Sequential Activity Profiling : Latent Dirichlet Allocation of Markov Chains

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Abstract

To provide a parsimonious generative representation of the sequential activity of a number of individuals within a population there is a necessary tradeoff between the definition of individual specific and global representations. A linear-time algorithm is proposed that defines a distributed predictive model for finite state symbolic sequences which represent the traces of the activity of a number of individuals within a group. The algorithm is based on a straightforward generalization of latent Dirichlet allocation to time-invariant Markov chains of arbitrary order. The modelling assumption made is that the possibly heterogeneous behavior of individuals may be represented by a relatively small number of simple and common behavioral traits which may interleave randomly according to an individual-specific distribution. The results of an empirical study on three different application domains indicate that this modelling approach provides an efficient low-complexity and intuitively interpretable representation scheme which is reflected by improved prediction performance over comparable models.

I. INTRODUCTION

The now commonplace ability to accurately and inexpensively log the activity of individuals in a digital environment makes available log files of user activity which may be employed in characterizing individual specific behavioral profiles. To achieve this it is necessary to induce space efficient representations, or profiles, of individuals from the available traces of each individuals logged activity. Most often, such recordings take the form of streams of discrete symbols ordered in time, for example a web-site log-file stores the time-ordered sequence of site specific web-pages (a finite set) visited by individuals having entered the site.

The modelling of time dependent sequences of discrete symbols from a dictionary $\mathcal{S}$ employing m'th order Markov chains (typically m=1 or m=2) has been extensively studied in a number of domains, most notably in statistical language modelling [15]. Recent attention has turned to modelling web browsing behavior [4], [7], [1] and somewhat related, web-page pre-fetch prediction [22] as well as bio-sequence analysis [11]. If there is a requirement to capture long range temporal dependencies within the sequences observed then a higher-order m'th order Markov model can be employed, however these suffer from an $\mathcal{O}(|\mathcal{S}|^{m+1})$ growth in the number of model parameters which require to be estimated from the available data. In statistical language modelling the size of the symbol dictionary (number of unique words in language) may be of the order of tens of thousands elements. As an example if there are 5×10^3 unique words defined in the language (a very small number in language modelling terms), reliably estimating the $(5 \times 10^3)^3 = 12.5 \times 10^{10}$ parameters which define a 2'nd order Markov model becomes a formidable challenge. Many methods have been developed to effectively deal with this exponential rise in the number of free parameters, such as linearly interpolating higher order with lower order models [15]. In addition approximating the full long term dependencies with linear mixtures of pairwise lower order transition models have been developed in [19] and subsequently employed in [23], [24]. Other approaches to capturing longer term dependencies have been presented in [25] and functions of first order Markov chains such as the Hidden Markov Model (HMM) [18], [11] successfully capture longer term dependencies, however, inevitably these come with an increased computational cost.

The representation provided by such models is global in the sense that a single monolithic generating process is assumed to underlie all observed sequences. However, to capture the possibly heterogeneous nature of a set of observed sequences a model with a number of differing generating processes needs to be considered. This is particularly important in user or customer behavior modelling, where the sequential activity of a number of individuals within a group needs to be efficiently modelled. Indeed the notion of a heterogeneous population, characterized for example by occupational mobility and consumer brand preferences, has been captured in the *Mover-Stayer* model [10]. This model is a discrete time stochastic process that is a two component mixture of first-order Markov chains, one of which is degenerate and possesses an identity transition matrix characterizing the *stayers* in the population. The original notion of a two-component mixture of Markov chains has recently been extended to the general form of a mixture model of Markov chains in [5]. The main motivation in developing this mixture model was the visualization of the class structure inherent in the browsing patterns of visitors to a commercial web-site [5]. In such a mixture representation each class of users is characterized by their shared common prototypical behavior, and therefore such mixture models will not be appropriate for identifying the shared behavioral patterns which are the basis of multiple relationships between users and groups of users which may yield a more realistic model of the behaviors exhibited by the population as a whole.

In this paper we propose a dynamic user[1] model, for individuals within a group, that explicitly captures the assumption that there exists a common set of behavioral traits which can be estimated from all observed user activity. In addition each user is defined by a personalized distribution of the probability of exhibiting these traits and each of these forms the individual user profiles within the group. This is a computationally attractive model, as relatively simple structural characteristics may be assumed at the generative level. For example consider a small set of simple first-order Markov Chains (MC) which combine to generate sequences by interleaving in various proportions of participation. Clearly the sequences that result from this combined interleaving will be more complex than sequences generated by any of the chains taken individually. This is the case as the overall sequences may exhibit transitions that are present in any of the available generators in the set.

We employ this construction as a generative model to 'explain' complex heterogeneous user behavior of a number of individuals in terms of a compact set of structurally simple common behavioral patterns along with their user-specific proportions of participation (interpreted as the users' individual profiles over the basis set) and propose to estimate both of these from sets of user trace recordings. This is much more parsimonious than creating separate models for each individual, a task which may be beset with statistical estimation problems if there is only a small amount of available logged activity for an individual. At the same time such a representation can possibly account for more complex behavior at the level of each individual than any single global model of the same order. The resulting model is thus a distributed dynamic model which represents an effective tradeoff between individual-specific and a general group-level behavior model.

The technical aspects of defining such a model, benefit from the recent developments in distributed parts based modelling of static vectorial data [12], [21], [8], [3], [16], [9], with various applications including image decompo-

[1]The term *user* is employed in this context to mean an individual using a resource, such as, someone who visits and browses a web-site, or someone who regularly uses a telephone service

sition [12], document modelling, information retrieval [8], [3], [16] and collaborative filtering [9]. The consistent generative semantics of the recently introduced latent Dirichlet allocation (LDA) [3] will be adopted and by analogy with [16] the resulting model will be referred to as a simplicial mixture of Markov chains. A somewhat related idea of decomposing event sequences has been proposed within the independent component analysis framework in [14], however the independence assumption is not made here.

II. Simplicial Mixtures of Markov Chains

We define a sequence of L symbols $s_L, s_{L-1}, \cdots, s_1, s_0$, such the symbol emitted at time t is s_0, symbol s_1 is emitted at the previous time $t-1$ and s_L is observed at time $t=0$. This sequence of symbols, denoted by $\mathbf{s}$, can be generated from a dictionary $\mathcal{S}$ by an m'th order discrete time invariant Markov chain k which has initial state probability $P_1(k)$ and has $|\mathcal{S}|^{m+1}$ state transition probabilities denoted by $T(s_m, \cdots, s_1 \rightarrow s_0|k)$. The number of times that the symbol s_0 follows from the state defined by the m-tuple of symbols $s_m, \cdots, s_1$ within the sequence is given as $\mathcal{N}(s_m, \cdots, s_1 \rightarrow s_0)$ and so the probability of the sequence of symbols under the k'th Markov process of order m is $P(\mathbf{s}|k) = P_1(k) \prod_{s_m=1}^{|\mathcal{S}|} \cdots \prod_{s_0=1}^{|\mathcal{S}|} T(s_m, \cdots, s_1 \rightarrow s_0|k)^{\mathcal{N}(s_m, \cdots, s_1 \rightarrow s_0)}$. We employ `Start` and `Stop` states in each symbol sequence $\mathbf{s}_n$ and incorporate the initial state distribution of the `Start` state as the transition probabilities from this state within the $|\mathcal{S}|^m \times |\mathcal{S}|$ dimensional state transition matrix T_k. We denote the set of all state transition matrices $\{\mathrm{T}_1, \cdots, \mathrm{T}_k, \cdots, \mathrm{T}_K\}$ as $\mathbf{T}$. Suppose that we are given a set of symbolic sequences $\{\mathbf{s}_n\}_{n=1:N}$ over a common finite state space, each having different length L_n. In contrast to cluster models for sequences which try to model inter-sequence heterogeneities, our intuition is that in sequences over a common finite state space, provided they are sufficiently long it is sensible to look for several randomly interleaved generating processes, some of which might be common to several sequences. To account for this idea, we will adopt the LDA [3] modelling strategy. We will employ general m'th-order Markov models here, however other appropriate models would equally be possible to assume. The complete generative semantics of LDA allows us to describe the process of sequence generation where mixing components $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_k, \cdots, \lambda_K]$ are K-dimensional Dirichlet random variables and so are drawn from the $K-1$ dimensional simplex defined by the Dirichlet distribution $\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\alpha})$ with parameters $\boldsymbol{\alpha}$. These are then combined with the individual state-transition probabilities T_k, which are model parameters to be estimated, and yield the symbol transition probabilities $T(s_m, \cdots, s_1 \rightarrow s_0|\boldsymbol{\lambda}) = \sum_{k=1}^{K} T(s_m, \cdots, s_1 \rightarrow s_0|k)\lambda_k$. The overall probability for a sequence $\mathbf{s}_n$ under such a mixture, which we shall now refer to as a simplicial mixture of Markov chains [16], denoted as $P(\mathbf{s}_n|\mathbf{T}, \boldsymbol{\alpha})$ is equal to

$$\int_{\triangle} P(\mathbf{s}_n|\mathbf{T}, \boldsymbol{\lambda}) \mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\alpha}) d\boldsymbol{\lambda} = \int_{\triangle} d\boldsymbol{\lambda} \mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\alpha}) \prod_{s_m=1}^{|\mathcal{S}|} \cdots \prod_{s_0=1}^{|\mathcal{S}|} \left\{ \sum_{k=1}^{K} T(s_m, \cdots, s_1 \rightarrow s_0|k)\lambda_k \right\}^{\mathcal{N}_n(s_m, \cdots, s_1 \rightarrow s_0)} \tag{1}$$

Each sequence will have its own expectation under the Dirichlet mixing coefficients and so the ability of such a representation to model intra-sequence heterogeneity emerges naturally. It should be noted here that in the case where no memory is assumed in the generating process, sometimes referred to as a *zero*'th order Markov model, then (1) reduces to the multinomial LDA model as originally detailed in [3].

It may be interesting to observe that if in (1) the mixing coefficients were constrained to be drawn exclusively from the vertices of the simplex then the summation within (1) becomes a selector for the $k'th$ generator and the expectation with respect to the Dirichlet distribution becomes an expectation over the distribution of probability mass allocated to each vertex of the simplex, i.e. the integral over the simplex reduces to a weighted summation over the number of possible vertices

$$P(\mathbf{s}_n) \quad = \quad \sum_{k=1}^{K} P(k) \prod_{s_m=1}^{|\mathcal{S}|} \cdots \prod_{s_0=1}^{|\mathcal{S}|} T(s_m, \cdots, s_1 \rightarrow s_0|k)^{\mathcal{N}_n(s_m,\cdots,s_1 \rightarrow s_0)} \tag{2}$$

For the case where $m = 1$ then the mixture of Markov chains proposed in [5] is recovered. Indeed, we now see that for the model represented in (2), for each observed sequence, only one Markov process will be responsible for the generation of a whole sequence.

A. Inference and Parameter Estimation

The detailed derivation of the inference and parameter estimation algorithm for the case where $m = 0$ i.e. a multinomial distribution over a *bag-of-words* can be found in [3]. Extending the detailed derivation developed in [3] to arbitrary order Markov chains now requires multiple indices, despite this the generalization is straightforward. As detailed in [3] exact inference within the LDA framework is not possible , however the likelihood can be lower-bounded by introducing a sequence specific variational free parameter $\boldsymbol{\gamma}_n$ and applying Jensen's inequality such that

$$\log P(\mathbf{s}_n|\mathbf{T}, \boldsymbol{\alpha}) \geq \mathsf{E}_{\mathcal{D}\boldsymbol{\gamma}_n} \left[\log \left\{ P(\mathbf{s}_n|\mathbf{T}, \boldsymbol{\lambda}) \frac{\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\alpha})}{\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\gamma}_n)} \right\} \right] \tag{3}$$

where $\mathsf{E}_{\mathcal{D}\boldsymbol{\gamma}_n}$ denotes expectation with respect to $\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\gamma}_n)$. Employing the following abbreviated notation $\mathcal{N}_n^{m\cdots 0} \equiv \mathcal{N}_n(s_m, \cdots, s_1 \rightarrow s_0)$, $T_{m\cdots 0,k} \equiv T(s_m, \cdots, s_1 \rightarrow s_0|k)$, and introducing the additional variational parameter $Q_n^{m\cdots 0,k}$ then the above (3) can be further lower-bounded by noting that

$$\log P(\mathbf{s}_n|\mathbf{T}, \boldsymbol{\lambda}) \geq \sum_{s_m=1}^{|\mathcal{S}|} \cdots \sum_{s_0=1}^{|\mathcal{S}|} \sum_{k=1}^{K} \mathcal{N}_n^{m\cdots 0} Q_n^{m\cdots 0,k} \log \left\{ \lambda_k \frac{T_{m\cdots 0,k}}{Q_n^{m\cdots 0,k}} \right\} \tag{4}$$

where $\sum_{k=1}^{K} Q_n^{m\cdots 0,k} = 1$ and for reasons of tractability it is assumed to be independent of $\boldsymbol{\lambda}$, it will however depend on the parameter of the posterior Dirichlet. As in [3] by employing (4) in (3), expanding and evaluating $\mathsf{E}_{\mathcal{D}\boldsymbol{\gamma}_n} [\log \lambda_{kn}] = \psi(\gamma_{kn}) - \psi(\sum_{k'} \gamma_{k'n})$, where ψ denotes the digamma function, then solving for $Q_n^{m\cdots 0,k}$ and γ_{kn} and finally combining yields the following multiplicative iterative update for the sequence specific variational free parameter $\boldsymbol{\gamma}_n$,

$$\gamma_{kn}^{t+1} = \alpha_k + \exp\{\psi(\gamma_{kn}^t)\} \sum_{s_m=1}^{|\mathcal{S}|} \cdots \sum_{s_0=1}^{|\mathcal{S}|} \mathcal{N}_n^{m\cdots 0} \frac{T_{m\cdots 0,k}}{\sum_{k'=1}^{K} T_{m\cdots 0,k'} \exp\{\psi(\gamma_{k'n}^t)\}} \tag{5}$$

Solving for the transition probabilities and combining with the fixed point solutions for each $Q_n^{m\cdots 0,k}$ yields the following iteration

$$\tilde{T}_{m\cdots 0,k} = T_{m\cdots 0,k}^t \sum_{n=1}^{N} \mathcal{N}_n^{m\cdots 0} \frac{\exp\{\psi(\gamma_{kn}^t)\}}{\sum_{k'=1}^{K} T_{m\cdots 0,k'}^t \exp\{\psi(\gamma_{k'n}^t)\}} \quad ; \quad T_{m\cdots 0,k}^{t+1} = \frac{\tilde{T}_{m\cdots 0,k}}{\sum_{s_0'=1}^{|\mathcal{S}|} \tilde{T}_{m\cdots 0',k}} \tag{6}$$

The parameters of the prior Dirichlet distribution $\boldsymbol{\alpha}$ given the variational parameters $\boldsymbol{\gamma}_n$ are estimated using standard methods [20], [3]. Note that both (5) and (6) require an elementwise matrix multiplication and division so these iterations will scale linearly with the number of non-zero state-transition counts. This presentation of the required iterations is convenient here both from the point of showing the algorithmic scaling of the method and also in highlighting the close relationship of the LDA modelling framework with Probabilistic Latent Semantic Analysis (PLSA) (or the so called aspect model) [8] in the next subsection. We will show that these two methods are instances of the same theoretical model and differ only in the estimation procedure adopted.

B. Relation with the Aspect Model

While approximation methods can only guarantee local maximization of a lower bound on the true likelihood, it is however relatively simple to compute the maximum argument of the true posterior without actually computing the posterior density. This is the so called *maximum a posteriori* (MAP) estimation technique, frequently employed in latent variable models and their network implementations [2]. MAP estimators are notoriously prone to overfitting, especially where there is a paucity of available data [26]. However, MAP estimators are useful e.g. when the task is simply to analyze a given data set, as they provide the most probable hypothesis given the data [17]. It is also known that if sufficient data is available, then the MAP estimate reaches the Maximum Likelihood estimate [2]. We will show that a MAP estimate of LDA under a uniform Dirichlet prior yields exactly PLSA [8] (for the zero-th order case), both being instances of the same theoretical model. As an additional insight, we will also highlight the similarity of these two methods at the algorithmic level, both yielding iterations of multiplicative form similar to the 'parts based modelling' technique of Non-negative Matrix Factorisation [12].

The posterior probability of the random variable $\boldsymbol{\lambda}$ given the observed sequence $\mathbf{s}_n$ and current parameters is $P(\boldsymbol{\lambda}|\mathbf{s}_n, \mathbf{T}, \boldsymbol{\alpha})$ so the MAP estimate for $\boldsymbol{\lambda}$ is

$$\boldsymbol{\lambda}_n^{MAP} = \underset{\boldsymbol{\lambda}}{\text{argmax}} \ \log\{P(\boldsymbol{\lambda}|\mathbf{s}_n, \mathbf{T}, \boldsymbol{\alpha})\} = \underset{\boldsymbol{\lambda}}{\text{argmax}} \ \log\{P(\mathbf{s}_n|\boldsymbol{\lambda}, \mathbf{T})\} + \log\{\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\alpha})\}$$

Adding a Lagrange multiplier to enforce the constraint that $\boldsymbol{\lambda}_n^{MAP}$ is a sample point from a Dirichlet random variable, then solving for each $\boldsymbol{\lambda}$ yields the following convergent series of updates for λ_{kn}^t where the superscript denotes the t'th iteration, and as in [12], for each observed sequence in the sample a MAP value for the variable $\boldsymbol{\lambda}$ is iteratively estimated by the following multiplicative updates

$$\tilde{\lambda}_{kn} = \lambda_{kn}^t \sum_{s_m=1}^{|\mathcal{S}|} \cdots \sum_{s_0=1}^{|\mathcal{S}|} \mathcal{N}_n^{m\cdots 0} \frac{T_{m\cdots 0,k}}{\sum_{k'=1}^K T_{m\cdots 0,k'} \lambda_{k'n}^t} + (\alpha_k - 1) \ ; \quad \lambda_{kn}^{t+1} = \frac{\tilde{\lambda}_{kn}}{L_n + \sum_k (\alpha_k - 1)} \tag{7}$$

where $L_n = \sum_{s_m \cdots s_0} \mathcal{N}_n^{m\cdots 0}$ denotes the length of the sequence $\boldsymbol{s}_n$. Once the MAP values $\boldsymbol{\lambda}_n^{MAP}$ for each $\mathbf{s}_n$ are obtained then the maximum likelihood estimation of the transition probabilities yields the multiplicative iteration

$$\tilde{T}_{m\cdots 0,k} = T_{m\cdots 0,k}^t \sum_{n=1}^N \mathcal{N}_n^{m\cdots 0} \frac{\lambda_{kn}^{MAP}}{\sum_{k'=1}^K T_{m\cdots 0,k'}^t \lambda_{k'n}^{MAP}} \ ; \quad T_{m\cdots 0,k}^{t+1} = \frac{\tilde{T}_{m\cdots 0,k}}{\sum_{s_{0'}=1}^{|\mathcal{S}|} \tilde{T}_{m\cdots 0',k}} \tag{8}$$

Observe that as a special case of the 0-th order model, specifically if employing the maximum entropy Dirichlet prior (i.e. when $\alpha_k = 1, \forall \;\; k = 1 : K$) we recover exactly the PLSA algorithm. As each $\boldsymbol{\lambda}_n^{MAP}$ is a Dirichlet sample point, it then defines a multinomial distribution, the k-th dimension of $\boldsymbol{\lambda}_n$ is viewed in PLSA as $P(k|n)$. To make the relation to the previously outlined variational approach more evident on the algorithmic level, note that the MAP estimation can be seen as defining the bound (3) using the MAP estimator, such that $\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\gamma}_n) = \delta(\boldsymbol{\lambda} - \boldsymbol{\lambda}_n^{MAP})$, where $\boldsymbol{\gamma}_n = \boldsymbol{\lambda}_n^{MAP}$, in which case (3) is equal to $\log P(\mathbf{s}_n|\mathbf{T}, \boldsymbol{\lambda}_n^{MAP}) + \log \mathcal{D}(\boldsymbol{\lambda}_n^{MAP}|\boldsymbol{\alpha}) + \mathcal{H}^{\delta}$ where $\mathcal{H}^{\delta}$ denotes the entropy of the delta function around $\boldsymbol{\lambda}_n^{MAP}$ (which can be discarded in this setting as it does not depend on the model parameters, although it amounts to minus infinity).

C. Prediction with Simplicial Mixtures

The predictive probability of observing symbol s_{next} given the n'th sequence of L symbols $\mathbf{s}_n = \{s_{Ln}, \cdots, s_1\}$, generated by an individual, based on a simplicial mixture of m'th order Markov chains is given as

$$P(s_{next}|\mathbf{s}_n) = \int_{\triangle} P(s_{next}|s_m, ..., s_1, \boldsymbol{\lambda}) P(\boldsymbol{\lambda}|\mathbf{s}_n) d\boldsymbol{\lambda} \tag{9}$$

$$= \sum_{k=1}^{K} T(s_m, \cdots, s_1 \rightarrow s_{next}|k) \mathsf{E}_{P(\boldsymbol{\lambda}|\mathbf{s}_n)}\{\lambda_k\} \tag{10}$$

hence it is achieved by performing prediction on each 'basis'-transition separately and then combining the results in a user-specific manner as defined by the expectation $\mathsf{E}_{P(\boldsymbol{\lambda}|\mathbf{s}_n)}\{\lambda_k\}$. Note also that from (9) despite m-th order Markov chains forming the basis of the representation, the resulting simplicial mixture is not m-th order Markov with any global transition model. Rather it approximates the individual specific m-th order models whilst keeping the generative parameter set compact. A simplicial mixture of m-th order Markov chains embodies the m-th order information of each individual's past behavior in the user-specific latent variable estimate.

Employing the variational Dirichlet approximation then the following approximation can be employed in the above predictive distribution

$$\mathsf{E}_{P(\boldsymbol{\lambda}|\mathbf{s}_n)}\{\lambda_k\} \approx \mathsf{E}_{\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\gamma}_n)}\{\lambda_k\} = \frac{\gamma_{kn}}{\sum_{l=1}^{K} \gamma_{ln}}$$

If we employ the MAP approximation for the Dirichlet distribution then the required expectation can be approximated as

$$\mathsf{E}_{P(\boldsymbol{\lambda}|\mathbf{s}_n)}\{\lambda_k\} \approx \mathsf{E}_{\delta(\boldsymbol{\lambda} - \boldsymbol{\lambda}_n^{MAP})}\{\lambda_k\} = \lambda_{kn}^{MAP} \tag{11}$$

where λ_{kn}^{MAP} is the k-th dimension of $\boldsymbol{\lambda}_n^{MAP}$.

In a mixture model, due to the delta function prior, equation (2) , the predictive distribution is

$$P(s_{next}|\mathbf{s}_n) = \sum_{k=1}^{K} P(s_{next}|s_m, ..., s_1, k) P(k|\mathbf{s}_n) = \sum_{k=1}^{K} T(s_m, \cdots, s_1 \rightarrow s_{next}|k) P(k|\mathbf{s}_n) \tag{12}$$

Note that the posteriors $P(k|\mathbf{s}_n)$ are typically sharp, moreover the model insists that only one component is responsible for symbol emission and sequence generation. In consequence, a mixture of m-th order Markov models

is not m-th order at the global level, as is noted in [5], however at the level of each cluster or prototypical behavior the representation is still m-th order.

In all cases, given a new sequence $\boldsymbol{s}_{new}$, the symbol s_{next} which is most likely to be predicted from the model as a suggested continuation of the sequence, is the maximum argument of $P(s_{next}|\mathbf{s}_n)$. In the next section we consider practical examples where it is reasonable to assume Markovian dynamics at the level of individual behavior, however the heterogeneity of individuals makes even sophisticated global prediction models inefficient.

III. Experiments : Distributed Modelling of Sequential Activity

Three different types of sequential activity are now modelled in the following sections. The first illustrates the utility of the simplicial mixture of Markov chains in modelling the usage and interaction of a number of individuals with a wordprocessor software package. The second example considers modelling the sequential usage of a telephone service by a large group of individuals and finally the web browsing activity of visitors to a commercial website is studied. A brief description of the three collections of sequences is now provided.

A. Collections of Sequences Considered

1) Word Processor Command Usage: The first collection of sequential activity used in this study consists of the sequences of wordprocessor commands which were issued by a number of individual users during daily working sessions over a period of time. During a session of wordprocessor usage it is possible that a user-specific number of distinct tasks requiring particular sequences of commands may be undertaken, for example the creation and formatting of a table or the insertion and editing of an embedded object. In such a case there may be intra-sequence heterogeneity over interleaved common dynamic patterns which may not be modelled adequately by a mixture model. The sequences were acquired by monitoring the day to day usage of a wordprocessor package by more than 20 individuals at the MITRE corporation [13] over a period of 12 months[2]. Sequences of interactions that were logged during each session having less than three commands issued were discarded and after this trimming there remained 1,460 individual sequences. There are a total of 169 unique commands, however if we observe the ranked distribution of usage of the 169 commands on a $\log$ scale we can observe that there is a steep drop in the frequency of usage of certain commands ranked lower than 22 of 169, Figure(1). Due to this the twenty-two most commonly used commands are retained, as shown in Figure (1), and the remaining 147 are grouped together and listed as `other`. For the purposes of modelling there is a total of 23 symbols within the dictionary which correspond to the set of twenty two most frequently used commands with one symbol representing all those commands which are rarely or never issued. This data set will be referred to as WORD from now on.

2) Telephone Usage Modelling: The ability to model the usage of a telephone service[3] is of importance at a number of levels, e.g. to obtain a predictive model of customer specific activity and service usage for the purposes of service provision planning, resource management of switching capacity, identification of fraudulent usage of services. A representative description can be based on the distribution of the destination numbers dialled

[2]http://athos.rutgers.edu/ml4um/datasets/owl-data-info.html

[3]This data will be made publicly available to allow replication of the reported experiments and enable further investigation.

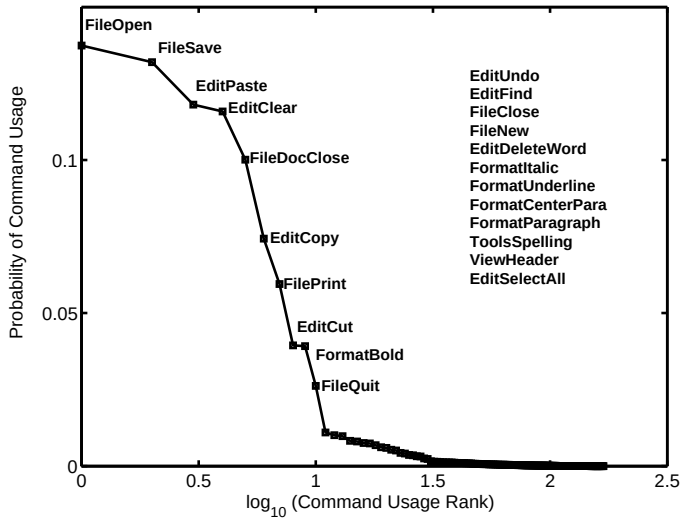

Fig. 1. The ranked normalized frequency of usage of the 169 available wordprocessor commands. Each of the ten most frequently used commands is listed at the relevant rank-point on the graph, with the remaining twelve most frequently issued commands being listed in the chart body.

and connected by the customer, in which case a multinomial distribution over the dialling codes can be employed. One method of encoding the destination numbers dialled by a customer is to capture the geographic location of the destination, or the mobile service provider if not a land based call. This is useful in determining the potential demand placed on telecommunication switches which route traffic from various geographical regions on the service providers network. Two weeks of transactions from a UK telecommunications operator were logged during weekdays, amounting to 36,492,082 and 45,350,654 transactions in each week respectively. All transactions made by commercial customers in the Glasgow region of the UK were considered in this study. This amounts to 1,172,578 transactions from 12,202 high usage customers in the first week considered and 1,753,304 transactions being made in the following week. The mapping from dialling number to geographic region or mobile operator was encoded with 87 symbols amounting to a possible 7,569 symbol transitions. Each customers activity is defined by a sequence of symbols defining the sequence of calls made over each period considered and these are employed to encode activity in a customer specific generative representation.

3) Web Page Browsing: The third data set used in this study is a selected subset of the msnbc.com user navigation collection employed in [5]. Sequences of users who visited at least 9 of the overall 17 page categories (frontpage, news, tech, local, opinion, on-air, misc, weather, msn-news, health, living, business, msn-sports, sports, summary, bbs, travel) have been retained, this selection criteria is motivated by the observation that there would be little scope in trying to model interleaved dynamic behavior in observables which are too short to reveal any intra-sequence heterogeneity. The resulting data set, referred to as WEB, totals 119,667 page requests corresponding to 1,480 web browsing sessions, thus being comparable in size to WORD however having fewer states and state transitions.

TABLE I

The performance of various models in terms of perplexity. The rows indicate the type of model, either Global, Mixture or Simplicial mixture. The columns indicate the order of the Markov chain, 0 - zero'th order, 1- first-order, 2 - second-order, 3 - third-order. The entries give the mean value $\pm$ one standard error of the perplexity computed over ten-folds and the value in brackets corresponds to the number of components in each of the mixture models. The best result is highlighted in bold.

	0	1	2	3
Global	14.09$\pm$0.35	6.70$\pm$0.24	6.36$\pm$0.23	7.07$\pm$0.26
Mixture	9.52$\pm$0.48 (90)	6.38$\pm$0.26 (20)	6.49$\pm$0.24 (2)	-
Simplicial	9.13$\pm$0.45 (90)	**5.91$\pm$0.24** (80)	6.04$\pm$0.2 (10)	-
HMM	-	6.80$\pm$0.20 (50)	-	-

B. Results

1) Word Processor Command Usage: In this experiment a range of global and mixture models were assessed for predictive performance. The most basic representation was a zero'th-order Markov chain, in short a MAP estimated [4] multinomial distribution over the twenty three commands. First, second and third order Markov chains (global models) were then assessed for predictive performance by computing the out-of-sample perplexity. In this experiment perplexity is measured, under each model, in the standard manner, computed as the exponential of the negative normalized (normalized by the number of observed symbols) log-likelihood obtained on out-of-sample sequences i.e.

$$\exp\left\{-\frac{1}{\sum_{m=1}^{N_{test}} L_m}\sum_{m=1}^{N_{test}}\log P(\mathbf{s}_m)\right\} \tag{13}$$

and due to the small number of available sequences this was estimated using ten-fold cross validation.

From the first row of Table (I) it can be observed how the estimated perplexity varies under differing orders of global Markov models. Moving from a zero'th order to a first-order model accounts for a halving (from 14.09 to 6.70) of the achievable perplexity under the model. It is therefore clear that taking into account the temporal nature of the sequences has a substantial effect on the predictive description of the data. Looking further we observe that a second-order model delivers a very slightly lower perplexity than the first-order model, however it is not statistically significant at the 5% level, as tested using a parametric t-test and a non-parametric Wilcoxon Rank-Sum test. The third-order model exhibits a degree of overfitting which is somewhat expected given that 12,167 state transition probabilities require to be estimated (in comparison to 529 state transition probabilities in the first-order model).

We now consider fitting mixture models to this collection of sequences and assessing their predictive performance. In all mixture models naive random initialization of the parameters was employed and parameter estimation was halted when the in-sample likelihood did not improve by more than 0.001%, no annealing or early stopping

[4] Standard Laplace smoothing was adopted.

TABLE II

A listing of five of the most probable transitions from three of the transition matrices of a ten component simplicial mixture of first-order Markov chains.

Component 2	Component 1	Component 8
FileDocClose→FileQuit	FormatItalic→FormatBold	EditClear→EditClear
FilePrint→FileClose	FormatUnderline→FormatItalic	EditCut→EditPaste
FileOpen→FileNew	FormatCenterPara→FormatCenterPara	EditCopy→EditCut
FileNew→FileDocClose	FormatItalic→FormatItalic	EditSelectAll→EditCopy
FileQuit→FileOpen	FormatUnderline→FormatUnderline	EditSelectAll→ToolsSpelling

was utilized, fifteen randomly initialized parameter estimation runs for each model were performed. In estimating the basis-transitions MAP smoothing with a constant Dirichlet parameter greater than 1 has been utilized, similarly to [5] which guarantees that the basis transitions are ergodic.

Initially both zero'th order mixture models (Naive Bayes mixture model) and simplicial mixture models employing the MAP estimator (PLSA) are considered here. The number of factors (dimensionality of the Dirichlet random variable, aspects in PLSA parlance, or classes for the mixture model) in each model ranged from 2 up to 100 elements. The order of the mixture model (number of factors) which gave the lowest out-of-sample perplexity was chosen and the performance along with the corresponding number of factors (in brackets) is listed in Table (I). In the case of the zero'th order model we observe that the mixture model substantially improves over the single global representation and that the MAP estimated simplicial mixture model (PLSA in this case) provides an improvement over the mixture model which is, however, statistically insignificant at the 5% level (employing the t-test). We shall observe in subsequent experiments that employing the variational estimation procedure and relaxing the uniform prior assumption, improved solutions can be obtained as observed in [3] when modelling text based documents.

If we now consider mixtures of first-order Markov models we note from the second column of Table (I) that the best mixture model achieves a lower perplexity than the global model, the difference however is statistically insignificant at the 5% level. On the other hand the simplicial mixture of first-order Markov chains yields a statistically significant lower value of perplexity than the global and best performing mixture model. A range of hidden Markov models were also assessed on this data and the best performing model achieved similar performance as the global model.

The second-order mixture models performance can be seen to be slightly inferior (though statistically insignificant) to the global model, whilst the simplicial mixture model indicates a robustness to overfitting on this data, which may be improved by employing more efficient estimation methods than the MAP estimator. To further illustrate the manner in which the simplicial representation represents the observed sequences, five of the most probable state transitions are listed for three of the component transition matrices of a ten component first-order model. It is illustrative that the transitions correspond to activities associated with the generic commands `file, format, edit`.

This experiment indicates that the only statistically significant improvement in perplexity over the global first-

order Markov model is obtained by a MAP estimated simplicial mixture of Markov chains, thus indicating the potential of such an approach to modelling sequential activity of a group of individuals. This performance can of course be improved by employing more efficient estimation methods such as those developed in [3], [16]. The following experiment considers a substantially larger collection of logged user activity and assesses whether any practically significant improvement can be achieved when employing simplicial mixtures.

2) Telephone Usage Modelling: As with the collection of sequential data of the previous section a reduction in perplexity (measured on the logged activity from the second week) of 53% is achieved when replacing a zero'th order global model with a global first-order model indicating the importance of the temporal content in the sequences. In this experiment the ability of the models to correctly predict the next symbol s_{next} given a sequence $\mathbf{s}_m$ is assessed by employing both the predictive perplexity defined as

$$\exp\left\{-\frac{1}{N_{test}}\sum_{m=1}^{N_{test}}\log P(s_{next}|\mathbf{s}_m)\right\} \tag{14}$$

and, in addition, the predictive accuracy under a 0-1 loss, i.e. given a number of previously unobserved truncated sequences, the number of times the model correctly predicts the symbol which follows in the sequence is then counted.

The number of components for the models considered ranged from 2 up to 200. On this data set the parameters of a global first-order Markov chain (bigram), mixtures of first-order Markov chains [5], and simplicial mixtures of first-order Markov chains (using both the MAP and variational (Variational Bayes - VB) estimation procedures) are estimated using the first week of customer transactions and the predictive capabilities of the models are assessed on the transactions from the following week. The results are summarized in Figure 2, from the predictive perplexity measures it is clear that the simplicial representation provides a statistically (tested at the 5% level using a t-test) and practically significant reduction in perplexity over the global and mixture models. This is also reflected in the levels of prediction error under each model, however the mixture models tend to perform slightly worse than the global model. As expected the MAP estimated simplicial model performs slightly worse than that obtained using VB [3]. This also provides an additional insight as to why LDA models improve upon PLSA, as they are in fact both the same model using different approximations to the likelihood, refer to [26] for an illustrative discussion on the weaknesses of MAP estimators. As a comparison to different structural models hidden Markov models with a range of hidden states were also tested on this data set the best results obtained were for a ten state model which achieved a predictive perplexity score of (mean$\pm$standard-deviation) 11.119 ± 0.624 and fraction prediction error of 0.674 ± 0.959, considerably poorer than that obtained by the models considered here.

In addition to the predictive capability of a simplicial representation of a customers activity the cost of encoding such a representation can be assessed by measuring the entropy rate [6] of each of the constituent first-order transition matrices which act as a basis in the representation of the individual specific generative process. The left hand plot of Figure (3) shows the distribution of the entropy rates for the transition probabilities in twenty factor simplicial and mixture models, the results are obtained from fifty randomly initialized estimation procedures. The entropy rates for the simplicial mixture are significantly lower than that of a mixture model indicating that the basis of each representation describes a number of simpler processes. The transition matrices of the three dominant

factors defining the behavior of the customer considered in Figure (3) are shown in Figure (4). These have a clear interpretation in terms of customer activity, the transition matrix corresponding to factor 3, generates sequences which have many transitions from regions 35 & 40 to 75 to 80. Symbols 35 to 40 correspond to locations within Glasgow whilst 75 to 80 correspond to mobile service providers. The second factor shows finite probabilities of transition in region 40 to 60 as well as 10 & 20 corresponding to Scottish regions, whilst the final transition matrix gives high probability of making calls to one specific mobile provider. The profile of each customer can then be defined by the distribution of the expected Dirichlet variable (Figure 3) given the 'basis' transition matrices (Figure 4).

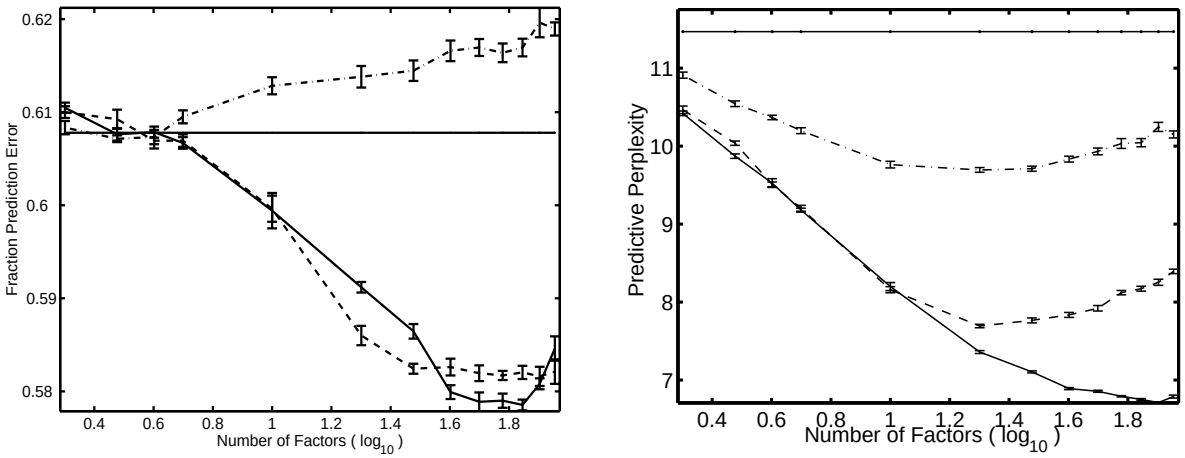

Fig. 2. The left hand plot shows the percentage of incorrect predictions against the number of model factors and the right hand plot charts the predictive perplexity of each model against model order for the PHONE dataset. The global first-order Markov chain is represented by a solid straight line, the dashed line represents the MAP estimated simplicial model, the solid line represents the VB estimated simplicial model and the dash-dot line represents the mixture model. The error bars represent one standard error.

3) Web Page Browsing: The final experiment demonstrated considers the WEB data set. The results of ten-fold cross-validated predictive perplexities again show statistically significant improvement obtained with the VB-estimated simplicial mixture. The results are summarized in Figure 5. Five of the estimated transition factors of a twenty-factor model are shown in Figure 6, demonstrating once more that the proposed model creates a low entropy and an easily interpretable dynamic factorial representation. The numbers on the axes on these charts correspond to the 17 page categories enumerated earlier and the average strength of each of these factors amongst the full set of twenty factors computed as $\frac{1}{N}\sum_{n=1}^{N} \mathsf{E}_{\mathcal{D}(\boldsymbol{\lambda}|\boldsymbol{\gamma}_n)}\{\lambda_k\}$ is also given above each chart. We can see that a behavioral feature manifested is a keen interest to visit pages about 'news' along with a quite dynamic transition model (left hand chart) which characterizes around 12% of the behavioral patterns of the entire user population under consideration while static state-repetition (second chart) or an almost exclusive interest in viewing the homepage (last chart) etc represent also relatively strong common characteristics of browsing behavior. The

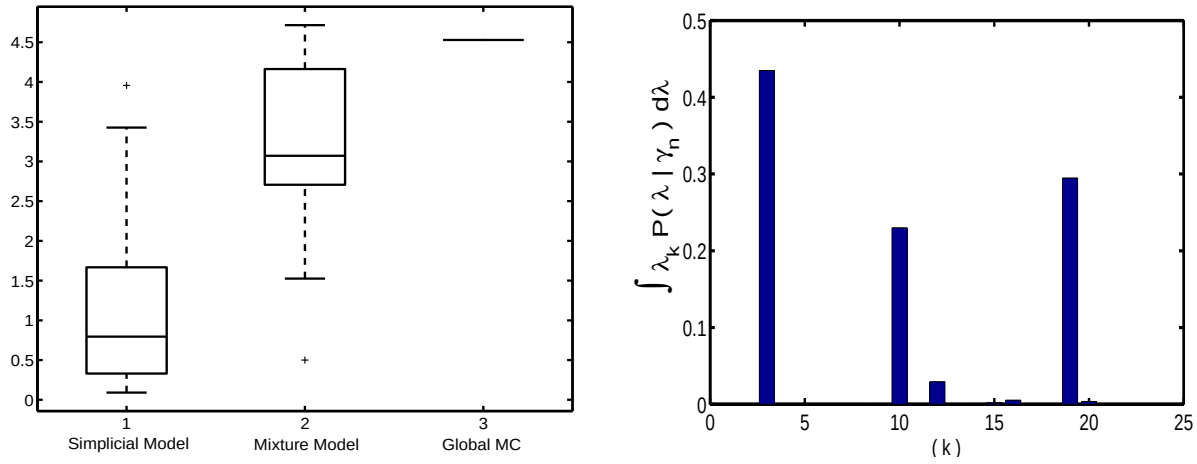

Fig. 3. The left hand plot shows the distribution of entropy rates for the transition matrices of a twenty factor mixture and simplicial mixture models (VB). The right hand plot shows the expected value of the Dirichlet variable under the variational approximation for one customer indicating the levels of participation in factor specific behaviors.

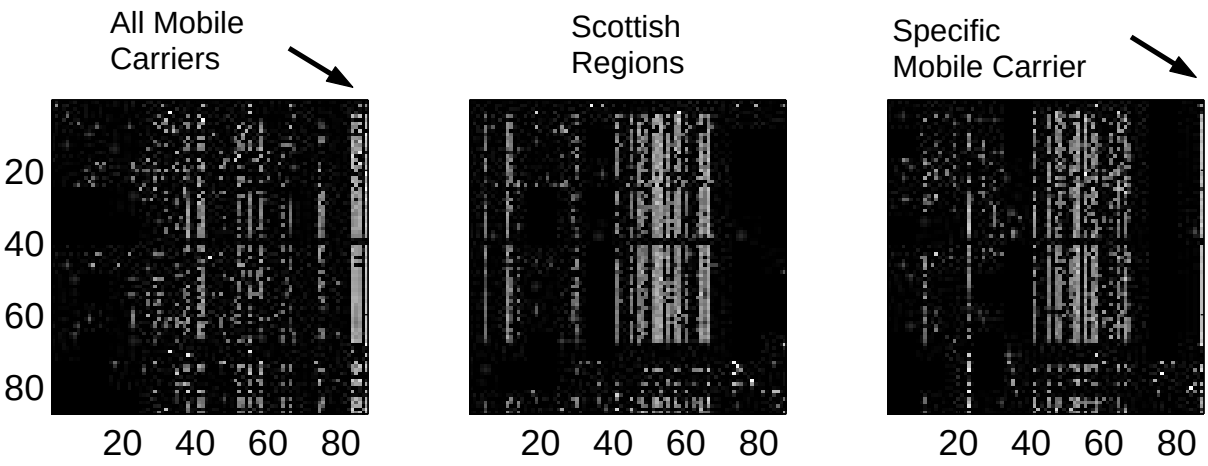

Fig. 4. The state transition matrices corresponding to factors 3, 10 & 19 for the customer under consideration.

distribution of the entropy rates of the full set of these twenty basis-transitions in comparison to those obtained from the mixture model is given on the right hand plot of Figure 5. Clearly, the coding efficiency of a simplicial mixture representation is significantly (statistically tested) superior. Note also these basis-transitions embody correlated transitions (transitions which appear in similar dynamical contexts and so have similar functionality), as can be seen from the multiplicative nature of the equations used for identifying the model. It is not surprising then that state repetitions or transitions which express focused interest in one of the topic categories appear together on distinct factors. We can also see a joint interest in msn-news and msn-sport being present together on the 4-th chart of Figure 6 — indeed, as the prefix of these page categories also indicates, these are related page categories.

Transitions produced by mixtures of MCs, found to be visually similar to those listed for simplicial mixtures

on Figure 6, are given on Figure 7. The state repetition probabilities are notably high on all parameter transitions. This is because by their construction, mixtures tend to partition the users and represent average behaviors of the identified groups. Clearly, in this case, prototypical users of all groups exhibit a behavioral feature characterized by state repetitions. By contrast, the simplicial mixture does not partition users but extracts behavior features instead. These features may be common to several users or groups of users.

Before concluding, it may also be worth mentioning that our intuition that simplicial mixtures are more appropriate for modelling observation sequences that are sufficiently long and diverse, such that their intra-sequence heterogeneity can be exploited, has also been confirmed in our experiments. While simplicial mixtures perform consistently superior on 'rich' observation sequences, they may become poor when the typical sequence length is very short — in such cases mixtures appear to be more appropriate. We have also found that simplicial mixtures are much more robust against small number of sequences compared to mixtures.

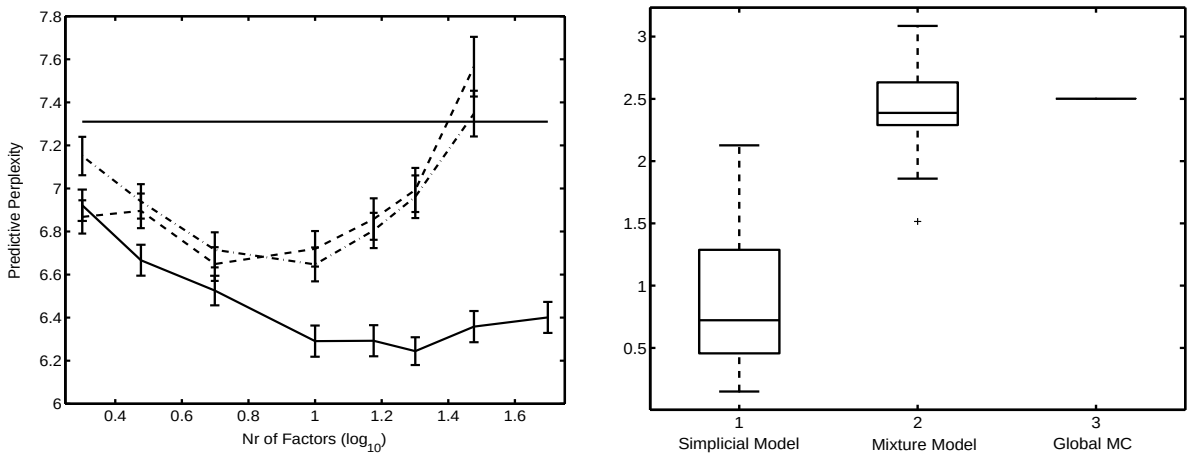

Fig. 5. The left hand plot is the predictive perplexity for the WEB data (the straight line corresponds to a global first-order Markov chain. As before, the dashed line represents the MAP estimated simplicial model, the solid line represents the VB estimated simplicial model and the dash-dot line represents the mixture model. The left hand hand plot is the distribution of entropy rates.

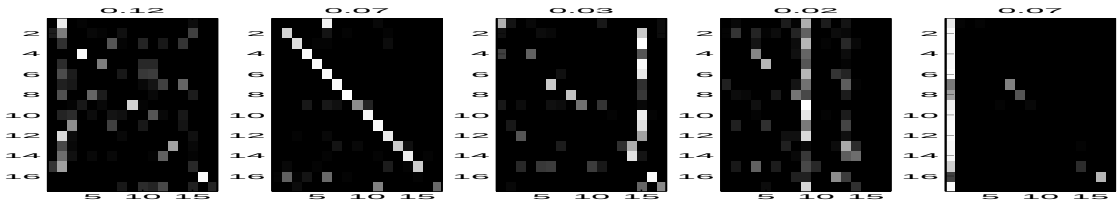

Fig. 6. State transition matrices of selected factors from a twenty-factor run produced by Simplicial Mixtures of MCs on the WEB data set.

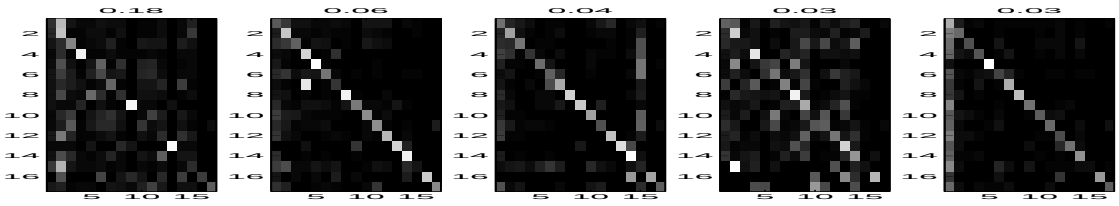

Fig. 7. Selected state transition matrices produced by mixture of MCs on a twenty-component run on the WEB data set.

IV. Conclusions

This paper has presented a linear time method to model finite-state sequences of discrete symbols which may arise from user or customer activity traces. The main feature of the proposed approach has been the assumption that heterogeneous user behavior may be 'explained' by the interleaved action of some structurally simple common generator processes and we have related this representation to several existing models. An empirical study conducted on three collections of logged user activity demonstrated that the proposed approach yields an efficient representation, revealed by both objective measures of prediction performance, low entropy rates, and interpretable representations of the user profiles provided. In spite of its computational simplicity it has been observed that a simplicial mixture of first-order Markov chains is capable of outperforming a global Hidden Markov Model in terms of prediction performance.

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