angles. (2) Suspension characteristics are in the linear range. (3) Excluding air resistance. (4) Front roll center is the same as rear one and set as the coordinate origin as shown in Fig. 1. The forward direction is the x-axis, the inward direction is the y-axis and the vertical direction is the z-axis. Thus vehicle steering motion model is simplified as a 4-DOF dynamics model which includes lateral motion, yaw motion, roll motion and steering motion.



(a) Top view (b) Rear view

Fig.1 Vehicle steering motion model

The Force Balance Equation of y-axis Direction

The External Force of y-axis Direction. The total lateral force of y-axis direction comes from two aspects when the vehicle of independent suspension travels on a curve: one is the tire extraversion lateral force and cornering force, and the other is the roll camber force caused by additional wheel angle with the reason of compartment rolling of the front and rear independent suspension as shown in Fig. 1 [7,9].

The total external force of y-axis direction is

$$Y = Y_\beta \beta + Y_{\omega_r}\omega_r + Y_\phi \phi + Y_\delta \delta + k_1 \Lambda_1 \tag{9}$$

where, β is the side slip angle of mass-center, ω_r is vehicle yaw rate, ϕ is the carriages roll angle, δ is the front wheel steering angle, Λ_1 is the front toe angle, Y_β is the lateral reaction force of the ground caused by unit side slip angle, Y_{ω_r} is the lateral reaction force of the ground caused by unit yaw rate, Y_δ is the lateral reaction force of the ground caused by unit roll angle, Y_δ is the lateral reaction force of the ground caused by unit front wheel steering angle.

$$Y_\beta = -(K_1 + K_2) \tag{10}$$

$$Y_{\omega_r} = \frac{1}{v}(-aK_1 + bK_2) \tag{11}$$

$$Y_\phi = \left(\frac{\partial Y_{1\gamma}}{\partial \phi} + \frac{\partial Y_{2\gamma}}{\partial \phi}\right) \tag{12}$$

$$Y_\delta = K_1 \tag{13}$$

The Inertial Force of y-axis Direction. When a vehicle steers at a speed of v the inertial force along the y-axis direction F_y can be expressed as[10]

$$F_y = M(\dot{v} + \omega_r v) - M_s(h_2 - h_1)\ddot{\phi} \tag{14}$$

When β is small the tangential acceleration is expressed as $\dot{v} = v\dot{\beta}$. And Eq. 14 can be rewritten as

$$F_y = M(v\dot{\beta} + \omega_r v) - M_s(h_2 - h_1)\ddot{\phi} \tag{15}$$

where, $\dot{\beta}$ is the mass-center angular velocity of the slip angle, M_s is the mass of suspension parts, $\ddot{\phi}$ is the carriages roll angular acceleration, h_1 is the front and rear suspension roll center height, h_2 is the mass-center height of suspension parts.

The Force Balance Equation of y-axis Direction. According to that $\sum F_y - Y = 0$ the force balance equation of y-axis direction is expressed as

$$Y_\beta \beta + Y_{\omega_r}\omega_r + Y_\phi \phi + Y_\delta \delta + K_1 \Lambda_1 = M(v\dot{\beta} + \omega_r v) - M_s(h_2 - h_1)\ddot{\phi} \tag{16}$$

The Torque Balance Equation Around x-axis

The Moment of Inertia Around x-axis. When the vehicle is turning carriages as the suspension part will roll around the x-axis. The moment of inertia is expressed as

$$\sum M_{xg} = I_X \ddot{\phi} - I_{XZ}\dot{\omega}_r + M_s v(h_2 - h_1)(\dot{\beta} + \omega_r) \tag{17}$$

where, $\sum M_{xg}$ is the carriages' moment of inertia around the x-axis, I_X is the carriages rotational inertia around the x-axis, I_{XZ} is the carriages' product of inertia around the x-axis and z-axis.

The External Torque of Suspension Parts. The external torque of carriages around x-axis is

$$\sum M_{xw} = G_s\phi(h_2 - h_1) - (k_{\phi_1} + k_{\phi_2})\phi - \left(\frac{\partial L_1}{\partial p} + \frac{\partial L_2}{\partial p}\right)p \tag{18}$$

where, $\sum M_{xw}$ is the carriages' external torque around the x-axis, p is the carriages roll angular velocity, $\frac{\partial L_1}{\partial p}$ is the roll angle damping of front suspension, $\frac{\partial L_2}{\partial p}$ is the roll angle damping of rear suspension, k_{ϕ_1} is the front inclination stiffness, k_{ϕ_2} is the rear inclination stiffness.

$$p = \dot{\phi} \tag{19}$$

According to that $\sum M_{xg} - \sum M_{xw} = 0$ the torque balance equation around x-axis can be expressed as

$$G_s\phi(h_2 - h_1) - (k_{\phi 1} + k_{\phi 2})\phi - \left(\frac{\partial L_1}{\partial p} + \frac{\partial L_2}{\partial p}\right)p = I_X\ddot{\phi} + I_{XZ}\dot{\omega}_r - M_s v(h_2 - h_1)(\dot{\beta} + \omega_r) \tag{20}$$

The Torque Balance Equation Around z-axis

The Moment of Inertia Around z-axis. Vehicles do the yaw movement around z-axis while steering, the moment of inertia can be expressed as

$$\sum M_{zg} = I_Z\dot{\omega}_r + I_{XZ}\ddot{\phi} \tag{21}$$

where, $\sum M_{zg}$ is the vehicle yaw moment around the z-axis, I_Z is the vehicle rotational inertia around the z-axis, I_{XZ} is the vehicle product of inertia around the x-axis and z-axis.

The External Torque Around z-axis. When the vehicle is turning the external torque around z-axis consists of three parts [11]. (1) The torque of z-axis caused by lateral force of the front and rear axles. (2) Aligning torque generated by the front and rear tire cornering (Here it can be ignored because it's too small). (3) The torque generated by the rolling resistance of the left and right wheels.

The total external torque around z-axis can be expressed as

$$\sum M_{zw} = M_\beta\beta + M_{\omega_r}\omega_r + M_p p + M_\phi\phi + M_\delta\delta \tag{22}$$

where, $\sum M_{zw}$ is the vehicle total external torque around the z-axis, M_β is the torque around the z-axis caused by unit vehicle side slip angle, M_{ω_r} is the torque around the z-axis caused by unit yaw rate, M_p is the torque around the z-axis caused by unit roll angular velocity, M_ϕ is the torque around the z-axis caused by unit roll angle, M_δ is the torque around the z-axis caused by unit front wheel steering angle.

$$M_\beta = -aK_1 + bK_2 + N_1 + N_2 \tag{23}$$

$$M_{\omega_r} = -\frac{1}{v}\left(a^2K_1 + b^2K_2\right) \tag{24}$$

$$M_p = \left(\frac{\partial L_1}{\partial p} + \frac{\partial L_2}{\partial p}\right)\mu \tag{25}$$

$$M_\phi = aE_fK_1 - bE_rK_2 \tag{26}$$

$$M_\delta = -K_1a + N_1 \tag{27}$$

where, N_1 is the front wheel aligning torque coefficient, N_2 is the rear wheel aligning torque coefficient, E_f is the front wheels roll steering coefficient, E_r is the rear wheels roll steering coefficient.

According to that $\sum M_{zg} - \sum M_{zw} = 0$ the torque balance equation around z-axis can be expressed as

$$M_\beta\beta + M_{\omega_r}\omega_r + M_pp + M_\phi\phi + M_\delta\delta = I_Z\dot{\omega}_r + I_{XZ}\ddot{\phi} \tag{28}$$

Dynamics Equation of Steering System. Simplifying the steering system as the rotation around the front wheel kingpin and ignoring frictional torque and viscous drag torque of the system the force input kinetic equation can be expressed as [10]

$$\begin{aligned}&\left(iI_s\cos\alpha\right)\dot{\omega}_r - \left(iI_s\sin\alpha\right)\ddot{\phi} - \left(i^2I_s + I_\omega\right)\ddot{\delta} + k_\omega\dot{\delta} - 2K_1D_\omega\frac{a}{v}\omega_r\\&-2K_1D_\omega\beta + 2K_1D_\omega E_f\phi + \left(2K_1D_\omega + C_\omega\right)\delta = iT\end{aligned} \tag{29}$$

where, D_ω is the front wheels aligning torque, I_s is the steering wheel rotational inertia, I_ω is the front wheels rotational inertia around kingpin, C_ω is the stiffness of the resistance to the corner of front wheels when the steering wheel is free, k_ω is the equivalent drag coefficient of the front wheels around kingpin when the steering wheel is free, α is the angle of the steering column with the z-axis, i is the total transmission ratio of the steering system, T is the input torque on the steering wheel by the driver.

Dynamics Equations of the Whole Vehicle System. Eq.16, Eq.20, Eq.28 and Eq.29 constitute the dynamics equations of the whole vehicle system while steering.

Numerical Simulation

Take the sample vehicle as a study object and give the steering wheel a force step input of $iT = 530N \cdot m$ to simulate the condition of turning. The vehicle yaw rate ω_r, mass-center sideslip angle β, carriages roll angle ϕ and front wheel steering angle δ are decided as the solution variables. The dynamics equations of the whole vehicle system can be solved with MATLAB under two conditions (One is considering the tire nonlinear impact and the other is not) by K_1 and K_2 being replaced by k_1 and k_2 respectively in the dynamics equations. Setting the speed as 90 km/h and 120 km/h the effects of tire characteristics on turning stability of the vehicle can be calculated and compared. Table 1 shows the parameters of the sample vehicle and the simulation results are shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5.

Table 1. Parameters of the sample vehicle

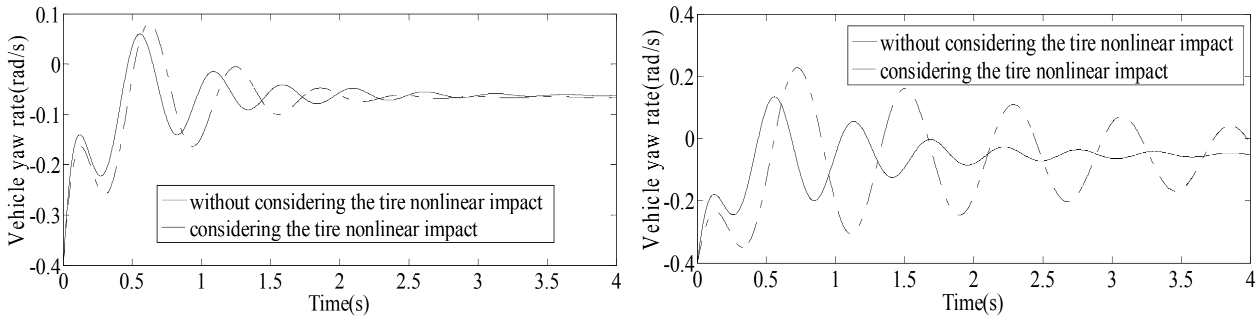
M(kg)	Λ_1 (deg)	b(m)	k_{ϕ_1} (N·m·rad^{-1})	k_{ϕ_2} (N·m·rad^{-1})	$\frac{\partial L_2}{\partial p}$ (N·m·s·rad^{-1})	$\frac{\partial L_1}{\partial p}$ (N·m·s·rad^{-1})
3018	3	1.88	100500	32700	3400	3400

Continued

a(m)	I_Z (kg·m^2)	I_X (kg·m^2)	I_{XZ} (kg·m^2)	k_1 (N·rad^{-1})	k_2 (N·rad^{-1})	I_s (kg·m^2)	M_s (kg)
1.84	10437	1960	0	29890	50960	0.054	2685

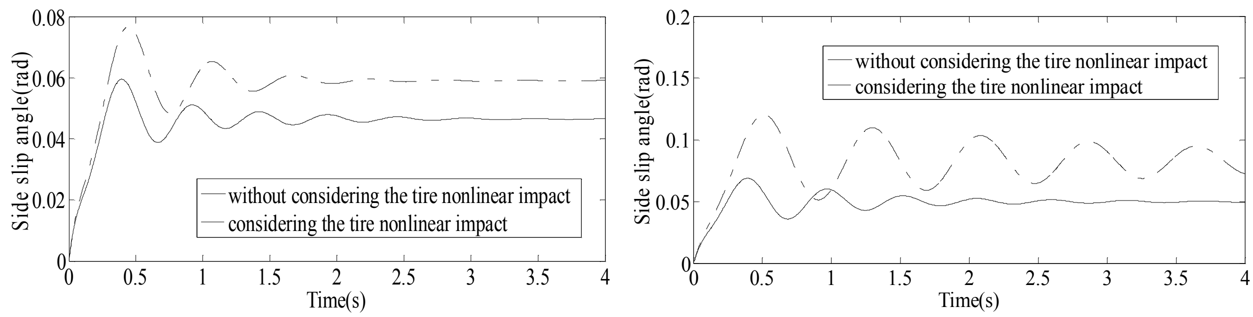
Continued

I_w (kg·m^2)	k_w (N·m·s·rad^{-1})	C_w(N·m·s·rad^{-1})	α (deg)	E_r	μ	E_f	i	h (m)
3.93	294	0	58.5	0	0.6	-0.114	24.9	0.488



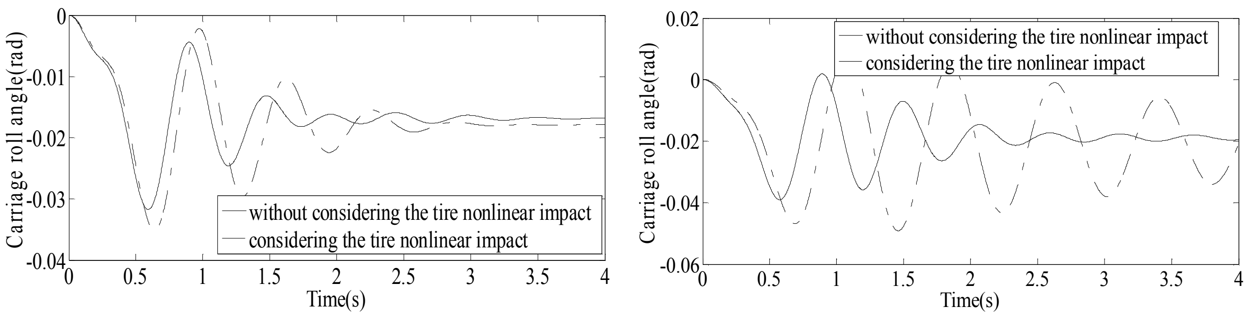
(a) $v = 90$ km/h (b) $v = 120$ km/h

Fig. 2 Yaw rate simulation results



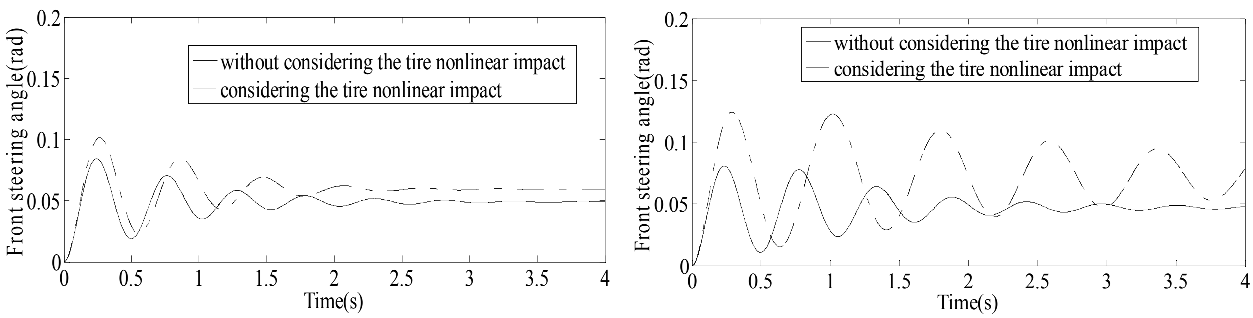
(a) $v = 90$ km/h (b) $v = 120$ km/h

Fig. 3 Mass-center sideslip angle simulation results



(a) $v = 90$ km/h (b) $v = 120$ km/h

Fig. 4 Carriages roll angle simulation results



(a) $v = 90$ km/h (b) $v = 120$ km/h

Fig. 5 Front wheel steering angle simulation results

It can be seen from Fig.2 that the vehicle yaw rate curve has nearly no difference between that two conditions when the sample vehicle turns at speed of 90 km/h. But a big difference appears at the speed of 120 km/h. This shows that the higher the speed is the more obvious the tire non-linear characteristics affect.

It can be seen from Fig. 3 that the mass-center sideslip angle curve gets a big difference between that two conditions when the sample vehicle turns both at speed of 90 km/h and 120 km/h. And the higher the speed is the more different it is.

It can be seen from Fig. 4 that the carriages roll angle curve has poor difference between that two conditions when the sample vehicle turns at speed of 90 km/h. But a big difference establishes at speed of 120 km/h. Affected by tire nonlinear characteristics the carriages roll angle will go through a long time to tend to a stable value.

It can be seen from Fig. 5 that the front wheel steering angle curve keeps no difference between that two conditions when the sample vehicle turns at speed of 90 km/h. But the tire nonlinear characteristics have an important influence to the steering stability when it's at speed of 120 km/h.

Conclusions

(1) In consideration of the effects of tire nonlinear characteristics at high speed a 4-DOF dynamics model for vehicles is established to analyze and compare the response of the vehicle in high-speed steering.
(2) Each state response of the vehicle is larger with considering the effects of tire nonlinear characteristics than without considering the effects of tire nonlinear characteristics. And as the speed increasing the differences become more obvious.
(3) The theoretical results can provide theoretical support for the safety control of heavy vehicles in curve driving.

Reference

[1] Cui Sheng-min, Li Man-tian: Journal of Harbin Institute of Technology Vol. 34(2002), p.396. In Chinese.
[2] Qu Qiu-zhen, Liu Yan-zhu: Chinese Quarterly of Mechanics Vol. 21(2000), p.38. In Chinese.
[3] Yung-Hsiang Judy Hsu, Shad Laws, Christopher D. Gadda: *A method to estimate the friction coefficient and tire slip angle using steering torque* (ASME International Mechanical Engineering Congress and Exposition, USA, 2006).
[4] Yin Hao, Zhao You-qun, Xu Pei-min :Transactions of the Chinese Society for Agricultural Machinery Vol. 38(2007), p. 23-26. In Chinese.
[5] Masato Abe, in: *Movement and manipulation of the vehicle*, edited by Machinery Industry Press, Beijing, 1998, in press. In Chinese.
[6] Zhao You-qun, Yin Hao, Zhang Gui-xia: Mechanical Science and Technology Vol. 24(2005), p. 1138. In Chinese.
[7] Wei Dao-gao, Wang Xiao-feng, Jin Dafeng: Automotive Engineering Vol. 31(2009), p. 33. In Chinese.
[8] Wei Dao-gao, Li Ke-qiang, Wang Xiao-feng: Transactions of the Chinese Society for Agricultural Machinery Vol. 38(2007), p.19. In Chinese.
[9] Wei Dao-gao, Wang Xiao-feng, Jin Dafeng: Journal of Tsinghua University(Science and Technology) Vol. 46(2006),p. 724. In Chinese.
[10] Guo Kong-hui, in:*Vehicle handling dynamics*, edited by Jiangsu science and Technology Press , Nanjing, 2011,in press. In Chinese.
[11] Ki Baek Kim: IEEE Transactions on Automatic Control Vol. 49(2004), p. 797.