Decision Feedback Detection Using Modulation Constraints for MIMO Rayleigh Fading Channels

Narayan Prasad and Mahesh K. Varanasi

Abstract — **We propose a new technique for designing decision feedback detectors (DFDs) wherein the per-symbol decision rules are obtained by exploiting modulation constraints. We present our results in the context of the multi-input, multi-output (MIMO) Rayleigh fading channel with N receive and K transmit antennas. Three cases are considered to illustrate our technique where all the transmitters employ real-valued pulse-amplitude modulation (PAM), or phase-shift keying (PSK), or complex quadrature amplitude modulation (QAM). In each case, the corresponding per-symbol decision rule is obtained by imposing the modulation constraint on a generalized likelihood function maximization problem. For the case when all transmitters employ PAM, we show that the resulting DFD (the PAM-DFD) is equivalent to the decorrelating decision feedback detector (D-DFD) when the latter is used on a modified received statistic. We derive the exact joint and symbol error probability (JEP, SEP) for the PAM-DFD and show that the (possibly fractional) diversity orders of both the JEP and SEP are equal to $N - \frac{K-1}{2}$. This greatly improves on the diversity order of $N-K+1$ of the conventional D-DFD which does not exploit the real modulation constraint. We also derive the PSK-DFD and the QAM-DFD for the cases when all transmitters employ PSK and QAM, respectively, and show that these detectors result in significant performance improvements over the conventional D-DFD as well.**

I. System Model

The discrete time model of an uncoded wireless communication system in a flat fading environment with N receive and K transmit antennas ($N \geq K$) is given by

$$\mathbf{y} = \mathbf{H}\mathbf{W}^{1/2}\mathbf{b} + \mathbf{v}\,. \tag{1}$$

$\mathbf{y}$ is the $N \times 1$ received vector and $\mathbf{b} = [b_1, \cdots, b_K]^T$ contains the K transmitted symbols. The symbol b_k is transmitted by the k^{th} transmitter and belongs to a normalized (to unit average energy) constellation $\mathcal{F}_k$ of size M_k. The fading is described by the $N \times K$ matrix $\mathbf{H}$ having independent, identically distributed (i.i.d.), zero-mean, unit variance complex normal elements. The random matrix $\mathbf{H}$ is assumed to be perfectly known to the receiver and unknown to the transmitters. The $N \times 1$ vector $\mathbf{v}$ represents the additive noise at the receiver and has i.i.d., zero-mean, complex normal elements each with variance σ^2. $\mathbf{W}$ is a $K \times K$ diagonal matrix such that the average power transmitted through transmitter i is w_i, the i^{th} diagonal element of $\mathbf{W}$.

II. Decision feedback detection under modulation constraints

In the following we obtain the decision rule for the transmitter detected first, and the decision rules for the subsequent transmitters are derived in a similar manner after feedback. In particular, we propose the maximization of the generalized likelihood function under the modulation constraints. Without such constraints, the maximum generalized likelihood decision rule for transmitter 1 would be

$$\hat{b}_1 = \arg\min_{x_1 \in \mathcal{F}_1} \min_{\mathbf{x}_{\bar{1}} \in \mathbb{C}^{K-1}} ||\mathbf{y} - \mathbf{H}_1\sqrt{w_1}x_1 - \mathbf{H}_{\bar{1}}\mathbf{W}_{\bar{1}}^{1/2}\mathbf{x}_{\bar{1}}||^2$$

The authors are with the ECE department, University of Colorado, Boulder, CO 80309. E-mail: {prasadn, varanasi}@dsp.colorado.edu. This work was supported in part by NSF Grants CCR-0112977 and ANIR-9725778.

where $\mathbf{H}$ is partitioned as $\mathbf{H} = [\mathbf{H}_1 \ \mathbf{H}_{\bar{1}}]$. This unconstrained minimization results in the conventional decorrelating DFD (D-DFD) (aka the nulling/cancelling receiver). In an earlier work [1], the exact JEP and SEP obtained by the conventional D-DFD were derived and the diversity orders of both JEP and SEP were shown to be $N-K+1$.

In the case where all transmitters employ PAM symbols, it is suggested that the decision rule with the real constraint be

$$\hat{b}_1 = \arg\min_{x_1 \in \mathcal{F}_1} \min_{\mathbf{x}_{\bar{1}} \in \mathbb{R}^{K-1}} ||\mathbf{y} - \mathbf{H}_1\sqrt{w_1}x_1 - \mathbf{H}_{\bar{1}}\mathbf{W}_{\bar{1}}^{1/2}\mathbf{x}_{\bar{1}}||^2\,.$$

It can be shown [2], that the resulting PAM-DFD is equivalent to the D-DFD operating on the model $\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{W}^{1/2}\mathbf{b} + \tilde{\mathbf{v}}$ where $\tilde{\mathbf{y}} = [\mathrm{Re}\{\mathbf{y}\}^T \ \mathrm{Im}\{\mathbf{y}\}^T]^T$ and $\tilde{\mathbf{H}} = [\mathrm{Re}\{\mathbf{H}\}^T \ \mathrm{Im}\{\mathbf{H}\}^T]^T$. Further the exact JEP and SEP can be derived and the diversity orders of both can be shown to equal $N - \frac{K-1}{2}$. Note that the *real* constraint imposed to derive the PAM-DFD results in a substantial diversity order gain when compared to the conventional D-DFD.

We next consider the case when all transmitters use PSK modulation. We suggest the decision rule with the new constraint to be

$$\hat{b}_1 = \arg\min_{x_1 \in \mathcal{F}_1} \min_{\mathbf{x}_{\bar{1}} : \mathbf{x}_{\bar{1}}^{\dagger}\mathbf{x}_{\bar{1}} = K-1} ||\mathbf{y} - \mathbf{H}_1\sqrt{w_1}x_1 - \mathbf{H}_{\bar{1}}\mathbf{W}_{\bar{1}}^{1/2}\mathbf{x}_{\bar{1}}||^2\,.$$

This minimization problem has a convex objective function but a non-convex constraint. An analytical solution can be obtained using the results in [3] and a low-complexity implementation based on the Newton-Raphson method is presented in [2].

Finally, when all transmitters employ QAM modulation, the decision rule proposed is

$$\hat{b}_1 = \arg\min_{x_1 \in \mathcal{F}_1} \min_{\mathbf{x}_{\bar{1}} : \mathbf{x}_{\bar{1}}^{\dagger}\mathbf{x}_{\bar{1}} \leq A} ||\mathbf{y} - \mathbf{H}_1\sqrt{w_1}x_1 - \mathbf{H}_{\bar{1}}\mathbf{W}_{\bar{1}}^{1/2}\mathbf{x}_{\bar{1}}||^2$$

where A is a constant determined by the QAM constellations. The convex minimization involved can be solved using the Kuhn-Tucker conditions.

References

[1] N. Prasad and M. K. Varanasi, "Analysis of decision feedback detection for MIMO rayleigh fading channels and optimum allocation of transmitter powers and QAM constellations," in *Proc. Allerton Conf. on Comm., Control, and Comput.*, Monticello, IL, Oct. 2001, University of Illinois.

[2] N. Prasad and M. K. Varanasi, "Decision feedback detection using modulation constraints for MIMO rayleigh fading channels," under preparation.

[3] S. Boyd and L. Vandenberghe, *Convex Optimization*, Course notes for Convex Optimization with Engineering Applications, www.stanford.edu/class/ee364, 2001.

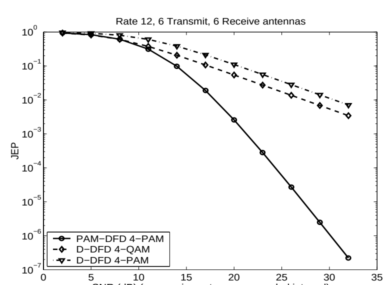

Fig. 1. JEPs for a six-transmit, six-receive antenna system at total rate 12 bits per channel use and $\mathbf{W} = \mathbf{I}$. The JEPs using the conventional D-DFD when all transmitters transmit 4-PAM and 4-QAM respectively are plotted. Also plotted is the JEP using the PAM-DFD when all transmitters transmit 4-PAM.