

## Shaking Moment Optimum Balancing of Planar Mechanisms

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**Abstract.** A new method of shaking moment balancing of force balanced linkages is presented in this paper. The shaking moment balancing is gained based on the angular momentum principle. The method of synthesizing a dyad to balance the shaking moments is given, two kinds of structure types are calculated, all the maximum absolute values of the shaking moments are decreased by more than 75 % and the fluctuation values of those are reduced by more than 68 %.

### Introduction

During machines run, all components produce inertia forces and moments. The forces and moments may cause noise, wear, vibration and make the input crank wind up. In 1971, Berkof and Lowen dealt with the shaking moment optimization [1]. In 1973, Berkof represented a complete force and moment balance of an inline four-bar linkage [2]. In 1982, Begci gave the method to completely balancing shaking force and moment of linkages using balance idler loop [3]. In 1985, Huang presented the method of synthesis of a dyad to balance inertia input torque [4]. In 1999, Esat and Bahai dealt with the complete force and moment balance of planar linkages mechanisms [5]. In 2001, Arakelian and Dahan dealt with the partial shaking moment balancing of fully balanced linkages [6]. In 2010, the method with active balancer based on planar linkages was presented [7].

As mention before, balance of shaking moment of planar linkages is usually accomplished by means of appropriately chosen counterweights or attached motive components. The method of choosing appropriate counterweights is usually used for four-bar linkages. The method of attaching motive components has been taken to balance shaking moment of four-bar and six-bar linkages. But it usually makes the mechanism become very complex.

In this paper, a new method of synthesizing a dyad is taken to balance shaking moment of six-bar linkages, and good results are obtained with minimum number of attached components.

### Angular Momentum of Mechanisms

The shaking moment of a mechanism with one-*DOF* is equal to the derivative of the total angular momentum with respect to time, that is

$$M_s = -dH_o / dt \quad (1)$$

The angular momentum of the six-bar linkage is

$$H_o = \sum_{i=1}^5 (m_i(x_i \dot{y}_i - y_i \dot{x}_i) + I_{st} \dot{\phi}_i) \quad (2)$$

Where  $M_s$  and  $H_o$  are shaking moment and angular momentum of a mechanism about origin point of the inertial reference system respectively.  $x_i$  and  $y_i$  are coordinates of the mass center of  $i$  th link with respect to the origin o.  $I_{si}$  is centered mass moment of inertia of the  $i$  th link.  $\dot{\phi}_i$  is angular velocity of  $i$  th link.

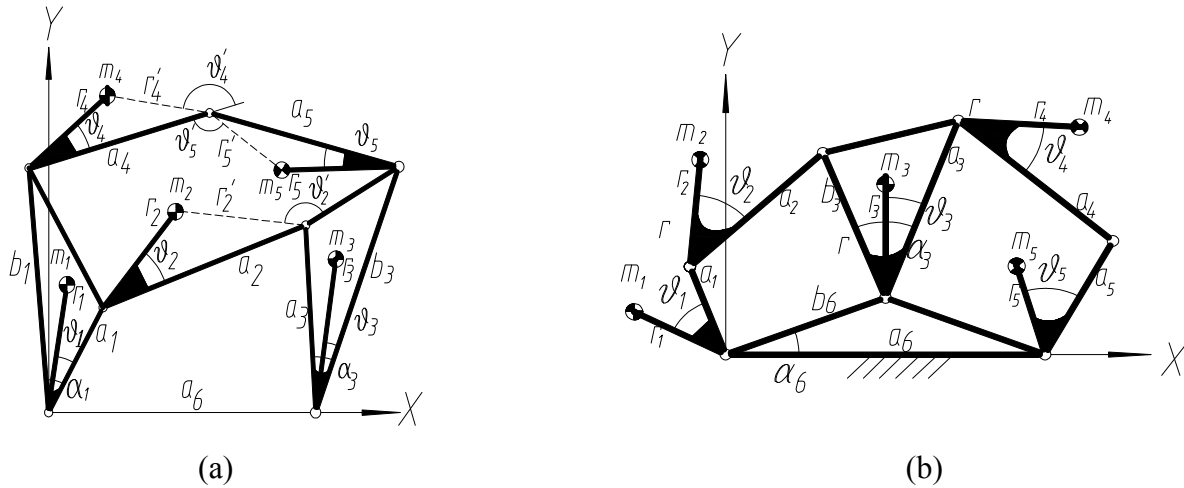


Fig. 1 Two original six-bar linkages

Six-bar linkages can be classified as Watt’s and Stephenson’s types. In both types of linkages a linkage is selected, respectively, Fig. 1. It is easy to get the condition of shaking force balance of the linkage in Fig.1(a) by means of the method of linearly independent mass vectors, that is

$$\begin{aligned}
 m_1 r_1 e^{j\theta_1} &= m_4 b_1 e^{j(\theta_4 - \alpha_1)} r'_4 / a_4 + m_2 a_1 e^{j\theta_2} r'_2 / a_2 \\
 m_3 r_3 e^{j\theta_3} &= m_5 b_3 e^{j\theta_5} r'_5 / a_5 + m_2 a_3 e^{j(\theta_2 + \alpha_2 + \pi)} r'_2 / a_2 \\
 m_5 r_5 e^{j\theta_5} &= m_4 a_5 e^{j(\theta_4 + \pi)} r_4 / a_4
 \end{aligned}
 \tag{3}$$

The dimensions and parameters given and determined by equation (3) are a listed in Table 1. Here, all are SI units. The symbols see Fig. 1.

Using the dimensions and parameters listed in Table 1 we calculated the angular momentums of the two linkages, during the input cranks rotate round  $2\pi$ .

Table 1 Parameters of the six-bar linkages

	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$r_1$	$r_2$	$r_3$	$r_4$	$r_5$
1st	.02	.16	.09	.10	.15	.13	.062	.08	.09	.071	.074
2nd	.02	.16	.06	.06	.08	.19	.10	.06	.037	.069	.069
	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$m_1$	$m_2$	$m_3$	$m_4$	$m_5$	
1st	$\pi$	0	3.33	0.79	3.9	.001	.008	.0045	.005	.0075	
2nd	$\pi$	0	$3\pi/2$	$-\pi/6$	$-\pi/6$	.001	.008	.010	.003	.008	
	$I_{s1}$	$I_{s2}$	$I_{s3}$	$I_{s4}$	$I_{s5}$	$b$		$\alpha$			
1st	.018	.14	.12	.08	.135	$b_1 = .03$	$b_3 = .12$	$\alpha_1 = .79$	$\alpha_3 = .19$		
2nd	.018	.14	.16	.07	.07	$b_3 = .09$	$b_6 = .12$	$\alpha_3 = \pi/2$	$\alpha_6 = \pi/12$		

### Method of Shaking Moment Balance

From equation (1) we know that the shaking moment equals or approximates zero if the angular momentum equals or approximates a constant. So we may attach a rotating roll to the mechanism, the fluctuation of the angular momentum of which can offset the fluctuation of that of the original mechanism, that is

$$H_o + H_r = C_o \quad (4)$$

where  $H_o$  is the angular momentum of the original mechanism,  $H_r$  is the angular momentum of the attached component,  $C_o$  is an appropriately selected constant.

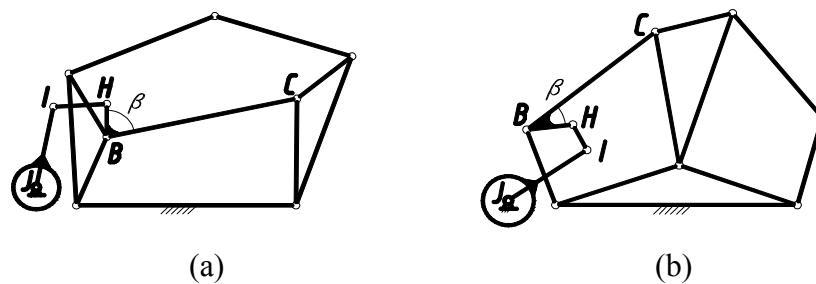


Fig.2 Mechanisms attached with dyads

A added dyad  $HIJ$  with a roll  $IJ$  which is housed to the ground in a revolute pair may be used to balance the shaking moment, Fig.2. Point  $H$  is connected to the coupler.

If the mass centre of link  $IJ$  is at the point  $J$ , the angular momentum of  $IJ$  is

$$H_r = I_r \dot{\phi}_r \quad (5)$$

where  $H_r$  is the required angular momentum of link  $IJ$ ,  $I_r$  is the centered mass moment of inertia of  $IJ$ ,  $\dot{\phi}_r$  is the angular velocity of  $IJ$ .

When the link  $HI$  is short enough and its mass is much smaller than that of  $IJ$ , the error existing as neglecting the angular momentum of  $HI$  is very small. When a dyad synthesized appropriately is attached to a six-bar linkage, the total angular momentum of the linkage can approximate invariable, so that the shaking moment of the linkage can approximate zero.  $H_o$  in equation (4) may be written as

$$H_o = I_o \dot{\phi}_1 \quad (6)$$

Where  $\dot{\phi}_1$  is angular velocity of input crank  $I_o$  is equivalent mass moment of inertia obtained from the total original mechanism. From equation (2),  $H_o$  is a position function of mechanism with constant input angular velocity  $\dot{\phi}_1$  so  $I_o$  is a position function of the mechanism as well. The dyad  $HIJ$  does not affect the balance of shaking force, as the centre of mass of  $IJ$  is fixed. Substituting equations (5) and (6) into (4) yield

$$\dot{\phi}_r I_r = C_o - I_o \dot{\phi}_1 \quad (7)$$

Equation (7) may be rewritten as

$$\dot{\phi}_r / \dot{\phi}_1 = (C - I_o) / I_r \quad (8)$$

The influence coefficient  $G_r$  is

$$G_r = \dot{\phi}_r / \dot{\phi}_1 = (C - I_o) / I_r \quad (9)$$

From equation (9)

$$\phi_r = \int_0^{\phi_1} \frac{C - I_o}{I_r} d\phi_1 \quad (10)$$

$I_r$  can not be chosen free after the constant  $C$  is selected, if we want the input crank rotates

$2\pi$  while the roll rotates  $2\pi$  also, thus

$$I_r = \frac{1}{2\pi} \int_0^{2\pi} (C - I_o) d\phi_1 \quad (11)$$

Hence, the problem of shaking moment balance has been transformed into a problem of mechanism synthesis. To mechanism attaches a roll, which rotates around a fixed

shaft and is driven by the original mechanism. The motion of the roll should satisfy equation (9). From equation (9)

$$G_{r\max} = (C - I_{o\min}) / I_r \quad (12)$$

$$G_{r\min} = (C - I_{o\max}) / I_r \quad (13)$$

Solving the system of simultaneous equations, yields

$$C = (G_{r\max} I_{o\max} - G_{r\min} I_{o\min}) / (G_{r\max} - G_{r\min}) \quad (14)$$

Constant  $C$  can be calculated by equation (14) after selecting appropriate  $G_{r\max}$  and  $G_{r\min}$ .

## Two Examples

Shaking moment balances of the six-bar linkages in Fig.1 are taken for examples. The curve 1 and curve 2 are shaking moments of the two linkages before and after balancing respectively, Fig.

3. In table 2,  $M_{smb}$  and  $M_{sna}$  are maximum absolute values of  $M_s$  before and after balancing

respectively.  $\Delta M = (M_{sma} - M_{smb}) / M_{smb}$ .  $\Delta F = (M_{afa} - M_{afb}) / M_{afb}$ .  $M_{sfb}$  and  $M_{sfa}$  are fluctuation

values of  $M_s$  before and after balancing respectively. The fluctuation value of shaking moment

is the difference of the maximum value and the minimum value. The others are parameters of the dyad  $HIJ$ . All are SI units. Type of the dyad  $HIJ$  is +1 or -1 depending on the order  $HIJ$  to

be counterclockwise or clockwise respectively.  $\beta$  is the angle  $HBC$ .

The results show that the maximum absolute values of shaking moment are decreased by more than 75% and the fluctuation values of which are reduced by more than 68%.

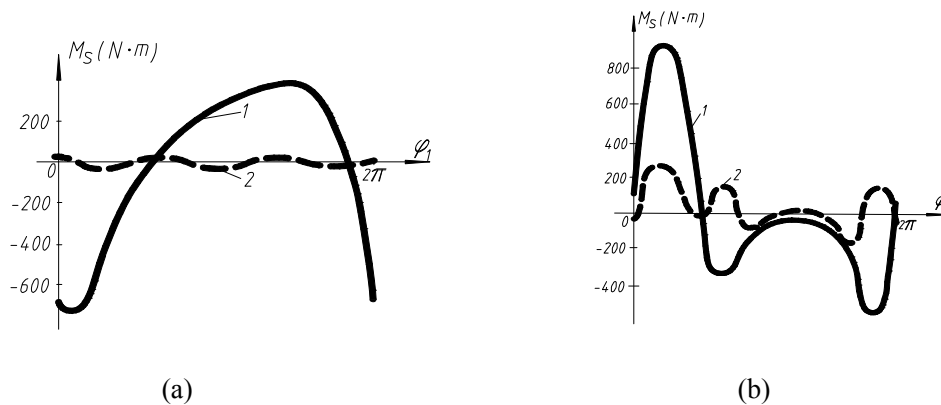


Fig. 3 The shaking moment curves before and after balancing

Table 2 Shaking moments of mechanisms and parameters of dyads

	$M_{smb}$	$M_{sma}$	$\Delta M\%$	$M_{sfb}$	$M_{sfa}$	$\Delta F\%$	$BH$	$HI$	$IJ$	$\beta$	$x_j$	$y_j$	$I_r$	type
1st	74.54	3.46	-95.4	115.2	6.3	-94.5	.0028	.0231	.0135	0.28	.0086	-0.0001	2.43	+1
2nd	93.43	22.35	-76.1	151.7	48.5	-64.3	.0063	.0026	.0198	-1.4	.0037	-0.0016	4.31	-1

## Conclusion

The method synthesizing a two-link dyad to compensate the fluctuation of angular momentum of the original mechanism and make it approach all invariable is feasible for shaking moment balance. A good result has already achieved, a large percentage has been cut down. From the analysis we know that even more complex mechanisms can also be balance using this method.

The balance method of synthesis a dyad has some advantages such as, it is good in effect of balance and simple in construction of the balance attachment. No matter how complex the original mechanism may be, the only necessary is two-link dyad to attach with. The more complex the original mechanism, the more evident the advantage is.

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