RESEARCH STATEMENT

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My primary area of research is image processing and vision analysis. During the past two decades, this field has attracted many mathematicians' interest by the pioneering works [5, 9, 11]. The mathematical aspects include nonlinear PDEs, Calculus of Variations, Scientific Computing and Geometric Image Analysis. I have worked under the guidance of Prof. Jakie Shen especially on several aspects of segmentation problem. The goal is to partition a given image into a collection of objects, for high-level tasks such as object detection, recognition, and tracking. My future plan is to continue to study related problems and I am also interested in working on new problems including wavelets, multiscales, and stochastic methods. In what follows, I provide a brief description of the problems I have worked on, and explain plans for future work.

1. RESEARCH SUMMARY

1.1. *Multiphase segmentation: Multiphase Image Segmentation via Modica-Mortola Phase Transition*

The multiphase segmentation problem arises when a given image has more than two objects to be identified. The problem is also called the minimal partition problem, as formulated by Mumford and Shah [9]. In this work [6], we propose a new multiphase segmentation model based on the celebrated phase transition model of Modica and Mortola [8], bridging image segmentation to the phase transition problem in material sciences and fluid mechanics.

First of all, different phases in material sciences are characterized by a few key phase variables such as densities and tensions, while in image and vision analysis, distinct "object" segments are similarly characterized by some key *visual features* such as intensities, orientations. Secondly, the difficulties in dealing with sharp interfaces emerging from both material phase transitions and image segmentation share the very same roots – the characteristic complexities in handling free boundaries and their geometry.

With K objects to be segmented in the given image u_o, our new model is

$$E_\epsilon[z|u_o] = \int_\Omega \left[\epsilon|\nabla z|^2 + \frac{1}{\epsilon}\sin^2 \pi z\right] dx + \lambda \sum_{k=0}^{K-1} \int_\Omega |C_k - u_o|^2 \mathrm{sinc}^2(z-k)\, dx, \tag{1}$$

where $C_k = \int_\Omega u_o \,\mathrm{sinc}^2(z-k)\, dx / \int_\Omega \mathrm{sinc}^2(z-k)\, dx$.

The proposed sine-sinc model(1) outputs a single multiphase distribution from which each individual segment or phase can be easily extracted. The existence of its minimizers is proved via the direct methods in the calculus of variations and its Γ-convergence behavior is shown using the result in Modica and Mortola [8] and the stability property of Γ-convergence under continuous perturbations.

Since the model is not quadratic nor convex, for computation we adopted the convex-concave procedure (CCCP) that has been developed in the literature of both computational nonlinear PDEs [4] and neural computation [17]. Figure 1 shows the satisfying performance of the proposed model and its algorithm on an MRI brain image.

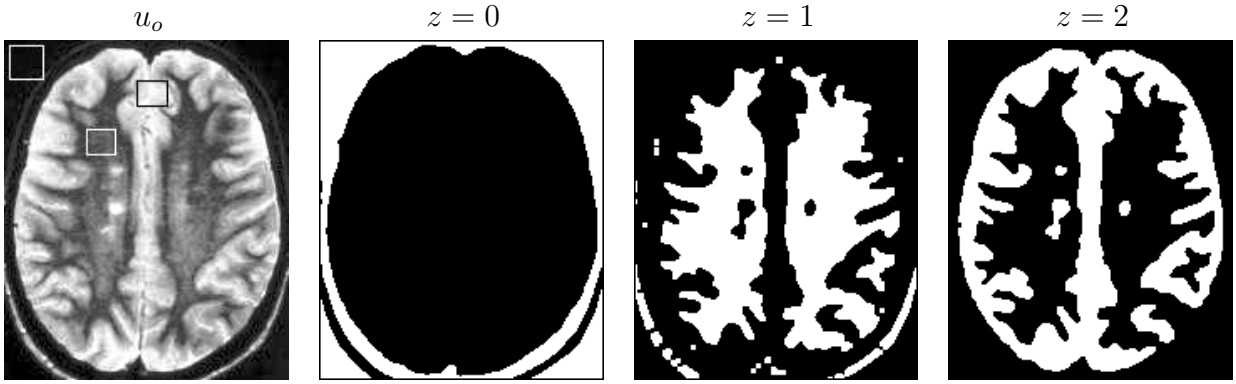

FIGURE 1. The performance of the proposed Modica-Mortola sine-sinc model to the segmentation of a MRI brain image. Shown on top of the original image (upper left) are the three "seed" phase patches which are often easily supervised by a radiologist.

1.2. *Segmentation with missing boundaries: First-Order Modeling and Stability Analysis of Illusory Contours*

In visual cognition, illusions help elucidate certain intriguing latent perceptual functions of the human vision system, and their proper mathematical modeling and computational simulation are therefore deeply beneficial to both biological and computer vision. *Illusory contours* are such a well known class of visual illusions.

Inspired by existent prior works Sarti, Malladi, and Sethian [12] and Zhu and Chan [18, 19], we proposed a first-order energy-based model for analyzing and simulating illusory contours in [7]: with existing boundaries Γ_{re} and missing boundaries Γ_{im},

$$E_o[\Gamma] = E_o[\Gamma \mid \alpha, \beta] = \alpha \int_{\Gamma_{\mathrm{re}}} ds + \beta \int_{\Gamma_{\mathrm{im}}} ds, \tag{2}$$

where s denotes the arc-length parameter and $0 < \alpha \ll \beta$.

The lower complexity of the proposed model(2) facilitates rigorous mathematical analysis on the detailed geometric structures of illusory contours. We characterized local minima of (2), and showed that the famous illusory contours Kanizsa's triangle and square are local minimizers. After being asymptotically approximated by classical active contours, the proposed model is then robustly computed using the celebrated level-set method of Osher and Sethian [10] with a natural supervising scheme. Figure 2 shows the computational result applied to the Kanizsa triangle.

FIGURE 2. Kanizsa' illusory triangle as a stable local minimum to the proposed contour energy using the level-set method of Osher and Sethian.

1.3. *Light-adaptive segmentation: Weberized Mumford-Shah Model with Bose-Einstein Photon Noise*

Light adaptivity is a remarkable feature of human visual perception. Entering a relatively dimmer room from an outdoor stroll in the sun, few people ever feel much difference in image interpretation, although physically the average ambient light intensities have undergone a major change. Contributing to such remarkable flexibility is a famous law in psychology and psychophysics known as *Weber's Law*. For dynamic vision systems of either mobile robots or surveillance cameras, light adaptivity then becomes indispensable due to the constant changes of incident light in a dynamic environment.

Towards a more faithful model of human visual segmentation, the work [15] introduced the light adaptivity feature into the celebrated Mumford–Shah segmentation model [9], as inspired by Weber's Law. To be consistent with the intensity interpretation of images in the weberization procedure, the traditional gray scale shift-invariant Gaussian noise model is accordingly replaced by Bose–Einstein photon noise. We thereby proposed the following new model:

$$E[u, \Gamma \mid u_0] = \alpha \mathcal{H}^1(\Gamma) + \beta \int_{\Omega \setminus \Gamma} \frac{|\nabla u|^2}{u^2} dxdy + \lambda \int_{\Omega} \left(\ln u + \frac{u_0}{u} \right) dxdy, \tag{3}$$

where u_0 is given image, $\mathcal{H}^1$ denotes one-dimensional Hausdorff measure.

We explained why the classical Mumford–Shah model lacks light adaptivity, and why its "weberized" version can more faithfully reflect human vision's superior segmentation capability in a variety of illuminance conditions. In particular, Ambrosio-Tortorelli's Γ-convergence approximation theory [1, 2] is adapted and stable numerical algorithms are developed for the associated pair of nonlinear Euler-Lagrange PDEs. The synthetic image example in Figure 3 is to verify and highlight the light adaptivity of our new model.

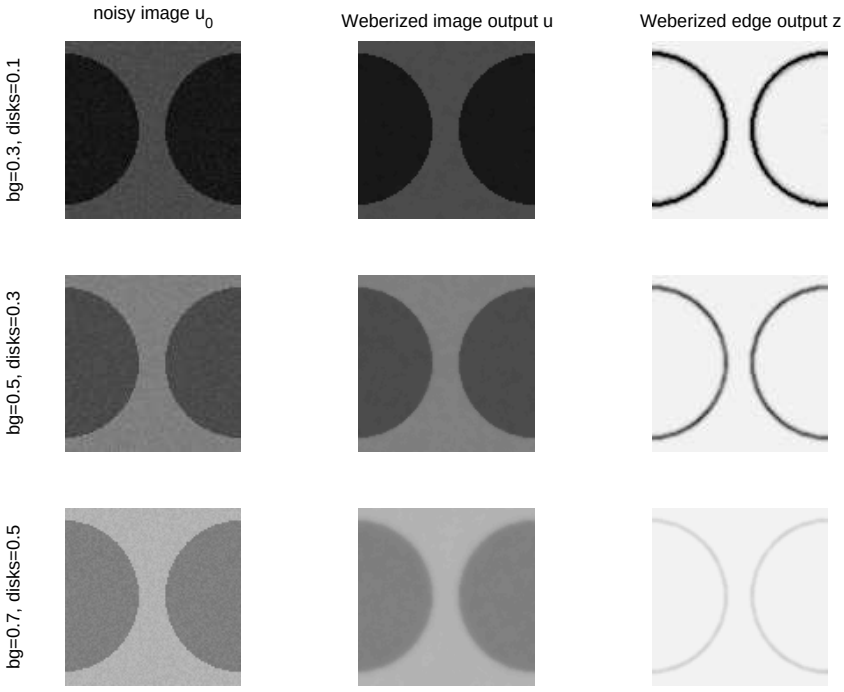

FIGURE 3. Performance via a synthetic image: light adaptivity of the weberized Mumford–Shah model. The edge functions z are distinct and have relative strengths.

1.4. *Geometric and Stochastic Analysis of Reaction-Diffusion Patterns*

After Turing's ingenious work [16], reaction-diffusion patterns have been extensively studied in terms of modeling and analysis of pattern formations in both chemistry and biology. But most patterns have only been discovered, identified or explained by human vision and intelligence.

Inspired by the recent advancement in mathematical image and vision analysis, the paper [14] developed both geometric and stochastic tools and frameworks for identifying, classifying, and characterizing common reaction-diffusion patterns and pattern formations. Various statistical tools were developed for the monitoring of pattern formations and for the identification and classification of matured patterns. Important geometric measures such as total areas, total variations, and total curvatures were employed to compute the key geometric features of common patterns such as average radius and width.

2. FUTURE PLANS

There remain many interesting and significant problems to be addressed and studied in the area of image processing and vision analysis, in both theory and computation. I would like to continue my research related on segmentation, deblurring and inpainting. To be more specific, I explain the problems which I am currently working on or particularly interested in.

- *Blind deconvolution with singular measures*: To recover a sharp image from its blurry observation is the problem called deblurring or deconvolution and a typical problem in inverse problems. In this project, Prof. Shen and I consider blurring due to camera shake or photo takers' motion(for example, in a moving car) during exposure. One key characteristic of this problem is that the blur kernel is singular. Unlike a Gaussian kernel, since the blurring process is along a curve, we have to deal with singular measures. Currently we are developing an appropriate model and investigating its mathematical behavior. After this is done, we are planning to develop its computational algorithm.
- *Inpainting via phase transition models*: Inpainting is the process of filling in missing or damaged regions with the use of available surrounding information. Assuming a given image is quantized and discrete, it is seemingly reasonable to apply phase transition models in material sciences and fluid dynamics. By inspiring work Bertozzi, Esedoglu and Gillette [3], I am planning to develop a new inpainting model. I have already tested a second-order model to binary synthetic images. I am planning to apply to multiphase images and develop a fourth-order model to approximate curvature information.
- *Segmentation vs pattern classification*: Pattern classification aims to classify data (patterns) based on either a priori knowledge or on statistical information extracted from the patterns. Especially finding decision boundaries is very similar to finding edges in image processing, although one is a statistical/probabilistic problem and the other is a deterministic problem. In image processing terminology, pattern classification may be viewed as 'segmentation with large inpainting'. Recently there is an interesting work of Shen [13], which interpreted a variational segmentation problem as a probabilistic problem. Thus, it is very interesting to apply segmentation and inpainting models to pattern classification. Applying pattern classification tools to image processing is also interesting.

I am also interested in real imaging problems arising from engineering and physics, since they have strong connection with my research area and share common properties. I eagerly look forward to continuing my research in related research problems.

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