# Modeling Bluetooth Piconet Performance

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***Abstract*—The performance of a single Bluetooth piconet is analyzed using the theory of M/G/1 queues with vacations. Analytical results for probability distributions of packet access time and service cycle time are derived. Two scheduling policies are modeled and compared: exhaustive service was found to perform better, but limited service does not incur the risk of starvation. A hybrid scheme known as $k$-limited scheduling is shown to provide a reasonable tradeoff between performance and fairness. Results were confirmed through simulations.**

***Index Terms*—Bluetooth, Bluetooth piconet, exhaustive service scheduling, limited service scheduling, queueing analysis, queues with vacations.**

## I. Introduction

BLUETOOTH appears to be a feasible solution for short range ad hoc networking [1], [2], yet performance analyses of Bluetooth networks, esp. at the media access control (MAC) level, are still scarce. A number of issues have yet to be resolved, including the actual scheduling policy, which is not prescribed by the current Bluetooth specification [1]. The performance of different scheduling algorithms has been analyzed through discrete event simulation techniques (e.g., [3], [4]). An analytical approach based on the theory of queues with vacations [5] seems more appropriate, but the existing results on polling systems cannot be directly applied to the master-driven TDD communication mechanism used in Bluetooth: some authors analyze unidirectional traffic only [6], others use exhaustive service for downlink traffic and limited service for uplink traffic, [7] and optimum polling schemes such as SLQ [8] require that the master knows the status of slaves' queues.

The present paper describes analytical modeling of the Bluetooth bidirectional polling system in a single piconet with one master device and $m-1$ slave devices, as shown in Fig. 1. We consider two scheduling policies: under limited service, the piconet master polls each slave only once and then moves on to the next one; and under exhaustive service, the master stays with the slave as long as there are packets to be transmitted and moves on only when both queues are empty. These two may be considered to be limiting cases for the set of policies that may be applied in Bluetooth networks. We also consider a hybrid scheme known as $k$-limited service, in which the master stays with a slave for at most $k$ frames; due to space limitations, only the simulation results for this case will be presented. None of these policies requires that the piconet master knows the status of the slaves' queues and, therefore, does not incur additional signaling overhead between the master and the slaves.

Manuscript received June 30, 2002. The associate editor coordinating the review of this paper and approving it for publication was Prof. C. Douligeris.

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Digital Object Identifier 10.1109/LCOMM.2002.807435

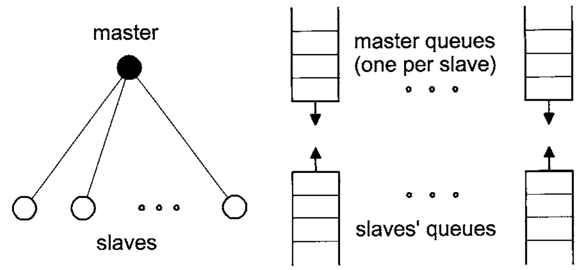



Fig. 1. A single piconet and its queueing model.

Although we have assumed buffers of infinite size, small-size buffers may be more common in practice. However, Bluetooth implies the use of low-power devices; as the data traffic will be generated by the applications running on those same devices, packet arrival rates will be low and buffer overflows will be rare, even with small buffers.

The operation of a piconet may be characterized with two main performance indicators: access delay $W_a$, the time a data packet has to wait in the uplink queue of the source device before it is serviced and end-to-end delay $W_e$, the time from the moment a packet enters the uplink queue at the source device, to the time it arrives at its destination device.

We assume that each device generates bursts (batches) of packets that follow a Poisson distribution with arrival rate $\lambda$. All packets within a burst have the same destination node and the distribution of destination nodes is assumed to be uniform. Then, burst arrival rates for the channel queues will be $\lambda_u = \lambda$ and $\lambda_d = \lambda(m-2)/(m-1) + \lambda/(m-1) = \lambda$ for the uplink and downlink queues, respectively.

The probability distribution of length of the packet burst may be described with the probability generating function (PGF) $G_b(x) = \sum_{k=0}^{\infty} b_k x^k$, where $b_k$ is the probability that the burst will contain exactly $k$ packets; the mean value of the burst length is $\overline{B} = G_b'(1)$. The equivalent Laplace-Stieltjes transform (LST) of the probability distribution may be obtained by substituting the variable $x$ with $e^{-s}$ [5]; for example, the LST of the packet burst length PDF is $G_b^*(s) = \sum_{k=0}^{\infty} b_k e^{-ks}$.

The probabilities of packets being one, three and five slots long are $p_1$, $p_3$ and $p_5 = 1 - p_1 - p_3$, respectively. The corresponding probability generating function (PGF) is $G_p(x) = p_1 x + p_3 x^3 + p_5 x^5$. First and second moments of the packet length distribution are equal to $\overline{L} = G_p'(1)$ and $\overline{L^2} = G_p''(1) + G_p'(1)$, respectively and its LST is $G_p^*(s) = p_1 e^{-s} + p_3 e^{-3s} + p_5 e^{-5s}$.

## II. Performance of Limited Service

If $X_c$ denotes the length of the piconet cycle time, the probabilities that the channel queues are not empty will be $P_d =$

$\lambda_d \overline{BX}_c$ and $P_u = \lambda_u \overline{BX}_c$ for master and slave queues, respectively. The durations of the downlink and uplink communications may be described with the following PGFs:

$$G_d(x) = (P_d p_1 + (1 - P_d))\, x + P_d p_3 x^3 + P_d p_5 x^5 \quad (1)$$
$$G_u(x) = (P_u p_1 + (1 - P_u))\, x + P_u p_3 x^3 + P_u p_5 x^5 \quad (2)$$

The PGF for the piconet service cycle time (i.e., the time for the piconet master to service all of its slaves once) will be $G_{X_c}(x) = (G_d(x)G_u(x))^{m-1}$. The mean value of the service cycle time may be obtained as $\overline{X}_c = G'_{X_c}(1)$ and its second moment as $\overline{X}_c^2 = G''_{X_c}(1) + G'_{X_c}(1)$.

In single server-multiple client systems, when the server finds an empty client queue, it goes on to service other clients—i.e., it takes a vacation, which lasts until the next visit to this client [5]. In a Bluetooth piconet, a server vacation starts when the master, after having polled a slave, moves on to next one. The duration of the vacation period $V_l$ may be described with the following PGF:

$$G_{V_l}(x) = x G_d(x) \left(G_d(x) G_u(x)\right)^{m-2} \quad (3)$$

and its first and second moments are $\overline{V}_l = G'_{V_l}(1)$ and $\overline{V}_l^2 = G''_{V_l}(1) + G'_{V_l}(1)$, respectively.

We are now able to calculate the LST for distribution of the access delay at the slave queue as [5]

$$W_a^*(s) = \frac{1 - V_l^*(s)}{s\overline{V}_l} \cdot \frac{s\left(1 - \lambda \overline{BX}_c\right)}{s - \lambda + \lambda G_b\left(X_c^*(s)\right)} \cdot \frac{1 - G_b\left(X_c^*(s)\right)}{\overline{B}\left(1 - X_c^*(s)\right)} \quad (4)$$

where $V_l^*(s)$ and $X_c^*(s)$ denote the LST of the probability distributions of vacation time and cycle time, respectively. The average access delay at the slave is then calculated as $\overline{W}_a = -W_a^{*\prime}(0)$, or

$$\overline{W}_a = \frac{\lambda \overline{B}\, \overline{X}_c^2}{2\left(1 - \lambda \overline{B}\, \overline{X}_c\right)} + \frac{\overline{B}^{(2)}\, \overline{X}_c}{2\overline{B}\left(1 - \lambda \overline{B}\, \overline{X}_c\right)} + \frac{\overline{V}_l^2}{2\overline{V}_l} \quad (5)$$

where $\overline{B}^{(2)} = E[B(B-1)] = G''_b(1)$ is the second factorial moment of burst length distribution.

Furthermore, the burstiness of the traffic in the downlink queue will differ from the one in the slave queue because the bursts from different source slaves might become interleaved in the same downlink queue, which will in turn lead to an equivalent decrease in burst length. Exact analysis of this phenomenon is fairly involved, so we will only present an approximate model for the decrease of the burst length. The probability that two packet bursts from different slaves will not have the same destination and hence will not be interleaved in the same downlink queue, will be $1 - 1/(m-1)^2$. Therefore, the equivalent mean burst length at the master (downlink) queue will be

$$\frac{1}{\overline{B}_m} = \frac{1}{\overline{B}}\left(1 - \frac{1}{(m-1)^2}\right) + 1 \cdot \frac{1}{(m-1)^2} \quad (6)$$

In order to maintain the same server utilization under decreased burst length, the burst arrival rate has to be scaled so that $\lambda_{dm}\overline{B}_m = \lambda_d \overline{B}$.

The queueing delay at the piconet master may be described with an expression similar to (4), except that $\lambda$ is replaced with $\lambda_{dm}$ and mean burst length is replaced with $\overline{B}_m$. The LST of the end-to-end delay is $W_e^*(s) = W_a^*(s) W_m^*(s)$ and, consequently, the mean value of the end-to-end delay is $\overline{W}_e = \overline{W}_a + \overline{W}_m$.

## III. Performance of Exhaustive Service

Under exhaustive service scheduling, a number of frames (packet sent in downlink, followed by another in uplink) may be exchanged between the master and a single slave during a single visit to that slave. The exchange ends when both queues are empty, i.e., when the slave responds with a NULL packet to a POLL packet sent by the master [1]. The actual number of frames is equal to the number of packets in the downlink or the corresponding uplink queue, whichever is larger, plus one for the POLL-NULL sequence. Let $X_{ms}$ denote the time to service a single channel, i.e., the time to empty both channel queues for one particular slave. Its PGF is

$$X_{ms}(x) = e^{(\lambda_u + \lambda_d) X_c(x)(G_b(G_p(x)) - 1)} \cdot e^{|\lambda_u - \lambda_d| X_c(x)(G_b(x) - 1)} \cdot x^2 \quad (7)$$

Let $X_c$ denote the piconet service cycle time; its PGF is $X_c(x) = X_{ms}^{m-1}(x)$. Then, the PGF of the server vacation time is

$$G_{V_e}(x) = x^2 X_{ms}^{m-2}(x) e^{\lambda_d X_c(x)(G_b(x G_p(x)) - 1)} \quad (8)$$

and its first and second moments are $\overline{V}_e = G'_{V_e}(1)$ and $\overline{V}_e^2 = G''_{V_e}(1) + G'_{V_e}(1)$, respectively.

The LST for the access delay probability distribution is

$$W_a^*(s) = \frac{1 - V_e^*(s)}{s\overline{V}_e} \cdot \frac{1 - G_b\left(G_p^*(s)\right)}{\overline{B}\left(1 - G_p^*(s)\right)} \cdot \frac{s\left(1 - 2(m-1)\lambda \overline{B}\, \overline{L}\right)}{s - 2(m-1)\lambda + 2(m-1)\lambda G_b\left(G_p^*(s)\right)} \quad (9)$$

which translates into the mean access delay of the form

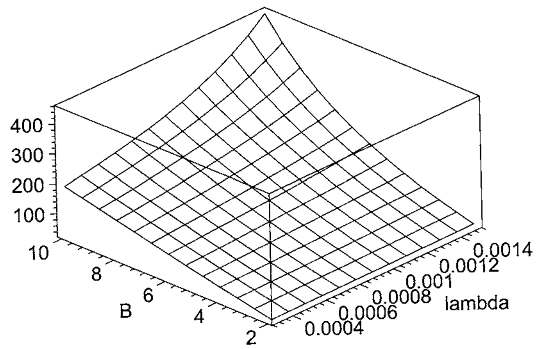
$$\overline{W}_a = \frac{2(m-1)\lambda \overline{B}^2\, \overline{L}^2 + \overline{B}^{(2)}\, \overline{L}}{2\overline{B}\left(1 - 2(m-1)\lambda \overline{B}\, \overline{L}\right)} + \frac{\overline{V}_e^2}{2\overline{V}_e} \quad (10)$$

The LST of delay at the master queue has the same form as that of access delay, $W_m^*(s) = W_a^*(s)$ and, consequently, the mean value of the end-to-end delay is $\overline{W}_e = 2\overline{W}_a$.

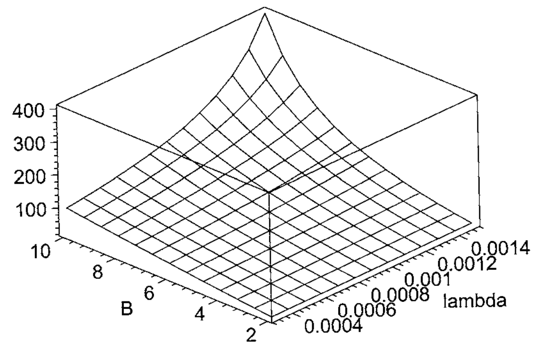
## IV. Performance Comparison

We have performed both theoretical analysis as presented in the previous Section and simulation using the Artifex Petri Net simulation engine [9]. We analyzed the piconet with a maximum size of $m = 8$, with geometrically distributed packet burst size. Mean packet length was $\overline{L} = 3$ and $p_1 = p_2 = p_3 = 1/3$.

Analytical results for end-to-end delays as functions of burst arrival rate $\lambda$ and mean burst size $\overline{B}$, are shown in Fig. 2(a) and (b) for limited and exhaustive service, respectively. Delays are expressed in Bluetooth time slots $T = 625\ \mu$s. Burst arrival rates are scaled according to the rated piconet capacity; for example, parameter values in the far corner of both diagrams in Fig. 2 correspond to offered load of $\rho = m(\lambda_u + \lambda_d)\overline{B}\,\overline{L} = 0.72$. Ratios of delays are shown in Fig. 3(a) and (b), where lines denote analytical solutions and diamonds correspond to simulation results.

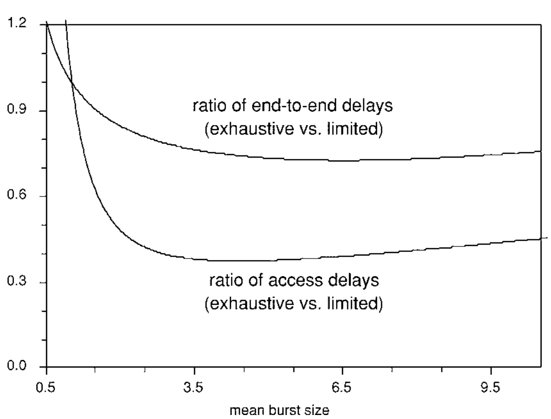



(a)

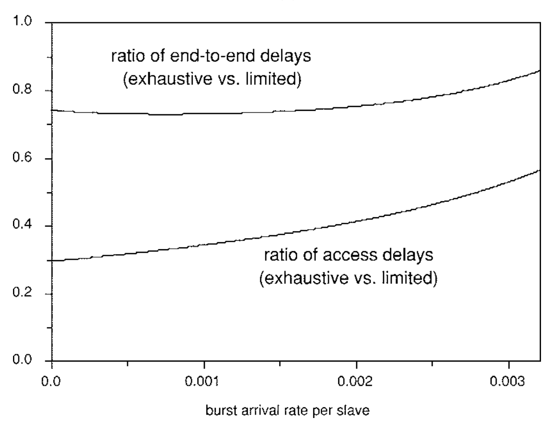



(b)

Fig. 2. End-to-end delay (in units of $T = 625\ \mu$s) vs. packet burst arrival rate $\lambda$ and mean burst size $\overline{B}$.




(a)




(b)

Fig. 3. Performance of different scheduling policies: simulation results.

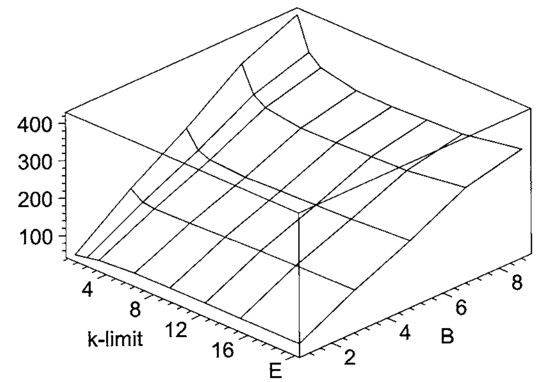



Fig. 4. End-to-end delay (in units of $T = 625\ \mu$s) versus $k$-limit and mean burst size, $\lambda\overline{B} = 0.015$.

As can be seen, delay behaves in the same manner for both scheduling policies, although exhaustive service clearly performs better in a wide range of traffic parameters. The difference is in the range of 20%–25% and it decreases only at very high packet arrival rates and/or long burst sizes (which is of little practical value, as the end-to-end delays will be unacceptably long anyway). In particular, the shape of Fig. 2(b) suggests that exhaustive service is less influenced by bursty traffic than its limited counterpart. Similar results have been reported by others [3], [4], [10], but through simulation only, without the theoretical foundation provided in this letter.

Since exhaustive service offers better performance than limited service but may lead to starvation, we have also investigated the performance of a hybrid policy known as $k$-limited scheduling (limited service corresponds to $k = 1$, while exhaustive service corresponds to $k \to \infty$). The results obtained through simulation are shown in Fig. 4, where $E$ stands for exhaustive service. We observe that most of the improvement associated with exhaustive service may be obtained with values of $k$ as low as 3 to 5, while still providing a sufficient degree of fairness. It may be interesting to note that $k$-limited service actually outperforms the exhaustive service for a range of values of mean burst length; the exact analysis of this phenomenon is the subject of our current research.

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