Noncoherent Decorrelative Detection for Nonorthogonal Multipulse Modulation over the Multiuser Gaussian Channel

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***Abstract*—This paper introduces the problem of noncoherent detection for nonorthogonal multipulse modulation in the context of the synchronous multiuser Gaussian channel. Each user sends an M-ary information symbol by transmitting one of M possibly nonorthogonal waveforms. Furthermore, the M signals of one user are allowed to be correlated with the signals of all other users. A key idea proposed here is that of a noncoherent decorrelative receiver front end. Like its counterpart in single-pulse modulation, this front end eliminates multiuser interference. It therefore reduces the multiuser detection problem into decoupled single-user problems over equivalent noise-enhanced single-user channels. Each equivalent single-user channel is rather general and can be described as one where the waveforms employed are not only correlated, but are also of unequal energies. Several new results pertaining to the design and analysis of optimum and suboptimum noncoherent detectors for this single-user channel are obtained. In the multiuser channel, these detectors constitute the post-decorrelative processing units for each user.**

***Index Terms*—Code division multiaccess, error analysis, Gaussian channels, multipulse modulation, multiuser channels, noncoherent detection.**

I. Introduction

THE need for noncoherent detection arises in a variety of applications where there may be oscillator phase instability, uncertain and rapid changes in propagation delay of the transmitted signal, fading, etc. The two common modulation techniques that lend themselves to noncoherent detection are differential phase-shift keying (DPSK) and orthogonal multipulse modulation (OMM). DPSK is a linear modulation scheme in which the modulated signal is the output of a linear filter driven by differentially encoded data. OMM, a simple example of which is frequency shift keying (FSK), is seen as a nonlinear modulation scheme where an M-ary symbol is transmitted by sending one of M orthogonal, equienergy waveforms. DPSK is suitable for bandwidth-efficient communications, whereas OMM is more appropriate for power-efficient communication where the energy per bit required to achieve a given error rate can be lowered with increasing values of the size of the symbol alphabet [4].

Noncoherent multiuser detection was introduced in the context of DPSK modulation for the synchronous Gaussian channel in [5] and for the asynchronous Gaussian channel in [6]. In those papers, phase-independent bilinear detectors were analyzed, among which the noncoherent decorrelating detectors were shown to be optimal in near–far resistance. Subsequently, there have been several papers on multiuser detection for DPSK modulation. For instance, with the decorrelating detector used to produce tentative first-stage decisions, the two-stage noncoherent DPSK multiuser detector was proposed in [7] by extending the idea of the coherent two-stage detector of [8] to the noncoherent DPSK case.

In this paper, we consider a general form of multipulse modulation. The waveforms employed by each user are neither orthogonal nor of equal energy. We will refer to this modulation scheme as nonorthogonal multipulse modulation (NMM). In a multiuser channel, several NMM transmitters send data simultaneously so that the receiver observes a superposition of multiple NMM waveforms. The users are also allowed to employ sets of waveforms which are mutually correlated. We will, hence, refer to it as correlated waveform multiple access (CWMA) (some readers may choose to read this as code division multiple access (CDMA), which is a more common, but less appropriate name). The carrier phase associated with each user's transmission is modeled as being uniformly distributed random variable. The background noise is assumed to be additive, white, and Gaussian. Fully noncoherent and near–far resistant detection is sought.

There are compelling reasons to consider the design of receivers for NMM in both single and multiuser channels. For instance, it may be difficult, and often impossible, to ensure, by a suitable choice of transmitted waveforms, that after passing through a distorting channel, the *received* waveforms be strictly orthogonal. In multiuser channels, correlation between sets of waveforms of different users can also arise because of the same reason. Furthermore, in cellular FDMA or TDMA systems that may even manage, with careful system design, to eliminate interuser interference within a cell, the intercell interference resulting from frequency reuse can cause

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the base station to observe a CWMA-NMM-like waveform. Another reason for studying CWMA-NMM systems is as follows. In the context of single-pulse modulation CWMA systems, correlations between signals of different users, when properly designed, can yield significantly higher bandwidth efficiencies than orthogonal signals, thereby giving rise to the bandwidth-efficient multiple-access (BEMA) strategy [9]. It is possible that nonorthogonal multiuser multipulse signaling will allow us to similarly optimize this class of systems for resource-efficient operation. Before such an optimization can be performed, however, it is necessary to develop analytical tools for the systematic design and analysis of high-performance receivers for CWMA-NMM systems.

The remainder of this paper is organized as follows. In Section II, the NMM-CWMA system model is introduced. In Section III, the simple but key idea of noncoherent decorrelation for NMM-CWMA is described. Like its linear modulation counterpart, this noncoherent decorrelator simplifies the problem by reducing the NMM-CWMA channel into K decoupled channels, each of which can be effectively regarded as a single-user NMM channel. The remainder of this paper consists of the design and analysis of optimum and suboptimum noncoherent detectors for such a single-user channel. These detectors then constitute the post decorrelative processors for each user in the multiuser channel. In particular, in Section IV, an optimum decision rule for the effective single-user channel is derived. However, this rule may be too complicated to implement in practice and, moreover, its performance analysis is intractable. In Section V, we use the generalized likelihood ratio test (GLRT) and analyze the resulting detector in terms of error probability and asymptotic efficiency. The GLRT detector is also shown to be near–far resistant. In contrast, the conventional detector is near–far limited. The GLRT detector has the advantage of not requiring the knowledge of the energies of the transmitted waveforms. When such knowledge is available, however, an asymptotic approximation of the optimum decision rule derived in Section VI is more appropriate. The performance analysis of this asymptotically optimum detector is carried out and the pair-wise error probability is characterized in terms of a new integral. In Section VII, we present numerical results, and Section VIII concludes this paper.

The results reported in this paper were presented earlier in part in [1]–[3]. For other papers on noncoherent multiuser detection, the reader is referred to [10]–[15]. These papers consider the general NMM-CWMA system, with the exception of [10] and [11], which are applicable to a rather specific system in which OMM is used and realized through Walsh–Hadamard coding, while direct-sequence spread-spectrum (DS-SS) signaling is used at the coded bit level. The detector in [11] for the OMM-DS-SS model therein applies the phase-independent decorrelator of [16] at the coded bit level. The results in [13] on GLRT detection are similar to those in [3, Sec. IV]. A successive cancellation approach was adopted in [10], which is, however, highly susceptible to multiple-access interference and error propagation effects. A systematic approach to noncoherent decision feedback detection was introduced more recently for the NMM-CWMA system in [15]. Finally, the fundamental performance limits for NMM-CWMA systems over fading channels were studied in [14], which includes the asymptotic analysis of bit error probability of the optimum noncoherent multiuser detector.

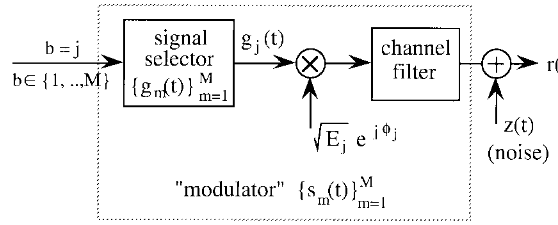

Fig. 1. NMM for a single-user channel with additive noise.

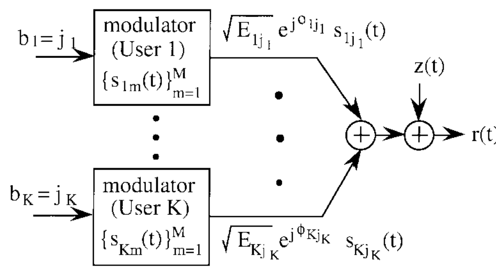

Fig. 2. A multiuser channel with NMM and additive noise.

II. SYSTEM MODEL

We consider a signaling scheme where each of the K users transmits one of M nonorthogonal signals to send $\log_2 M$ bits of information. After passing through an additive white Gaussian noise (AWGN) channel, the superposition of the K signals arrives in symbol synchronism at the receiver. The low-pass received signal becomes

$$r(t) = \sum_{k=1}^{K} \sqrt{E_{kj_k}} e^{j\phi_{kj_k}} s_{kj_k}(t) + z(t), \qquad t \in [0, T] \quad (1)$$

with $z(t)$ having a noise power spectral density (one-sided) of σ^2, and $s_{kj_k}(t)$, $j_k \in \{1, 2, \cdots M\}$ being the M linearly independent, complex, equiprobable, unit energy and time-limited signature signals of user k. E_{kj_k} and ϕ_{kj_k} denote the energy and phase of the j_kth signal of user k. It is assumed that the phases remain constant over one signal interval and are independent and uniformly distributed random variables on $[0, 2\pi]$.

The received signals $\{s_{kl}(t)\}_{l=1}^{M}$ result from another set of signals $\{g_{kl}(t)\}_{l=1}^{M}$ after having passed through a linear filter channel. The multipulse modulator for a single user is depicted in Fig. 1. The multiuser channel model that results from K such modulators transmitting simultaneously is depicted in Fig. 2.

III. NONCOHERENT DETECTION

A discrete-time model for the problem can be obtained by extracting the sufficient statistics. These are obtained from $r(t)$ as the sampled outputs of a bank of MK matched filters, matched to each of the signature signals $s_{kj_k}(t)$, when each is driven by the received signal $r(t)$. The output of the filter bank is an MK-dimensional complex vector $\mathbf{y}$ given as

$$\mathbf{y} = \int_0^T r(t)\mathbf{s}^*(t)\,dt \tag{2}$$

with

$$\begin{aligned}\mathbf{s}(t) &= (\mathbf{s}_1^T(t),\, \mathbf{s}_2^T(t),\, \cdots \mathbf{s}_K^T(t))^T\\ \mathbf{s}_k^T(t) &= (s_{k1}(t)s_{k2}(t)\cdots s_{kM}(t)), \qquad k \in \{1, \cdots K\}.\end{aligned}$$

The problem of optimum joint detection of the information transmitted by all users involves evaluating the *a posteriori* probabilities associated with the M^K hypotheses. Such a computation involves the multidimensional integration of the conditional likelihood function over the uniformly distributed random phases of the K users. This problem is analytically intractable. Suboptimum strategies must be found.

A. Conventional Detection

The optimum detector for the single-user OMM channel has been suggested, and certain approximations of its performance over what is essentially a multiuser NMM channel have been obtained in the literature on spread-spectrum communications [17], [18]. This detector computes the magnitudes of the M matched filter outputs with these filters matched to each of the M signals of that user, and then decides in favor of the signal i according to the largest magnitude of the first M terms of the vector $\mathbf{y}$ [19]. When it is used for the multiuser channel, it will be called the *conventional detector*.

Since the modulation scheme is neither OMM, nor the channel free of multiple-access interference, the conventional detector is strictly suboptimal. Each matched filter output contains interference terms from all users (including the user in question) due to the nonorthogonality of the intra and interuser signature signals. Moreover, these terms depend on the phases of the interfering signals. Consequently, an analysis of the average error probability requires the evaluation of $(K-1)$-fold integrals over the unknown phases and, since this problem is intractable, approximate analysis is required [17], [18]. However, it is possible to compute the conditional error probability conditioned on those interfering phases. This is sufficient to show that the conventional detector is near–far limited.

B. Noncoherent Decorrelation for Nonlinear Modulation

In linear (single-pulse) modulation, the signal space spanned by the superposition of the transmitted signals is independent of the symbols sent. Decorrelation therefore involves the multiplication of the vector of sampled matched filter outputs by the inverse of the correlation matrix of the signature waveforms [20], [21]. Carrier phase-dependent, as well as phase-independent, decorrelation for coherent detection was defined for passband channels in [16]. Phase-independent decorrelation for noncoherent detection of differentially encoded signals was found in [5] and [6]. The phase-independent decorrelator of [16] was applied at the bit level to the OMM-DS-SS model in [11]. The common theme among these decorrelators is that they decompose the multiuser channel into K decoupled single-user channels.

In a NMM-CWMA system, where information is transmitted by each user by choosing one of M signals, the superposition of signals transmitted by the K users lies in a signal subspace that depends on the particular choice of information symbols transmitted. This subspace is one of M^K K-dimensional subspaces depending on which of the M^K realizations of the K M-ary symbols are transmitted. How then must one decorrelate the interfering users when it is not known what signals those users are transmitting? The answer is as follows: view the signal space as being an expanded signal space spanned by the MK signals, M each of all users. The idea here is to formally write the received signal $r(t)$ in a form representing a psuedolinear modulation scheme. We therefore introduce the dummy vector $(\kappa_{k1},\, \kappa_{k2},\, \ldots,\, \kappa_{kM}) \triangleq \boldsymbol{\kappa}_k^T \in F$ with the set F given as

$$F = \{(1\;0\cdots 0),\, (0\;1\;0\cdots 0),\, \cdots,\, (0\cdots 0 1)\} \tag{3}$$

which enables us to write (1) as

$$r(t) = \sum_{k=1}^{K}\sum_{m=1}^{M} \kappa_{km}\sqrt{E_{km}}e^{j\phi_{km}}s_{km}(t) + z(t) \qquad t \in [0, T]. \tag{4}$$

Thus, (2) becomes

$$\mathbf{y} = \mathbf{R}\mathbf{A}\boldsymbol{\kappa} + \mathbf{n} \tag{5}$$

where $\mathbf{n}$ is a zero-mean complex Gaussian random vector with covariance matrix $\sigma^2\mathbf{R}$ with the KM-dimensional correlation matrix $\mathbf{R}$ and the rest of the symbols in (5) defined as

$$\begin{aligned}\mathbf{R} &= \int_0^T \mathbf{s}^*(t)\mathbf{s}^T(t)\,dt \;\in \mathbb{C}^{MK\times MK}\\ \boldsymbol{\kappa} &= (\boldsymbol{\kappa}_1^T,\, \boldsymbol{\kappa}_2^T,\, \cdots,\, \boldsymbol{\kappa}_K^T) \;\in F^K\\ \mathbf{A} &= \mathrm{diag}(\mathbf{A}_1,\, \mathbf{A}_2,\, \cdots,\, \mathbf{A}_K) \;\in \mathbb{C}^{MK\times MK}\\ \mathbf{A}_k &= \mathrm{diag}\Big\{\sqrt{E_{k1}}e^{j\phi_{k1}},\, \cdots,\, \sqrt{E_{kM}}e^{j\phi_{kM}}\Big\}.\end{aligned} \tag{6}$$

The psuedolinear model of (5) now paves the way for applying some of the techniques known in multiuser detection for linear modulation. If we assume that all KM signals $s_{km}(t)$ are linearly independent, we can apply the phase-independent $\mathbf{R}^{-1}$ transformation. The new decision statistic $\mathbf{z}$ becomes

$$\mathbf{z} = \mathbf{R}^{-1}\mathbf{y} = \mathbf{A}\boldsymbol{\kappa} + \boldsymbol{\eta} \tag{7}$$

where $\boldsymbol{\eta}$ is zero-mean Gaussian with covariance matrix $\sigma^2\mathbf{R}^{-1}$. Note that $\mathbf{z}$ is also a sufficient statistic. However, in order to decouple the multiuser channel into K single-user channels, an information loss operation is performed. We base the decision as to which signal is transmitted by user k only on the M outputs $\mathbf{z}_k$, corresponding to user k, and thus neglect the correlation in the noise vector $\boldsymbol{\eta}$. Without loss of generality, we can restrict the analysis to user 1. The decision vector of interest is

$$\mathbf{z}_1 = \mathbf{A}_1\boldsymbol{\kappa}_1 + \boldsymbol{\eta}_1. \tag{8}$$

In the next section, we derive the optimum decision rule based on these M outputs. If the signals are linearly dependent, $\mathbf{R}^{-1}$ does not exist, and we replace $\mathbf{R}^{-1}$ by $\mathbf{R}^{+}$, the Moore–Penrose generalized inverse [22]. Assuming that the M signals of the first user are linearly independent, i.e., none of them lie in the subspace spanned by the other $MK-1$ signals, the model in (8) continues to hold because the first M rows are $\mathbf{R}^{+}\mathbf{R}$ are the first M unit row vectors (see [21, Lemma 1] for a proof).

We remark here that, when the OMM-DS-SS model of [10] is viewed as a particular case of the NMM-CWMA model of this paper, the MK-dimensional decorrelator is easily seen to be equivalent to the sequence of MK-dimensional per-bit decorrelations of [11]. In this degenerate case, however, the Gaussian noise vector in the discrete-time model (8) has independent, identically distributed elements so that the standard maximum magnitude detector and its error rate formula [19], [23] can be directly applied (see the Appendix for more details).

In the more general CWMA-NMM model, however, in the post-decorrelative model of (8), there is arbitrary correlation in the noise vector $\boldsymbol{\eta}_1$. This requires the analysis of NMM in effective single-user channels where the M signals are allowed to be correlated and of unequal energy. The remainder of this paper deals with precisely such an analysis.

IV. Optimum Decision Rule

Let H_i denote the hypothesis that user 1 transmits the ith signal $s_{1i}(t)$. As we assume equal *a priori* probabilities for the M signals, the minimum error probability rule coincides with the maximum-likelihood (ML) test, leading to the optimum detector ϕ^O (the superscript O stands for optimum),

$$\phi^O\colon \hat{\imath}^O = \arg\max_i \{f(\mathbf{z}_1|H_i)\} \tag{9}$$

with f being the conditional probability density function (pdf) of $\mathbf{z}_1|H_i$. In order to evaluate $f(\mathbf{z}_1|H_i)$, we first compute the conditional pdf $f(\mathbf{z}_1|H_i, \phi_{1i})$ and then average over the phase. Given H_i and ϕ_{1i}, $\mathbf{z}_1$ is complex Gaussian with mean $(0\cdots 0 \;\; \sqrt{E_{1i}}e^{j\phi_{1i}} \;\; 0\cdots 0)^T$, with the nonzero entry being at position i, and covariance matrix $\sigma^2\mathbf{Q}_1$ where $\mathbf{Q}_1$ is the upper left $M\times M$ submatrix of $\mathbf{R}^{-1}$ (or $\mathbf{R}^{+}$ in the case of noninvertible $\mathbf{R}$). Since this matrix is not diagonal, we have here a single-user noncoherent detection problem for correlated M-ary signals. Furthermore, even if we assume that the received energies of the M signals of the first user were identical, the effective single-user channel that results after the decorrelation operation corresponds to one with correlated signals and *unequal* energies. After substituting the conditional density $f(\mathbf{z}_1|H_i)$ into (9), we obtain

$$\begin{aligned}
\hat{\imath}^O &= \arg\max_i \Bigg\{ \frac{1}{2\pi}\int_0^{2\pi} \exp\Bigg(-\frac{1}{\sigma^2}(\mathbf{z}_1^\dagger \mathbf{P}\mathbf{z}_1 + E_{1i}P_{ii}) \\
&\qquad + 2\frac{\sqrt{E_{1i}}}{\sigma^2}\operatorname{Re}\Bigg\{\exp(j\phi_{1i})\sum_{m=1}^{M}(P_{im}z_{1m})^*\Bigg\}\Bigg)d\phi_{1i}\Bigg\} \\
&= \arg\max_i \Bigg\{ \exp\left(-\frac{E_{1i}P_{ii}}{\sigma^2}\right)\frac{1}{2\pi} \\
&\qquad \cdot \int_0^{2\pi}\exp\left(2\frac{\sqrt{E_{1i}}}{\sigma^2}\left|\sum_{m=1}^{M}P_{im}z_{1m}\right|\cos(\phi_{1i}+\alpha_i)\right)d\phi_{1i}\Bigg\} \\
&= \arg\max_i \Bigg\{ \exp\left(-\frac{E_{1i}P_{ii}}{\sigma^2}\right) \\
&\qquad \cdot I_0\left(2\frac{\sqrt{E_{1i}}}{\sigma^2}\left|\sum_{m=1}^{M}P_{im}z_{1m}\right|\right)\Bigg\} \\
&= \arg\max_i \{\Lambda_i^O\}
\end{aligned} \tag{10}$$

where $\Lambda_i^O \triangleq -(E_{1i}P_{ii})/\sigma^2 + \ln I_0(2(\sqrt{E_{1i}}/\sigma^2)|\sum_{m=1}^{M}P_{im}z_{1m}|)$ with $\mathbf{P}_1 \triangleq \mathbf{Q}_1^{-1}$, and P_{ij} is the (i, j)th element of $\mathbf{P}_1$. In the second equality, we introduced the phase $\alpha_i = \arg\{\sum_{m=1}^{M}(P_{im}z_{1m})^*\}$. In the third equality, we used $I_0(x)$, the modified Bessel function of the first kind of order zero defined as $\int_0^{2\pi}\exp(x\cos(\phi))\,d\phi$. If $\mathbf{R}$ is positive definite, so are $\mathbf{Q}_1$ and $\mathbf{P}_1$ [22]. This remains true even when $\mathbf{R}$ is singular when our linear independence assumption is satisfied.

As for implementation, a bank of M linear filters matched to the decorrelating signals which are the first M waveforms in the vector $\mathbf{R}^{-1}\mathbf{s}^*(t)$ can be used, followed by the decision algorithm described by (10). The resulting decorrelative optimum detector is shown in Fig. 3 [the decorrelating filters are denoted as $h_1(t), \cdots, h_M(t)$]. Note that its front-end structure is the same as that of a conventional detector, but it does require the more complicated decision rule of (10).

The optimum decision rule for the single-user NMM channel described by (8) extends previously known results. In particular, the special case of orthogonal (set $\mathbf{Q}_1 = \mathbf{I}$) and equienergy M-ary modulation (set $E_{1i} = E_1$) reduces to the well-known optimum detector that chooses the hypothesis that corresponds to the matched filter output with maximum magnitude [19]. The more general case of orthogonal signaling with unequal energies is also known [23]. Binary correlated but equienergy modulation can also be found in the standard single-user communication literature [19, pp. 308–313].

The problem with the optimum decision rule (10) is that it may be somewhat complicated to implement because of the presence of the Bessel function. Note also that it requires the knowledge of the energies of the transmitted signals for its implementation. Moreover, the symbol error probability analysis of the optimum detector appears to be intractable.

Two suboptimum approaches are considered in the remainder of this paper. For channels where the energies E_{1m} cannot be assumed known, a suboptimum detector based on the generalized likelihood ratio test is derived and its performance is evaluated. Assuming known energies E_{1m}, an asymptotic (as $\sigma\to 0$) approximation of the optimum detector is proposed. This asymptotically optimum detector is much simpler to implement. An upper bound on the symbol error probability is also obtained. Incidentally, this upper bound also serves to bound from above the error rate of the optimum detector.

V. Generalized Likelihood Ratio Test Detector

In this section, we assume that the complex amplitudes of the signals are unknown. Consequently, the application of the GLRT requires the best estimate of $A_{1i} \in \mathbb{C}$, assuming H_i is true. These estimates are then used in a likelihood ratio test as if they were correct [24], which results in the detector

$$
\begin{aligned}
\phi_D^G &= \arg\max_i \max_{A_{1i}} \{ f(\mathbf{z}_1 | A_{1i}, H_i) \} \\
&= \arg\max_i \max_{A_{1i}} \Bigg\{ -|A_{1i}|^2 P_{ii} \\
&\quad + 2\,\mathrm{Re}\Bigg\{ A_{1i} \left(\sum_{m=1}^{M} P_{im} z_{1m} \right)^* \Bigg\} - \mathbf{z}_1^\dagger \mathbf{P}_1 \mathbf{z}_1 \Bigg\} \\
&= \arg\max_i \max_{A_{1i}} \Bigg\{ -P_{ii} \left| A_{1i} - \frac{1}{P_{ii}} \sum_{m=1}^{M} P_{im} z_{1m} \right|^2 \\
&\quad + \frac{1}{P_{ii}} \left| \sum_{m=1}^{M} P_{im} z_{1m} \right|^2 \Bigg\} \\
&= \arg\max_i \{ \Lambda_i^G \}
\end{aligned} \tag{11}
$$

where $\Lambda_i^G \triangleq (1/P_{ii}) |\sum_{m=1}^{M} P_{im} z_{1m}|^2$. The second equality results from substituting $f(\mathbf{z}_1 | A_{1i}, H_i)$ into the right-hand side of the first equality and from the exponential function being monotonic in its argument. The last equality results from doing the inner minimization, for which we obtain $(A_{1i})_{\max} = 1/P_{ii} \sum_{m=1}^{M} P_{im} z_{1m}$. The superscript G stands for GLRT.

Incidentally, the particular case of orthogonal and equienergy signaling (with $\mathbf{Q}_1 = \mathbf{I}$ and $E_{1i} = E$) results in the usual (maximum magnitude) optimum detector so that this detector has a new interpretation as a GLRT detector, in addition to being the minimum error probability rule. Note that, when the energies are unequal, however, the minimum error probability rule changes, but the GLRT rule remains unchanged and is, hence, suboptimal. The implementation of the decorrelative GLRT detector is similar to that of the optimum detector as shown in Fig. 3 with Λ_i^O replaced by Λ_i^G in the decision rule.

The GLRT decision rule is analyzed in terms of error probability in the next section.

A. Error Probability Bounds

Given any hypothesis H_i, an error occurs if for any H_j, $\Lambda_j^G > \Lambda_i^G$. Thus, the error probability can be union bounded as

$$
P^G \leq \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{M} \mathrm{Prob}(\Lambda_j^G > \Lambda_i^G) \triangleq \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{M} P_{H_i \to H_j}^G \tag{12}
$$

where the pair-wise error probability $P_{H_i \to H_j}^G$ is implicitly defined. A lower bound can be obtained by bounding the probability of the union of error events by a single error event,

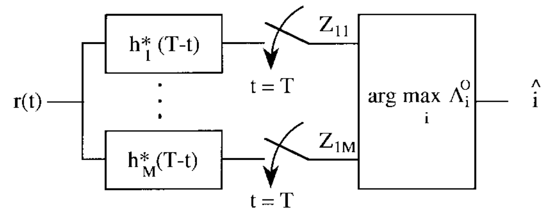

Fig. 3. An implementation of the post-decorrelative optimum detector.

so that

$$
P^G \geq \frac{1}{M} \sum_{i=1}^{M} P_{H_i \to H_{j(i)}}^G \geq \frac{1}{M} P_{H_i^* \to H_{j(i^*)}}^G \tag{13}
$$

where $j(i) \in \{1, \cdots, M\} - \{i\}$ for each i is chosen to yield the tightest single error event lower bound $P_{H_i^* \to H_{j(i^*)}}^G$ on

$$
\mathrm{Prob}\left(\bigcup_{\substack{j=1 \\ j \neq i}}^{M} (\Lambda_j^G > \Lambda_i^G) \right)
$$

and i^* is the value of i for which $P_{H_i^* \to H_{j(i^*)}}^G$ is the largest among the probabilities $P_{H_i \to H_{j(i)}}^G$.

Each of the pair-wise error probabilities $P_{H_i \to H_j}^G$ takes on the form

$$
P_{H_i \to H_j}^G = \mathrm{Prob}(|X_i|^2 - |X_j|^2 < 0) \tag{14}
$$

with $X_i \triangleq \sqrt{P_{jj}} \sum_{m=1}^{M} P_{im} z_{1m}$. The above probability is a special case of a more general probability obtained in [19, Appendix B]. Applying that result, we obtain

$$
\begin{aligned}
P_{H_i \to H_j}^G =& Q\left(\frac{f_{G1}^{ij}}{\sigma}, \frac{f_{G2}^{ij}}{\sigma} \right) - \frac{1}{2} I_0 \left(\frac{f_{G1}^{ij} f_{G2}^{ij}}{\sigma^2} \right) \\
& \cdot \exp\left(-\frac{(f_{G1}^{ij})^2 + (f_{G2}^{ij})^2}{2\sigma^2} \right)
\end{aligned} \tag{15}
$$

where we define

$$
\begin{aligned}
f_{G1}^{ij} &= \sqrt{\frac{1}{2} v^{ij} (\alpha_1^{ij} v^{ij} - \alpha_2^{ij})} \\
f_{G2}^{ij} &= \sqrt{\frac{1}{2} v^{ij} (\alpha_1^{ij} v^{ij} + \alpha_2^{ij})}
\end{aligned} \tag{16}
$$

with $\alpha_1^{ij} \triangleq E_{1i} P_{ii}^2 P_{jj} (P_{ii} P_{jj} - |P_{ij}|^2)$ and $\alpha_2^{ij} \triangleq E_{1i} P_{ii} (P_{ii} P_{jj} - |P_{ij}|^2)$. Furthermore, the parameter v^{ij} is defined as $[(P_{ii} P_{jj} - |P_{ij}|^2) P_{ii} P_{jj}]^{-(1/2)}$. In (15), the function $Q(\cdot, \cdot)$ is the Marcum Q function, which has the integral representation

$$
Q(a, b) = \int_b^\infty x \exp\left(-\frac{x^2 + a^2}{2} \right) I_0(ax)\, dx. \tag{17}
$$

The union bound is, of course, exact in the case of binary modulation.

B. Asymptotic Efficiency

In this section, we characterize the asymptotic (high SNR) behavior of the error probability so as to capture the performance degradation due to the interfering users rather than the background noise [6], [25]. The asymptotic efficiency of a multiuser detector for user l can be defined as

$$\eta_l = \sup\left\{0 \le r \le 1;\ \lim_{\sigma\to 0} \frac{P_l^M(E_l, \sigma)}{P^S(rE_l, \sigma)} < \infty\right\} \tag{18}$$

where $P_l^M(E_l, \sigma)$ is the error probability of the multiuser detector for user l and $P^S(x, \sigma)$ is the error probability in the single-user case with energy per symbol equal to x.

As each user employs nonorthogonal signals in the multiuser channel, the corresponding single-user channel involves M-ary signaling with correlated signals. To arrive at analytical results, we assume that the received energies of all M signals of user 1 are equal to E_1. The error probability for user 1 can be upper and lower bounded in the same way as in (12) and (13), with $P^G_{H_i\to H_j}$ in the last equality replaced by

$$\begin{aligned} P^S_{H_i\to H_j} =& Q\left(\frac{f_{S1}^{ij}}{\sigma}, \frac{f_{S2}^{ij}}{\sigma}\right) - \frac{1}{2} I_0\left(\frac{f_{S1}^{ij} f_{S2}^{ij}}{\sigma^2}\right) \\ &\cdot \exp\left(-\frac{(f_{S1}^{ij})^2 + (f_{S2}^{ij})^2}{2\sigma^2}\right) \end{aligned} \tag{19}$$

for $i, j \in \{1, \cdots, M\}$ with

$$\begin{aligned} f_{S1}^{ij} &= \sqrt{\frac{E_1}{2}\left(1 - \sqrt{1 - |R_{ij}|^2}\right)} \\ f_{S2}^{ij} &= \sqrt{\frac{E_1}{2}\left(1 + \sqrt{1 - |R_{ij}|^2}\right)} \end{aligned} \tag{20}$$

with R_{ij} being the (i, j)th element of the correlation matrix $\mathbf{R}$. The pair-wise error probability for user l is evidently obtained by replacing E_1 by E_l and R_{ij} by $R_{i+(l-1)M, j+(l-1)M}$.

The evaluation of the asymptotic efficiency as defined in (18) requires the limit of the ratio of the GLRT detector error probability to that of the single-user channel optimum detector. By replacing the numerator by the union upper (lower) bound and the denominator by the lower (union upper) bound on the error probability for the single-user channel optimum detector, it is possible to compute a lower (upper) bound on the asymptotic efficiency. The numerator and the denominator in both these computations involve sums of functions that are the difference between the Marcum Q function and the product of an exponential function and the modified Bessel function of the zeroth order. The key to the evaluation of the limit of the ratio of these composite functions is to obtain asymptotically tight upper and lower exponential bounds for each function in the sums. Those problems were solved in [6, Appendix], with the following upper and lower bounds (for $f_2 > f_1$) on functional expressions showing up in the conditional error probabilities of the single- and multiuser case given in (15) and (19):

$$\begin{aligned} &Q\left(\frac{f_1}{\sigma}, \frac{f_2}{\sigma}\right) - \frac{1}{2} I_0\left(\frac{f_1 f_2}{\sigma^2}\right) \exp\left(-\frac{f_1^2 + f_2^2}{2\sigma^2}\right) \\ &\quad < \exp\left(-\frac{(f_2 - f_1)^2}{2\sigma^2}\right)\left(\frac{f_1}{f_2} - \frac{c\sigma}{\pi\sqrt{f_1 f_2}}\right) \end{aligned} \tag{21}$$

$$\begin{aligned} &Q\left(\frac{f_1}{\sigma}, \frac{f_2}{\sigma}\right) - \frac{1}{2} I_0\left(\frac{f_1 f_2}{\sigma^2}\right) \exp\left(-\frac{f_1^2 + f_2^2}{2\sigma^2}\right) \\ &\quad > \frac{1}{2}\frac{c\sigma}{\pi\sqrt{f_1 f_2}} \exp\left(-\frac{(f_2 - f_1)^2}{2\sigma^2}\right) \end{aligned} \tag{22}$$

where c tends to $\sqrt{\pi/2}$ for small σ.

Now, to find the lower bound on asymptotic efficiency, the pair-wise error probabilities of the union upper bound of the GLRT detector in the numerator are further bounded by their upper bounds given in (21), while the pair-wise error probabilities in the lower bound of the single-user optimum detector in the denominator are further lower bounded using (22). With these, both the numerators and the denominators are sums of exponential functions, so that in computing the maximum value of r for which the limit of the ratio is finite, we need only retain the exponential term (in both the numerator and denominator) that decays the slowest. Matching the exponents, therefore, a lower bound on asymptotic efficiency is obtained. Similarly, by working with the ratio of the lower bound on error probability of the GLRT detector to the union upper bound on the single-user optimum detector, it is possible to find an upper bound on the asymptotic efficiency by following the procedure outlined for the lower bound on asymptotic efficiency above. Since the upper and lower bounds for the GLRT detector and the single-user optimum detector are exponentially tight, it can be shown that the upper and lower bounds on the asymptotic efficiency are coincident and, hence, equal to the asymptotic efficiency. The final result is simple

$$\eta_1 = \frac{\min_{i,j} \max^2\left\{0, f_{G2}^{ij} - f_{G1}^{ij}\right\}}{\min_{i,j} \max^2\left\{0, f_{S2}^{ij} - f_{S1}^{ij}\right\}} \tag{23}$$

where f_{G1}^{ij}, f_{G2}^{ij} and f_{S1}^{ij}, f_{S2}^{ij} are given by (16) and (20), respectively.

C. Near–Far Resistance

An important issue in multiuser communication is the near–far problem [6], [21]. With asymptotic efficiency defined as in (18) and with the error rate in the multiuser channel interpreted as being averaged over the phase of the desired user, but conditioned on the phases of the interfering users, we define the notion of near–far resistance as follows. The worst case asymptotic efficiency over the signal amplitudes and phases of the interfering signals is called *near–far resistance* defined for noncoherent detection (as in [5]) as

$$\overline{\eta}_l = \inf_{\substack{E_k \ge 0 \\ \phi_k \in [-\pi, \pi] \\ k \ne l}} \eta_l \tag{24}$$

where E_k and ϕ_k denote the energy and the phase of the kth user. For $\overline{\eta}_l \ne 0$, the error probability will show an exponential decay independently of the energies and phases of the interfering users. Owing to the decorrelation operation, however, the asymptotic efficiency of the GLRT detector is equal to its near–far resistance.

That the GLRT detector is, indeed, near–far resistant is made evident by simplifying (23) for the equal-energy case where $E_{1i} = E_1$ so that

$$\eta_1 = \frac{\min_{i,j}\left\{P_{ii}\left(1 - \frac{|P_{ij}|}{\sqrt{P_{ii}P_{jj}}}\right)\right\}}{\min_{k,l}\{1 - |R_{kl}|\}} > 0 \tag{25}$$

where we assumed, for notational simplicity, that all energies involved are equal. From (25), and using the positive definiteness of the matrix $\mathbf{P}_1$, it is obvious that the near–far resistance is always strictly positive. Optimality in near–far resistance is still an open problem.

In contrast, it can be shown that the near–far resistance of the conventional detector is identically equal to zero. The proof is left as an exercise for the reader. The idea is similar to the proof of the near–far limitedness of the conventional detector for DPSK modulation in [5].

VI. The Asymptotically Optimum Detector

In this section, we reconsider the optimum decision rule (10), assuming known signal energies. We first obtain an asymptotically tight series expansion for the modified Bessel function, which yields a simpler decision rule. For large arguments x, consider the following series expansion for the modified zeroth-order Bessel function [26]:

$$I_0(x) = \frac{\exp x}{\sqrt{2\pi x}}\left[1 + \mathcal{O}\left(\frac{1}{x}\right)\right] \tag{26}$$

where $\mathcal{O}(\cdot)$ denotes the order of decrease in x. Applying this expansion to (10) and neglecting the square root in the denominator against the exponential function in the numerator for high SNR, one obtains

$$\phi^{AO}\colon \hat{\imath}^{A0} = \arg\max_i \left\{-P_{ii}E_{1i} + 2\sqrt{E_{1i}}\left|\sum_{m=1}^{M} P_{im}z_{1m}\right|\right\} \tag{27}$$

where AO stands for AO asymptotically optimum. The error probability can be upper bounded in the same way as in (12), where $P^{G}_{H_i\to H_j}$ is replaced by

$$P^{AO}_{H_i\to H_j} = Prob(|Y_j| - |Y_i| > c_{ij}) \tag{28}$$

with

$$Y_i = \sqrt{E_{1i}}\sum_{m=1}^{M} P_{im}z_{1m}, \qquad Y_j = \sqrt{E_{1j}}\sum_{m=1}^{M} P_{jm}z_{1m} \tag{29}$$

and $c_{ij} = 2^{-1}(E_{1j}P_{jj} - E_{1i}P_{ii})$. Given H_i, Y_i, and Y_j are Gaussian with mean and covariance matrix

$$e^{j\phi_{1i}}\begin{pmatrix} E_{1i}P_{ii} \\ \sqrt{E_{1i}E_{1j}}P_{ji}\end{pmatrix}, \quad \sigma^2\begin{pmatrix} E_{1i}P_{ii} & \sqrt{E_{1i}E_{1j}}P_{ij} \\ \sqrt{E_{1i}E_{1j}}P^*_{ij} & E_{1j}P_{jj}\end{pmatrix} \tag{30}$$

respectively. Consequently, the evaluation of (28) amounts to computing the probability that one Rician random variable exceeds another (correlated) Rician random variable by some constant. Although the form of (28) looks similar to the one of (14), its analysis is far more complicated because of the nonzero threshold c_{ij}. Defining $R_i = |Y_i|$ and $R_j = |Y_j|$, the conditional error probability becomes the double integral

$$P^{AO}_{H_i\to H_j} = \int_{R_i=0}^{\infty}\int_{R_j=R_i+c_{ij}}^{\infty} f(R_i, R_j)\, dR_i\, dR_j. \tag{31}$$

In order to obtain the joint pdf $f(R_i, R_j)$, we first express the problem as one for the four real Gaussian random variables Y_{iR}, Y_{iI}, Y_{jR}, Y_{jI}, with $Y_i = Y_{iR} + jY_{iI}$ and $Y_j = Y_{jR} + jY_{jI}$, a way of processing proposed in [27] for a problem with equal variances of the four real random variables.

As we assumed that the original complex Gaussian process $z(t)$ is circular symmetric, i.e., $E[z^2(t)] = 0$, so are Y_i and Y_j. As a consequence, the variance of the two variables is equally distributed among the real and imaginary part, and the real and imaginary part are uncorrelated and, thus, independent.

Since $z(t)$ is the low-pass equivalent of a bandpass process, it also follows that $E[Y_{iR}Y_{jR}] = E[Y_{iI}Y_{jI}]$ and $E[Y_{iR}Y_{jI}] = -E[Y_{iI}Y_{jR}]$ [28]. Therefore, $(Y_{iR}Y_{iI}Y_{jR}Y_{jI})^T$ is a Gaussian vector with mean

$$\begin{pmatrix} \tilde{E}_{1i}\cos(\phi_{1i}) \\ \tilde{E}_{1i}\sin(\phi_{1i}) \\ B_{ij}\cos(\phi_{1i} + \psi_{ji}) \\ B_{ij}\sin(\phi_{1i} + \psi_{ji})\end{pmatrix} \tag{32}$$

and covariance matrix

$$\mathbf{K}_{ij} = \frac{\sigma^2}{2}\begin{pmatrix} \tilde{E}_{1i} & 0 & B_{ij}c(\psi_{ji}) & B_{ij}s(\psi_{ji}) \\ 0 & \tilde{E}_{1i} & -B_{ij}s(\psi_{ji}) & B_{ij}c(\psi_{ji}) \\ B_{ij}c(\psi_{ji}) & -B_{ij}s(\psi_{ji}) & p_{ij}\tilde{E}_{1i} & 0 \\ B_{ij}s(\psi_{ji}) & B_{ij}c(\psi_{ji}) & 0 & p_{ij}\tilde{E}_{1i}\end{pmatrix} \tag{33}$$

with $c(\cdot)$ and $s(\cdot)$ standing for $\cos(\cdot)$ and $\sin(\cdot)$, ψ_{ji} being the phase of P_{ji}, i.e., $P_{ji} = |P_{ij}|e^{j\psi_{ji}}$ and $\tilde{E}_{1i} = E_{1i}P_{ii}$, $B_{ij} = |P_{ij}|\sqrt{E_{1i}E_{1j}}$, and $p_{ij} = E_{1j}P_{jj}/E_{1i}P_{ii}$. We now perform a coordinate transformation to polar coordinates and make two further substitutions

$$\begin{aligned} Y_{iR} &= R_i\cos(\theta_i), & Y_{iI} &= R_i\sin(\theta_i) \\ Y_{jR} &= R_j\cos(\theta_j), & Y_{jI} &= R_j\sin(\theta_j) \\ \theta'_i &= \theta_i - \phi_{1i}, & \theta'_j &= \theta_j - \phi_{1i} - \psi_{ji}. \end{aligned}$$

The pdf in terms of the new variables R_i, R_j, θ'_i, θ'_j takes on the following form:

$$\begin{aligned} f(R_i, R_i, \theta'_i, \theta'_j) &= \frac{R_iR_j}{4\pi^2A_{ij}}e^{-\tilde{E}_{1i}/\sigma^2}\exp\left\{-\frac{\sigma^2\tilde{E}_{1i}}{4A_{ij}}(p_{ij}R_i^2 + R_j^2)\right. \\ &\quad \left. + \frac{\sigma^2B_{ij}R_iR_j}{2A_{ij}}\cos(\theta'_j - \theta'_i) + \frac{2R_i}{\sigma^2}\cos(\theta'_i)\right\} \end{aligned} \tag{34}$$

with $A_{ij} = \sqrt{\det(\mathbf{K}_{ij})} = (1/4)\sigma^4(p_{ij}\tilde{E}^2_{1i} - B^2_{ij})$. By integrating first over θ'_j and then over θ'_i, we obtain the desired pdf in R_i and R_j

$$f(R_i, R_j) = \frac{R_i R_j}{4\pi^2 A_{ij}} e^{-\tilde{E}_{1i}/\sigma^2} \exp\left\{-\frac{\sigma^2 \tilde{E}_{1i}}{4A_{ij}}(p_{ij}R_i^2 + R_j^2)\right\} \cdot I_0\left(\frac{\sigma^2 B_{ij} R_i R_j}{2A_{ij}}\right) I_0\left(\frac{2R_i}{\sigma^2}\right). \tag{35}$$

With this, the pdf becomes independent of ϕ_{1i} and ψ_{ji}.

We can now go back to the expression for the conditional error probability $P^{AO}_{H_i \to H_j}$ given in (31). The integral over R_j can be solved by invoking the definition of the Marcum Q function in (17), so that the pair-wise error probability can be expressed as

$$P^{AO}_{H_i \to H_j} = \frac{(p_{ij} - \lambda_{ij}^2)}{4\pi^2} e^{-2\rho_i} \cdot \int_{x=0}^{\infty} xQ\left(\lambda_{ij}x, x + \sqrt{\frac{\rho_i}{p_{ij} - \lambda_{ij}^2}}(p_{ij} - 1)\right) \cdot e^{-(x^2/2)(p_{ij} - \lambda_{ij}^2)} I_0\left(2x\sqrt{\rho_{ij}(p_{ij} - \lambda_{ij}^2)}\right) dx \tag{36}$$

with $\lambda_{ij} = B_{ij}/\tilde{E}_{1i}$, $\rho_i = \tilde{E}_{1i}/2\sigma^2$, and $x = \sigma\sqrt{\tilde{E}_{1i}} R_i/\sqrt{2A_{ij}}$. Note that $\tilde{E}_{1i}$ ($\tilde{E}_{1i}p_{ij}$) can be viewed as the effective signal energy of the ith (jth) signal of user 1. The integral in (36) is new to our knowledge.[1] It can be evaluated numerically. The union upper bound of (12) and the lower bound in (13) can be used with each pair-wise error probability replaced by $P^{AO}_{H_i \to H_j}$ given in (36) to obtain upper and lower bounds on the error probability of the asymptotically optimum detector.

Incidentally, the upper bound thus obtained can also be seen as an upper bound for the error probability of the optimum decision rule in (10), so that

$$P^O \le \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \ne i}}^{M} P^{AO}_{H_i \to H_j} \tag{37}$$

with each pair-wise error probability $P^{AO}_{H_i \to H_j}$ given by (36).

We conclude our analysis with open questions. It would seem that the asymptotic efficiencies (and, hence, near–far resistances) of the asymptotically optimum rule in (36) and the optimum decision rule in (10) must be equal. Obtaining the asymptotic efficiency of the asymptotically optimum detector has proved to be a difficult problem. Finally, decorrelative detection for linear modulation, in both the coherent and differentially coherent cases, has the property of being optimally near–far resistant [5], [16], [21]. This may also be true of the post-decorrelative optimum and asymptotically optimum detectors given by (10) and (27) for nonorthogonal multipulse modulation.

[1] Stripping the new integral of the details of the specific problem at hand, we define the five-variable integral as $P(c_1, c_2, \cdots, c_5) = \int_0^\infty xQ(c_1 x, c_2 x + c_3)e^{-c_4 x^2} I_o(c_5 x)\, dx$.

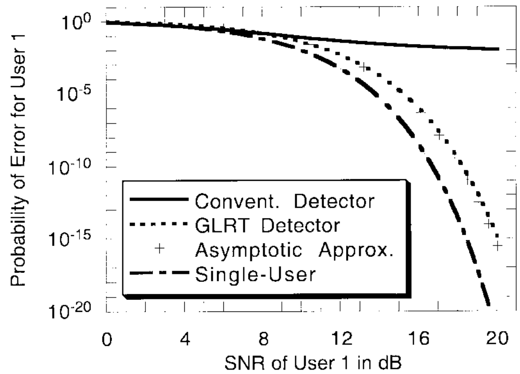

Fig. 4. An error probability comparison of the conventional and post-decorrelative GLRT and asymptotically optimum detectors in a four-user channel with quaternary modulation and lightly correlated signals. The minimum error probability in a single-user channel is given as a reference.

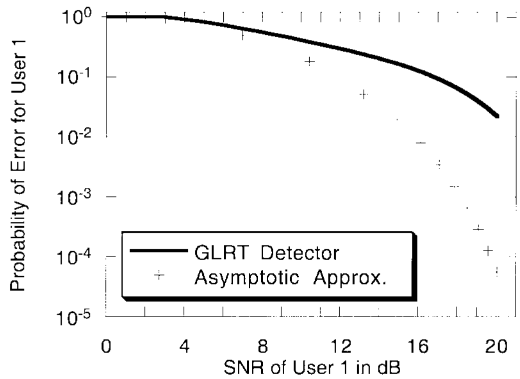

Fig. 5. An error probability comparison of the post-decorrelative GLRT and asymptotically optimum detectors for a four-user channel with quaternary modulation and heavily correlated signals.

VII. Numerical Examples

We present numerical results for a four-user case with quaternary modulation ($M = 4$). The 16 signals (and, hence, the 16-dimensional correlation matrix), four for each user, are generated using one of the signal design algorithms obtained in [29]. In particular, that algorithm yields equienergy signals that maximize the determinant of their correlation matrix (a measure related to performance) under a maximum rms bandwidth constraint using the Hadamard method described in [29, Sec. II] ([29, Fig. 4] gives a plot of how such signals might look). In Fig. 4, a lightly correlated signal set (high bandwidth) is considered and in Figs. 5 and 6, a heavily correlated (small bandwidth) signal set is chosen.

In Fig. 4, the symbol error rates for the single-user optimum detector in a single-user channel, the union upper bounds on the error probabilities of the GLRT decorrelative detector and the asymptotic optimum decorrelative detector, and the symbol error rate of the conventional detector are depicted. In the case of the conventional detector, perfect power control is assumed, in that the signal energies of all signals of all users are identical. The phases of all users were arbitrarily set equal to zero. The single-user error probability for the single-user channel serves as an absolute lower bound for achievable error rate in the multiuser channel. It is seen that the

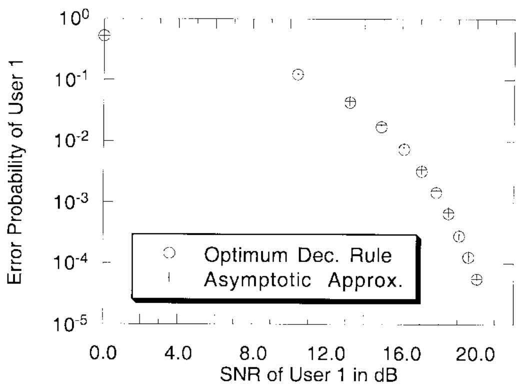

Fig. 6. Error probabilities of the post-decorrelative optimum and asymptotically optimum detectors via simulations.

GLRT and the asymptotically optimum decorrelative detectors show a dramatic improvement in performance relative to the conventional detector. The difference between these two suboptimum decorrelative detectors does not appear to be significant in this example.

As one increases the signal correlations, the gap between the multiuser detectors proposed and the single-user case widens due to the decorrelation operation. The latter leads to a decrease of the effective SNR of the desired user unless all signals are lightly correlated. This occurs because of projecting the received signal onto the orthogonal complement of the interfering user signal space. This implies in our context that, as M increases, in order to obtain satisfactory performance, bandwidth must suitably be increased to have sufficiently lightly correlated signals. Furthermore, for higher correlations, the spread of the effective signal strengths of the M signals per user becomes larger. In such cases, the asymptotically optimum decorrelative rule has a significantly superior performance in comparison to the GLRT decorrelative detector. This is depicted in Fig. 5. For an error probability of 10^{-4}, for example, the SNR gap between the two rules is 5.6 dB, whereas the comparable SNR gap for light correlated signals in Fig. 4 is only 0.1 dB. This example shows that, for highly correlated signals, there is a lot to be gained by the correct and simple use (via the asymptotically optimum detector) of the knowledge of the received energies when such knowledge is available at the receiver.

Finally, in Fig. 6, we compare the performances of the decorrelative optimum and asymptotically optimum detectors. The optimum rule cannot be analyzed analytically, hence, its performance is obtained with simulations. It was expected that the asymptotically optimum rule would closely hug the performance of the optimum rule for high SNR ratios. The figure, in addition, reveals that the so-called asymptotically optimum rule is almost optimal, even in the low SNR cases. The error rates of the two detectors are hardly distinguishable even, at 0 dB. The signal correlation matrix was chosen to be the same as in Fig. 5.

VIII. Conclusions

A systematic approach to the problem of noncoherent multiuser detection for M-ary nonorthogonal multipulse modulation was introduced. The simple but important idea of viewing nonorthogonal multipulse modulation as psuedolinear modulation was proposed, which allows the specification of noncoherent decorrelation for NMM. Restricting attention to only the outputs of the decorrelator corresponding to a particular user, the optimum, the asymptotically optimum, and the suboptimum GLRT decision rules were derived and analyzed. The post-decorrelative GLRT detector offers the advantage of being energy independent and fares almost as well as the post-decorrelative optimum detector for lightly correlated signals. For signature signal sets with higher correlations, however, the latter exhibits a significantly better performance than the post-decorrelative GLRT detector.

Noncoherent decorrelation works well if the correlations of the signature signal set are sufficiently low. For higher correlations, the reduction of the effective SNR ratio can be quite severe. More elaborate multiuser detection schemes such as noncoherent multistage and decision feedback must be used for higher performance. The results of this paper have served as a starting point for a systematic approach to such research [15].

Appendix

We describe an example of NMM-CWMA model where each user employs OMM. Such a model was studied in [10] and [11]. In particular, the signal $s_{kj}(t)$ has the form $\sum_{i=1}^{M} b_j(i)p_k(t - iT/M)$ for $k = 1, \cdots, K$ and $j = 1, \cdots, M$ with $b_j(i) \in \{+1, -1\}$ denoting the ith bit of the jth M-length Walsh–Hadamard code word, and with $p_k(t)$ denoting the signature waveform of the kth user of duration T/M. The energy of the pulse $p_k(t)$ is normalized to be $1/M$. The sufficient statistics for the problem are the sequence of M vectors of length K, each obtained by matched filtering to K matched filters $\{p_k(t)\}_{k=1}^{K}$, followed by sampling at the appropriate time instants. The equivalent discrete-time model for each such K-length vector is $\mathbf{q}(i) = \mathbf{VWb}(i) + \epsilon(i)$ where $\mathbf{V}$ is the correlation matrix between the pulses $\{p_k(t)\}_{k=1}^{K}$ with the (k, l)th element denoted as V_{kl} and $\mathbf{W}$ is a diagonal matrix of complex amplitudes of the K users that are assumed to be constant over the M coded bits and independent of the particular code word transmitted, and $\mathbf{b}(i)$ is the vector of ith coded bits transmitted by the K users. At the coded bit level, this modulation technique is a simple BPSK single-pulse modulator. The per-bit single-pulse decorrelating transformation $\mathbf{V}^{-1}$ is applied to $\mathbf{q}(i)$ and the decision on the kth user is based on the M-length sequence of kth outputs of this decorrelator. This sequence is then correlated with the M Walsh–Hadamard codes for each user and the M outputs of these correlations for each user admit the M-length discrete-time model $\mathbf{x}_k = W_k\mathbf{u}_j + \phi_k$ where j denotes the code word transmitted by the kth user and $\mathbf{u}_j$ is the jth unit vector and ϕ_k is a zero-mean white Gaussian noise vector with covariance $\sigma^2 M^{-1}(\mathbf{V}^{-1})_{kk}\mathbf{I}$.

When viewed in the context of the more general NMM-CWMA model, the above OMM technique yields the following. The $MK \times MK$ correlation matrix $\mathbf{R}$ as we define it in (6) of the code-word-level signals $s_{jk}(t)$ takes the particular striped form wherein, if it were to be partitioned into $M \times M$ blocks, the (k, l)th block could be expressed as a scalar multiple of the identity matrix given as $MV_{kl}\mathbf{I}$ because the code words of the common Walsh–Hadamard code used for each user are orthogonal. The $MK \times MK$ decorrelation operation defined in (5) would result in the OMM model $\mathbf{x}_k = W_k\mathbf{u}_j + \phi_k$ for the kth user by considering the kth subvector of length M of the output of the decorrelator. The covariance of the Gaussian noise vector ϕ_k is proportional to the kth diagonal block of $\mathbf{R}^{-1}$. It is easy to show that $\mathbf{R}^{-1}$ has the same striped structure of $\mathbf{R}$ with the (k, l)th $M \times M$ block given by $M^{-1}(\mathbf{V}^{-1})_{kl}\mathbf{I}$ so that the model $\mathbf{x}_k = W_k\mathbf{u}_j + \phi_k$ is identical to the model described at the end of the previous paragraph. This is equivalent to the single-user OMM model for which the error analysis of the optimum detector can be found in standard texts [23, eq. (3.4.29)] by replacing $\sigma^2 = N_0$ in that formula with $\sigma^2 M^{-1}(\mathbf{V}^{-1})_{kk}$. Hence, the asymptotic efficiency is simply $M((\mathbf{V}^{-1})_{kk})^{-1}$.

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