Research Statement: Multiresolution in Graphics and Simulation **Eitan Grinspun, Feb 2004**

Introduction and Summary

The primary goal of my research is to develop *simple* computational models for simulating the behavior of physical systems. I draw on ideas from graphics, applied mathematics, differential geometry, and engineering.

My research shows that simple approaches need not sacrifice accuracy or performance. Simple approaches lead to more insight, wider adoption by the scientific computing community, and the opportunity to pursue more ambitious applications to mechanical engineering, medicine, biology, and computer graphics. With this philosophy in mind I am focusing on two key aspects of simulation. First, to improve *scalability* of simulators with respect to large datasets and high geometric complexity, I am developing a framework for **adaptive multiresolution simulation**. Second, I am developing **discrete models**: geometric descriptions of physical phenomena distilled into their most fundamental form.

Multiresolution simulation For improved scalability it is highly desirable to run *adaptive* simulations, allocating computational resources where they can be most profitably used. Implementing adaptive methods robustly is generally very difficult. Alleviating this difficulty was the focus of my doctoral dissertation, which presents a general framework for adaptive **multiresolution simulation**[3]. *Multiresolution* refers to decomposing geometry into coarse shapes superimposed with progressively finer details. While earlier adaptive approaches were problem-specific, this method is broadly applicable, domain- and dimension-independent (see Figure 1). Most importantly, the implementation is simple.

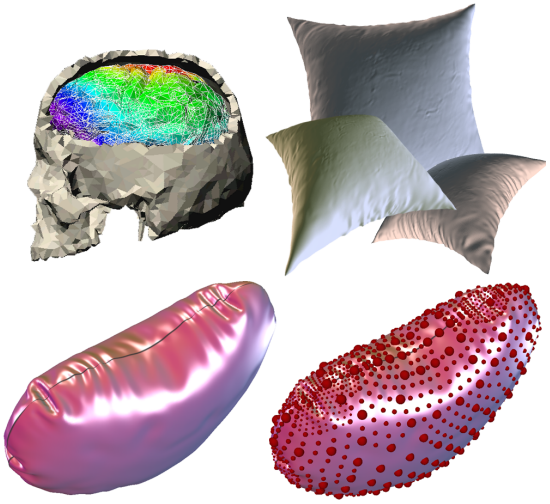


Figure 1: *Example applications which benefit from our multiresolution simulation framework: (top-left) surgery simulation, (top-right) modeling wrinkles on cloth, (bottom) rapid inflation of airbag.*

The formalization of a unified framework for multiresolution simulation opens the door to many exciting research directions, such as **interactive simulation**. To achieve interactive rates, I am collaborating with **parallel computing** expert Prof. Karamcheti (NYU) in an effort to understand the complementary roles of multiresolution and parallelism. As an alternative approach toward interactivity, I am also exploring **streaming implementations** of numerical solvers using commodity graphics processors (**GPUs**)[1].

Advances in multiresolution simulation also have impact on geometric modeling applications. For instance, in computer-aided geometric design (CAGD) as well as in physical simulation it is often necessary to determine the regions where a surface self-intersects. We introduced the first such technique for **subdivision surfaces**[6], a class of surfaces which has become popular in computer graphics, by exploiting the natural multiresolution structure of subdivision theory.

Discrete models Recently I have turned my attention to **discrete differential geometry** (DDG), a new field led by the efforts of Profs. Desbrun (USC), Marsden (Caltech), and Bobenko (TU Berlin) among others. My belief is that DDG is key to deriving *simple*, robust models of physical materials. Recently we applied DDG to modeling the physics of thin, flexible objects. Simulation of so-called *thin shells* was previously avoided by the graphics community because earlier models were difficult to implement. This situation changed after the publication of our work on **discrete shells**[4], as derivative projects have sprung up at various research institutions.

Unifying my research efforts in physical simulation is the philosophy that simple solutions are the most insightful and practical. I am very excited by the pursuit of this ideal via projects in multiresolution simulation, interactive scientific computation, geometric modeling and discrete differential geometry. I am equally enthusiastic about exploring new projects along the same vein, as described below. The following is an elaboration on some of the ideas, projects and plans mentioned above.

Multiresolution simulation

Many interesting phenomena, such as crushing and crumpling of a car body, exhibit features at different scales. We simulate the behavior of these systems *adaptively*, beginning with a coarse geometrical representation and adding finer geometrical details where/when interesting behavior occurs (see Figure 2). Adaptivity is often considered difficult to implement and is thus avoided. This is unfortunate, as compared to simulating at a very fine resolution, adaptivity promises reduced computational cost and improved accuracy.

CHARMS My doctoral dissertation presents a framework for adaptive simulation that alleviates many of the implementation difficulties of earlier approaches. Bringing together ideas from multiresolution graphics and classical simulation, we developed a novel paradigm: *multiresolution simulation*. In this framework, geometrical data is represented as a superposition of *basis functions*, which in turn are sorted by levels of resolution. Thus a geometrical model is represented by a coarse model and additional layers of progressively finer details. Our conforming, hierarchical, adaptive refinement methods (CHARMS) have demonstrated applications to engineering, surgical and biomechanical simulation, as well as computer animation. CHARMS was well-received in the graphics community (see the Siggraph publication[5]) and in the engineering community[7], reaffirming that graphics research can introduce ideas to sister sciences.

Numerical solvers for multiresolution systems Currently we use standard numerical techniques for solving the equations posed by multiresolution simulation. As prior work on *hierarchical preconditioning* has demonstrated, performance could be greatly improved by using numerical solvers that are explicitly tailored to the separate levels of resolution of CHARMS (*e.g.*, multigrid solvers).

Furthermore, currently we evolve the state of the simulation synchronously. Recent developments in asynchronous variational time integrators (AVIs) are very promising[8]. In a continuing collaboration with Prof. Marsden (Caltech) we seek to learn how the temporal adaptivity of AVIs complements the spatial adaptivity of CHARMS.

Interactive implementations A CHARMS-based simulator could empower an interactive scientific computing environment with applications to rapid design cycles in engineering, steerable simulations in computational fluid- and solid-mechanics, and real-time medical applications such as surgical assistance or training. Our goal is that user input (*e.g.*, changing the thickness of an airplane wing in a wind tunnel simulator) immediately results in feedback from a coarse-level simulation while progressively more detailed results arrive later in time.

Parallel/streaming architectures With interactive simulation as our goal, we have begun to investigate affordable, high-powered architectures for numerical computing. We are working on a parallel implementation of CHARMS, in which the "multiresolution-aware" computational cluster explicitly harnesses the level-of-detail decomposition.

At Caltech we pursued an alternative approach, treating a GPU as a high-performance streaming processor. We implemented two basic, broadly useful, computational kernels: a sparse matrix conjugate gradient solver and a regular-grid multigrid solver. These solvers performed efficiently, as demonstrated in a geometric flow and fluid simulation applications.

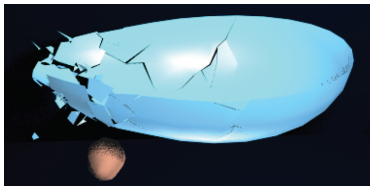
Discrete models

Recently I became interested in the application of discrete differential geometry to the simulation of thin, flexible *shells*, such as sheet metal in car bodies, airbags, leaves, and skin. Whereas *classical* differential geometry deals with operators (*e.g.*, shape operator) defined over smooth surfaces (more generally, manifolds), its counterpart, DDG, defines these operators over *discrete surfaces*, in our case triangle meshes. Our efforts to develop discrete models for computer graphics and simulation run parallel to efforts by pioneers of DDG, in particular work on *discrete mechanics*[9].

Discrete shells Already this research direction is displaying promise—in July we published a simple formulation for *discrete thin shells* demonstrating (see Figure 3) realistic animations of a falling hat[4].

Recently we began a collaboration with applied mathematician Prof. Erik Demaine (MIT) who is using our simulation framework to study computational origami (with potential applications to protein folding).

Our discrete model of thin shells is parameterization-independent—we measure lengths, angles, and in general only quantities which do not depend on parameterizing (*i.e.*, assigning a local coordinate frame to) the surface. This is attractive from a theoretical perspective, a pedagogical perspective (insight into the model is often hidden by parameterization), and an implementation perspective (debugging is often easier).



Discrete shape operator Recently at NYU we have begun developing a novel *discrete shape operator* which is at the same time both simple to implement yet versatile, with applications to elastic and plastic deformations, brittle and ductile fracture (shown in the figure), and adaptive simulation of thin shells[2, 10]. This project may eventually bridge the gap between simulation methods used in engineering—complex, difficult to implement, but based on sound theoretical foundations—and those used in graphics—fast, easy, but rather ad-hoc in design. Furthermore, we believe that our discrete shells model may serve to formally explain *why* seemingly ad-hoc approaches from graphics produce such nice results.

Summary and Future Directions

My research area is inherently interdisciplinary, drawing on ideas from graphics, applied mathematics, differential geometry, and engineering to develop *simple* computational models for physical simulation. In particular I have developed a unified framework for adaptive multiresolution simulation, key to the design of scalable simulators for high geometric-complexity scientific computing applications. The most important result is a set of *simple* algorithms that apply uniformly to a broad range of settings. This work has been well-received by both the graphics and engineering communities, and it leads naturally to the pursuit of interactive large-scale scientific computing environments. The design of such environments requires consideration of systems issues. To that end I have started collaborations toward both parallel and streaming implementations of multiresolution simulators. While these investigations are at their infancy, the preliminary results are very promising.

I believe that to truly reap the benefits of multiresolution, all components of the simulator must take advantage of the separation of geometry into levels-of-detail. I plan to incorporate into my framework a multiresolution-aware numerical solver based on the ideas of multigrid methods. I would also like to explore the role of temporally-adaptive schemes within my simulation framework. Thus I am in communication with colleagues at Caltech pioneering the field of asynchronous variational integrators.

While multiresolution simplifies the task of building a scalable adaptive simulator, I am also interested in simplifying the underlying physical models that are being simulated. To that end I am pursuing discrete models of shells, solids, fluids and other physical phenomena. Discrete models are easy to implement and they offer geometric insight into the underlying physics. They also tend to be robust as a problem scales *down* in size, which is important for computer graphics and games applications, and is also of theoretical interest. The emerging field of discrete differential geometry shows promise of providing tools for developing such discrete models, as evidenced by our preliminary results on the simulation of deforming and fracturing thin shells.

So far, however, adaptive simulation of discrete models has been ad-hoc. Fortunately, multiresolution offers a clean, elegant framework for adaptive simulation. By applying multiresolution techniques to discrete approaches, I believe that we will be able to tackle implementations of even more ambitious simulations. However, the question of whether the paradigms of multiresolution simulation and discrete models can coexist and complement each other remains open, and of great interest to me.

As all these pieces come together, I look forward to interactive simulation applications, across all arenas of scientific computing, including medicine, engineering, and graphics. I eagerly anticipate new collaborations with doctors, biologists, and mechanical engineers as domain experts and users of these applications.

References

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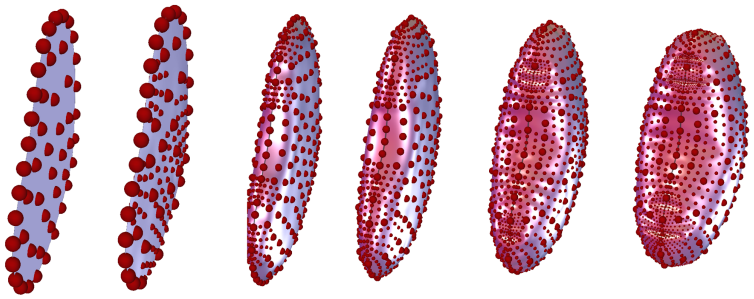

Figure 2: *Simulation run within our CHARMS framework: rapid (2ms) inflation of metal-foil balloon.*

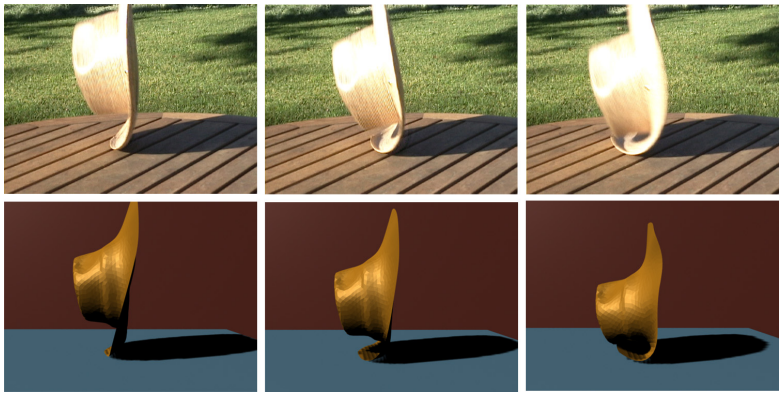

Figure 3: *Discrete constitutive model for thin shells: real footage vs. simulation of a hat. (top) A real hat is dropped on a table, and (bottom) our simulation captures the bending of the brim.*