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Research on the repeater optimization based on Voronoi model

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Abstract: This paper propose a two-tiered network in which lower-power users communicate with one another through repeaters, which amplify signals and retransmit them, have limited capacity, and may interfere with one another if their transmitter frequencies are close and they share the same private-line tone. Motivated by cellular networks, this paper gives a naive solution where the number of repeaters and their positions can be obtained analytically. In a circular area with radius 40 miles, 12 repeaters can accommodate 1,000 simultaneous users. This paper further propose an iterative refinement algorithm consisting of three fundamental modules that draw the Voronoi diagram, determine the centers of the circumscribed circles of the Voronoi regions, and escape the local optimum by using external optimization. The algorithm obtains a solution with 11 repeaters, which we prove to be the absolute minimum. For 10,000 users, it uses 104 repeaters, better than the naive solution's 108.

1.Model

We use a cellular network of equal-size regular hexagons centered at repeaters. The pattern is the Voronoi diagram of the repeater sites.

We show how to obtain a solution for $R \geq 2\Phi$ and $N = 1,000$ by using Table 1. The user density is $\rho = N / \pi\Phi^2 \approx 0.1989$. To make sure that each point is covered by at least one repeater, the edge length r_h of the regular hexagon should not exceed the communication range r. In addition, according to the capacity constraint, r_h should satisfy $\frac{3\sqrt{3}}{2} r^2 \rho \leq C$

These two conditions determine the longest possible edge length; to cover as large circle as possible, we always use the longest r_h. For our case, r = 15.28 mi, and according to above, $_c$ should be 15.28 mi. The circle to be covered has radius $\Phi = 40mi$. Since

$$\frac{\sqrt{19}}{2} \approx 2.18 < \frac{\Phi}{r_h} \approx \frac{40}{15.28} \approx 2.62 < \sqrt{7} \approx 2.65$$

According to Table 1, 12 repeaters are sufficient (but 11 won't do) (Figure 2). (We consider only $R \geq 2\Phi$, for which frequencies and PL tones are easily arranged; in our example, we use 3 PL tones.) The algorithm for the naive solution is to find the longest allowable r_h and search Table 2 (extended as necessary).

However, extending the table is not easy (at least by hand). We get analytical results, for up to 121 circles, for two particular cases: the covered circle's center is at the center of the central hexagon, or it is at the intersection of three more central hexagons (Table 1).

Table 1 The intersection of three more central hexagons

Cells	Radius of largest coverable circle	Cells	Radius of largest coverable circle	Cells	Radius of largest coverable circle
1	$\sqrt{3}/2$	3	1	7	2
6	$\sqrt{3}$	13	$3\sqrt{3}/2$	12	$\sqrt{7}$
19	$\sqrt{13}$	18	3	31	$5\sqrt{3}/2$
27	4	37	5	36	$\sqrt{21}$
55	$7\sqrt{3}/2$	48	$\sqrt{43}$	61	$\sqrt{43}$
60	6	85	$9\sqrt{3}/2$	75	7
91	8	90	$\sqrt{57} \approx 7.5$	121	$\sqrt{91} \approx 9.5$

Applying this approach to the case $N = 10,000$, we have $\rho = N / \pi\Phi^2 \approx 1.989$ and $r_h = 4.8miles$. Since $8 < \Phi / r_h < 9$, according to Table 2, 108 repeaters are sufficient. Since this number is smaller than 324, it is easy to arrange them when $R \geq 2\Phi$.

We present algorithms: to adaptively place the repeaters so as to solve the circle-covering problem with the fewest circles, to assign receiver frequencies, transmitter frequencies, and PL tones.

1. Randomly place M_0

2. Determine the Voronoi region of each repeater.

3. Determine the circumscribed circle of each Voronoi region.

4. Calculate the coordinates of the center of each circumscribed circle.

5. Calculate the distance from each repeater's current location to the center of its circumscribed circle. Sum up the distances over all repeaters and compare the sum to a threshold ζ. If the sum is less, the current locations are considered to be converged; otherwise, move each repeater to the center of its circumscribed circle and return to Step 2.

6. Check if the number of users in each Voronoi region is less than the Repeater's capacity C, and if the radius of the circumscribed circle is less than the communication radius of users. If so, stop the algorithm and output the current solution. Otherwise, go to Step 7.

7. If the number of external optimization operations is less than a threshold T_c, pick up the repeater with the smallest Voronoi region, move it to a random position and go to Step 2; and advance the count of external optimization operations. Otherwise, add one more repeater, randomize the positions of all repeaters, and go back to Step 2.

External optimization is a method to escape a local optimum by changing the individual with the least fitness (here, the repeater with the smallest Voronoi region).

This algorithm does not change locations of repeaters as obtained from the first algorithm nor add new repeaters. It tries to maximize the reachable area of users just by rearranging the frequencies and PL tones of repeaters.

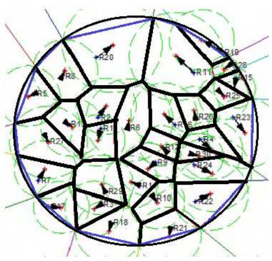

Figure1

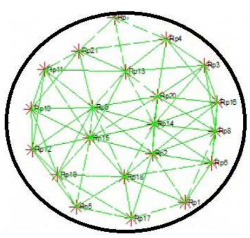

Figure2

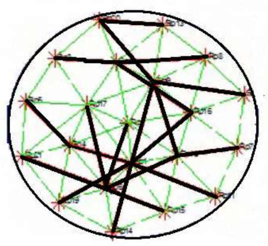

Figure3

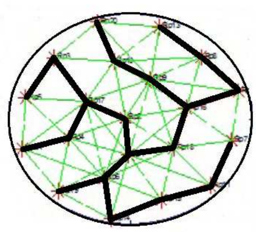

Figure4

Figure 1. Example of application of the algorithm for repeater placement. The labeled plus signs are the original locations of the repeaters, and the black asterisks are their final ocations; arrows show the movements. Voronoi regions are outlined by black (thick) line segments, and circumscribed circles are drawn in green (dashed) arcs (with red asterisks for their centers).

The red asterisks represent repeaters, and a green line connects two repeaters if they can communicate (that is, are less than R apart).

Maximum spanning tree (left) and the minimum spanning tree (right). The black (thick) lines are the edges in the spanning trees and the green (thin) lines are the other edges in the graph.

Resulting routings after the removal of edges corresponding to the spanning trees in the red (dashed) lines are the removed edges, while the black (thick solid) lines are the preserved ones. Repeaters connected by a blue line share the same tone.

1. Construct a graph G to represent the relation that two repeaters can transmit to each other: If the distance between them is less than R , adds an edge between them.

2. Find the minimum spanning tree (MST) T or the maximum spanning tree (MST) T0 of G . From the illustration in Figure 2, we see that edges in the minimum spanning tree will not intersect. If we build the transmission paths along the MST, the signals received by repeaters are fewer than for the maximum spanning tree. However, since the distance between two adjacent repeaters along the MST tree is the shortest, the received signals will be much stronger. So the assignment based on the MST is suitable for communication in a local area. In contrast, most edges in the MST intersect with one another, the number of signals in many areas is very large, and signals can cover larger areas. However, this situation increases the chance of interference. Depending on purpose, one can choose either spanning tree to continue the algorithm.

3. Remove edges from the tree. For any node i with degree $k>3$, delete $k-2$ edges. Then the node i will be apart from $k-2$ connected components. Let the size of component j be SC_j , then the method of removing edges can be presented as follows: Find $k-2$ edges to remove in order to minimize $\sum_{a,b}|SC_a - SC_b|$. The results can be found in Figure 3.

4. After Step 3, we may have several signal routes that do not connect. Assign a different PL tone to each route. Then for the repeaters in each route, assign transmitting frequency and receiving frequency. Make sure that the transmitting frequency of a repeater is the receiving frequency of the repeater's neighbors in the same route. (See Figure 4.).

2. Simulation

N=1,000 , $R = 85.45$mi

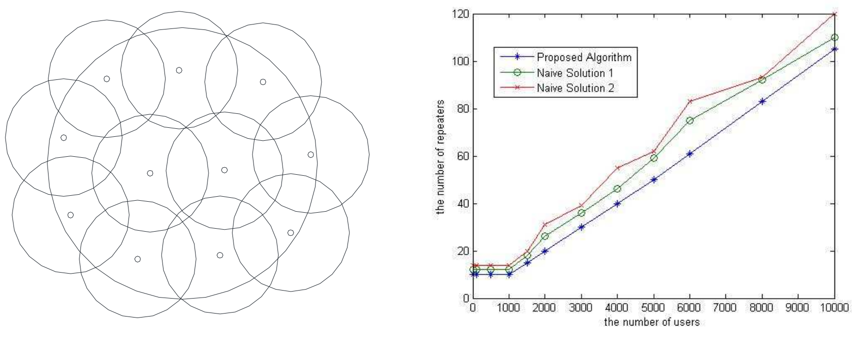

Figure5

Solution with 11 repeaters obtained by our algorithm.

Figure 5 shows a solution with 11 repeaters obtained by our algorithm. The maximal Voronoi area is 560.56, the user density is 0.1989, and thus the largest capacity demand is 112, smaller than the repeater's capacity $C = 119$. We prove that 11 repeaters is optimal: 10 repeaters with radius no more than 15.28 miles cannot cover a circle with radius 40 miles.

Let $r(n)$ be the maximum radius of a circular disc that can be covered by n closed unit circles, then $r(n) = 1 + 2\cos\left(\frac{2\pi}{n-1}\right)$. For $n = 8$, $n = 9$ and $n = 10$. According to the Lemma,

$$r(10) = 1 + 2\cos\left(\frac{2\pi}{9}\right) \approx 2.53 < \frac{40}{15.28} \approx 2.62$$

Comparison to fewest required repeaters obtained by our algorithm vs. the naive solution. The largest coverable circle in the naive Solution 1 is centered at the center of the central hexagon, while the largest coverable circle in naive Solution 2 is centered at the intersection of the three more central hexagons.

The repeaters' locations are the same as earlier, but the frequencies and PL tones are different.

To test the effectiveness of the secondary algorithm, we randomly pick 100,000 ordered pairs of points (u, v) inside the considered area and see in how many pairs u can send to v. The answer is 90,708. So, given a user, the probability that the system can satisfy this user's requirement to communicate with any other user at random is about 91%.

3. Conclusion

Compared with the related model for sensor wireless networks and mo- bile communication networks, our model is more general. Our algorithm is effective and efficient: It runs much faster than the simulated annealing approach and is better able escape the local optimum than another iterative refinement algorithm Our algorithm does not require any specific geographical features of the considered area, while many efficient circle-covering algorithms work well only for squares.

4. Acknowledgement

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References

[1] Akyildiz, I.F., W. Su, Y. Sankarasubramaniam, and E. Cayirci. 2002. Wire- less sensor networks: A survey. Computer Networks 38: 393–422.

[2] Aurenhammer, F. 1991. Voronoi diagrams—A survey of a fundamental geometric data structure. ACM Computing Surveys 23: 345–405.

[3] Bak, P., and K. Sneppen. 1993. Punctuated equilibrium and criticality in a simple model of evolution. Physical Review Letters 71: 4083.

[4] Balanis, C.A. 2005. Antenna Theory: Analysis and Design. 3rd ed. New York: Wiley.

[5] Boettcher, S., and A.G. Percus. 2001. Optimizationwith extremal dynamics. Physical Review Letters 86: 5211.

[6] Das, G.K., S. Das, S.C. Nandy, and B.P. Sinha. 2006. Efficient algorithm for placing a given number of base stations to cover a convex region. Journal of Parallel and Distributed Computing 66: 1353.