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The Mesoscopic Investigation on Moduli Prediction in Visco-Elastic Particle Reinforced Composites

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Abstract. The visco-elastic mechanism of particles reinforced composites has been investigated through revised Eshelby equivalent inclusion theory. A visco-elastic model is applied. Furthermore, by introducing Heaviside step function and Laplace transform, the creep constitutional equation related to strain rate effect is achieved. Finally, by equivalent inclusion theory, introducing secant modulus, the material moduli with time and volume fraction concerning Glass/ED6 particles reinforced materials have been given. The results show that the visco-elastic property of composite material is mainly determined by the visco-elastic behavior of the matrix, which meet experiment results well. It can be concluded from the results that there exits close relationship between the inclusion shape, volume fraction and loading path.

Introduction

For most metallic composite materials, when entering viscous and plastic phase, the Young's modulus can no longer retain constant. Visco-plastic problem in with the four parameter fluid model, which Maxwell model concatenates Voigt Model, has been studied by Jun Liang and Shanyi Du. In the present work, researches based on a new visco-elastic model, which is more close to the practical material, have been investigated[1,2,3,4].

Fundamental Equations

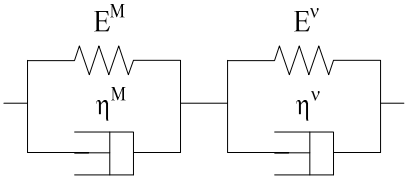

Fig. 1 Four Parameter Visco-Elastic Solid Model

The Laplace transform can transfer partial differential equations with time element and coordinate conversion into ordinary differential equations, which is widely used in elastic visco-plastic mechanics. The Laplace transform is

$$F(s)=\int_0^\infty f(t)e^{-st}dt\,. \tag{1}$$

Based on the principle of correspondence and by Laplace transform, the visco-elastic differential constitutive equation in this model can be transformed into

$$\sigma + \frac{\eta_1 + \eta_2}{E_1 + E_2}\dot{\sigma} = \frac{E_1 E_2}{E_1 + E_2}\varepsilon + \frac{E_1\eta_2 + E_2\eta_1}{E_1 + E_2}\dot{\varepsilon} + \frac{\eta_1\eta_2}{E_1 + E_2}\ddot{\varepsilon} \tag{2}$$

With creep test, $\sigma = \sigma_0 H(t)$, and by inverse Laplace transform, let $\tau_1 = (q_1 + q_2)/q_0$, there is

$$\varepsilon = \sigma_0 \exp(-\frac{t}{\tau_1})(\frac{p_1}{q_1 + q_2} - \frac{1}{q_0}) + \frac{\sigma_0}{q_0} \tag{3}$$

, where

$$p_1 = \frac{\eta_1 + \eta_2}{E_1 + E_2}, q_0 = \frac{E_1 E_2}{E_1 + E_2}, \quad q_1 = \frac{E_1\eta_2 + E_2\eta_1}{E_1 + E_2} \text{ and } q_2 = \frac{\eta_1\eta_2}{E_1 + E_2} \tag{4}$$

Effective Modulus Prediction

Assume that the distant field homogeneous stress acts on the boundary of composites. Then, the mean stress in matrix is

$$\sigma^M = \sigma^A + \tilde{\sigma} = L^M(\varepsilon^A + \tilde{\varepsilon}) \tag{5}$$

, where L^M is Young's Modulus in matrix, $\tilde{\varepsilon}$ is the perturbation strain caused by the interaction of spherical inclusions.

Similarly, the mean stress in spherical inclusions is

$$\sigma^F = \sigma^A + \tilde{\sigma} + \sigma' = L^F(\varepsilon^A + \tilde{\varepsilon} + \varepsilon') = L^M(\varepsilon^A + \tilde{\varepsilon} + \varepsilon' - \varepsilon^*) \tag{6}$$

, where ε^* is the equivalent eigenstrain of spherical inclusions.

By Eshelby equivalent inclusion theory [4], there is

$$\varepsilon' = S\varepsilon^* \tag{7}$$

, where S, which is related to matrix elasticity and the shape of inclusions, is the tetradic Eshelby tensor. From (5), (6), (7), there is

$$\sigma' = L^M(S - I)\varepsilon^* \tag{8}$$

In Eshelby's theory, the volumetric mean stress in composites is equal to the distant field homogeneous stress, namely,

$$\sigma^A = (1 - f)\sigma^M + f\sigma^F \tag{9}$$

, where f is the volume fraction of reinforcement. From (6), (7) and (8), we have

$$\tilde{\sigma} = -f\sigma', \tilde{\varepsilon} = -f(S - I)\varepsilon^* \tag{10}$$

Then, it can be obtained as

$$\varepsilon^* = A\varepsilon^A \tag{11}$$

where $A = \{L^M + (L^F - L^M)[fI + (1 - f)S]\}^{-1}(L^M - L^F)$.

Homogeneous tension stress σ_{11}^{A} acting on x, which is along the fiber and by Hook's law, Young's modulus is

$$\frac{E_{11}}{E_0}=\frac{\varepsilon_{11}^{A}}{\varepsilon_{11}^{A}+f\varepsilon_{11}^{*}}=\frac{1}{1+f(a_1-2\gamma_0 a_2)} \tag{12}$$

Pure shear stress σ_{12}^{A} acting on the plane perpendicular to x and by Hook's law, the shear modulus is

$$\frac{\mu_{12}}{\mu_0}=\frac{\varepsilon_{12}^{A}}{\varepsilon_{12}^{A}+f\varepsilon_{12}^{*}}=\frac{1}{1+fa_3} \tag{13}$$

Homogeneous tension stress acting on x and by Hook's law, Poisson's ratio is

$$\gamma_{12}=-\frac{\varepsilon_{22}^{A}+f\varepsilon_{22}^{*}}{\varepsilon_{11}^{A}+f\varepsilon_{11}^{*}}=\frac{\gamma_0-f[a_2-\gamma_0(a_1+a_2)]}{1+f(a_1-2\gamma_0 a_2)} \tag{14}$$

By Laplace transform, there is

$$\frac{E^{LT}}{E_M^{LT}}=\frac{1}{1+f(a_1^{LT}-2\gamma_0^{LT}a_2^{LT})},\quad \frac{\mu^{LT}}{\mu_M^{LT}}=\frac{1}{1+fa_3^{LT}},$$

$$\gamma^{LT}=\frac{\gamma_M^{LT}-f[a_2^{LT}-\gamma_M^{LT}(a_1^{LT}+a_2^{LT})]}{1+f(a_1^{LT}-2\gamma_M^{LT}a_2^{LT})} \tag{15}$$

a_1, a_2 and a_3 are parameters determined by Eshelby tensor, Lamè coefficients and volume fraction.

Suppose that the granular reinforcements and visco-elastic matrix are isotropic, then, the Young's modulus can be expressed as

$$E_M^{LT}=\frac{q_0+q_1 s+q_2 s^2}{p_1 s}=\frac{E_0^M E_0^{\upsilon}+(E_0^M\eta_0^{\nu}+E_0^{\upsilon}\eta_0^M)s+\eta_0^M\eta_0^{\nu}s^2}{(\eta_0^M+\eta_0^{\nu})s} \tag{16}$$

Based on the correspondence principle, visco-elastic property corresponds with elastic property in Laplace transform. Thus, the visco-elasticity can be obtained from mesomechanical elastic methods.

Results

The mechanical property of Glass/ED6 composite can be seen from the following figures.

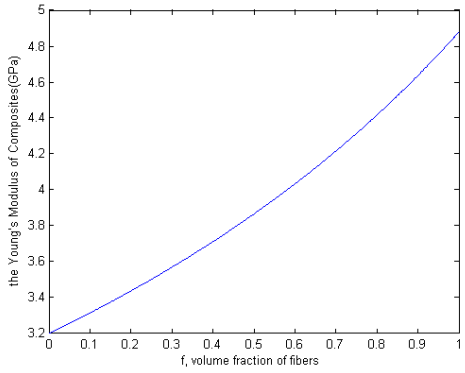

Fig.2 Variation of the Young's modulus with f

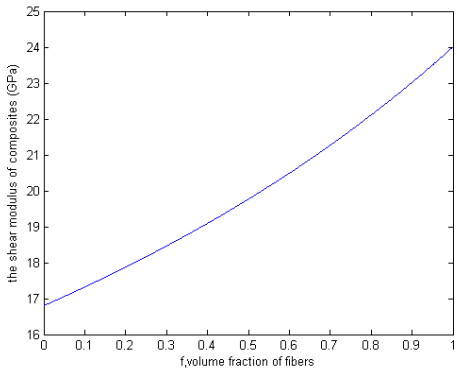

Fig.3 Variation of the shear modulus with f

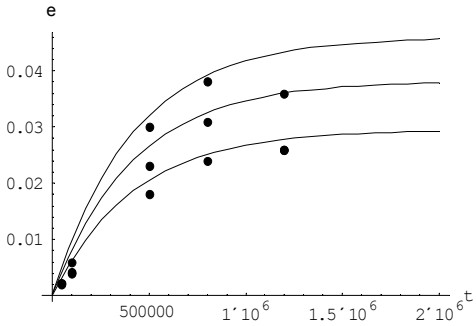

Fig.4 Comparison between Experiment and Theoretical Creep Curves

It can be seen that the Young's modulus and shear modulus of the composites increases in direct proportion with volume fraction of fibers. From the figure we can find that in the initial stage, the composite exhibits instantaneous elasticity, which is in good agreement with the experimental results. Then, instable creep stage occurs and finally, it reaches the stable creep stage, which fits the experimental data well.

Conclusions

The visco-elastic performance of particle reinforced composites is mainly determined by the visco-elasticity of matrix, which is closely related to the inclusion shape and volume fraction. This work will provide theoretical basis to the practical engineering applications such as material design and performance prediction of composites.

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