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KEYFRAME CONTROL OF FLUID SIMULATION USING SKELETAL PARTICLES

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Abstract

Using physically based simulation to create fluid animation has widely been exploited in computer graphics community for several decades since the plausible fluid animation can not be created solely by means of conventional animation techniques. For computer graphics applications, however, existing mathematical models from the field of computational fluid dynamics are too computationally intensive and time-consuming. Due to the fact that highly accurate results are not crucial in visualizing computer-generated fluids, any modifications can be done to achieve the acceptable visual realism using less computational time. Many developments have been purposed to model fluid simulation for computer graphics applications. However, there still has the need in a robust, flexible and controllable model for artistic purposes since animators want to be able to specify the type of effects and control the simulation to reach their goals in interactive time. Nowadays, this problem remains challenging and is continuously gaining the attention among computer graphics researchers.

In this research, we propose the novel technique to model a controllable fluid simulation for computer graphics uses by coupling meshfree fluid simulation, Reeb graph and Radial Basis Function. Smoothed Particle Hydrodynamics (SPH) is chosen to approximate flow dynamics because of its Lagrangian nature, which is analogous to particle systems commonly used in computer graphics software. Since our aim is to animate fluids which can be seen in daily lives, such as air, water and viscous substances, the system is formulated to be isothermal and modeled by using Navier-Stokes equations. Animators can control the simulation by either specifying three dimensional objects as keyframes or scripting the path of the flow. Then, the skeleton and contours of user-specified targets are being extracted by constructing Reeb graph. Our control mechanism bases on a set of path-specified particles called skeletal particles which are generated by using geometrical information of target animation and moved in accordance with the skeleton of a user-specified object. These particles are used to induce the mass and velocity of fluid particles while moving towards their destination. In order to simplify the calculation, Radial basis functions (RBF) are used to approximate skeletal particles motion while custom-designed kernels are used to approximate physical properties and motion of fluid particles. The weight for controlling density distribution of the skeletal particles are optimized by using sequential quadratic programming (SQP) to ensure that the resulting animation is matched with the specified target shape.

With our control scheme, the plausible fluid animation of various flow patterns can be created using less computational resources than previous algorithms. Moreover, our system is easy for animators to impose any animation techniques, such as keyframing and path control on the fluid simulation. Keywords: Suggest approximately 4 – 6 keywords for indexing purposes.

Keywords: Computational Fluid Dynamics, Meshfree methods, Smoothed Particle Hydrodynamics, Reeb graph, Radial Basis Function, Sequential Quadratic Programming.

1. Introduction

Fluids, such as water and air, are amorphous substances which have characteristic movement and continuously deformable shape. Entertainment industry always has high demand for the computer-generated motion of fluids due to their commonality to all of us. However, plausible fluid animation can not be created solely by means of conventional animation techniques, such as keyframing or procedural control since their motion is ruled by a number of complicatedly related parameters. For this reason, physically based modeling is used in animating fluids. There are various physically based modeling techniques for fluid simulation which can be classified into 2 main frameworks; that is grid-based and particle-based methods. Both methods have been used to simulate fluids and deformable objects in computer graphics for several decades. Nowadays, the use of physically based fluids is widespread. Fluid animation with various levels of visual realism can be seen in many films, TV commercials and computer games. This prevalent use results from the advances in modeling techniques, which are specifically developed for computer graphics applications as shown in previous works by Stam (1999) Fedkiw et. Al (2001) and Enright et. Al (2002). Although more visually plausible fluid animation presenting natural phenomena is in its prosperous state, an imaginary fluid motion that satisfies artistic purposes is still be an open problem since controlling fluid simulation is tedious and usually ends up with unpredictable results if no adaptation to classical modeling techniques is made.

1.1 Related works

In recent years, many techniques have been purposed to control fluid simulation. Almost of them are based on grid-based simulation. Foster and Metaxas (1997) used embedded controllers to methodically specify and control a 3D fluid animation. Treuille et al. (2003) proposed a gradient-based optimization with multiple shooting schemes for controlling smoke simulation through user-specified keyframes. Although this method provides fine-grain control over smoke simulation, there still have some visual artifacts due to local minima; some parts of the desired shape are missing. In addition, the optimization process is time consuming especially for the large problem with numerous control parameters. To handle with large fluid simulation, McNamara et al (2004) used adjoint method to compute exact derivatives of control parameters to improve the efficiency of the optimization process. Meanwhile, Fattal et al. (2004) purposed a novel technique to control the density of fluid by adding custom-designed driving-forces and a smoke-gathering term into the Navier-Stokes equations. Smoke-gathering term was designed to solve the problem of numerical diffusion, but it leads to an unnatural evolution of smoke. Another alternative method based on the level sets was introduced by Shi et. al (2005) and Rasmussen et al. (2004) Shi et. al (2005) created smoke shapes that resemble user-specified objects by imposing velocity constraints on the smoke boundary during simulation. Both the smoke region and the target object are represented by implicit functions and the velocity constraints are derived from shape matching process. Rasmussen et al. (2004) used dense particles to control the velocity and viscosity of the fluid in their local neighborhoods, the level set of the liquid surface and the velocity divergence. Despite grid-based method yields impressive fluid phenomena, there are several limitations from using grid. One of major drawbacks is visual artifact due to numerical diffusion. This problem can cause volume loss which is easily detectable. Furthermore, it is computationally intensive and slow because the calculation needs to be done over the entire space. In addition, direct control over these particles is not provided since massless particles are used as tracker and passively moved in velocity field. Consequently, grid-based simulation can not be easily adapted to create a flexible model for animating scripted motion.

Recently, there are an increasing number of works applying particle-based scheme since particle-based methods inherently provide controllability by actively moved particles. Angelidis et. al (2005) purposed high level method to control smoke simulation by using vortex filaments primitives. Vortex primitives, which can induce smoke velocity field, are used to control, define or edit the flow that defined in the vorticity space. However, there still has problem when simulate large scale problem. This work is later modified by introducing harmonic analysis and principal component analysis to handle more complex flow (2006). This scheme yields impressive results for both control and uncontrolled problems using much less time than those using grid-based scheme. Shi et al. (2005) purposed a technique to synthesize and control the motion of lightweight objects in gaseous medium by combining motion synthesis with large scale optimization. A data-driven synthesis based on stylistic motion planning is used to generate initial trajectories of these objects, while SPH and user-defined requirements are used during optimization to obtain the plausible path. Pighin et. al (2006) purposed a novel method to convert Eulerian to Lagrangian representation in order to edit flow. A set of pathlines that best represent the flow is selected, and then a group of massless tracker following selected pathlines is assigned to be advected particles, which are then used to edit the flow. Advected particles are served as center of Radial Basis Function and then used as a calculation point in SPH. Thurey et. al (2006) used low-pass filter to decompose velocity field of the flow in order to apply force constraint only in coarse scale. This work yields impressive results for both grid-based and particle-based model.

1.2 Our contributions

We purpose a novel method to model a controllable fluid simulation which provides animators flexibility and ease in creating imaginary fluid animation. We use Smoothed particle hydrodynamics (SPH), which is one of particle-based methods for simulating fluid, as an underlying model for our simulation since it can be coupled easily into various animation techniques due to the familiarity in using particle systems. Fluid particles can be controlled explicitly by either specifying three dimensional objects as keyframes pr scripting the flow path. Our control mechanism is based on the motion of a set of control particles generated from geometrical information of user-specified keyframes. These particles are called “skeletal particles” since they are determined from the skeleton and other information provided by the specified objects’ Reeb graph. Skeletal particles can control fluid simulation by exerting local forces on fluid particles while moving toward their destination. The use of skeletal particles in controlling can effectively reduce the computation cost since it does not require much extra computational overhead. Moreover, it is easy for animators who have no experience about physically based modeling.

The remainder of this paper is organized as follow. An overview of our framework is briefly reviewed in Section 2. Smoothed Particle Hydrodynamics and our control mechanism are explained in Section 3 and 4 respectively. Some results from our model are discusses in Section 5. Section 6 concludes this work and notes the future works.

2. Overview

The overview of our system is described in figure 1. Users can specify three-dimensional objects as the input to the system. Reeb graphs are then being constructed from the user-specified objects. This geometrical information, which is composed of the skeleton and contours, is used in generating a set of control particles named ‘skeletal particles’. The number and destination of skeletal particles are determined by the amount of nodes in the objects’ Reeb graph and maximum arc length specified by users. Skeletal particles can control the simulation by inducing the mass and velocity of fluid particles during simulation. To reduce the computational overhead, our control mechanism uses Radial Basis Function applied from the field of artificial neural network to optimize the density distribution of skeletal particles. By using ridge regression with RBF, we are able to match the position of randomly placed skeletal particles to the specified position in Reeb graph. In addition, the optimum weight of controlling and the cut-off radius of skeletal particles, which are also used in the next stage, are obtained. This stage can be viewed as the sketch of the animation because users can adjust the resolution of the animation by changing the amount of skeletal particles used. After the desired density distribution is archived, the number of fluid particles within the volume influenced by a skeletal particle can be specified and we are able to initialize the fluid simulation. The control forces generated from the motion of skeletal particles, which are composed of the attraction and velocity forces, are added as the external force terms in Navier-Stokes equation. During simulation, each fluid volume is forced to follow a skeletal particle to its destination. At time step t, the optimum weight of controlling and the cut-off radius of skeletal particles obtained from the fitting stage are assigned to SPH system to approximate the attraction and velocity forces. Using skeletal particles to exert control forces on fluid particles can greatly reduce the computation while preserving the plausible fluid animation.

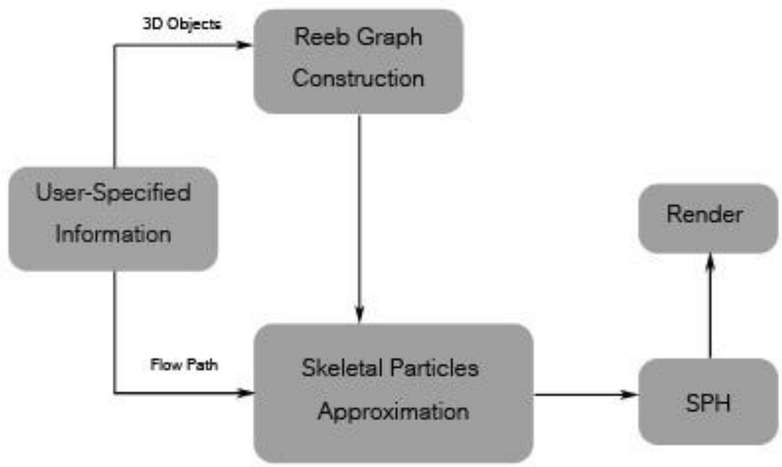

Figure 1. System overview

The detailed descriptions on Smoothed Particle Hydrodynamics with custom-designed kernels and the control mechanism based on skeletal particles are described in the following sections.

3. Smoothed Particle Hydrodynamics for Fluid Flow

Smoothed particle hydrodynamics (SPH) is an interpolation method to approximate field quantities from discrete sample points. In SPH, fluid volume is represented as a collection of particles which are moved under influence of forces. SPH has been widely used for astrophysical problems and fluid dynamics, and has recently gained its popularity among computer graphics researchers. For more details about SPH, we refer the readers to the work from Monaghan (2005).

Since our system is isentropic, there are two equations used in describing fluid flow: Navier-Stokes equation and the continuity equation. Navier-Stokes Equation states that the momentum of the flow is always conserved. It can be written in term of the substantial derivative as follow,

$$\rho \frac{D\vec{v}}{Dt} = -\nabla P + \mu \nabla^2 \vec{v} + \vec{F} \tag{1}$$

Where $\vec{v}, \rho, P, \mu$ are the fluid velocity, density, pressure and the kinematic viscosity, respectively, and $\vec{F}$ is the external body force.

From equation 1, the right hand side implies that there are three forces that responsible for the motion of fluid; pressure force, viscous force and external body force. Pressure and viscous forces are the internal forces resulted from the motion of fluid. The SPH approximation of these internal forces are formulated as follow,

$$\vec{f}_i^{\,pressure} = -\sum_{j \in N} m_j \frac{p_i + p_j}{2\rho_j} \nabla W(\vec{x}_i - \vec{x}_j, h) \tag{2}$$

$$\vec{f}_i^{\,visc} = \mu \sum_{j \in N} m_j \frac{\vec{v}_j - \vec{v}_i}{\rho_j} \nabla^2 W(\vec{x}_i - \vec{x}_j, h) \tag{3}$$

Where N is the total number of the neighboring particles within the smoothing length, h of particle i. m_j is the mass of fluid particle j. W is the smoothing kernel with compact support.

The external forces are composed of control forces generated from skeletal particles fluid motion, surface tension force and gravitational force.

$$\vec{F}_i = \vec{f}_i^{\,surface} + \vec{f}_i^{\,control} + \vec{f}_i^{\,gravity} \tag{4}$$

$$\vec{f}_i^{\,gravity} = \rho_i \vec{g} \tag{5}$$

Where $\vec{g}$ is the gravitational acceleration. The derivation of surface tension and control forces are being described in section 3.2 and 4, respectively.

Another equation that is used to describe fluid flow is Continuity equation. It states that fluid mass is always conserved during flow.

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \tag{6}$$

The SPH approximation of Continuity equation is formulated as follow,

$$\frac{d\rho_i}{dt} = \rho_i \sum_{j\in N} \frac{m_j}{\rho_j}(\vec{v}_j - \vec{v}_i)\nabla W(\vec{x}_i - \vec{x}_j, h) \tag{7}$$

The modified ideal gas state equation is used to approximate the pressure of the fluid. By adjusting a stiffness parameter, k, the range of fluid from viscous to stiff gas can be simulated. The modified ideal gas state equation is written as,

$$P_i = k(\rho_i - \rho_0) \tag{8}$$

Where ρ_i is the current density, ρ_0 is the initial density and k is a stiffness parameter.

3.1 The Smoothing Kernels

The 6th degree polynomial kernel suggested by Muller et. al (2003) is used as the default kernel in order to simplify the calculation. This kernel is designed to reduce the computation by not requiring the square root calculation. Moreover, it can increase stabilities of the model and enable larger time step.

$$W_{poly6}(\vec{r}, h) = \frac{315}{64\pi h^9}\begin{cases}(h^2 - \|\vec{r}\|^2)^3 & 0 \le \|\vec{r}\| \le h \\ 0 & otherwise\end{cases} \tag{9}$$

Where $\vec{r} = \vec{x}_i - \vec{x}_j$

The above kernel is used in all calculation except for computing pressure and viscous forces since it can cause particle clustering under high pressure and produce negative velocity field, which can cause artifacts and instability when using small number of fluid particles. Thus, the spiky and viscosity kernel are used to compute the pressure and viscous forces, respectively.

$$W_{spiky}(\vec{r},h) = \frac{15}{\pi h^6}\begin{cases}(h-\|\vec{r}\|)^3 & 0 \le \|\vec{r}\| \le h \\ 0 & otherwise\end{cases} \tag{10}$$

$$W_{visc}(\vec{r},h) = \frac{15}{2\pi h^3}\begin{cases}-\frac{\|\vec{r}\|^3}{2h^3}+\frac{\|\vec{r}\|^2}{h^2}+\frac{h}{2\|\vec{r}\|}-1 & 0 \le \|\vec{r}\| \le h \\ 0 & otherwise\end{cases} \tag{11}$$

3.2 Surface Tension Force

Surface tension, an effect within the surface layer of a liquid, is caused by the attractive forces between liquid molecules. These intermolecular forces are equal in all directions in the bulk of the liquid, so the net force is zero. On the other hand, the attractive forces at liquid surface are unbalance, so the surface molecules are subject to an inward force of molecular attraction. The surface tension forces act in the direction of the surface normal towards the fluid and tend to minimize surface curvature.

The surface of liquid can be found by using an additional field called color field, c_s.

$$c_s(\vec{x}_i - \vec{x}_j) = \sum_{j\in N} m_j \frac{1}{\rho_j} W(\vec{x}_i - \vec{x}_j, h) \tag{12}$$

The surface normal field is defined as the gradient of the color field

$$\vec{n} = \nabla c_s \tag{13}$$

The surface curvature, κ, is measured by the divergence of $\vec{n}$

$$\kappa = \frac{-\nabla^2 c_S}{\|\vec{n}\|} \tag{14}$$

The surface traction is defined as follow

$$t_{surface} = \sigma\kappa \frac{\vec{n}}{\|\vec{n}\|} \tag{15}$$

Finally, the surface tension force is evaluated by multiplying the surface traction by a normalized scalar field $\delta_S = \|\vec{n}\|$ which has non-zero value near the surface.

$$\vec{f}_i^{\,surface} = \sigma\kappa\vec{n} = -\sigma\nabla^2 c_S \frac{\vec{n}}{\|\vec{n}\|} \tag{16}$$

Where σ is the tension coefficient which depends on the two fluids forming the surface. The surface tension force is evaluated only at locations where $\|\vec{n}\|$ exceeds a specified threshold.

4. Fluid Control

This research aims to provide users flexibility in controlling a particle-based fluid simulation that requires less time and computational resource while preserving plausibility of fluid flow. Our main contribution is the use of skeletal particles, which are a set of control particles generated from geometrical information of user-specified keyframes. The amount and position of skeletal particles are determined using Reeb graph of the keyframe. Typically, the positions of skeletal particles are at the nodes, critical points and along the arcs in the Reeb graph whose length are exceed the minimum threshold distance specified by users, but in some cases that require better surface representation, they can also be placed on the contour of the object. Skeletal particles are used to control fluid simulation by locally exerting attraction and velocity force on fluid particles while moving toward their predestined position along the object skeleton. The details on skeletal particles motion and how they are able to control fluid simulation are described in the following subsections.

4.1 Radial Basis Functions Model of Skeletal Particles

In order to simplify the calculation, Radial Basis Function is used in fitting the sample particles distribution to the skeletal particles distribution. Let $c_i(t)$ be the position of a skeletal particle at time t. RBF representation centered at this particle is defined by,

$$\phi_i(t, \vec{x}) = \exp(-\frac{\|\vec{x} - \vec{c}_i(t)\|^2}{h_i(t)^2} \tag{17}$$

Where $h_i(t)$ is the radius of the particle. For our problem, we choose Gaussian kernel as the basic function because of its radial symmetry and compact support. RBF is then used to approximate the density at time step t by summing over the contribution of each particle within its local area as,

$$\tilde{\rho}(t,\vec{x}) = \sum_{i=1}^{N} W_\rho(t)\phi_i(t,\vec{x}) \tag{18}$$

Where N is the total number of skeletal particles and $W_\rho(t)$ is the density weight, which must be positive since it is used to describe physical quantities.

To fit RBF model to the skeletal particles distribution specified from the objects' Reeb graph, the sum of square error between these two value is minimized by,

$$Arg\min_{\{h_i(t),W_i(t)\}_{i=1}^{N}} \sum(\rho(t)-\tilde{\rho}(t))^2 + \mu\sum W_\rho(t)^2 \tag{19}$$

Where μ is a regularization parameter, which is estimated from generalized cross validation criteria.

4.2 Control Forces

A set of skeletal particle generated at previous stages can be used to easily control fluid motion by exerting local forces on each fluid particle. These local forces are the attraction force and the velocity force. The attraction force can pull fluid particles toward a skeletal particle within the same region, while the velocity force can be used to modify the velocity of fluid particles since they are influenced by the velocity of a skeletal particle.

At time step t, skeletal particle is given by its position, x_{ci}, velocity, v_{ci}, and cut-off radius, h_c, whose value is obtained from the value of influence radius calculated in the previous stage.

A scale factor, which is used to preserve the natural fluid behavior by scaling down the attraction force when an influence region of a skeletal particle is covered with all fluid particles, is define as,

$$\alpha_i = 1 - \min(1, \Sigma V_i W(\|\vec{x}_{ci} - \vec{x}_i\|, h_c) \tag{20}$$

Where V_i is the volume of fluid particle i. $\|x_{ci} - x_i\|$ is Euclidean distance between a skeletal particle and a fluid particle.

The attraction forces exerted by skeletal particles on a fluid particle is formulated as,

$$f_a = W_a \sum_i \alpha_i \frac{\vec{x}_{ci} - \vec{x}_i}{\|\vec{x}_{ci} - \vec{x}_i\|} W(\|\vec{x}_{ci} - \vec{x}_i\|, h_c) \tag{21}$$

Where W_a is a weight that defines the strength of the attraction force.

The second force is the velocity force. It is used to modify the velocity of the fluid particles, according to the skeletal particles motion. A velocity force exerted by skeletal particles on a fluid particle can be written as,

$$f_v = W_v \sum_i (\vec{v}_{ci} - \vec{v}_i) W(\|\vec{x}_{ci} - \vec{x}_i\|, h_c) \tag{22}$$

Where W_v is a constant that defines the influence of the velocity force.

In conclusion, the total control force acting on a fluid particle i is the sum of the attraction and velocity force according to the motion of skeletal particles. By integrating the total force into Navier-Stokes equation, SPH simulation can easily handle control problem without additional overhead since the flow of fluid is depended on skeletal particles motion.

5. Results and discussion

In this research, we purpose a novel method to create a controllable fluid simulation by coupling SPH, Reeb graph and Radial Basis Function. Our control technique is based on the motion control particles which are named skeletal particles after their origin. Fluid particles can be controlled explicitly by either specifying three dimensional objects into the system as keyframes or scripting the flow path. Figure 2 and 3 show the keyframed fluid resembled the shape of the hand.

To create the keyframed smoke in figure 2, the position of skeletal particles is specified at the nodes in object's Reeb graph and along the arcs whose length do not exceed a user-specified maximum threshold. The number of the skeletal particles is adjustable in order to let users roughly preview the results before working on the final animation. In the case of creating this imaginary smoke shape, the skeletal particles are not place along the contours of the hand shape in order to prevent the sharp surface which looks unnatural.

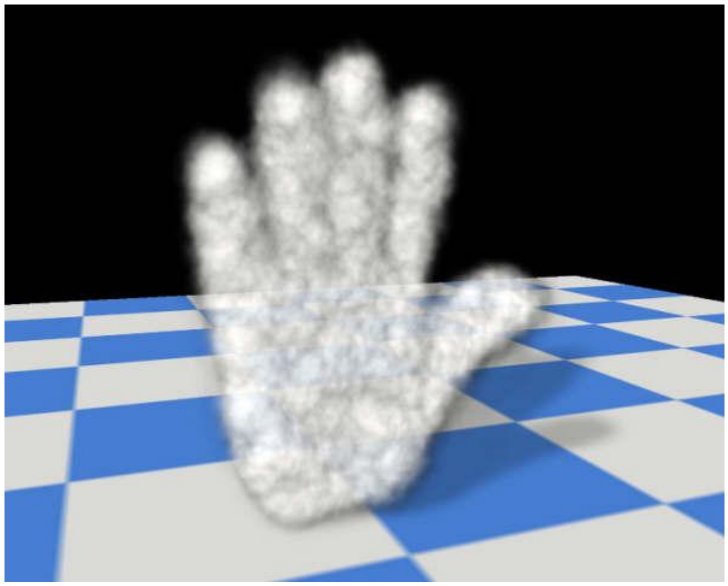

Figure 2. Keyframed smoke forming a hand shape.

However, for the problem of creating hand shape viscous liquid, the skeletal particles are also placed along the contours of the specified object to make the plausible liquid surface.

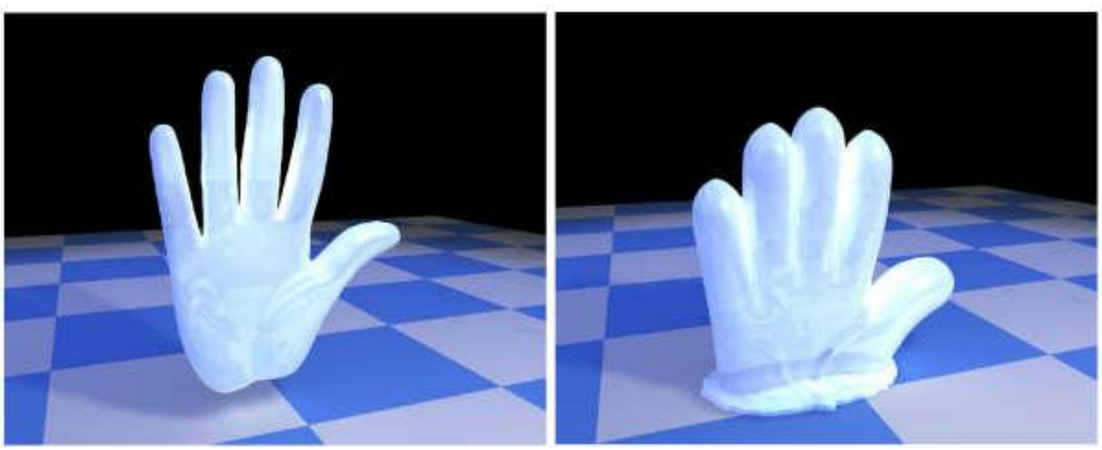

Figure 3. a) Keyframed viscous liquid hand b) Interaction with the floor

The path-specified smoke is shown in figure 4. In this case, the arbitrary number of skeletal particles is also chosen based on geometry of the target shape. However, there is the difference in the rule of skeletal particles motion since they can be freely scripted.

Figure 4. Path-specified smoke forming English alphabets

Using skeletal particles to exert control forces on fluid particles can greatly reduce the computation and prevent local minima that can cause visual artifacts in the resulting animation. Moreover, it offers flexibility to create imaginary fluid animation with various levels of details.

6. Conclusion

In this research, we purpose a novel method to control a particle-based fluid simulation by using a set of control particles called skeletal particles. Fluid particles can be controlled explicitly by either specifying three dimensional objects into the system as keyframes or scripting the flow path. Radial Basis Function is used to ease the calculation of skeletal particles motion and determine the optimum number of fluid particles, the influenced radius of skeletal particles, as well the total weight of controlling that satisfy the target keyframes. There are several advantages from using skeletal particles. First, the computational overhead from applying control to the fluid simulation can be reduced significantly. Although the fluid animation created using our method may not yield high physical accuracy, it is still convincing to see since the flow is approximated by physically based method. In addition, it can provide more flexibility to create fluid animation with adjustable shape because users can move the position of skeletal particles to deform previous shape of fluid.

However, exerting local control forces on fluid particles can cause visual artifact due to averaging fluid velocity with skeletal particles velocity. This can cause the missing of small scale details of the resulting fluid animation Moreover, it would be more effective if the number and destination of skeletal particles can be chosen automatically. Vector quantization may be applied to the simulation to improve the process of skeletal particles generation. By applying vector quantization on the user-specified objects, the skeletal particles can be automatically placed in the area that better represent the geometry of the target objects.

In our future works, we plan to reduce the visual artifacts of this purposed method by applying RBF to approximate fluid flow and filter out low pass frequency from fluid velocity. By using low pass filtering, the detail of fluid animation can be improved since control forces are applied only on the low 3frequency part of the velocity field.

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