AN EXAMPLE OF PERIODIC VARIATION IN FOREST SAMPLING

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INTRODUCTION

IN a recent paper [1948 (2)] the relative merits of random and systematic strip sampling of forests for the estimation of timber volumes have been discussed. No purely theoretical reasons for always preferring one system of sampling can be adduced. The choice of a sampling scheme must depend on accumulated experience of the variation normally encountered in forests, as well as upon the particular circumstances of the investigation in progress. The precision of an estimate based upon a systematic sample cannot be assessed from the sample itself, except possibly by the addition of some supplementary sampling, though that precision will often be greater than for a random sample at the same intensity. These two factors, knowledge of the precision and securing maximum precision, must be considered before the choice is made.

Information on the gain in precision to be expected in systematic sampling is clearly important, and can only be obtained empirically. In the previous paper tentative conclusions were based upon detailed analyses of two areas (Mount Stuart in south India and Blacks Mountain in California), but it was recognized that far more evidence must be examined before any trustworthy estimate of the average gain could be made. To this end further sets of data from extensive forest enumerations are being analysed. One of these (Dehra Dun in north India) shows interesting features which did not occur in the earlier analyses, since it has a marked regular periodic component of variation. As was pointed out earlier [1947 (1)], such periodicity is very disturbing to the precision of systematic sampling: a general gain in precision for systematic sampling as compared with random may disappear, or even be converted into a loss, at certain sampling intensities. The purpose of this paper is to describe the results of a statistical analysis of sampling problems for these Dehra Dun data analogous to those of the previous paper: the terminology and methods have already been fully described [1947 (1), 1948 (2)], except for complications introduced by the periodicity.

THE DATA

Griffith [1947 (3)] has reported the enumeration of the sal (*Shorea robusta* Gaertn.) forests of the Dehra Dun Division of the United Provinces, northern India. The climate and floristics of this northern tropical moist deciduous forest are summarized in his paper. In 1947 a 20 per cent. enumeration of this area was made, using strips of 2 chains in width but enumerating only every fifth strip of the total number that could have been taken. The trees in each 5-chain length

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of strip were recorded separately, so giving recording units of 1 acre. Sahai [1947 (4)] has given a detailed account of the enumeration procedure. For the two chief species, *Shorea robusta* Gaertn. and *Terminalia tomentosa* W. et Arn, records were kept of the number of trees in 4-inch diameter classes from 12 inches to 24 inches, and over 24 inches. These were converted to volumes, by local volume tables. By the kindness of Dr. Griffith the total volumes for the two species, in each recording unit, have been made available to the writer.

The data used in this paper relate to 292 strips, containing 8,506 recording units. At the north-western end of the base line the strips were short, mostly containing only 5–15 units; almost all of strips 42–171 had 30–50 units and none less than 20; strips 176–192 were short (188 had only 2 units); and with the exception of the short strips 279–282 the remainder were almost all between 25 and 45 units long. As in the previous paper [1948 (2)], only the one-dimensional sampling problem has been considered, the mean volume of timber per strip being used for analysis, without any attempt to make allowance for differences in strip length.

THE ESTIMATES OF VARIANCES

For the Mount Stuart and Blacks Mountain forests examined previously, data from complete enumerations of all strips in the areas were available. For Dehra Dun the 292 strips are themselves only a 20 per cent. systematic sample from a complete series of 1,460 strips that might have been enumerated. Hence no comparisons can be made with the true mean volume for the forest. Instead, samples of less than 20 per cent. may be compared with the 20 per cent. results. In other words the 292 strips may be used as though they constituted a complete enumeration of some forest, and different schemes of sampling from them may be compared. The analysis may not give an entirely fair picture of the relative merits of random and systematic selection of samples from the whole forest, for these are largely dependent upon the correlations between adjacent and near-adjacent strips: the samples to be discussed will not be all possible samples of a particular type that might be drawn from the forest but only the restricted set contained within the 292 strips, and correlations between successive strips of the 292 will in general be lower than between adjacent strips.

If suitable records for analysis were plentiful, the Dehra Dun series might have been discarded because of the limitation to sub-sampling from a 20 per cent. sample. In fact, records that may be used in extensive studies of sampling variation are too few for any to be lightly put on one side. The Dehra Dun records have therefore been analysed, in the first instance exactly as described for the other two forests. For convenience in terminology, the sampling will be described as though the 292 strips were the whole of a forest, so that what is termed a sample of 1 in r is in reality a sample of 1 in $5r$ from the true forest of 1,460 strips but constrained to be selected only from the particular set of 292 strips enumerated.

The volume of timber averaged 1,028·4 cu. ft. per acre for the 292 strips (unweighted average of strip means). At the north-western end of the forest the volumes were greater, most strips having more than 1,500 cu. ft. per acre, and some over 2,000 cu. ft. per acre; at the south-eastern end values fell to about

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500 cu. ft. per acre. The variation was very great and erratic, the extremes being 2,900 cu. ft. and 120 cu. ft. per acre.

For unrestricted random sampling, calculation from the sum of squares of deviations of the 292 strips gives

$$s_{\mathrm{I}}^2 = 207{,}400$$

in units of (cu. ft. per acre)^2. Table I shows estimates of variance per strip for stratified random sampling in blocks of $2r(s_{\mathrm{II}}^2)$, stratified random sampling in blocks of $r(s_{\mathrm{III}}^2)$ and systematic sampling (s_{IV}^2). For random sampling, calculations were made for a number of values of r sufficient to give a reasonable representation of the relationship between the variance and r. Since s_{II}^2 for any r is the same as s_{III}^2 for $2r$, both variances can be represented on the same diagram (Fig. 1). For systematic sampling, using $k = 4$, it was impossible to go beyond $r = 58$, but for reasons which will soon appear many values of r not used for random sampling were analysed. Fig. 2 shows the relationship between s_{IV}^2 and r.

TABLE I. *Estimates of variance per strip for Dehra Dun, for sampling of 1 in r*

[In units of 1,000 (cu. ft. per acre)^2]

r	s_{II}^2	s_{III}^2	s_{IV}^2	r	s_{II}^2	s_{III}^2	s_{IV}^2
2	45·4	33·4	25·1	28	131·4	117·6	86·7
3	59·4	31·4	25·4	30	130·5	122·9	104·5
4	68·9	45·4	21·2	32	142·3	130·6	135·9
5	81·3	49·5	28·3	34	..	..	220·3
6	98·0	59·4	29·3	36	147·7	130·1	201·4
7	92·5	63·7	33·0	38	..	..	107·6
8	119·6	68·9	31·3	40	140·8	132·5	77·1
9	116·8	84·9	36·7	42	..	..	90·6
10	110·3	81·3	42·9	44	..	..	120·4
12	115·7	98·0	69·8	46	..	..	129·8
14	117·6	92·5	82·8	48	161·8	150·3	123·7
15	122·9	104·1	96·2	50	149·0	137·8	210·0
16	130·6	119·6	135·7	51	..	..	243·2
17	..	..	158·2	52	..	..	241·2
18	130·1	116·8	149·0	54	..	..	152·9
20	132·5	110·3	64·6	56	148·0	131·4	65·0
22	..	118·9	45·4	58	..	..	45·7
24	150·3	115·7	79·1	60	153·3	130·5	..
25	137·8	109·6	92·0				
26	..	..	85·1				

The random sampling variances show steady increases as r increases, and for large values of r there is little difference between s_{II}^2 and s_{III}^2. Over the range of r studied, representation of the dependence of s^2 on r by a linear relationship between their logarithms (which seemed satisfactory for Mount Stuart and

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Blacks Mountain) would obviously be insufficiently close to the truth. Indeed, no simple mathematical model seemed adequate to describe the relationship; a smooth curve could be drawn through the points of Fig. 1, however, and for the

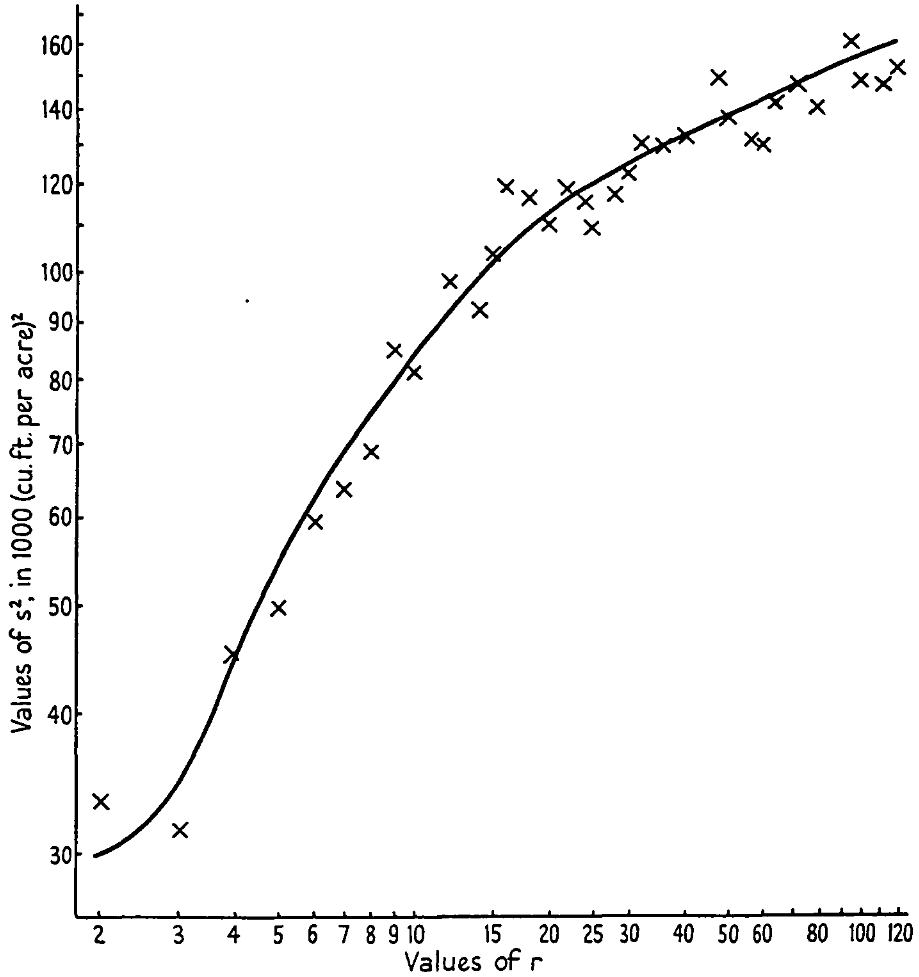

FIG. 1. Relationship between variance, s^2, and r, for random sampling at an intensity of 1 in r stratified in blocks of r strips (Type III); for sampling at the same intensity, but with stratification in blocks of $2r$ (Type II), read the figure for $2r$ instead of r. (The scales of s^2 and r are logarithmic.)

subsequent discussions of sampling efficiency variance estimates S_{II}^2 and S_{III}^2 have been read from this curve.

Inspection of Fig. 2, or of the last column of Table I, shows that the variance per strip in systematic sampling has a still more complicated relationship with r. In general, s_{IV}^2 increases as r increases; superimposed on this trend, however, are large increases in the neighbourhood of $r = 17$, $r = 34$, and $r = 51$, and decreases in the neighbourhood of $r = 22$, $r = 40$, and $r = 58$. This is strongly indicative

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of a regular periodic variation in the volume per strip, with period about 17. Periodicity might indeed have been suspected from results presented by Griffith. In his report on the Dehra Dun data [1947 (3)] he showed the means for the eight possible $2\frac{1}{2}$ per cent. enumerations and the sixteen possible $1\frac{1}{4}$ per cent. enumerations, or 1 in 8 and 1 in 16 systematic samples in the terminology of this paper. Not only were the 1 in 16 means very much more widely dispersed than the 1 in 8 (the range from highest to lowest being 300 instead of 100, a change much greater than would be expected on halving the intensity of the sampling), but of the 1 in 16 samples the six whose means exceeded the general mean were consecutive. The phenomenon is emphasized rather more strongly by examining the seventeen 1 in 17 samples, because 17 seems closer to the true period. Table II shows the means of each of the seventeen systematic samples at this intensity. These present an even greater dispersion: the difference between the highest and the lowest is 535, and the seventeen values are much more discrepant than would be stratified random samples at the same intensity. Moreover the sequence of means clearly suggests a regular cycle of variation, samples 5–9 being particularly high and samples 13–17 particularly low.

TABLE II. *Mean volume per strip for 1 in 17 systematic samples of Dehra Dun (in cu. ft. per acre)*

Sample beginning with strip no.	*Mean volume*	*Sample beginning with strip no.*	*Mean volume*
1	943	11	974
2	953	12	913
3	1,078	13	871
4	1,119	14	872
5	1,267	15	887
6	1,185	16	779
7	1,314	17	879
8	1,282	General mean	1,028
9	1,101		
10	1,072		

Standard error for mean of 1 in 17 stratified random samples (Type III) would be approximately 80.

Undoubtedly then, the 292 strips of Dehra Dun show periodicity, with a sequence of rise and fall in volume of timber repeated every seventeen strips (i.e. every 85 strips of the whole area). The regularity is obscured by other sources of variation; it is not apparent when the whole set of 292 strip values is read but is made clear by the averaging process that produced Table II. No explanation of this phenomenon can be offered. Of course, it could arise from a regular variation in some soil or topographical feature, but inquiry from those who know the region has failed to disclose any tendency for hills and valleys to occur regularly at 170-chain intervals over the 36 miles of forest sampled! There may be some reason connected with the nature of the region or with the method of sampling,

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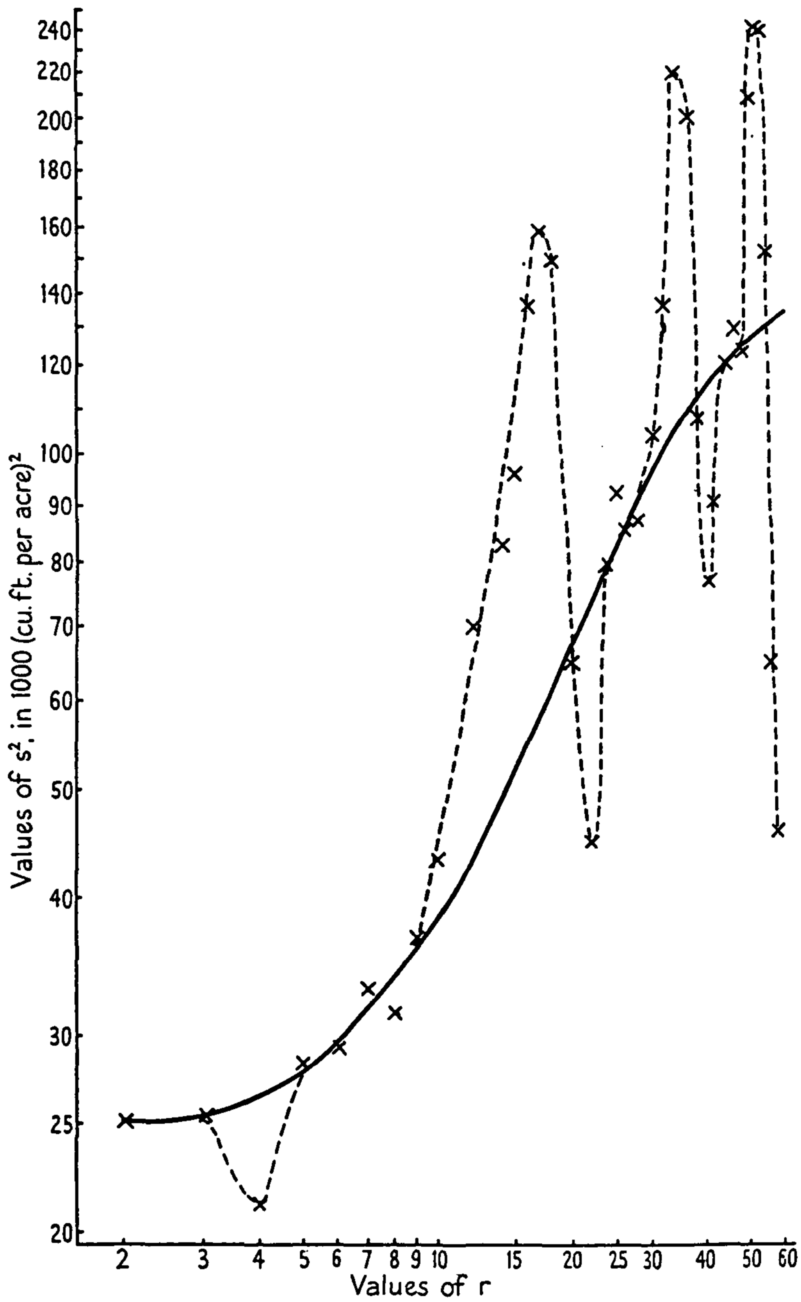

FIG. 2. Relationship between variance, s^2, and r, for systematic sampling at an intensity of 1 in r (Type IV). The continuous curve represents what is described in the text as the non-periodic relationship, the broken curve the relationship allowing for the periodic component. (The scales of s^2 and r are logarithmic.)

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but it is unlikely that any simple explanation of this kind can be found. Alternatively, the periodicity may be in a sense a chance or accidental effect. The complexity of interactions between trees growing close to one another, generally classified under the single heading of 'competition', makes the timber volumes on neighbouring strips dependent upon one another, so that the series of strip values is 'autocorrelated'. In a short series—and statistical theory suggests that series very much longer than 292 terms may be 'short' in that they are liable to behave atypically—the true character of the autocorrelations may fail to be completely manifested, and may appear by chance in over-simplified form. It is scarcely to be believed that the type of competition in growth present in the Dehra Dun forests necessarily produces a simple periodicity of length 170 chains, but the chance circumstances of this particular region may have happened to give prominence to this component of a much more complex pattern.

Further speculation on the origin of the periodicity seems useless. The chief concern in this paper is to consider its effect on sampling. In Fig. 2 the steady increase in variance as r increases is disturbed by prominent maxima when r is nearly a multiple of the period, 17. As is well known, systematic sampling is of low precision if the interval between successive units is in step with a periodicity, so that all the units of one sample tend to be near a peak or all near a trough of the cycle. On the other hand, if the interval between units is about 1·5, 2·5, 3·5,... times the period, the variance may be expected to be excessively low, since a strip that happens to be at a peak of the cycle will be followed by one in a trough, and vice versa. An effect of this kind is apparent in Fig. 2, the variance being low near $r = 22, 40, 58$; these minima are less pronounced than the maxima, and occur rather earlier than the half-period, presumably because of some asymmetry in the form of the cycle.

A smooth curve has been drawn in Fig. 2, as in Fig. 1, entirely by eye, to represent the general trend of the points, but points which appear to have been affected by the periodicity have been ignored. This might seem to introduce a strong subjective element into the drawing of the curve. In fact, the curve has been drawn to have a form similar to that of Fig. 1, concave upwards for low values of r and convex upwards for large r, for it is reasonable to suppose that the variance will not increase indefinitely as r is increased. With this restriction, and a concave portion extending rather farther than in Fig. 1, a number of the points indicate a curve very satisfactorily. The curve may be regarded as a representation of the variance in systematic sampling if there were no periodicity, and it is in some respects more suitable for comparison with variances in random sampling, in a general evaluation of the precision of the two types of sampling, than would be a curve which allowed for the periodic component.

The data seem insufficient to indicate any mathematical model for the periodic variation: without this, any discussion of the precision of sampling when allowance is made for periodicity must be little better than speculation. In order to provide smoothed values for variances, a modified curve has also been drawn in Fig. 2 so as to pass as closely as possible through the points affected by the periodicity. This curve has been constructed to have maxima at $r = 17, 34, 51$, and minima

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at 5–7 units after the maxima, or at $r = 22, 40, 58$; between the regions where the periodicity has a marked effect, the curve is taken to be identical with the first curve. A difficulty arises at $r = 4$, for which s_{IV}^2 was appreciably lower than for $r = 2$ and $r = 3$. Is this merely chance variation from the non-periodic curve, or is it another manifestation of sub-normal variation where the sampling interval exceeds a multiple of 17 by 4–5 units? The non-periodic curve may be drawn so very satisfactorily through the points for $r = 2, 3, 6$ that the second explanation seems more plausible, and for the periodic curve a minimum has been incorporated at $r = 4$. The periodic curve must not be regarded as more than a qualitative indication of the effects of the periodicity. Even if the already extensive calculations were increased so as to give values of s_{IV}^2 for other intensities of sampling, the curve for so complex a relationship could scarcely be determined sufficiently exactly from these data to give a satisfactory quantitative assessment; from the empirical curve in Fig. 2, smoothed values of s_{IV}^2 may be derived, on which may be based a discussion of the periodicity in qualitative terms.

In all that follows, the empirical variances shown in Table I will be replaced by smoothed values, for which

$$S_{I}^2 = 207{,}400,$$

S_{II}^2 and S_{III}^2 are read from Fig. 1, and S_{IV}^2 is read from Fig. 2. Unless the contrary is explicitly stated, S_{IV}^2 relates to the non-periodic curve.

THE COMPARISON OF SAMPLING PROCEDURES

Random and systematic sampling may now be compared for Dehra Dun in several different ways, exactly as was done previously for Mount Stuart and Blacks Mountain [1948 (2)].

The percentage efficiency of systematic sampling relative to stratified random is shown in Fig. 3. Except for large values of r, S_{I}^2 is so much greater than S_{IV}^2 that the efficiency of systematic sampling relative to unrestricted random is high, being substantially over 300 per cent. when r is less than 20. Type II stratification compares more favourably with systematic, and Type III is better still, but even relative to these two types, systematic sampling is more than twice as efficient for most values of r less than 20. As was found for Mount Stuart and Blacks Mountain, at very low sampling intensities the difference in efficiency between systematic and either form of stratified sampling seems practically to disappear. A phenomenon not previously observed is the relative improvement in the efficiency of random sampling at very high intensities ($r < 8$). Dehra Dun in general shows a lesser relative efficiency for random sampling than the forests previously analysed. Even under the most unfavourable conditions neither Mount Stuart nor Blacks Mountain showed either form of stratified random sampling to be less than half as efficient as systematic.

Fig. 3 shows the relative efficiencies of three forms of sampling for a forest like Dehra Dun but without periodic variation. If the modified curve in Fig. 2 is used to give smoothed variances, S_{IV}^2, which include the periodicity, a new set of relative efficiencies may be calculated, and these are represented in Fig. 4. It must

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again be emphasized that the details of the curves are somewhat speculative, and only their qualitative characteristics can be regarded as trustworthy. The low efficiency of systematic sampling when the sampling interval is nearly a multiple

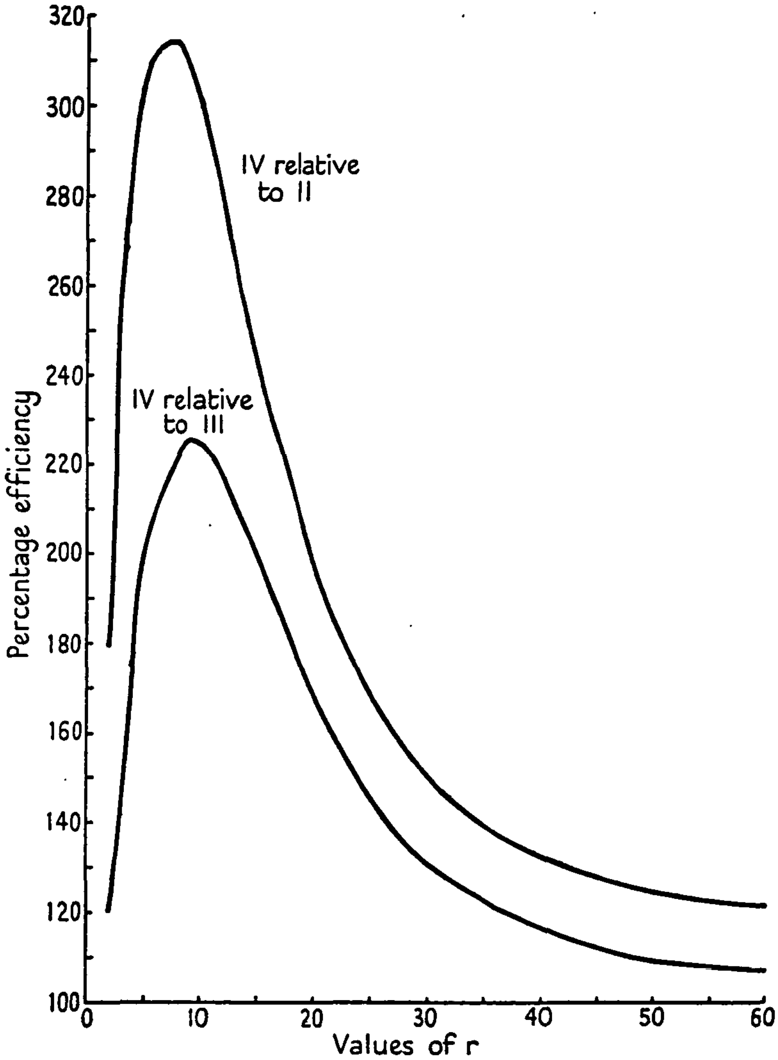

FIG. 3. Efficiency of systematic sampling, IV (non-periodic), relative to stratified random, II and III, at a sampling intensity of 1 in *r*. Upper curve: IV relative to II. Lower curve: IV relative to III.

of the period is to be expected on general principles, and Fig. 4 suggests that in the Dehra Dun forest periodicity was sufficiently marked to make the efficiency substantially less than for stratified random sampling at the same intensity. On the other hand, when the sampling interval exceeds a multiple of the period by a few units, the efficiency will be higher than in the absence of a periodic

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component. The dependence of the efficiency upon r is therefore represented by 'wavy' curves.

Fig. 5 shows the intensity of sampling necessary with any one of the three types of random sampling in order to obtain the same precision as by systematic sampling at a specified intensity. These are very like the corresponding curves for

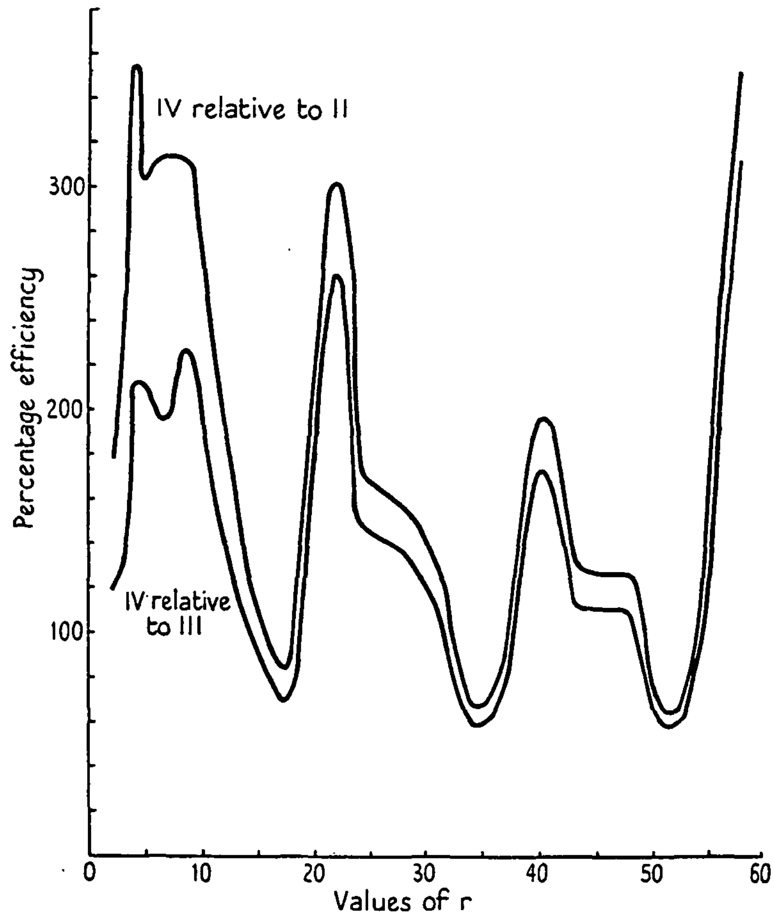

Fig. 4. Efficiency of systematic sampling, IV (periodic), relative to stratified random, II and III, at a sampling intensity of 1 in r. Upper curve: IV relative to II. Lower curve: IV relative to III.

Mount Stuart and Blacks Mountain. Inclusion of the periodic component would modify the curves and again produce maxima at 17, 34, 51. However, it is scarcely important to show that, whereas a 1 in 20 stratified sample would be as precise as a 1 in 17 systematic, to obtain the precision of a 1 in 22 systematic sample a 1 in 10 stratified would be needed! The modified curves are therefore not shown.

The percentage standard error per strip, adjusted for finite sampling, namely

$$\frac{100S\sqrt{(1-1/r)}}{1028\cdot 4},$$

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is shown in Fig. 6 for each of the four types of sampling. Again the curves are similar to those for the other forests, except that for Dehra Dun all the standard errors are rather larger and the advantage for systematic sampling more marked. The 'wavy' curve which would allow for the periodicity has not been drawn.

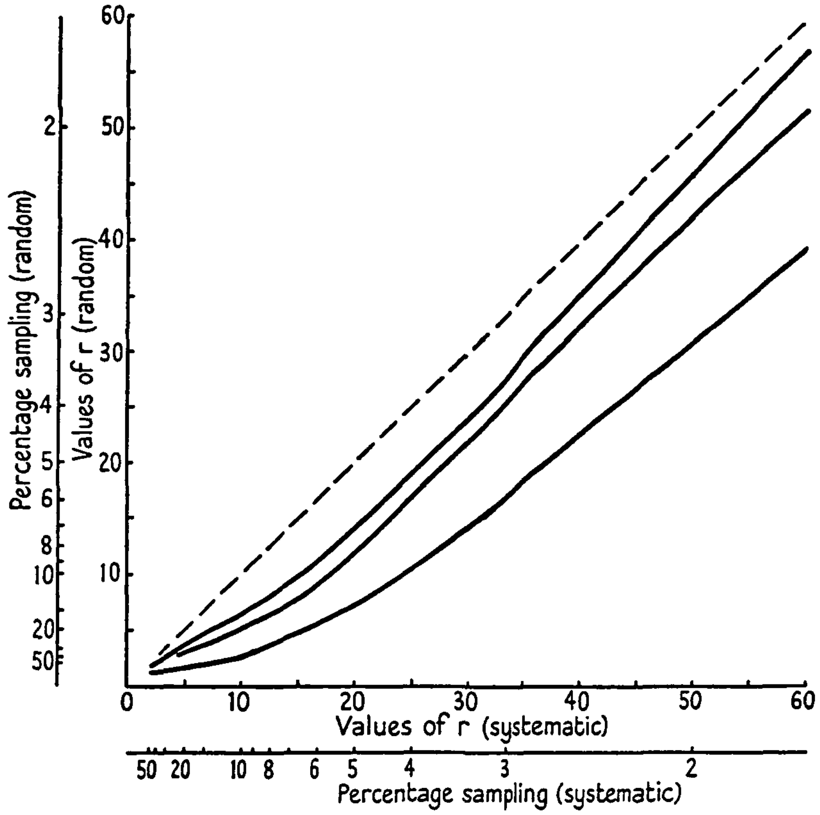

FIG. 5. Sampling intensity necessary with random selection, I, II, or III, in order to give same precision as a specified intensity with systematic selection, IV (non-periodic). Lower curve: Unrestricted random, I. Middle curve: Stratified in blocks of $2r$, II. Upper curve: Stratified in blocks of r, III.

(Broken line shows relationship as it would be for all types of sampling if volume per strip varied in a completely random manner, independently of position and of neighbouring strips.)

Table III shows the limits of error in estimation of mean volume, to be expected on an average in sampling at various intensities by each of the methods under discussion. The three sampling intensities chosen for illustration give total numbers of strips per sample about the same as in the illustrations used for Mount Stuart and Blacks Mountain. Stratification improves the precision very considerably, as compared with Method I, unrestricted random sampling. The two forms of stratified sampling do not differ greatly in their limits of error, but there is a further gain if systematic sampling is substituted for stratified random (especially for very intensive sampling), at least if no periodic component is present. The 1

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in 16 and 1 in 32 samples, however, happen to be intensities at which the effect of the periodicity is very noticeable, so that for the genuine Dehra Dun data (by contrast with the idealized non-periodic results) systematic sampling is no better than either form of stratified.

TABLE III. *Percentage limits of error for Dehra Dun*

Sampling intensity	1 in 8		1 in 16		1 in 32	
No. of strips in sample	36		18		9	
Probability of error	0·05	0·01	0·05	0·01	0·05	0·01
Sampling method:						
Unrestricted random (I)	13·5	17·8	19·8	26·0	28·5	37·4
Stratified random (II)	9·7	12·7	15·5	20·4	23·9	31·4
Stratified random (III)	8·1	10·7	14·2	18·6	22·3	29·3
Systematic (IV) non-periodic	5·5	7·2	10·2	13·4	19·8	26·0
Systematic (IV) periodic	5·5	7·2	16·0	21·1	23·1	30·3

DISCUSSION

Two aspects of the Dehra Dun results are of interest in the general study of forest sampling. In a previous paper [1948 (2)] tentative conclusions on the merits of different sampling methods were based on data from only two forests, though it was recognized that considerably more evidence was desirable. As has already been pointed out, Dehra Dun is not entirely satisfactory for this purpose, because the 292 strips for which records are available are not contiguous but are themselves a 20 per cent. systematic sample. The precision of any form of sampling from these strips might be expected to be less than for a corresponding sampling from 292 consecutive strips. Fig. 6 and Table III support this view: on comparison with the corresponding diagrams and tables for Mount Stuart and Blacks Mountain [1948 (2)], percentage standard errors and limits of error are seen to be everywhere greater. If the periodicity is disregarded, systematic sampling again proves more precise than any form of random sampling, and the advantage of systematic over either stratified random method is greater than for the other two forests. Fig. 5 shows that a 30–50 per cent. increase in sampling intensity may be needed in order to obtain a random sample as precise as a specified systematic sample. How far this is due to the non-contiguity of the strips cannot be said, but the conclusion tentatively advanced previously [1948 (2)], that suitable stratification would give results almost as precise as a systematic sample, must now be considered as possibly over-optimistic.

The second important feature of Dehra Dun is the periodicity. No certain explanation of this can be given, and it may be little more than an accidental circumstance that a comparatively short series of values should behave in the manner shown in Table II. If many statistical investigations of this kind, involving detailed analyses of series of records for systematic sampling at different intensities, were undertaken, occasionally chance alone would produce an appearance of

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periodicity. That the Dehra Dun records do show a period of 17 strips or 170 chains is a fact that inspection of Table I and Fig. 2 leaves in no doubt. In practice only a very unfortunate coincidence would lead to occurrence of a periodicity and

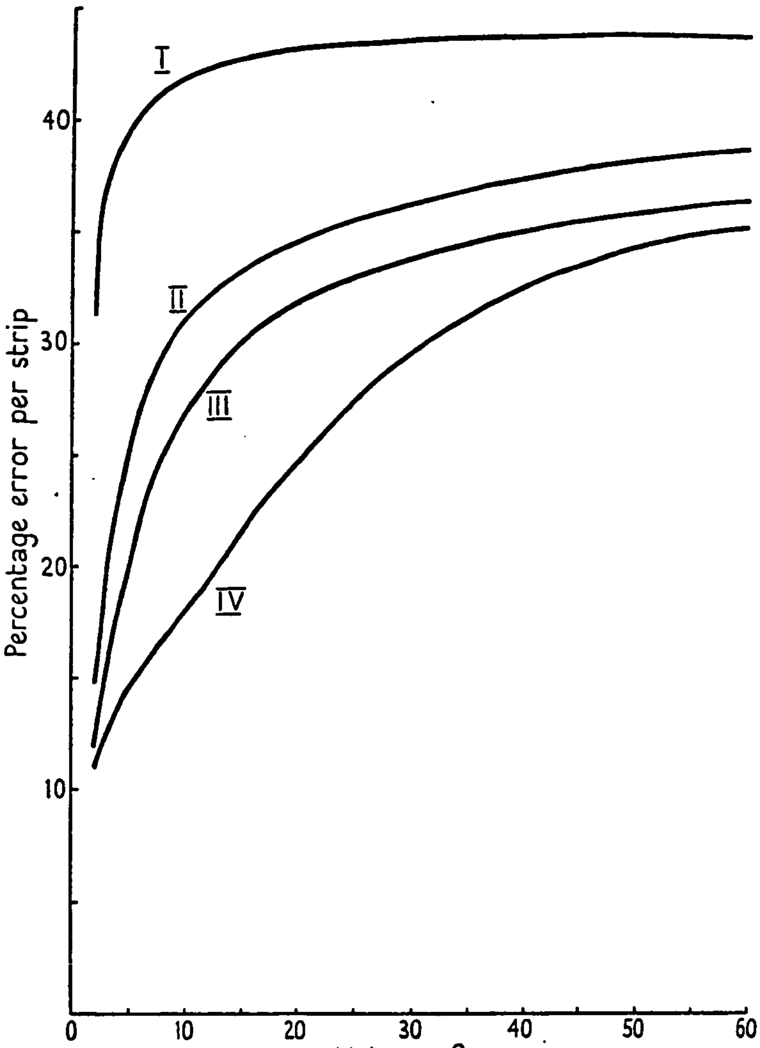

FIG. 6. Percentage error per strip for four sampling procedures, I, II, III (all random), and IV (systematic, non-periodic).

the choice of a multiple of the period as the sampling interval. (Of course, in a forest for which regular planting, or some exceptional topographical regularity, makes periodicity likely, care should be taken to avoid the use of any sampling interval which is a multiple of the expected period.) Such a coincidence might be accepted as one of the hazards of systematic sampling. It will be rare, but if it

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occurs the sample will be much less precise than was hoped—for Dehra Dun as low as for stratified random sampling (Method II), and perhaps relatively worse in other forests. Moreover, no clue to the presence of periodicity will be found in the sample itself: rather the contrary, indeed, for the constituent strips of the sample will tend to be in particularly close agreement with one another [1947 (1)].

It has already been emphasized [1947 (1), 1948 (2)] that a decision to adopt systematic sampling is in general an acceptance of the opinion that no internal estimate of sampling error is needed, for there is little hope of forming a valid estimate from the sample. The danger that precision may be upset by periodicity, and that the sample data may not arouse any suspicion of this, must now also be recognized. These are not necessarily condemnations of systematic sampling, the arguments for which have been adequately presented elsewhere. What is essential is that anyone who adopts systematic sampling in forestry should be fully aware of its dangers, as well as of its virtues, and should not (as so often in the past) accept facile arguments in its favour yet expect to obtain also the alternative merits of random sampling. Table III and corresponding tables in the previous paper make clear that, though systematic sampling tends to be more precise, a stratified random sample with two units per block will usually be of comparable precision and not so poor that its considerable merits must be disregarded because of any overwhelming superiority of the systematic scheme.

In an important paper [1948 (5)] Yates has recently discussed the theory of systematic sampling, and has examined several series of data by methods similar to those used here. The introduction and conclusions of his paper should be read by all concerned with the practice of sampling, even though they may find the main body of the theory beyond their mathematical competence. Yates states that his investigation 'confirms that no method of obtaining a really valid estimate of error from the sampling results themselves can be hoped for' in systematic sampling. He points out that, whereas 'random sampling can be applied with complete confidence to material about which nothing is previously known', if systematic sampling is adopted 'unsuspected periodicities in the material may lead to grossly misleading results'. If something is already known about the variability of the material to be sampled, or if the projected investigation is so extensive as to justify a preliminary study of some part of the material purely as an aid to the planning of the main sample, systematic selection of the sample becomes more worthy of consideration.

The next best thing to an unbiased estimate of the standard error of a systematic sample would be an upper limit to that error which was not so large as to be useless. For many practical purposes the investigator would be quite content to state that his error was not greater than some known quantity, provided that the limit was reasonably near to the true value. A suggestion from Yates indicates how such an upper limit might be obtained from the records of a single systematic sample, if the variation between strips (or other sampling units) were predominantly random. As has often been emphasized, however, timber volumes on successive strips of a forest are not randomly distributed. Even to obtain an upper limit for the error then requires some supplementary sampling. Yates has suggested either

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additional randomly located systematic samples over part of the material, or, again only for part of the material, taking additional sampling units at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of the interval between successive units of the main sample. He has illustrated the use of the supplementary data. It is doubtful whether this ingenious proposal can be used to much advantage in forest sampling, except for very large tracts of forest. If the estimates of the components of variance are to be trustworthy, they must be based upon a reasonable number of degrees of freedom, and therefore there will be a lower limit to the amount of supplementary sampling that is worth while, irrespective of the size of the whole investigation. Unless the main sample consists of 100 or more strips, the supplementary data required for the assessment of error are likely to be proportionally so great as to raise the question of whether a random sample, with an equal or lesser *total* number of strips, would not be both more economical and more satisfactory. If sampling intensities of the order of 5 per cent. are under consideration, this means that the whole forest must contain 2,000 or more strips before supplementary sampling for the assessment of the precision of a systematic sample becomes an economic proposition.

SUMMARY

The methods of a previous paper have been applied to the study of alternative methods of sampling certain Dehra Dun forests of northern India, for the estimation of timber volume. The data available are records of an enumeration of every fifth strip of these forests, and different schemes for sub-sampling from this sample have been considered.

The data show clear evidence of periodic variation, the period being about 170 chains in length. This has important effects on the relative precisions and efficiencies of different types of random and systematic sampling. In general, systematic sampling was more precise than any form of random sampling at the same intensity, but this property was violently upset if the interval between successive strips of the systematic sample was a multiple of the period. The importance of this to the practice of systematic sampling, and to the problem of assessing the standard error of an estimate based upon a systematic sample, has been discussed. The conclusions must be accepted with reservations, however, because the data on which they are based relate only to a 1 in 5 sample of the forest.

Recent work by Yates has given some theoretical guidance on how to assess an upper limit to the error of a systematic sample. Unfortunately, this seems unlikely to be of great use in forest sampling because of the amount of supplementary data required. Except perhaps for very large forests, the investigator is still forced to choose between a random sample, for which he can assess the precision, and a systematic sample, probably but not necessarily more precise than the random and failing to give any reliable information on its own precision.

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