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Differential Cross Section and N^* Spin Density Matrix of $\pi N \to \pi N^*$ and $SU(6)\otimes O(3)$ Structure

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On the basis of the symmetric quark model, $d\sigma/dt(\pi N \to \pi N^*)$ and its N^* spin density matrix for N^* which lies on N_α and N_γ trajectories are calculated by using the Barger-Phillips $\pi N \to \pi N$ amplitude as input. Agreement with experiment is satisfactory. It is predicted that the polarization distribution of the decay product N of N^* is determined mostly by the $SU(6)$ structure of N^* and behaves quite differently depending on different $SU(6)$ structures.

§ 1. Introduction

We calculated a differential cross section $(d\sigma/dt)(\pi N \to \pi N^*)$ and its N^* spin density matrix $\rho_{mm'}$ on the basis of the symmetric quark model in our preceding paper[1] (hereafter called I). There we first expressed $(d\sigma/dt)(\pi N \to \pi N^*)$ and $\rho_{mm'}$ in terms of $\pi N \to \pi N$ scattering amplitudes under the assumptions that the approximation of additivity and factorizability holds and that the inner quark wave function of N and N^* has the $SU(6)\otimes O(3)$ structure in the "least velocity frame" (where the larger value of the velocities of N and N^* takes the smallest value). Then we calculated $(d\sigma/dt)(\pi N \to \pi N^*)$ and $\rho_{mm'}$ numerically by adopting the harmonic oscillator wave function for the $O(3)$ part of the quark wave function (the harmonic oscillator parameter $\alpha = 0.63$ GeV) on the basis of our previous analysis[2] and by using the approximate $\pi N \to \pi N$ amplitudes of a simple exponential form (see (3·1)) as input amplitudes.

In this paper, using the Barger-Phillips amplitudes[3] as input $\pi N \to \pi N$ amplitudes which agree with experiment better than those used in I, we will calculate $(d\sigma/dt)(\pi N \to \pi N^*)$ and $\rho_{mm'}$ of N^* which lies on the N_α and N_γ trajectories. Further, we will calculate the angular distribution and polarization of the decay product N of N^*.

In § 2 necessary formulae among those obtained in I will be repeated and in § 3 the input $\pi N \to \pi N$ scattering amplitudes will be discussed. The obtained $(d\sigma/dt)(\pi N \to \pi N^*)$, $\rho_{mm'}$, the angular distribution and the polarization of final N will be shown and compared with experiment in § 4.

§ 2. N^* production cross section and spin density matrix in terms of $\pi N\to\pi N$ amplitudes

This paper treats of N^* which lie on N_α trajectory ($J^P=1/2^+, 5/2^+, 9/2^+$) and N_γ trajectory ($J^P=3/2^-, 7/2^-, 11/2^-$). In the quark model these resonances are classified into L-excited states with $J=L+1/2$ ($L=0, 2, 4$ for N_α and $L=1, 3, 5$ for N_γ), where L is the inner total orbital angular momentum.

The formulae obtained in I have the following forms for these N^*'s:

The $\pi^- p\to\pi^- N^*$ helicity amplitudes in the least-velocity frame (L.V.F.) are

$$\hat{T}^{N_\alpha}_{\mu'\mu}=\begin{array}{r} \\ \mu'=J \\ \vdots \\ \mu'=1/2 \\ \mu'=-1/2 \\ \vdots \\ \mu'=-J \end{array}\begin{array}{c} \begin{array}{cc} \mu=1/2 & \mu=-1/2 \end{array} \\ \begin{pmatrix} 0 & 0 \\ & \\ T_\xi(\frac{3}{9},\frac{5}{9}) & T_\eta(1,\frac{1}{3}) \\ -T_\eta(1,\frac{1}{3}) & T_\xi(\frac{3}{9},\frac{5}{9}) \\ & \\ 0 & 0 \end{pmatrix} \end{array}\times(-3)\sqrt{\frac{L+1}{2L+1}}\hat{F}_L^{N_\alpha}(\Delta_M), \qquad (2\cdot1)$$

$$\hat{T}^{N_\gamma}_{\mu'\mu}=\begin{array}{r} \\ \mu'=J \\ \vdots \\ \mu'=1/2 \\ \mu'=-1/2 \\ \vdots \\ \mu'=-J \end{array}\begin{array}{c} \begin{array}{cc} \mu=1/2 & \mu=-1/2 \end{array} \\ \begin{pmatrix} 0 & 0 \\ & \\ T_\xi(\frac{6}{9},\frac{4}{9}) & T_\eta(0,\frac{2}{3}) \\ -T_\eta(0,\frac{2}{3}) & T_\xi(\frac{6}{9},\frac{4}{9}) \\ & \\ 0 & 0 \end{pmatrix} \end{array}\times(-3)\sqrt{\frac{L+1}{2L+1}}\hat{F}_L^{N_\gamma}(\Delta_M). \qquad (2\cdot2)$$

(2·1) and (2·2) show that the N^*'s are produced with helicity $\pm1/2$. This property comes from the fact that the L of the proton is 0 and the total quark spin S of $N_\alpha(N_\gamma)$ is 1/2. (In the transition $N\to N^*$, L_z is conserved in the L.V.F. as is seen from Fig. 1. Since $L_z=0$ for N, L_z of N^* must be zero. So, J_z of

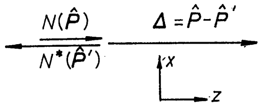

Fig. 1. Momenta of initial N and N^* in the L.V.F.

N^* coincides with S_z which is $\pm1/2$.) T_ξ and T_η are given as

$$\begin{aligned} T_\xi(\alpha,\beta)&=\alpha(C_M\mathcal{T}^{(+)}_{++}-S_M\mathcal{T}^{(+)}_{+-})+\beta(C_M\mathcal{T}^{(-)}_{++}-S_M\mathcal{T}^{(-)}_{+-}),\\ T_\eta(\alpha,\beta)&=\alpha(S_M\mathcal{T}^{(+)}_{++}+C_M\mathcal{T}^{(+)}_{+-})+\beta(S_M\mathcal{T}^{(-)}_{++}+C_M\mathcal{T}^{(-)}_{+-}), \end{aligned} \qquad (2\cdot3)$$

where $\mathcal{T}^{(\pm)}_{+\pm}$ are the π-quark scattering helicity amplitudes in the πN c.m.s. and can be written in terms of the $\pi N\to\pi N$ helicity amplitudes $T^{(\pm)}_{++}$ as

$$\mathcal{T}^{(+)}_{++}=\frac{1}{3\hat{F}_0^N(\Delta_m)}\{(3C_m^2+S_m^2)T^{(+)}_{++}-2C_mS_mT^{(+)}_{+-}\},$$

$$\mathcal{T}^{(+)}_{+-}=\frac{1}{3\hat{F}_0^N(\Delta_m)}\{-2C_mS_mT^{(+)}_{++}+(3S_m^2+C_m^2)T^{(+)}_{+-}\},$$
$$\mathcal{T}^{(-)}_{++}=\frac{1}{5\hat{F}_0^N(\Delta_m)}\{(3C_m^2+S_m^2)T^{(-)}_{++}+2C_mS_mT^{(-)}_{+-}\}, \tag{2·4}$$
$$\mathcal{T}^{(-)}_{+-}=\frac{1}{5\hat{F}_0^N(\Delta_m)}\{2C_mS_mT^{(-)}_{++}+(3S_m^2+5C_m^2)T^{(-)}_{+-}\},$$

and we use

$$T^{(\pm)}=\tfrac{1}{2}(\langle\pi^-p|T|\pi^-p\rangle\pm\langle\pi^-n|T|\pi^-n\rangle) \tag{2·5}$$

as the charge state of the scattering amplitudes.*)

$\hat{F}_L^{N_\alpha}(\Delta_M)$ $(\hat{F}_L^{N_\gamma}(\Delta_M), \hat{F}_0^N(\Delta_m))$ is the overlapping integral of the quark spatial wave functions of $N_\alpha(N_\gamma, N)$ with that of N when the momentum transfer is $\Delta_M(\Delta_m)$. We use the harmonic oscillator wave functions as model spatial wave functions on the basis of our previous analysis.[2] Then

$$|\hat{F}_L^{N^*}(\Delta)|^2=\frac{1}{(2L-1)!!}\left(\frac{\Delta^2}{3\alpha^2}\right)^L e^{-\Delta^2/3\alpha^2}\times\begin{cases}L!/(L!!)^2 & \text{for } N_\alpha,\\ (L+1)!/\{(L+1)!!\}^2 & \text{for } N_\gamma.\end{cases} \tag{2·6}$$

The value of the harmonic oscillator parameter α is taken as $\alpha=0.63$ GeV, which is determined in the same analysis.[2] Then our formulae have no adjustable parameter.

$\Delta_M(\Delta_m)$ is the 3-momentum transfer in the L.V.F. and its magnitude is

$$\Delta_M^2=\frac{(M+m)^2}{4Mm}\{(M-m)^2-t\}, \tag{2·7}$$

where m and M are the masses of N and N^*, respectively, and $t(s,u)$ is the usual Mandelstam variable.

C_M, S_M, C_m and S_m in (2·3) and (2·4) are the factors which come from the Wigner rotations of the Lorentz transformation between the L.V.F. and the πN c.m.s. and are given as**)

$$C_m=\sqrt{\frac{-t}{4m^2-t}}\frac{s+m^2-\mu^2}{\sqrt{\mathcal{S}_{m\mu}}}, \qquad S_m=\frac{2m}{\sqrt{4m^2-t}}\sqrt{\frac{\mathcal{S}_{m\mu}+ts}{\mathcal{S}_{m\mu}}},$$
$$C_M=\sqrt{\frac{1+\cos(\omega_M'-\omega_M)}{2}}, \qquad S_M=\sqrt{\frac{1-\cos(\omega_M'-\omega_M)}{2}}, \tag{2·8}$$

where

$$\cos\omega_M=\frac{1}{\sqrt{\mathcal{S}_{m\mu}\mathcal{T}_{mM}}}\frac{1}{2s}\{(s+M^2-\mu^2)\mathcal{S}_{m\mu}-(s+m^2-\mu^2)\vartheta_M\},$$

*) In I, we used $T^p=\langle\pi^-p|T|\pi^-p\rangle$ and $T^n=\langle\pi^-n|T|\pi^-n\rangle$ as charge states. From (2·3)~(2·5), $T_{\xi(\eta)}$ in (2·3) is related to $T_{x(y)}$ in (5·5) of I as $T_{\xi(\eta)}(\alpha,\beta)=T_{x(y)}((\alpha+\beta)/2,(\alpha-\beta)/2)$.

**) In I, only the approximate formulae for $s\gg M^2$, m^2, μ^2 are shown.

$$\sin\omega_M=\frac{1}{\sqrt{\mathcal{S}_{m\mu}\mathcal{T}_{mM}}}\frac{m}{\sqrt{s}}\{\mathcal{S}_{m\mu}\mathcal{S}_{M\mu}-\vartheta_M{}^2\}^{1/2},$$
$$\cos\omega_M'=\frac{1}{\sqrt{\mathcal{S}_{M\mu}\mathcal{T}_{mM}}}\frac{1}{2s}\{(s+m^2-\mu^2)\mathcal{S}_{M\mu}-(s+M^2-\mu^2)\vartheta_M\},\tag{2·9}$$
$$\sin\omega_M'=\frac{-1}{\sqrt{\mathcal{S}_{M\mu}\mathcal{T}_{mM}}}\frac{M}{\sqrt{s}}\{\mathcal{S}_{m\mu}\mathcal{S}_{M\mu}-\vartheta_M{}^2\}^{1/2},$$

$$\mathcal{S}_{m\mu}=\{s-(m+\mu)^2\}\{s-(m-\mu)^2\},\quad \mathcal{S}_{M\mu}=\{s-(M+\mu)^2\}\{s-(M-\mu)^2\},$$
$$\mathcal{T}_{mM}=\{t-(M+m)^2\}\{t-(M-m)^2\},\quad \vartheta_M=s(t-u)+(m^2-\mu^2)(M^2-\mu^2).\tag{2·10}$$

The differential cross section of $\pi N\to\pi N^*$ is given as

$$\frac{d\sigma}{dt}=\pi^3\frac{mM}{s}\frac{1}{P_c^2}\frac{1}{2}\sum_{\mu\mu'}|\hat{T}^{N*}_{\mu'\mu}|^2,\tag{2·11}$$

where P_c is the magnitude of the momentum of $N(\pi)$ in the πN c.m.s.

N^* spin density matrix in the Gottfried-Jackson frame is

$$\rho_{mm'}=\begin{array}{c} \\ m=J \\ \vdots \\ m=1/2 \\ \\ m=-1/2 \\ \vdots \\ m=-J \end{array}\begin{array}{c} \begin{array}{cccc} \quad & m'=1/2 & m'=-1/2 & \quad \end{array} \\ \left(\begin{array}{c|cc|c} 0 & 0 & 0 & 0 \\ \hline 0 & \dfrac{1}{2} & -i\dfrac{\mathrm{Im}(T_\xi T_\eta{}^*)}{|T_\xi|^2+|T_\eta|^2} & 0 \\ 0 & i\dfrac{\mathrm{Im}(T_\xi T_\eta{}^*)}{|T_\xi|^2+|T_\eta|^2} & \dfrac{1}{2} & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) \end{array}.\tag{2·12}$$

This density matrix has two features: (1) $\rho_{mm'}\neq0$ only for m, $m'=\pm1/2$. (2) $\rho_{mm'}$ hardly depends on $J=L+1/2$ (It depends on J only through the M in C_M and S_M) and is determined mostly by the $SU(6)$ structure of N^*.

§ 3. Input πN elastic scattering amplitudes

Putting the πN elastic scattering amplitudes into the formulae in § 2, we get the N^* production cross section and the spin density matrix. In I, we used the following approximate formula as input:

$$T^{(+)}_{++}=i\frac{\sqrt{s}P_c}{m}ae^{(b/2)t},\qquad T^{(+)}_{+-}=T^{(-)}_{++}=T^{(-)}_{+-}=0\,,\tag{3·1}$$

where a and b are constants and take the values of $a=1.6\,\mathrm{GeV}^{-2}$ and $b=7.8\,\mathrm{GeV}^{-2}$ for $16\,\mathrm{GeV}/c$ incident pion laboratory momentum k_L.

Barger and Phillips[3] obtained phenomenological $\pi N\to\pi N$ amplitudes by using the Regge-pole amplitudes. Their amplitudes reproduce the experimental values of $(d\sigma/dt)(\pi N\to\pi N)$ and the polarization very well. On the other hand the simple approximate formulae (3·1) give $(d\sigma/dt)(\pi N\to\pi N)$ which starts to deviate from the experimental value at $|t|\approx 0.5\,\mathrm{GeV}^2$ and becomes smaller by a factor of 3 at $|t|=1.0\,\mathrm{GeV}^2$. Furthermore (3·1) does not give the polarization.

Therefore, we use here the Barger-Phillips phenomenological amplitudes as input $\pi N\to\pi N$ amplitudes. Our helicity amplitudes are related to their invariant amplitudes as

$$T_{++}^{(\pm)}=\frac{1}{8\pi^2 m}\left[2mA^{(\pm)\prime}+\left\{s-m^2-\mu^2-\frac{(s-u)/2}{1-t/4m^2}\right\}B^{(\pm)}\right]\cos\frac{\theta_c}{2},$$
$$T_{+-}^{(\pm)}=\frac{1}{8\pi^2 m}\frac{1}{\sqrt{s}}\left[(s+m^2-\mu^2)A^{(\pm)\prime}\right.$$
$$\left.+\left\{m(s-m^2+\mu^2)-(s+m^2-\mu^2)\frac{(s-u)/2}{1-t/4m^2}\right\}B^{(\pm)}\right]\sin\frac{\theta_c}{2},\qquad(3\cdot2)$$

where θ_c is the scattering angle of π in the πN c.m.s. and

$$\cos\frac{\theta_c}{2}=\sqrt{\frac{\mathscr{S}_{m\mu}+ts}{\mathscr{S}_{m\mu}}},\qquad \sin\frac{\theta_c}{2}=\sqrt{\frac{-ts}{\mathscr{S}_{m\mu}}}.\qquad(3\cdot3)$$

Barger and Phillips assumed the Regge-pole form for $A^{(\pm)\prime}$ and $B^{(\pm)}$ and determined their parameters as follows:

$$A^{(+)\prime}=\sum_{i=P,P',P''}[-\gamma_i(\nu_0^2-\nu^2)^{\alpha_i/2}],$$
$$A^{(-)\prime}=\sum_{i=\rho,\rho'}[-\gamma_i\nu(\nu_0^2-\nu^2)^{(\alpha_i-1)/2}],$$
$$B^{(+)}=\sum_{i=P,P',P''}\beta_i\nu(\nu_0^2-\nu^2)^{(\alpha_i-1)/2},\qquad(3\cdot4)$$
$$B^{(-)}=\sum_{i=\rho,\rho'}[-\beta_i(\nu_0^2-\nu^2)^{(\alpha_i-1)/2}],$$

where

$$\nu=(s-u)/(4m),\qquad \nu_0=\mu+t/(4m),$$
$$\alpha_P=1+0.36t,\qquad \alpha_{P'}=0.56+0.86t,\qquad \alpha_{P''}=t,$$
$$\beta_P=42.5e^{2.45t},$$
$$\beta_{P'}=24.7e^{0.17t}\sin(\tfrac{1}{2}\pi\alpha_{P'})[\Gamma(1-\tfrac{1}{2}\alpha_{P'})]^2,$$
$$\beta_{P''}=49.8e^{2.81t},$$
$$\gamma_P=21.6e^{6.5t}+31.1e^{1.8t},$$
$$\gamma_{P'}=(22.2+16.9e^{0.84t})\sin(\tfrac{1}{2}\pi\alpha_{P'})[\Gamma(1-\tfrac{1}{2}\alpha_{P'})]^2,$$
$$\gamma_{P''}=0,$$
$$\alpha_\rho=0.55+t,\qquad \alpha_{\rho'}=t,$$
$$\beta_\rho=-(24.6+58.7e^{1.29t})\Gamma(1-\alpha_\rho)\sin(\tfrac{1}{2}\pi\alpha_\rho),$$

$$\beta_{\rho'} = -293.8te^{5.0t},$$

$$\gamma_{\rho} = 3.94(1+6.0t)e^{2.55t}\Gamma(-\alpha_{\rho})\sin(\tfrac{1}{2}\pi\alpha_{\rho}),$$

$$\gamma_{\rho'} = -74.8t(1+2.45t)e^{4.78t}$$

in the units of $\hbar=c=1$ and $\mathrm{GeV}=1$. The numerical results calculated by inserting the Barger-Phillips formula into (3·2) will be shown in the next section.

§ 4. Our model prediction and comparison with experiments

1. *Production differential cross section*

The calculated $(d\sigma/dt)(\pi^-p\to\pi^-N^*)$ for $k_L=16\,\mathrm{GeV}/c$ and $8\,\mathrm{GeV}/c$ are shown in Figs. 2 and 3.

$N_\alpha(5/2^+, 1688)$: Agreement with experiment is good for $|t|\gtrsim 0.1\,\mathrm{GeV}^2$. It is better than in I. The data referred to here[6] seems to flatten in $|t|\lesssim 0.1\,\mathrm{GeV}^2$, while the calculated values give a straight gradient. If such a flattening is es-

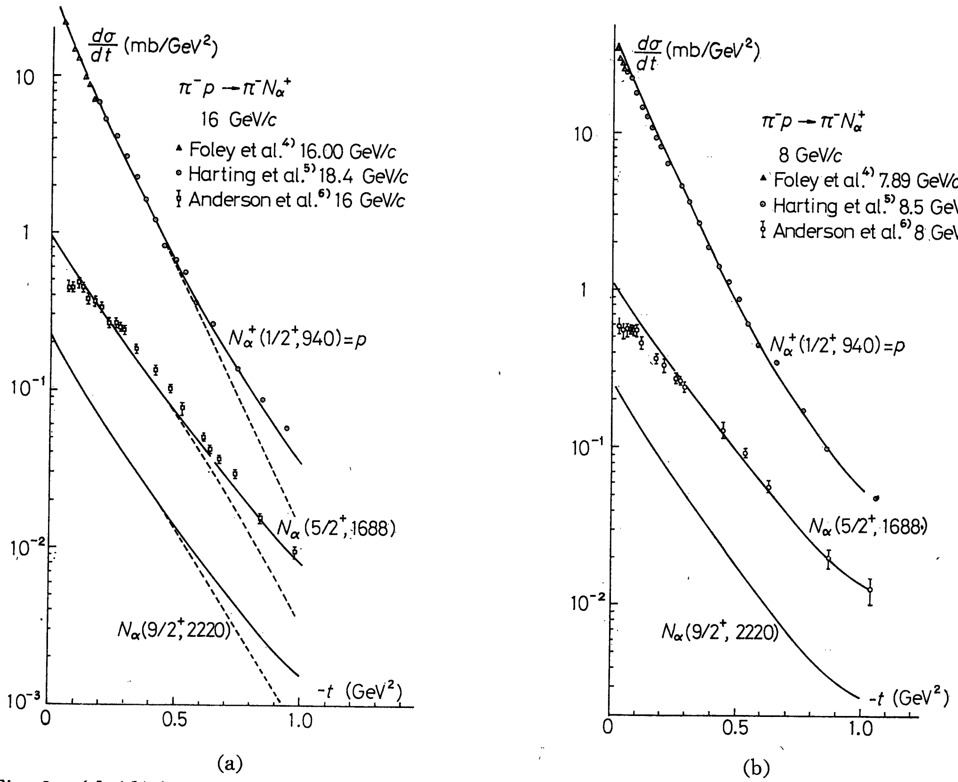

Fig. 2. $(d\sigma/dt)(\pi^-p\to\pi^-N_\alpha^+)$. The solid and the dashed curves are the calculated values by using the Barger-Phillips amplitudes (3·4) and the simple exponential form (3·1), respectively.

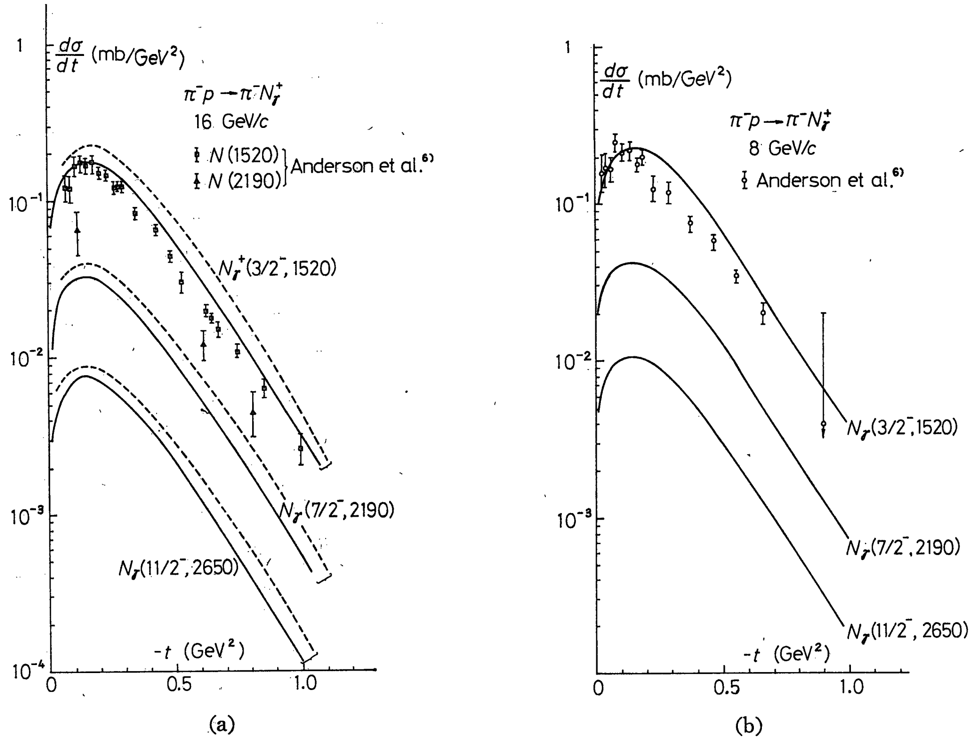

Fig. 3. $(d\sigma/dt)(\pi^- p \to \pi^- N_\gamma^+)$. The solid and the dashed curves are the calculated values by using the Barger-Phillips amplitudes (3·4) and the simple exponential form (3·1), respectively.

tablished,*) some new element must be introduced to our model.

$N_\gamma(3/2^-, 1520)$: Agreement with experiment is satisfactory.

$N_\gamma(7/2^-, 2190)$: An experiment exists only for $k_L = 16\,\text{GeV}/c$. The calculated value agrees with experiment within factor 2. Experimental data are not good enough to discuss the difference of factor 2.

As a whole, agreement with experiment seems to be satisfactory if we take into account that $SU(6) \otimes O(3)$ is an approximate symmetry.

We expect that experiments on $N_\alpha(9/2^+, 2220)$, $N_\gamma(11/2^-, 2650)$, etc., will be performed.

2. *N* spin density matrix*

One of the features of our model is that N^* is produced with helicity $\pm 1/2$ in the L.V.F. and also in the Gottfried-Jackson frame. Then the angular distribution of the decay product N of N^* in the Gottfried-Jackson frame is given as[8]

*) The experiment by Foley et al.[7] is consistent with a straight line.

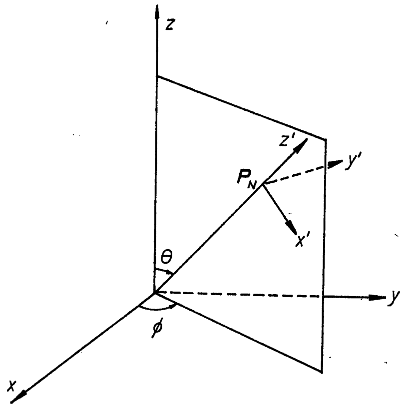

Fig. 4. The Gottfried-Jackson frame. N^* is at rest and the z-axis is taken to the direction of the initial N momentum and the x-axis is taken in the N^* production plane. The z'-axis is taken to the direction of the final N momentum p_N and the x'-axis is taken in the zz'-plane.

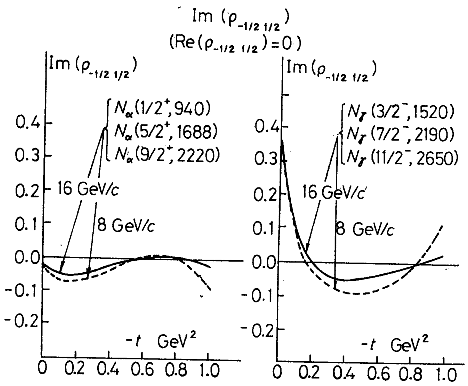

Fig. 6. Element $\rho_{-1/2\,1/2}$ of N^* spin density matrix.

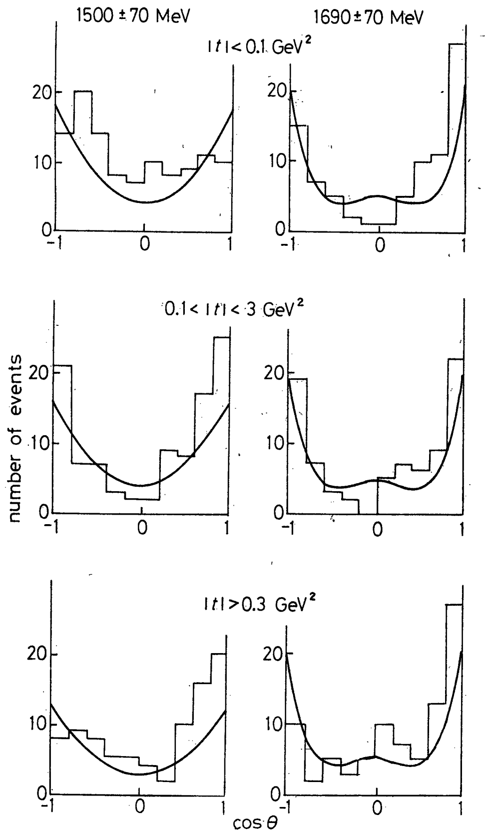

Fig. 5. Polar decay angle distribution in the Gottfried-Jackson frame for $N^*\to N\pi$. Data are taken from Oh et al.[9]

$$I(\theta, \varphi) = \frac{2J+1}{8\pi} \sum_{\mu=\pm1/2} (d^{(J)}_{1/2\mu}(\theta))^2$$

$$= \frac{1}{8\pi} \times \begin{cases} (3\cos^2\theta + 1) & \text{for } J=3/2\,, \\ \frac{3}{2}(5\cos^4\theta - 2\cos^2\theta + 1) & \text{for } J=5/2\,, \\ \frac{1}{8}(175\cos^6\theta - 165\cos^4\theta + 45\cos^2\theta + 9) & \text{for } J=7/2\,, \end{cases} \quad (4\cdot1)$$

where θ and φ are taken as in Fig. 4. It must be noted that $I(\theta, \varphi)$ depends on neither t nor $\rho_{-1/2\,1/2}$ and φ.*) The experimental data on the θ-distribution

*) Oh et al.[9] mention that N_α (1688) and N_γ (1520) are produced with helicity $\pm1/2$ and also their $I(\theta, \varphi)$ has φ-dependence. But these two statements contradict each other.

have been published for $N_\alpha(5/2^+, 1688)$ and $N_\gamma(3/2^-, 1520)$[9] and are consistent with (4·1) as is shown in Fig. 5.

The only one non-vanishing off-diagonal element of our model $\rho_{-1/2\,1/2}(=\rho^*_{1/2\,-1/2})$ is purely imaginary and $\mathrm{Im}(\rho_{-1/2\,1/2})$ is shown in Fig. 6. It is hardly dependent on $J=L+1/2$ (three curves lie within the breadth of the line) and is determined almost only by the $SU(6)$ structure of N^*. $\rho_{-1/2\,1/2}$ cannot be determined from the angular distribution of the final N but can be done from its polarization. The polarization is given as

$$\begin{aligned}
I(\theta\varphi)P_{z'} &= \mathrm{Im}(\rho_{-1/2\,1/2})\frac{2J+1}{2\pi}\sin\phi\, d^{(J)}_{1/2\,1/2}(\theta)\, d^{(J)}_{-1/2\,1/2}(\theta) \\
&= \mathrm{Im}(\rho_{-1/2\,1/2})\frac{\sin\phi\sin\theta}{4\pi}\begin{cases}(9\cos^2\theta-1) & J=3/2\,, \\ \frac{3}{2}(25\cos^4\theta-14\cos^2\theta+1) & J=5/2\,,\end{cases} \\
I(\theta\varphi)P_{x'} &= \mathrm{Im}(\rho_{-1/2\,1/2})\frac{2J+1}{4\pi}\sin\phi\{(d^{(J)}_{-1/2\,1/2}(\theta))^2-(d^{(J)}_{1/2\,1/2}(\theta))^2\} \\
&= \mathrm{Im}(\rho_{-1/2\,1/2})\frac{\sin\phi\cos\theta}{4\pi}\begin{cases}(-9\cos^2\theta+5) & J=3/2\,, \\ \frac{3}{2}(-25\cos^4\theta+26\cos^2\theta-5) & J=5/2\,,\end{cases} \\
P_{y'} &= -2\,\mathrm{Im}(\rho_{-1/2\,1/2})\cos\phi\,,
\end{aligned} \tag{4·2}$$

where x'- y'- and z'-axes are taken as in Fig. 4.

The final N produced in the N^* production plane is polarized to the direction normal to the production plane (the direction of $\boldsymbol{k}_L\times\boldsymbol{k}_L'$, where $\boldsymbol{k}_L$ and $\boldsymbol{k}_L'$ are the momenta of the incident and scattered pion in the lab. sys.) and has the polarization $P=-2\,\mathrm{Im}(\rho_{-1/2\,1/2})$. The final N which normally emerges to the production plane is polarized longitudinally and has the polarization $P=\pm 2\,\mathrm{Im}(\rho_{-1/2\,1/2})$ (the double signs correspond to $J=3/2$ and $5/2$, respectively).

It is to be noted that these polarizations are determined almost only by the $SU(6)$ structures of N^* and behave quite differently if the $SU(6)$ structures are different.

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Note added in proof: The off-diagonal element of the spin density matrix, $\rho_{-1/2\ 1/2}$, can be observed by the left-right asymmetry ($\varepsilon(\theta)$) of π for the case of the polarized target: $\varepsilon(\theta)=2\,\mathrm{Im}\,(\rho_{-1/2\ 1/2})P_I$, where P_I is the polarization of the target nucleon.