Thermal Radiation in Participating Media: The Past, the Present, and Some Possible Futures

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A review of major events in the development of the engineering treatment of radiative transfer in participating media is presented. This review is followed by a discussion of the major analytical methods presently used for this class of problems, along with their strengths and limitations. Some comments are added concerning important areas of research that remain to be completely treated, and some areas of potential future application.

Introduction and a Short History

The history of early measurements of spectral and total radiation as contained in the seminal references makes fascinating reading. Herschel's evident amazement at the discovery of energy outside the visible part of the spectrum (referring to it as "invisible light") (Herschel, 1800) and the delight expressed by Nobili and Meloni (1831) at the sensitivity of their thermopile radiometer still reverberate from the antique chronicles to bring excitement even to the modern reader. Nobili and Meloni note, following their measurements of the temperatures of over 400 different insects, that they observed the following law concerning caterpillars: "The caterpillars always possess a more elevated temperature than the butterflies or chrysalides." They also noted that the reflectivity of metals follows the same progression as does their electrical conductivity, an observation that antedates the formulation of the Hagen-Rubens relation by nearly seventy years.

The ingenuity and accuracy of experimental measurements by these early workers, plus the later benchmark measurements of the blackbody total and spectral emissive power by Lummer and Pringsheim (1897, 1899, 1901) over the range 300 to 1300°C, provided a precise touchstone for the later theoretical work of Planck and others. Barr (1960) has provided an interesting review of the early measurements in radiation.

Because of its impact on the development of modern physics, thermal radiation is arguably the most fruitful of all the elements in the study of heat transfer. It is well known that Planck, after a series of attempts to match classical theory with experimental measurements of the blackbody spectrum, was forced to hypothesize the existence of quantized energy states. This led him to the development of quantum theory (Planck, 1959). Albert Einstein, in building on Planck's ideas, showed that an alternative derivation of the blackbody spectral distribution required consideration of the presence of induced emission and the coupling of the equilbrium radiation field with the emission of radiation by matter (Einstein, 1916, 1917). The explanation of the observed line absorption and emission spectra of gases led Neils Bohr to develop his theory of the structure of the hydrogen atom (Bohr, 1913), further opening the door to the modern quantum interpretation of the structure of matter and its interaction with energy.

We are now so confident of the theoretical basis of radiative transfer that most engineering radiative transfer calculations are accepted without experimental confirmation.

Engineering Radiative Heat Transfer in Absorbing-Scattering-Emitting Media

By the 1920s, the need for design tools that could adequately predict radiative transfer in industrial furnaces was becoming obvious. Exchange between simply arranged surfaces was previously worked out by ray-tracing techniques, i.e., following the history of emitted radiation from one surface through its history of reflections among the surfaces, and attempting to find a closed-form solution for the resulting infinite series (Christiansen, 1883). This method had serious flaws when many surfaces were present, or an attenuating medium was present between the bounding surfaces.

The pioneering work by Hottel (1931, 1933) systematically developed a methodology for such applications. The absence of high-speed computational capability at that time caused Hottel to develop methods that were amenable to hand calculation or that made reference to auxiliary tools such as graphs of gas emittance. These graphs were constructed from data that were painstakingly measured and extrapolated to useful parameter ranges.

Hottel's early work on configuration or view factors showed insight into these useful quantities. His derivation of the "crossed-string" method for factors between parallel, infinitely long bodies, as well as his original derivation of many commonly used factors, are perhaps no longer accorded their due regard (Hottel, 1931, 1933, 1954).

A very significant paper by Poljak (1935) introduced the net-radiation method, which simplifies the formulation of radiative exchange in enclosures by casting the formulation in terms of the net radiative energy at a given surface; i.e., the difference between the incident radiative energy flux (irradiance) and the outgoing radiant energy flux (radiosity). This approach allows radiative exchange to be set up in terms of a closed set of linear equations, rather than through use of an infinite series obtained by ray-tracing.

Hottel continued development of analytical treatments of radiative transfer in enclosures, including the effects of medium absorption. His treatment of an absorbing isothermal gas by using the mean beam length is an extremely clever engineering approximation. The exposition of this method in the various editions of the text edited by McAdams (Hottel, 1954) was the major reference for radiative transfer analysis throughout the 1940s and well into the middle 1950s. Further extensions culminated in the zoning method (Hottel and Cohen, 1958; Hottel and Sarofim, 1967), which allows consideration of nonuniform gas temperatures. This method continues to influence practitioners of furnace design.

Innovative applications of thermal radiation principles outside the conventional engineering of power plants and furnaces during the 1950s and 1960s put increasing demands on the accuracy of radiative transfer calculations. The greatest

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impetus was probably the emergence of NASA and the need for heat transfer calculations applicable to new propulsion systems and space vehicles, and the parallel need to determine radiative transfer in very high and very low temperature systems. This forced consideration of spectral, directional, and temperature-dependent property variations and their effects on radiative transfer, and required analytical methods that could include these effects. In addition, more accurate data for surface properties and the properties of attenuating media were required. The data gathered by Hottel and others for the total emittance of common gases and gas mixtures found in combustion processes became inadequate, although still useful for some applications.

Also during the 1960s, the greater availability, speed, and capacity of the electronic computer turned researchers toward alternative methods of calculation that were previously impractical. Certain drawbacks to the zoning method had long been obvious, notably the inability to treat variations in the properties of the absorbing medium within the enclosure easily, the incompatibility of practical zone structures with the grid size needed for fluid mechanical/convection/conductive analysis, and the difficulty of treating geometries that stray from purely rectangular or cylindrical. Although it is possible using the zoning method to account for spectral effects through the "sum of gray gases" method (Hottel and Sarofim, 1967), results are not completely satisfactory. Also, zoning allows no accounting for anisotropic scattering.

The intensified interest and the need for greater sophistication in the treatment of engineering radiative transfer encouraged the publication of a number of texts devoted to the subject during this time, including those by Wiebelt (1966), Sparrow and Cess (1978, first edition 1966), Hottel and Sarofim (1967), Love (1968), and Siegel and Howell (1972, based on their NASA reports of 1968, 1969, and 1971).

Attempts to treat the spectral properties of absorbing gases during the late 1960s and early 1970s were based on the total emittance charts developed by Hottel, or were borrowed from atmospheric physics, where the so-called narrow-band models (Goody, 1968; Ludwig et al., 1973) had been developed and used. Edwards and his co-workers (Edwards and Menard, 1964; Edwards, 1968; Edwards and Balakrishnan, 1973) provided the basis for much more useful engineering approaches to modeling the spectral properties of the gases most often encountered in engineering problems by the development of "wide-band" models. These provide relatively simple expressions for the band emittance of these gases as a function of total and partial pressure, path length, and temperature. They also make the band properties of gas mixtures relatively easy to calculate.

The problem of incorporating other modes of heat transfer into radiation-dominated problems also began in the 1960s, bringing to light the difficulties in grid-size matching and convergence. The solutions presented by Viskanta and Grosh (1962a, 1962b) long remained benchmark solutions for the planar geometry when conduction and radiation are present.

Throughout the 1970s and up to the present, the search has continued for methods that could be applied broadly to multidimensional radiative transfer in enclosures, with the additional capability of handling spectral and anisotropic characteristics in absorbing-emitting-scattering media. Interest was strong in the older methods and their extensions, and some newer approaches showed promise, such as the finite-element methods described later. New solutions for standard problems have provided benchmarks for comparing existing and emerging methods for accuracy and ease of solution. Notable among these are the solutions for two-dimensional gray enclosures given by Crosbie and Schrenker (1985) and Crosbie and Farrell (1985) and the three-dimensional solutions in rectangular enclosures given by Larsen (1983) using the zonal method and by Menguc (1985) using the P-3 method.

Methods developed for other applications have long been appropriated for use in engineering radiative transfer. These include the Schuster–Schwarzschild [now more commonly called the two-flux model; see Schuster (1905), Schwarzschild (1906), Chandrasekhar (1944)], Milne–Eddington, discrete ordinate, (S_n), and diffusion methods from astrophysics; and Monte Carlo, differential, P–N, invariant embedding, and others from nuclear physics and engineering.

Many of the methods developed in atmospheric science, astrophysics, and nuclear engineering are not directly applicable to the enclosure problems found in engineering. This is for two reasons. First, intensity at the boundary is often given as a boundary condition in the nonengineering fields, while temperature and boundary absorptance are the usual engineering boundary conditions. Second, the radiative flux divergence term in the energy equation is required for solution of many engineering problems, and evaluation of this term requires that the local radiative intensity be integrated over the sphere of solid angles, as is outlined in a later section. The additional complexity required by the engineering boundary conditions and the need to have much more detailed knowledge of the radiative intensity field at each point in the medium makes some methods impractical for engineering application.

Each of the developing methods was successfully applied to simple one-dimensional gray gas problems. When they were extended to multidimensional geometries with nongray media with various boundary conditions, drawbacks emerged in

Nomenclature

a = linear absorption coefficient
$\overline{gg}$ = gas volume-to-gas volume exchange area in zoning method
$\overline{gs}$ = gas volume-to-surface exchange area in zoning method
h = enthalpy of medium
i' = radiative intensity
I' = radiative source function
k = thermal conductivity of medium
P = pressure
s = distance from origin of intensity
$\overline{sg}$ = surface-to-gas volume exchange area in zoning method
$\overline{ss}$ = surface-to-surface exchange area in zoning method
q = energy flux; energy transiting normal to a given area per unit time per unit area
q''' = volumetric internal energy generation rate
T = absolute temperature
V = volume
κ = optical depth
λ = wavelength
ρ = density of medium
σ = linear scattering coefficient
τ = time
Φ = dissipation function
ϕ = scattering phase function
ω = solid angle
Ω = albedo $= \sigma/(\sigma+a)$

Subscripts

b = blackbody
g = property of participating medium
i = incident
i, j = surface indices
r = radiative
s = scattering
γ, μ = volume indices
λ = spectrally dependent

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every case. No single method is now accepted as being the best for all problems. The greatest stumbling blocks remain those found in applying the zoning method: inability to treat nonuniform gas absorption and scattering coefficients easily; difficulty in matching the required grid size for radiation with that needed for concomitant computation of convection, conduction, or fluid mechanics (although many methods do much worse than the zoning method in grid matching); incorporation of spectral effects in an accurate yet tractable way; and complete treatment of anisotropic scattering.

Exact solution techniques for the integral equation of transfer were found for some simples cases. Crosbie and Dougherty (1981) have reviewed exact solution methods for some one-dimensional problems, and Viskanta and Menguc (1987) discuss recent multidimensional exact solutions. All of the exact solutions assume that the properties of the attenuating medium are spatially uniform. Lin (1987) has presented detailed formulations of the exact integral radiative transfer equation for various simple configurations in one- and multidimensional geometries.

Additional problems arise as more complicated problems are addressed. An example is the difficulty in obtaining converged numerical solutions to problems in which radiation interacts with chemical energy release in flame fronts (Chen et al., 1987). In these problems, the temperature gradients are very steep within the combustion zone, and the reaction rate depends strongly on the temperature. If multi-step chemistry is considered, the resulting very stiff equations are quite difficult to bring to convergence. In a sense, this is another manifestation of the problem of matching the numerical grid for radiation with that necessary for the other parts of the energy equation, in this case for combustion relations.

The Radiative Transfer Equation and the Energy Conservation Equation

To explore further the present state of investigation into solving radiative transfer problems, it is necessary to look in some detail at the governing relations involved in a general formulation.

The radiative transfer equation (RTE), which describes the spectral radiation intensity i'_λ in a particular solid angle ω and at a local optical depth κ_λ, can be cast in terms of the gradient in radiative intensity in an absorbing-emitting-scattering medium. This *differential form* of the RTE is

$$\frac{\partial i'_\lambda}{\partial \kappa_\lambda} + i'_\lambda(\kappa_\lambda) = I'_\lambda(\kappa_\lambda,\omega) \tag{1}$$

The optical depth κ_λ depends on the local linear spectral absorption coefficient $a_\lambda(s)$ and the local spectral linear scattering coefficient, $\sigma_\lambda(s)$, and is defined by

$$\kappa_\lambda(s) = \int_{s^*=0}^{s} (a_\lambda + \sigma_\lambda)\, ds^* \tag{2}$$

where s is the distance from a boundary to the local position of interest. The linear spectral coefficients describe the magnitude of the logarithmic reduction in spectral intensity due to absorption and scattering, respectively.

The I'_λ is the *source function*, which defines the intensity in direction ω at a local position s arising from the emission of radiation by the medium, plus the intensity scattered into the directon ω at the local position due to incoming intensity from all directions. The source function is defined by

$$I'_\lambda(\kappa_\lambda,\omega) = (1-\Omega_\lambda) i'_{\lambda b}(\kappa_\lambda) + \frac{\Omega_\lambda}{4\pi} \int_{\omega_i=4\pi} i'_\lambda(\kappa_\lambda,\omega)\phi(\lambda,\omega,\omega_i)\, d\omega_i \tag{3}$$

Here, $\Omega_\lambda = \sigma_\lambda/(\sigma_\lambda + a_\lambda)$ is the albedo, and ϕ is the scattering phase function, which defines the intensity incident from direction ω_i that is scattered into direction ω. Note that the term $i'_{\lambda b}$, the local blackbody intensity, depends explicitly upon local temperature of the medium, which is normally not given in engineering radiation problems; rather, it is the unknown of interest.

The intensity in equation (1) is both spectrally and directionally dependent, and the albedo is also spectrally dependent. The differential RTE can be formally integrated by using an integrating factor to give the *integrated form* of the RTE, which describes the intensity of radiation at some optical depth κ_λ in the medium in terms of the intensity reaching that position from other locations κ^*_λ in the medium and from the medium boundary at $\kappa_\lambda = 0$. Its form is

$$i'_\lambda(\kappa_\lambda) = i'_\lambda(0)\exp(-\kappa_\lambda) + \int_0^{\kappa_\lambda} I'_\lambda(\kappa_\lambda,\omega)\exp[-(\kappa_\lambda - \kappa^*_\lambda)]\, d\kappa^*_\lambda \tag{4}$$

The local divergence of the radiative flux can be determined from the local intensity (if it is assumed, as is usually the case, that the scattering phase function ϕ is independent of the direction of incidence ω_i) by the relation

$$\nabla \cdot q_r = \int_{\lambda=0}^{\infty} a_\lambda(\lambda)\left[4\pi i'_{\lambda b}(\lambda) - \int_{\omega=0}^{4\pi} i'_\lambda(\lambda,\omega,\kappa_\lambda)\, d\omega\right] d\lambda \tag{5}$$

For most engineering problems that include radiation, it is necessary to solve an energy conservation relation that explicitly provides the local temperature. The form of this relation depends on the particular forms of energy that must be treated in a given problem, but it can be written for a one-component fluid in the general form

$$\rho\frac{Dh}{D\tau} = \frac{DP}{D\tau} + \nabla\cdot(k\nabla T - q_r) + q''' + \Phi \tag{6}$$

Here, the term $\nabla \cdot q_r$ represents the divergence of the radiative flux, which must be found by use of equation (5). The form $(D/D\tau)$ represents the substantial derivative. The term $\rho(Dh/D\tau)$ represents contributions due to convection, $DP/D\tau$ expansion work, $\nabla \cdot k\nabla T$ conduction, q''' internal generation, and Φ is the dissipation function. Some, none, or all of these terms may be present in a given radiation problem.

The fact that the radiative flux divergence depends upon the temperature at each point throughout the medium and at the boundaries through equations (4) and (5), and that this temperature distribution is the objective of solving equation (6), is what makes these problems challenging. The problems are always implicit in temperature, and therefore require iterative solution unless the temperature distribution is specified.

Present Methods of Choice

A survey of the literature over the past few years plus discussions with other workers in the field show that some methods are receiving much attention, while others that appeared promising at one time have fallen out of favor. In some cases, extension of methods that were quite easy to apply in one-dimensional geometries has proven difficult or intractable in two and three dimensions. For other methods, the addition of important effects, such as anisotropy, nonuniform properties, or spectral effects, has proven to be the difficulty.

The following methods are attracting interest at present:

P-N Methods. The use of expansions of the local intensity in terms of spherical harmonics, with truncation to N terms in the series and substitution into the moments of the differential form of the equation of transfer, leads to the so-called *P-N* approximation. In general, the higher the value of N, the better

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will be the agreement with exact solutions. When $N=1$, this method is also called the differential approximation.

Usually, odd expansions (P-1, P-3, etc.) are used in the P-N method, for two reasons. First, the even terms are difficult to incorporate into useful engineering boundary condition formulations (Ratzel, 1981). The odd-order expansions work well in the so-called Marshak boundary conditions (Marshak, 1946), which have a physical interpretation in terms of the net boundary radiative flux for first order terms. Second, for one- and two-dimensional problems in many geometries, it is found that second-order terms are negligible or drop out of the formulation of the radiative transfer equation or its moments, indicating that second-order expansions often will provide only marginal increases in accuracy over first-order solutions.

Even-order expansions can be carried out if the boundary condition formulations proposed by Mark (1945) or their modified version used by Shokair and Pomraning (1981) are used. As expected, however, Shokair and Pomraning found that second order expansions provide little increase in accuracy over first-order.

According to Davison (1958), for higher order expansions ($N>7$), the Mark conditions provide better accuracy than the Marshak conditions. However, unless only the intensity of radiation is of interest in a particular problem, the use of expansions of order $N>3$ becomes very difficult. Since the intensity itself is seldom of interest in engineering radiative transfer problems (because the flux or its divergence is needed in the energy equation), the Marshak conditions are used most generally in engineering formulations.

The P-N method is now used widely because the differential form of the resulting transfer equations makes them compatible with the gridding requirements of the energy and fluid-mechanics equations often present in the numerical formulation of a given problem. It has usually been found that $N=3$ provides good accuracy in one-, two-, and three-dimensional problems (Higenyi, 1979; Bayazitoglu and Higenyi, 1979; Ratzel, 1981; Menguc, 1985; Menguc and Viskanta, 1985), while going to $N=5$ provides little additional increase in accuracy (Menguc and Viskanta, 1982).

Extension of the P-N method to include anisotropic scattering has made it useful in modeling combustion systems using coal, where anisotropic scattering is an important factor. Menguc (1985) used the delta-Eddington phase function in a P-3 solution in three dimensions.

Two-Flux and Discrete Ordinate Methods. The two-flux method is based on a simple physical model in which the positive-direction and negative-direction radiative intensity are each assumed isotropic over their respective hemisphere of solid angles. Anisotropic scattering can be considered, but is used as an integrated average fraction for forward and backscattering. The method works well in one-dimensional systems, where it is simple to apply. It has been used in analysis of packed beds (Brewster and Tien, 1982; Brewster, 1986); radiative transfer through fibers and powders (Wang and Tien, 1983; Tong and Tien, 1983); combustion in composite solids (Brewster and Patel, 1987); and porous layers with penetrating flow and an external radiation source (Lee and Howell, 1986). Viskanta (1982) has reviewed the pertinent literature on the two-flux model through 1982.

When the solid angles about a location are divided into more than the two directions with uniform intensities used in the two-flux method, the method is known as the multi-flux, discrete ordinate, or S_n method (Chandrasekhar, 1960; Lathrop, 1966). Fiveland (1984, 1987) has developed and implemented the discrete ordinate method as part of a general code for the analysis of heat transfer in coal-fired furnaces. Menguc and Viskanta (1987) present a comprehensive review of recent work using the method.

Finite Element Methods. Finite element solutions (Sokmen and Razzaque, 1987; Razzaque et al., 1983, 1984; Chung and Kim, 1984; Wu and Ferguson, 1981; Fernandes et al., 1981; Nice, 1983) have been applied to the nonlinear combined-mode problems of conduction and/or convection with radiation, including scattering and both known temperature and known heat flux boundary conditions. Most of these solutions have used the Galerkin finite element method. The finite element method in principle gives exact solutions within the errors introduced by the numerical solution itself; i.e., no approximations need be made in the formulation of a problem. Each finite element of the medium volume can have its temperature (or the fourth power of its temperature) described in terms of various degrees of accuracy. If each element is taken to be isothermal, then the method is equivalent to a zoning technique. Usually, the elements in two-dimensional problems have their temperature described by biquadratic functions, which allow a continuous temperature profile to be prescribed in the medium by matching the element boundary temperatures to the temperatures at the boundaries of adjacent elements. Higher order functions for the temperatures allow matching of temperature slopes at the element boundaries as well.

Sokmen and Razzaque (1987) and Chung and Kim (1984) have included isotropic scattering in their analysis of two-dimensional systems, and Sokmen and Razzaque also were able to include known-heat-flux boundary conditions in their analysis of enclosures with absorbing-emitting-scattering conducting media.

Much remains to be done to exploit the finite element approach. The method offers the possibility of high accuracy and a built-in match of the numerical grids for the radiation field and the energy equation. However, most solutions to date have used large grid size, because computer running time for this method tends to be long, especially for problems in which the conduction is relatively small in comparison with radiation. If a creative way can be found to use the generated temperature profiles within the individual elements to carry out analytically all or part of the radiative field integrals, the method could be greatly speeded up and perhaps used to develop radiative transfer subroutines to existing thermal analysis programs. Perhaps this can be done by the use of a partial integration of the general form of the integrals followed by numerical evaluation of the remaining simplified form.

Tan (1988) has proposed and applied the product-integration method (Baker, 1977) to radiative transfer problems. This approach is closely related to the finite element method but reduces the dimension of the required integrations, thus significantly reducing required computation time. This reduction occurs because for a given degree of approximation of the temperature (i.e., constant, linear, binomial, etc.) within each of n finite volume or surface elements, the number of calculations in zonal or finite element approaches increases as n^2, while in the product-integration method it increases as n. For example, Tan was able to carry out analysis of a square enclosure divided into 8×8 volume elements on an IBM XT computer, while the finite element analysis used by Razzaque et al. (1983, 1984) was limited to 4×4 elements by computer time requirements on a CDC Cyber system. Tan included linear-anisotropic scattering effects in analysis of a two-dimensional emitting-absorbing-scattering medium. This approach appears to hold great promise.

Zoning Method. Considerable literature continues to appear that is based on the zoning method, or that extends its capabilities. Naraghi and Chung (1985) developed an explicit matrix formulation for the zoning method that significantly reduces the programming necessary for computer solution. Larsen and Howell (1986) derived a method based on the same assumptions used by Hottel. Their method uses "exchange

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factors," which are physically measurable quantities related to radiative exchanges among surface and volume elements. These appear in the analysis in place of the "exchange areas" used in the zoning method, which can only be computed. They show the mathematical relation between exchange factors and exchange areas. Liu and Howell (1987) then measured exchange factors using a scale model of an enclosure filled with a near-isotropically scattering medium, and compared radiative transfer results with those calculated by the zoning method. Their approach allows treatment of enclosures that are not limited to rectangular or cylindrical shapes. The required experimental measurements, however, are quite tedious.

Naraghi et al. (1987) developed a continuous exchange factor method that improves the accuracy of the zoning method, but was difficult to program and execute. Naraghi (1988) has proposed a unified matrix formulation that reduces the computational time of the earlier work. The exchange factors defined in the work by Naraghi et al. (1987, 1988) differ from those used by either Hottel and Sarofim (1967) or Larsen and Howell (1985). In Larsen's work, the factors are defined for the absorbing medium in radiative equilbrium, and a separate gas–gas exchange factor is invoked to account for deviations from radiative equilibrium when energy sources or sinks are present in the absorbing/emitting medium. Gas-to-gas exchange for radiative equilibrium in the Larsen paper is thus included in the gas–surface, surface–gas, and surface–surface factors, and the gas–gas factor is not needed or calculated for such problems. In the Naraghi and Chung work, the gas–gas factor is used for all gas-to-gas exchange, whether the system is in radiative equilibrium or not. The original Hottel formulation, Larsen's work, and the work by Naraghi et al. (1987, 1988) each present different mathematical and, to some extent, physical interpretations of the transfer mechanism. There are probably other formulations that could be used as well. Each of the methods described has computational advantages in certain situations.

Sowell and O'Brien (1972), Vercammen and Froment (1980), and Larsen and Howell (1986) have all presented methods for normalizing and smoothing exchange areas so that round-off, truncation, or other errors in individually computed or measured values obey the required conservation relations. Sowell and O'Brien give a methodology for using the conservation relations on exchange areas, defined as

$$(4aV)_\gamma = \sum_{i=1}^{N} \overline{g_\gamma s_i} + \sum_{\mu=1}^{\Gamma} \overline{g_\gamma g_\mu} \tag{7}$$

$$A_i = \sum_{j=1}^{N} \overline{s_i s_j} + \sum_{\gamma=1}^{\Gamma} \overline{s_i g_\gamma} \tag{8}$$

to evaluate the $M=N+\Gamma$ exchange areas from the unique $M(M+1)/2$ exchange areas that must be specified after reciprocity is invoked. Here, $\overline{s_i s_j}$, $\overline{s_i g_\gamma}$, $\overline{g_\gamma s_j}$, and $\overline{g_\gamma g_\mu}$ are the surface-to-surface, surface-to-gas, gas-to-surface, and gas-to-gas exchange areas, respectively. This method assures that the set of exchange factors thus found will satisfy equations (7) and (8); however, if the unique factors used in the method have any computation or round-off errors, then there is no guarantee that the individual factors in the calculation are correct. Vercammen and Froment (1980) use a regression technique to smooth the factors. Larsen and Howell (1986) invoke error minimization through the method of Lagrangian multipliers using equations (7) and (8) as constraints.

Siddall (1986) presents a method for accurate calculation of direct exchange areas in rectangular geometries by a transformation that reduces the order of the integrations normally involved to a single numerical integration. He states that five-figure accuracy is readily obtained. Edwards and Balakrishnan (1972) present simplified expressions for exchange areas, while Rasmussen (1986) presents a fast approximate method of determining exchange areas between zones of finite size by formulating a distance correction between zone centers that is used in the element-to-element expressions.

Smith et al. (1981) and Farag and Allam (1981) have presented recent work on the "sum of gray gases" approach to incorporating spectral effects into the zone method. Steward and Kocaefe (1986) have used band emission data to generate total gas emittance and absorptance values for carbon dioxide, water vapor, and their mixtures, and provide algebraic expressions for these properties as functions of the temperature, pressure, and path length. Skocypec and Buckius (1984) have presented a method for determining the total emittance of a mixture of emitting gases and scattering particles, using wide-band models for the gas emittance. Goodwin and Ebert (1987) questioned the results presented by Skocypec and Buckius, and presented an alternative method based on individual spectral bands. The latter paper led to a lively debate (Skocypec and Buckius, 1987; Self, 1987), with final agreement that the total emittance of an emitting gas containing scattering particles could be expressed as

$$\epsilon_\lambda = \epsilon_{\lambda,g} + C\epsilon_{\lambda,s} \tag{9}$$

where ϵ_λ is the emittance of the mixture, $\epsilon_{\lambda,g}$ is the emittance of the gas, $\epsilon_{\lambda,s}$ is the emittance of the scattering medium, and C is a multiplier.

Monte Carlo Methods. Sufficiently low computer costs will allow solution of almost any problem if even an inefficient method is available to model a given problem exactly. We have had such a method for a long time: The Monte Carlo method can in principle be programmed to include an exact simulation of all important physical processes. The sole remaining difficulty with Monte Carlo, given sufficiently fast and cheap computation capability, is the grid-size incompatibility problem, in that the computational element size required for statistical accuracy in the Monte Carlo solution may not be compatible with the grid size necessary for numerical solution of the energy equation. Given enough computer power, that problem too can be overcome by taking a sufficiently small Monte Carlo computation element size (with a resulting increase in the number of statistical simulations required for accuracy) to match other gridding needs. Of course, some elegant grid-matching procedures may also emerge that will make Monte Carlo the method of choice in the near term.

Reviews of the method are available (Howell, 1968; Halton, 1970; Haji-Sheikh, 1988). Recent work on reducing computation time includes that by Mishkin and Kowalski (1983), who used the transient form of the energy equation to predict a temperature field, and then used the predicted temperature field in a Monte Carlo evaluation of the radiation field. The radiative flux divergence from the Monte Carlo solution was then introduced into the energy equation at each succeeding time step, and the procedure was repeated until convergence at a steady-state solution.

Recent applications of the Monte Carlo method have appeared that exploit its flexibility and power to examine difficult problems. A few of these can be cited to give the overall flavor of these applications. Slater et al. (1982), Bernes (1979), Meier and Lee (1978), Katkovskii et al. (1983), and Vlasov (1979) all have used Monte Carlo approaches to solve problems in which the medium was not assumed to be in local thermodynamic equilibrium (LTE). Carter et al. (1978) used the method to include the effects of polarization on radiative transfer. Egan and Hilgeman (1978) examined the spectral reflectance of particulates. Dunn (1983) has applied the method to inhomogeneous media. Gupta et al. (1983) include anisotropic scattering in their Monte Carlo analysis of coal furnaces with fly-ash. Lewis and Miller (1984) and Meier et al.

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(1978) present valuable information on Monte Carlo applications in scattering problems.

Some discussion of the possibilities for significant improvement in the calculation times for Monte Carlo solutions is presented later.

The Present: Emerging Directions

Radiation continues to emerge as an important and indeed necessary consideration in applications where it often was previously ignored. For example, turbine and diesel engines are approaching operating temperatures at which radiation must be explicitly treated (Menguc et al., 1985; Chang and Wang, 1987).

General combustion problems have attracted a major research effort, recently reviewed in detail by Viskanta and Menguc (1987) and, for the case of flames, by Faeth (1986). More specialized combustion systems are also being investigated in which radiation plays a primary role, such as in fluidized bed combustion (Brewster, 1986; Brewster and Tien, 1982) and combustion within highly porous foamed ceramics (Echigo et al., 1986; Chen et al., 1987; Tong et al., 1987; Yoshizawa et al., 1987).

Renewed interest in space power systems and the space station has also triggered interest in novel radiation problems, such as the use of radiative transfer from free flowing droplets of working fluid in a space power system to replace large space radiator assemblies (Siegel, 1987a, 1987b).

Echigo has exploited the idea of placing a highly emitting solid matrix (metallic screen or ceramic foam) into a flowing hot gas stream, thus converting a portion of the gas enthalpy into emitted radiation from the matrix. He has analyzed the application of this idea for recovering stack gas enthalpy in industrial furnaces (Echigo, 1982), combustion of materials with low energy content (Echigo et al., 1983; Yoshizawa et al., 1987), and for enhancing radiative transfer to water tubes in steam boilers (Echigo, 1985).

Much attention is being paid to treating anisotropic scattering because of the importance of this effect in atmospheric radiation transfer, furnaces with particulate matter (ash, soot) present, and other technologies. Almost without exception, a basic assumption in this work is that the individual particles scatter as if they were independent point scatterers; i.e., particle interactions do not affect the distribution of scattered intensity. The important papers by Tien and co-workers (Brewster and Tien, 1982; Cartigny et al., 1986; Yamada et al., 1986) definitively show the effect of wavelenth, particle number density and size, and particle scattering and absorption cross section, on the regions of dependent and independent scattering, and give confidence that the assumption of independent scattering is indeed justified in most (but not all) engineering situations. Buckius (1986) reviews much of the work that deals with the properties of scattering media, and outlines the scattering properties of various particles of regular and irregular shape for independent scattering.

Methods of solution that can incorporate anisotropic effects are being investigated and, at the same time, realistic scattering phase functions are being tried that will allow treatment of scattering without the complexity necessary if the complete Mie scattering phase functions are used. The delta-Eddington function (Joseph et al., 1976) and the Henyey–Greenstein approximation (Kamiuto, 1987; McKellar and Box, 1981) offer a reasonable degree of simplification while allowing good accuracy. The varous Dirac-delta phase function approximations have been reviewed by Crosbie and Davidson (1985). Lee and Buckius (1986) have shown that radiation scaling laws allow some anisotropic scattering problems to be reduced to, and solved as, nonscattering problems. Menguc and Viskanta (1986) have investigated radiative transfer in a cylindrical furnace by the P-3 method, including the effects of spectral absorption and anisotropic scattering (delta-Eddington) in the gas.

The interaction of turbulence with radiation in combustion is drawing attention. Gore et al., (1986, 1987) have examined the effect both experimentally and analytically for the case of a turbulent diffusion flame, with emphasis on finding how turbulence affects the emission of radiation from a flame. Significant enhancement (as much as 20 percent) over the emission predicted using mean properties was found for hydrogen/air flames when a stochastic turbulence model was invoked, and the predicted enhancement agreed with measurements. The inverse question of how much effect radiation has on the scale and intensity of turbulence in a flame has not been carefully investigated.

Fundamental studies of the nature of the radiative exchange process, and the implications of the Second Law of Thermodynamics when applied to radiative exchange, are drawing the attention of some researchers (Bejan, 1987; and Arpaci, in a series of papers including Arpaci (1987a, 1987b) and Arpaci and Selamet (1986)). Just where these studies will lead in terms of engineering applications is not yet clear, but they do give insight and unification to some puzzling results from earlier work that was carried out separately. For example, Arpaci (1987b) shows the relationship among the Rosseland, Eddington, and thin gas (Planck) mean absorption coefficients that arise in the various approximate forms of the equation of transfer, proceeding from a single general formulation based on the radiative stress tensor. Bejan uses a classical thermodynamic analysis of radiative exergy to show the possible work extraction from enclosed radiation under various constraints, and shows the interrelation of work by some earlier approaches to this problem by Petela (1964), Spanner (1964), and Jeter (1981).

Future Work

A few problems are not yet being widely addressed. For example, anisotropic scattering is universally treated as if the scattering were independent of incident angle. This is the case for many practical systems, such as those with particle suspensions, because the particles are randomly oriented even if individually they have a nonspherical form. However, in some important cases, such as in radiative transfer within fixed solid matrices such as foamed ceramics, the scattering material may have a fixed orientation. The scattering phase function then depends on the angle of incidence as well as the angle of reflection, lending a new degree of difficulty to the analysis of scattering. Methods for treating this situation have not been developed.

Numerical difficulties still exist in treating radiative transfer problems. Combustion problems present an energy equation that is extremely stiff, and in some cases the addition of radiation terms can exacerbate the problem, leading to slow or lack of convergence for these problems. Numerical techniques for rapid solution of this class of problem need development.

The burgeoning speed and capability of supercomputers, with their ability to use vector and parallel processing, may soon make many of the concerns about methods moot (Shih et al., 1986; Howell, 1985). Adaptation of radiative transfer methods to these new capabilities has just begun, and can be expected to have one of the major impacts on this field in the next few years. The analyst will need to choose the best methodology for multimode heat transfer calculations that takes advantage of parallel processing. One method of exploiting parallel processing is perhaps to solve the radiative transfer equation using an initial assumed temperature field on one processor while simultaneously solving the energy equation to compute the temperature field from an initial assumed radiation field on another. Then, the calculated

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radiation field and temperature from each parallel path are traded and used as new initial guesses, and iteration proceeds until convergence. This could significantly reduce computation time.

Denning (1985) has reviewed the state of the art in parallel processing, and says that we are presently in *stage I* of its use. Present computers make parallel processing available by having their operating systems access more than one processor, so that successive tasks need now await completion of prior tasks, or by having multiple memory banks, or by the use of "vector pipelines." Newer machines, called *reduced instruction set computers*, or RISC's, are structured so that their compilers carefully analyze programs in a way that keeps the pipeline of instructions busy much of the time. Such methods are effectively invisible to the user, but result in much faster execution time.

To use the parallel processing capability efficiently, software must be structured in such a way as to make use of the inherent features of the parallel processing system. If the software is structured with linear commands, then much of the benefit of parallel processing may be lost. Even if the program is structured carefully, the software must have the capability of *microtasking*, i.e., telling the computer to access a parallel processor for a given task.

Stage II of the parallel processing evolution will make use of such advanced languages as parallel PASCAL (Reeves et al., 1980) or OCCAM, a derivative of PASCAL. These languages allow the structuring of programs so that the information required for continuation of a given calculation is supplied from a parallel processor when available, and the calculation proceeds. Such a language allows parallel processing even on separate machines, as long as a communication link is established.

Denning (1985) notes that, although these systems are well along in their development, their application to practical engineering computations has not been exploited. Also, the newer languages such as LISP and APL have not been structured internally to take full advantage of parallel processing, nor to allow discrete program parts, which Denning calls "chunks," to be processed independently and then rejoined. Such restructuring will be *stage III*, which Denning believes will lead in turn to *stage IV*, when program-user interaction will allow the program to structure itself into chunks automatically.

Arvind (1980) discusses the use of high-level dataflow programs to control the flow of individual calculations dynamically to parallel processors as a program is executed. He believes that existing programs of this type probably are not useful for the classes of calculation typically found in fluid mechanics (and radiative transfer). Rather, careful structuring by the programmer with static mapping of results into memory that can be accessed by parallel processors when information is needed may be most efficient.

Monte Carlo methods in particular stand to improve significantly in reduced computer time. The use of multiprocessor systems to handle the multiple path trees describing the distribution of radiation in complex systems should reduce the computing time by the number of parallel processors used. Lord et al. (1983) discuss some attempts to use vectorized Monte Carlo approaches in quantum field theory by Moriarity (1981) and by Barkai (1981), and describe these as not very successful in terms of reduced computation time in comparison with standard computation. Barkai and Moriarity (1982) are more positive about their work in a later report. Pawley and Dove (1983) report on the use of parallel processors for molecular dynamics calculations using Monte Carlo, and Genz (1982) applies parallel processing in the evaluation of multiple integrals, both with some success. The latter two are closely related to the needs of radiative transfer calculations.

As multiprocessor computers become widely available, other approaches will undoubtedly emerge.

Another relatively untouched tool in radiative transfer is the use of computer graphics. Emery et al. (1988) have used some elements of computer graphics in engineering configuration factor analysis. Some work on configuration factor analysis as applied to architectural lighting problems shows the potential for even more sophisticated approaches (Cohen and Greenberg, 1985; Goral et al., 1984; Nishita and Nakamae, 1985; Cohen et al., 1986). This work uses increasingly complex models of surface radiosity (in some recent work using nondiffuse reflectance models) to depict surface appearance graphically under varying conditions of illumination; it is a simple step to compute numerical values concurrently for configuration factors. All of the shading and blocking problems common to radiant exchange within a multisurfaced enclosure must be treated in these programs, and the visual representation of shading and blocking can be shown directly in color. Thus, confidence in the results is much greater than when a canned numerical program giving only tabular results is used. Also, this work has led to extensive re-examination of ray-tracing methods by the computer graphics community (Arvo and Kirk, 1987; Bentley, 1975; Cook et al., 1984; Fujimoto et al., 1986; Fussell and Subramanian, 1988; Glassner, 1984; Kaplan, 1985; Kay and Kajiya, 1986; Maxwell et al., 1986), which may lead to efficient new tools for configuration factor analysis or other uses in radiative transfer. Although the work to date has concentrated on surface-to-surface exchange, some initial work is being directed to systems with participating media (Rushmeier and Torrance, 1987; Nishita et al. 1987).

Some Observations

Based on the above exposition of the growth of capability in the computation of radiative transfer, some observations may be in order. First, the community of workers in radiative transfer labors in two fairly distinct areas. These are the *development of more efficient methods* for treating radiative transfer within the context of engineering heat transfer problems, and the *application* of existing methods to technical problems of immediate interest.

Those working on method development often start with applications to problems with little technical interest, such as the ubiquitous solution of the heat transfer and temperature profile in a uniform gray gas in radiative equilibrium between infinite parallel plates at fixed temperatures. This may have had merit in the past, because few accurate methods existed to treat even such a simple geometry. Presently, however, the problem has little intrinsic interest, since the real questions about a new method are these: Can it be applied in multidimensional problems? Will it work well if the properties of the attenuating medium are nonuniform in space? Does the method lend itself to a match with the gridding requirements of related conduction/convection equations? Can spectral properties be easily incorporated? Can anisotropic scattering be treated?

If a new method cannot be shown to have advantages over existing techniques in at least one of these areas, then publication of another one-dimensional gray-gas solution is pointless, even if the mathematical technique is novel or elegant. No one will bother to use elegant solutions that cannot be applied to problems of technical interest.

Looking at the progress in engineering radiative heat transfer over the past half-century, as we have progressed from the treatment of simple enclosures with an isothermal absorbing-emitting medium of uniform composition, to the present day, when we can treat multidimensional enclosures with an absorbing-emitting-anisotropically scattering medium with spectrally dependent properties that vary within the

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enclosure (with, it must be admitted, some considerable difficulty), we can see the steady, continuing progress. New methods, computational tools, and applications continue to appear, and the study of radiation heat transfer remains a robust area for both applications and research.

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