Optimal QoS Tradeoff and Power Control in CDMA Systems

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Abstract— **Dynamic power control and scheduling strategies provide efficient mechanisms for improving performance of wireless communications networks. A common objective is to maximize throughput performance of a network or to minimize the total transmission power while satisfying quality-of-service (QoS) requirements of the users. The achievement of these objectives requires the development of medium access control (MAC) strategies that optimally utilize scarce resources in wireless networks. When developing such strategies, a good understanding of the structure of the feasibility region is essential. The feasibility region is defined as a set of all QoS requirements that can be supported by a network with all users active concurrently. Thus, the structure of this set shows when (if at all) scheduling strategies can improve network performance. In particular, if the feasibility region is a convex set, then concurrent transmission strategies are optimal and the optimal power allocation can be obtained efficiently via a convex optimization. Other important problems are how the total transmission power depends on QoS requirements and what the optimal QoS tradeoff is. In this paper, we address all these problems and solve them completely in some important cases. The purpose of this paper is to explore the interrelationship between QoS requirements and physical quantities such as transmission power. Although the results are obtained in the context of a power-controlled CDMA system, they also apply to some other communications systems. A key assumption is that there is a monotonous relationship between a QoS parameter of interest (such as data rate) and the signal-to-interference ratio at the output of a linear receiver.**

I. INTRODUCTION

Future generations of wireless networks will have to support a wide range of services. Data applications have fundamentally different quality-of-service (QoS) requirements and traffic characteristics than video or voice applications. This, together with the physical limitation of the mobile radio channel, presents fundamental technical challenges for system designers. On the other hand, some characteristics of mobile communication systems open up opportunities for improving the network performance. For instance, scheduling strategies that exploit the relative delay tolerance of data applications can be used to achieve performance gains. Furthermore, channel-aware scheduling algorithms such as the Proportional Fair algorithm for CDMA 1xEV-DO exploits channel fluctuations to improve the throughput performance. The use of appropriate MAC strategies (include power control and scheduling) is particularly important if a number of potential users with certain QoS requirements is too large to be supported by the network.

When developing such strategies, system designers can pursue different objectives. Two common examples are to maximize throughput performance and to minimize total transmission power. In doing so, they need to satisfy certain quality-of-service (QoS) requirements of the users, and hence are faced with a constrained optimization problem. A fundamental question that immediately arises is: Given channel state and power constraints, how many users a wireless network can support so that each user meets its QoS requirement ? This problem is one of the central problems in wireless network design since it addresses the question of admissibility of users in a communications system with certain QoS guaranteed. Another important issue is that of an optimal tradeoff between QoS requirements of the users. Here, the problem is how the QoS requirements depend on each other under an optimal resource allocation ? This is important for developing strategies to balance contrary demands of the users. A key ingredient in the development of such strategies is convexity of the feasibility region since it opens the door to a widely developed theory. The feasibility region is defined as a set of all QoS requirements that can be supported by a network with all users active concurrently. A better understanding of the structure of the feasibility region is essential for developing optimal MAC strategies [1], [2], [3]. The structure of this set sheds light on when smart scheduling strategies can improve system performance. In particular, if the feasibility region is a convex set, then concurrent transmissions (no scheduling) are optimal strategies and the optimal power allocation can be obtained efficiently via a convex optimization. On the other hand, when the feasibility is not convex, the effect on scheduling is that optimal scheduling strategy may involve timesharing over subsets of simultaneously transmitting users.

This paper addresses the problems mentioned above and solves them completely in some important cases. We also address the problem of how QoS requirements influence the minimum total transmission power necessary to support them. Once this question is answered, it is possible to estimate the cost (in terms of additional power) for increasing QoS of the users. Our goal is to provide tools for better understanding the interrelationship between QoS requirements and physical quantities such as transmission power. Obviously, a solution to any of these problems depends on the choice of QoS parameters of interest (data rate, service delay, etc.) and the underlying system model as well as on what mechanisms can be used to improve system performance. In this paper, we consider a power-controlled Code Division Multiple Access (CDMA) channel. However, we point out that our results also apply to some other communications systems. We assume that each user is demodulated using a linear receiver structure and that there is a one-to-one monotonic relationship between a

QoS parameter of interest and the signal to interference ratio (SIR) at the output of the linear receiver. Signature sequences assigned to the users are fixed or chosen at random from the set of all possible sequences. Consequently, we say that QoS requirements are feasible if there exists a power allocation for which each user meets its QoS requirement with all users transmitting concurrently.

The problem of power control has received much attention in recent years (see [4] and [5] for references). Power control is a central mechanism for resource allocation and interference management in wireless systems. Dynamic allocation of power is a popular strategy to combat detrimental effects of multipath fading [6]. This paper generalizes some of the results of [5] and [7] and also extends them to channels whose inputs are subject to power constraints. In communication networks like the uplink channel, there are individual power constraints on each user rather than the total power constraint. We show that it is sufficient to solve the problem of admissibility of the users in a channel with a total power constraint, and then examine if the optimal power allocation satisfies the individual power constraints. Finally, we point out that the structure of the feasibility region for MIMO (multiple-input-multiple-output) systems with scheduling and successive interference cancellation was investigated in [8] and [9]. Reference [10] investigated the structure of the feasible SIR region under the linear minimum mean square error receiver.

The paper is organized as follows: Section II introduces the channel model and defines the feasibility region. In Section III, we assume no power constraints and prove a sufficient condition for convexity of the feasibility region. Furthermore, we derive a simple upper bound on feasible QoS requirements that clearly indicates the optimal tradeoff between them. Later in Section V, this bound turns out to be tight for the 2-user case. Section IV proves a sufficient condition for convexity of the feasibility region when there is a total power constraint. Moreover, we investigate the behavior of the minimum total transmission power when QoS requirements approach the boundary of the feasibility region for channels without power constraints. Finally, in Section V, we consider the 2 user case.

II. Channel Model and the Feasibility Region

We consider a $K-$user power-controlled CDMA system. Assuming no inter-symbol interference and a linear receiver structure, SIR at the output of the k-th receiver is of the form [11], [12]

$$\begin{aligned}\mathrm{SIR}_k(p) &:= \frac{V_{k,k}p_k}{\sum_{\substack{l=1\\l\neq k}}^K p_l V_{k,l} + C_k\sigma^2}\\ &= \frac{p_k}{\sum_{\substack{l=1\\l\neq k}}^K p_l \frac{V_{k,l}}{V_{k,k}} + \frac{C_k}{V_{k,k}}\sigma^2},\ 1\le k\le K\,.\end{aligned} \tag{1}$$

Here and hereafter, p_k is the power at which the user k transmits, the product $p_l V_{k,l} > 0$ incorporates the influence of the l-th user on the output of the k-th receiver, σ^2 is the variance of additive white Gaussian noise, and $C_k \geq 1$ depends on the structure of the linear receiver[1]. For instance, if a bank of matched-filter receivers is used [11], then $C_k = 1, 1 \le k \le K$. Otherwise, by the Cauchy-Schwartz inequality, we have $C_k > 1$. Each SIR is a function of the vector

$$p := (p_1, \ldots, p_K) \in \mathbb{R}_+^K\,,$$

which is referred to as a power allocation[2]. Since we assumed no inter-symbol interference, $V_{k,l}$ depends in general on such quantities as path attenuation, aperiodic crosscorrelations of signature sequences (which in turn depend on processing gains), and the relative time offset with respect to a common clock signal [11], [13], [14]. We point out that if any of these quantities is time varying, then one could consider average values of SIR by averaging $V_{k,l}$ with respect to the time varying variables. For instance, if there is no path attenuation and the relative time offsets are random variables uniformly distributed on $[-T,T)$, where T is the inverse symbol rate, then Reference [13] (see also [14]) derived an average $V_{k,l}$ under a matched-filter receiver.

In this paper, we assume that there is a one-to-one relationship between SIR and a QoS parameter of interest. Common examples of QoS parameters are data rate and service delay. Thus, meeting QoS requirements is equivalent to achieving certain SIR levels. To be more specific, suppose that $q_k \in \mathbb{G} \subseteq \mathbb{R}, 1 \le k \le K$, is a QoS requirement for the user k and define

$$\gamma_k := \gamma(q_k), \quad 1 \le k \le K\,, \tag{2}$$

where $\gamma : \mathbb{G} \to \mathbb{R}_+$ is a bijective positive real-valued function. Throughout the paper, γ_k and $\gamma(q_k)$ are used interchangeably. Since γ is bijective, γ_k is a minimum SIR level that is necessary to satisfy the QoS requirement q_k. We refer to the positive real numbers $\gamma_1, \ldots, \gamma_K$ as SIR requirements. Consequently, in order to guarantee a certain quality-of-service for the users, we need to ensure that

$$\gamma_k \le \mathrm{SIR}_k(p),\ 1 \le k \le K\,, \tag{3}$$

which is equivalent to

$$1 \le \min_{1\le k\le K} \frac{\mathrm{SIR}_k(p)}{\gamma_k} = \min_{1\le k\le K} \frac{\mathrm{SIR}_k(p)}{\gamma(q_k)}\,. \tag{4}$$

One of the central problems in wireless network design is the problem of admissibility [12]. The problem is whether or not there exists a power allocation $p \in \mathbb{R}_+^K$ for which (3) holds, where the power allocation is often limited by a total power constraint

$$\|p\|_1 = \sum_{k=1}^K p_k \le P_{tot} < +\infty\,. \tag{5}$$

Any such a power allocation (whether subject to (5) or not) is called a valid power allocation [12]. If a valid power allocation exists, we say that $q := (q_1, \ldots, q_K) \in \mathbb{G}^K$ is feasible. A subset of $\mathbb{G}^K$ of all QoS requirements that allow for a

[1] We assume that the inner product between the receiver of the user k and its signature sequence is equal to 1.

[2] $\mathbb{R}_+^K$ denotes a K-dimensional space of positive real-valued vectors

valid power allocation is called the feasibility region. We use $\mathbb{F}_\gamma(P_{tot})$ and $\mathbb{F}_\gamma$ to denote feasibility regions for systems with and without power constraints, respectively, where the subscript γ is used to emphasize the dependence on the function γ. In a special case $\gamma(x) = x, x > 0$, the feasibility region is usually referred to as the feasible SIR region. Note that this case is of practical interest since for small values of SIR, the data rate achieved by a user can be often modeled as a linear function of its SIR [2].

III. The feasibility region for channels without power constraints

A necessary and sufficient condition for $q \in \mathbb{G}^K$ to be feasible can be immediately obtained by maximizing the right-hand side of (4) with respect to all power allocations. However, if p is not subject to any power constraint, then the maximum does not need to exist since $\mathbb{R}_+^K$ is not a compact set. Indeed, the right-hand side of (4) has no maximum over $\mathbb{R}_+^K$, and hence, if there are no power constraints, $q \in \mathbb{G}^K$ is feasible if and only if

$$1 < \sup_{p\in\mathbb{R}_+^K} \min_{1\le k\le K} \frac{\mathrm{SIR}_k(p)}{\gamma(q_k)} . \tag{6}$$

We point out that in contrast to some other strategies, (6) ensures that all users meet their individual QoS requirements. The price for this is that (4) is not continuously differentiable so that smooth optimization methods such as the gradient method cannot be used to solve this problem. Considering Perron-Frobenius theory, however, we can rewrite (4) in terms of the spectral radius of a certain non-negative matrix. To this end, let V be a $K \times K$ matrix, whose k,l-th entry is given by

$$(V)_{k,l} = \begin{cases} \frac{V_{k,l}}{V_{k,k}} & 1 \le k,l \le K, k \ne l \\ 0 & \text{otherwise} \end{cases} . \tag{7}$$

Furthermore, let

$$\begin{aligned} \Gamma := \Gamma(q) &= \mathrm{diag}(\gamma_1, \dots, \gamma_K) \\ &= \mathrm{diag}(\gamma(q_1), \dots, \gamma(q_K)) . \end{aligned} \tag{8}$$

Now if V is irreducible, then, for any q with $\gamma(q_k) > 0$ for each $1 \le k \le K$, ΓV is irreducible as well. Recall that a matrix $A \in \mathbb{R}^{K\times K}$ is said to be reducible if there exists a permutation matrix P such that $P^T A P = \left(\begin{smallmatrix} X & Y \\ 0 & Z \end{smallmatrix}\right)$ where X and Z are both square matrices. Otherwise, A is said to be irreducible. Assuming that V is irreducible, Perron-Frobenius theory implies that $q \in \mathbb{G}^K$ is feasible if and only if [15]

$$\rho(q) := \rho(\Gamma(q)V) < 1 , \tag{9}$$

where $\rho(q)$ is used to denote the spectral radius of ΓV. Since V is fixed, in all that follows, we consider the spectral radius as a map from $\mathbb{G}^K$ into $\mathbb{R}_+$. By (9), the spectral radius determines the boundary of the feasibility region in channels without power constraints, and hence we have

$$\mathbb{F}_\gamma = \{q \in \mathbb{G}^K : \rho(q) < 1\} .$$

A. Log-convexity of the spectral radius

In the following, we prove that the spectral radius of ΓV is a log-convex function of QoS requirements if $\gamma : \mathbb{G} \to \mathbb{R}_+$ defined by (2) is a log-convex function on $\mathbb{G}$. To this end, let $\hat{q} = (\hat{q}_1, \dots, \hat{q}_K) \in \mathbb{G}^K$ and $\check{q} = (\check{q}_1, \dots \check{q}_K) \in \mathbb{G}^K$ are two arbitrary vectors of QoS requirements. Given $\hat{q}, \check{q} \in \mathbb{G}^K$, define

$$q(\alpha) = (1-\alpha)\hat{q} + \alpha\check{q}, \quad \alpha \in [0,1] .$$

In other words, $q(\alpha)$ describes a straight line connecting $\hat{q}$ and $\check{q}$. Now we are ready to prove the following theorem.

Theorem 1: Suppose that V is irreducible and fixed. If $\gamma : \mathbb{G} \to \mathbb{R}_+$ defined by (2) is log-convex[3], then $\rho : \mathbb{G}^K \to \mathbb{R}_+$ is log-convex as well. In other words, if

$$\log\gamma((1-\alpha)\hat{x} + \alpha\check{x}) \le (1-\alpha)\log\gamma(\hat{x}) + \alpha\log\gamma(\check{x}) , \tag{10}$$

for all $\hat{x}, \check{x} \in \mathbb{G}$ and $\alpha \in [0,1]$, then

$$\log\rho(q) \le (1-\alpha)\log\rho(\hat{q}) + \alpha\log\rho(\check{q}) \tag{11}$$

for all $\hat{q}, \check{q} \in \mathbb{G}^K$ and $\alpha \in [0,1]$.

Proof: See Appendix VII-A ■

An immediate consequence of the theorem is that the feasibility region is a convex set if γ is log-convex. To see this, it is convenient to rewrite (11) as

$$\rho(q) \le \rho(\hat{q})^{1-\alpha}\,\rho(\check{q})^\alpha,\ \alpha \in [0,1] . \tag{12}$$

Since the right-hand side of the inequality above is convex, we have $\max\{\rho(\hat{q}), \rho(\check{q})\} \ge \rho(q(\alpha)), \alpha \in [0,1]$. In fact, any positive log-convex function is a convex one [16]. Consequently, if $\hat{q}$ and $\check{q}$ are feasible, then $q(\alpha)$ with $\alpha \in [0,1]$ is feasible as well since

$$1 > \max\{\rho(\hat{q}), \rho(\check{q})\} \ge \rho(q(\alpha)) .$$

We summarize this observation in a corollary.

Corollary 1: Suppose that V is irreducible. If $\gamma : \mathbb{G} \to \mathbb{R}_+$ is log-convex, then $\mathbb{F}_\gamma$ is a convex set.

Subsequently, we present some applications of our results.

B. Some applications of the theorems

Theorem 1 requires that the function $\gamma : \mathbb{G} \to \mathbb{R}_+$ is log-convex. Consequently, the result is quite general in the sense that it applies to a wide range of interesting cases. The following list shows three examples of such a function.

1. Let $q_k = \log(\gamma_k), 1 \le k \le K$, so that $\gamma(x) = \exp(x), x \in \mathbb{G} = \mathbb{R}$. Obviously, the exponential function is log-convex on $\mathbb{R}$ since $(\log\exp(x))'' = x'' = 0$ for all $x \in \mathbb{R}$ [4]. Thus, by Theorem 1, $\rho(q)$ is log-convex on $\mathbb{R}^K$ when SIR requirements are expressed in logarithmic scale. Furthermore, by Corollary 1, the feasible log-SIR region is a convex set. This case is of great practical interest since $\frac{1}{2}\log(1 + \mathrm{SIR}_k(p)) \approx \frac{1}{2}\log(\mathrm{SIR}_k(p))$ when $\mathrm{SIR}_k(p) \gg$

[3] A function is log-convex if it is log-convex on its domain

[4] $(f(x))''$ denotes the second derivative of $f(x)$ with respect to x.

1. Thus, at large values of SIR, $\frac{1}{2}\log(\mathrm{SIR}_k(\gamma_k)$ is a reasonable approximation to the Shannon capacity (using independent decoding).

2. Another basic performance measure is the effective bandwidth of a user [12], [17]. Accordingly, users with given SIR requirements are admissible in a CDMA system if and only if the sum of their effective bandwidths is smaller than the processing gain of the system. The effective bandwidth of a user is a monotonic function of its SIR requirement and depends on the multiuser receiver employed. Assuming the linear MMSE receiver, the effective bandwidth of each user, say user k, becomes minimal and is equal to $\frac{\gamma_k}{1+\gamma_k}$ [17]. Now let $q_k = \log\frac{\gamma_k}{1+\gamma_k}, 1 \le k \le K$, and call the corresponding feasibility region the feasible log-EB region. In this case, we have
$$\gamma(x) = \frac{\exp(x)}{1-\exp(x)},\ x < 0\,,$$
and hence
$$\big(\gamma(x)\big)'' = \Big(\log\frac{\exp(x)}{1-\exp(x)}\Big)'' = \frac{\exp(x)}{(\exp(x)-1)^2}.$$
Since the right-hand side of this equation is positive for all $x < 0$, γ is log-convex on $\mathbb{G} = (-\infty, 0)$. Consequently, ρ is log-convex on $(-\infty, 0)^K$ and the feasible log-EB region is a convex set. We point out that a set of feasible effective bandwidths (expressed in logarithmic scale) under any linear receiver is a subset of the feasible log-EB region. However, in a special case when signature sequences assigned to the users form a Welch-Bound-Equality sequence set, the feasible log-EB region coincides with a set of feasible effective bandwidths under a scaled matched-filter receiver [12].

3. Another example is $q_k = \frac{B}{R_k\gamma_k}, 1 \le k \le K$, where R_k denotes a given data rate and B is the bandwidth, which is the same for all users. In this case, q_k is called the effective spreading gain (ESG) of the user k and is used as a performance measure in multi-rate CDMA channels [5]. For given data rates and fixed bandwidth, the objective is to minimize ESG of the users, which ensures the best possible quality-of-service expressed as a product of the data rate and SIR. Obviously, since $\gamma(x) = 1/x, x > 0$, is log-convex on $\mathbb{G} = \mathbb{R}_+$, we conclude that ρ is log-convex on $\mathbb{R}_+^K$ and the feasible ESG region is a convex set.

Figure 1 depicts $\rho(q(\alpha))$ as a function of $\alpha \in [0,1]$ for these three examples of the function $\gamma : \mathbb{G} \to \mathbb{R}_+$ and some given $\hat{q}, \check{q} \in \mathbb{G}^K$. We assumed that $V_{k,l} = 1/N$ for each $1 \le k, l \le K, k \ne l$. The simulation confirms the results of the theoretical analysis. Also note that in this example there exists $\alpha_0 \in (0,1)$ so that $\rho(q(\alpha)) > 1$ for all $\alpha > \alpha_0$, and hence in those cases the corresponding QoS requirements are not feasible.

Remarkably, an application of Theorem 1 shows that some interesting feasibility regions are convex sets. Unfortunately, since $\gamma(x) = x, x > 0$, is not log-convex on $\mathbb{R}_+$, Theorem 1 cannot be applied to the feasible SIR region. Actually, it is conjectured that the non-feasible SIR region is a convex set but this seems to be an open problem [18]. Since $\log\rho$ depends logarithmically on the SIR requirements of the users and $\gamma(x) = x, x > 0$, is a log-concave function on $\mathbb{R}_+$, one might be inclined to prove this conjecture by showing that $\rho(q)$ is log-concave. However, because of the supremum operator in (28), this approach cannot work. To see this, note that if γ is log-concave on $\mathbb{R}_+$, then
$$\log\gamma((1-\alpha)\hat{x} + \alpha\check{x}) \ge (1-\alpha)\log\gamma(\hat{x}) + \alpha\log\gamma(\check{x})$$
holds for all $\hat{x}, \check{x} \in \mathbb{R}_+$ and $\alpha \in [0,1]$. Proceeding essentially as in the proof of Theorem 1, we obtain
$$\log\rho(q(\alpha)) \ge \sup_{A\in\mathcal{S}}\Big[(1-\alpha)\Big(\sum_{\substack{k,l=1\\k\ne l}}^{K} u_k a_{k,l}\log\big(\frac{V_{k,l}\hat{\gamma}_l}{a_{k,l}}\big)\Big) + \alpha\Big(\sum_{\substack{k,l=1\\k\ne l}}^{K} u_k a_{k,l}\log\big(\frac{V_{k,l}\check{\gamma}_l}{a_{k,l}}\big)\Big)\Big]\,.$$
Now to prove the conjecture, one would have to exchange the supremum and sum but this, instead of a lower bound, always gives an upper bound. Consequently, log-concavity cannot be exploit to prove convexity of the non-feasible SIR region.

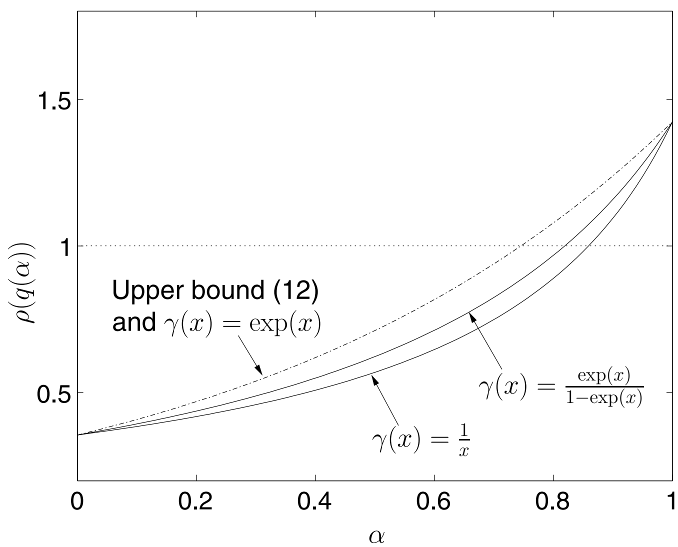

Fig. 1. The spectral radius $\rho(q(\alpha))$ as a function of $\alpha \in [0,1]$ for some given $\hat{q}$ and $\check{q}$.

C. An upper bound on feasible SIR requirements

The problem of convexity of the non-feasible SIR region is left open. Instead, we derive an upper bound on feasible SIR requirements, from which a superset of the feasible SIR region follows. The complement of this superset turns out to be a convex set. Obviously, this does not solve the original problem but the bound provides a convenient necessary condition for the feasibility of the SIR requirements. Note that using (2), this bound immediately yields an upper bound on feasible QoS requirements.

Representation (28) is a start point of our analysis. Let $\rho(V)$ be the spectral radius of V and $\hat{A}$ a doubly stochastic matrix so that

$$\log\rho(V) = \sum_{\substack{k,l=1\\k\neq l}}^{K} \hat{u}_k \hat{a}_{k,l} \log\big(\frac{V_{k,l}}{a_{k,l}}\big), \tag{13}$$

where $\hat{u} = (\hat{u}_1, \ldots, \hat{u}_K)$ is the left eigenvector of $\hat{A}$. Thus, when $\Gamma = I$, $\hat{A}$ is the matrix for which the supremum in (28) is attained. First we prove the following lemma.

Lemma 1: Suppose that V is irreducible. Let $\hat{A}$ and $\hat{u}$ be as above. Then, we have

$$\hat{u}_k = y_k \cdot x_k,\ 1 \le k \le K,$$

where y and x are the left and the right eigenvector of V, respectively, so that $y^T x = 1$ and

$$V x = \rho(V)\, x \qquad\qquad V^T y = \rho(V)\, y.$$

Proof: It follows from [19] that $\hat{A}$ is

$$(\hat{A})_{k,l} = \hat{a}_{k,l} = \frac{V_{k,l} x_l}{\rho(V)\, x_k}.$$

By definition, we have $\hat{A}^T u = u$. Combining this with the equation above yields

$$u_k = \sum_{l=1}^{K} \hat{a}_{l,k} u_k = \frac{1}{\rho(V)} \sum_{l=1}^{K} \frac{V_{l,k} x_l}{x_k} u_l,\ 1 \le k \le K,$$

or, equivalently,

$$\rho(V) \cdot \frac{u_k}{x_k} = \sum_{l=1}^{K} V_{l,k} \frac{u_l}{x_l},\ 1 \le k \le K.$$

Hence, the left eigenvector is $y_k = \frac{u_k}{x_k}$. Since $y^T x = 1$, we have $\sum_k u_k = 1$, and hence the proof is complete. ■

Now we are in a position to prove the upper bound.

Theorem 2: Let V be irreducible and assume that $\gamma(x) = x$. If $\gamma_1, \ldots, \gamma_K$ are feasible, then

$$\prod_{l=1}^{K} (\gamma_l)^{x_l y_l} < \frac{1}{\rho(V)}, \tag{14}$$

where, as in Lemma 1, y and x are the left and the right eigenvector of V, respectively.

Proof: Let $\hat{A}$ and $\hat{u}$ be defined by (13). Substituting $\hat{u}$ and $\hat{A}$ into (28) yields

$$\log\rho(q) \ge \sum_k \sum_{l\neq k} \hat{u}_k \hat{a}_{k,l} \log V_{k,l} + \sum_k \sum_{l\neq k} \hat{u}_k \hat{a}_{k,l} \log\gamma_l - \sum_k \sum_{l\neq k} \hat{u}_k \hat{a}_{k,l} \log\hat{a}_{k,l}$$

By (13), the first and the third term on the right hand is equal to $\log\rho(V)$ so that

$$\begin{aligned}\log\rho(q) &\ge \log\rho(V) + \sum_k \sum_{l\neq k} \hat{u}_k \hat{a}_{k,l} \log\gamma_l \\ &= \log\rho(V) + \sum_{l=1}^{K} \hat{u}_l \log\gamma_l \\ &= \log\rho(V) + \log\prod_{l=1}^{K} (\gamma_l)^{\hat{u}_l}.\end{aligned}$$

Hence, by Lemma 1,

$$\prod_{l=1}^{K} (\gamma_l)^{x_l y_l} \le \frac{\rho(q)}{\rho(V)}.$$

But, if $\gamma_1, \ldots, \gamma_K$ are feasible, we have $\rho(q) < 1$, and the theorem follows. ■

It may be easily verified that the complement of (14) is a convex set. Now consider a simple 2-user case with $V = \left(\begin{smallmatrix} 0 & \varrho \\ \varrho & 0 \end{smallmatrix}\right)$. Clearly, we have $\rho(V) = \varrho$ and $x = y = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. Thus, by Theorem 2, we obtain

$$\frac{1}{\sqrt{\gamma_1\gamma_2}} < \varrho\ .$$

It turns out (see Section V) that this bound is tight. Thus, in case of a channel with 2 users, Theorem 2 is a necessary and sufficient condition for the feasibility of SIR requirements.

The bound in (14) still requires the computation of the eigenvectors and the spectral radius. We show that if all non-diagonal elements of V are positive, this computation can be avoided completely at the expense of tightness of the bound. If $V_{k,l} > 0, 1 \le k, l \le K, k \neq l$, then we can substitute A given by

$$a_{k,l} = \begin{cases} \frac{1}{K-1} & k \neq l \\ 0 & k = l \end{cases}$$

into (28). Since the corresponding eigenvector of A is $u = (\frac{1}{K}, \ldots, \frac{1}{K})$, we obtain

$$\begin{aligned}\log\rho(q) &\ge \frac{1}{K(K-1)} \sum_k \sum_{l\neq k} \log V_{k,l} \\ &\quad - \frac{1}{K(K-1)} \sum_k \sum_{l\neq k} \log\frac{1}{K-1} \\ &\quad + \frac{1}{K(K-1)} \sum_k \sum_{l\neq k} \log\gamma_l \\ &= \frac{1}{K(K-1)} \sum_k \sum_{l\neq k} \log V_{k,l} \\ &\quad + \log(K-1) + \frac{1}{K} \sum_{l=1}^{K} \log\gamma_l\ .\end{aligned}$$

Now since (9) must hold, we conclude that if $\gamma_1, \ldots, \gamma_K$ are feasible, then

$$\prod_{l=1}^{K} (\gamma_l)^{\frac{1}{K}} < \frac{1}{(K-1)\prod_{\substack{k,l=1\\k\neq l}}^{K} (V_{k,l})^{\frac{1}{K(K-1)}}}. \tag{15}$$

Consequently, in this special case, the bound of Theorem 2 takes the simple form in (15). Note that neither the eigenvectors nor the spectral radius need to be calculated.

IV. The feasibility region for channels with power constraints

In case of CDMA channels with any power constraints, (6) or, equivalently, (9) is still a necessary condition but not a sufficient one. This is because we additionally need to satisfy power constraints. Thus, all the results obtained in the previous section do not carry over to channels in which there are power constraints. Throughout this section, we assume that p is subject to (5) for some given $P_{tot} > 0$. However, in view of many wireless communications networks, it is justified to ask if a total power constraint is sufficient to consider. In addition to a total power constraint, which is important to limit inter-cell interference and to reduce human exposure to electromagnetic radiation, users must often operate under individual power constraints. Alternatively, there can be only individual power constraints on each user. Later in this section, we show that it is sufficient to solve the problem subject to a total power constraint, and then verify whether or not the optimal solution satisfies individual power constraints. In contrast, the problem of convexity of the feasibility region under individual power constraints is not considered in this paper.

If there is a total power constraint as in (5), it is easy to see that the right-hand side of (4) attains its maximum on the subset of $\mathbb{R}^K_+$. Consequently, $q \in \mathbb{G}^K$ is feasible if and only if

$$1 \leq \max_{\substack{p\in\mathbb{R}^K_+ \\ \|p\|_1\leq P_{tot}}} \min_{1\leq k\leq K} \frac{\mathrm{SIR}_k(p)}{\gamma(q_k)} \,. \tag{16}$$

This max-min problem can be reformulated in terms of the minimum total power $P_{\mathrm{m}} : \mathbb{F}_\gamma \to \mathbb{R}_+$ defined to be

$$P_{\mathrm{m}}(q) := \min_{\substack{p\in\mathbb{R}^K_+ \\ \gamma(q_k)\leq \mathrm{SIR}_k(p), 1\leq k\leq K}} \|p\|_1 < +\infty \,. \tag{17}$$

In other words, P_{m} is a minimum total power for which all users satisfy their QoS requirements, and hence any power allocation p^* such that $\|p^*\|_1 = P_{\mathrm{m}}(q) < +\infty$ is called an optimal power allocation. Now we can conclude that $q \in \mathbb{G}^K$ is feasible if and only if the minimum total power $P_{\mathrm{m}}(q) < +\infty$ exists and satisfies the total power constraint

$$P_{\mathrm{m}}(q) \leq P_{tot} \,. \tag{18}$$

Furthermore, it follows that the feasibility region $\mathbb{F}_\gamma(P_{tot})$ is

$$\mathbb{F}_\gamma(P_{tot}) = \{q \in \mathbb{G}^K : P_{\mathrm{m}}(q) \leq P_{tot}\} \,.$$

Note that for a given P_{tot} the boundary of the feasibility region follows from $P_{\mathrm{m}}(q) = P_{tot}$.

A. Characterization of the optimal power allocation

Now we identify the optimal power allocation p^*. Using (1), we can rewrite (23) to obtain

$$p^*_k - \gamma_k \sum_{l=1}^{K} p^*_l \frac{V_{k,l}}{V_{k,k}} = \sigma^2 \gamma_k \frac{C_k}{V_{k,k}},\ 1 \leq k \leq K.$$

In the matrix form, this becomes

$$(I - \Gamma V)p = \sigma^2 \Gamma C 1, \tag{19}$$

where 1 denotes the vector of K ones and $C := \mathrm{diag}\big(C_1/V_{1,1}, \ldots, C_K/V_{K,K}\big)$. Consequently, an optimal power allocation must be a positive solution to (19). Obviously, there exists a positive solution to (19) if and only if (9) holds, and then the solution is unique [15]. Thus, if (9) is satisfied, the optimal power allocation p^* exists and follows from (19) to yield

$$p^* = \sigma^2 (I - \Gamma V)^{-1} \Gamma C 1. \tag{20}$$

B. Feasibility conditions in systems with individual power constraints

As already mentioned, in addition to a total power constraint, there are sometimes individual power constraints on each user. In such a case, $q \in \mathbb{G}^K$ is feasible if and only if

$$1 \leq \max_{\substack{p\in\mathbb{R}^K_+ \\ p_k\leq P_k, 1\leq k\leq K \\ \|p\|_1\leq P_{tot}}} \min_{1\leq k\leq K} \frac{\mathrm{SIR}_k(p)}{\gamma(q_k)} \tag{21}$$

for some given $P_k, 1 \leq k \leq K$. Comparing (16) with (21) reveals that in the latter case, we additionally need to satisfy $p_k \leq P_k$ for each $1 \leq k \leq K$, where P_k is an individual power constraint of the user k. Alternatively, one can drop the total power constraint and require the individual power constraints be satisfied. In this case, $q \in \mathbb{G}^K$ is feasible if and only if

$$1 \leq \max_{\substack{p\in\mathbb{R}^K_+ \\ p_k\leq P_k, 1\leq k\leq K}} \min_{1\leq k\leq K} \frac{\mathrm{SIR}_k(p)}{\gamma(q_k)} \,. \tag{22}$$

Now consider the following theorem.

Theorem 3: Problem (21) has a solution if and only if the optimal power allocation p^* with $\|p^*\|_1 \leq P_{tot}$ exists and $p^*_k \leq P_k$ for each $1 \leq k \leq K$. Problem (22) has a solution if and only if p^* with $\|p^*\|_1 < +\infty$ exists and $p^*_k \leq P_k$ for each $1 \leq k \leq K$.

Proof: It may be easily verified that if $\|p^*\|_1 = P_{\mathrm{m}}(q)$, then [20]

$$\gamma_k = \mathrm{SIR}_k(p^*),\ 1 \leq k \leq K. \tag{23}$$

Now let $p^\dagger \in \mathbb{R}^K_+$ and $p^\ddagger \in \mathbb{R}^K_+$ be optimal solutions of (21) and (22), respectively. Then, we have $\gamma_k = \mathrm{SIR}_k(p^\dagger) = \mathrm{SIR}_k(p^\ddagger), 1 \leq k \leq K$, as well. This is because if we had $\gamma_k < \mathrm{SIR}_k(p^\dagger)$ for some $1 \leq k \leq K$, we could allocate $\tilde{p} = (\tilde{p}_1, \ldots, \alpha\tilde{p}_k, \ldots, \tilde{p}_K)$ with $\alpha = \gamma_k/\mathrm{SIR}_k(p^\dagger) < 1$ to obtain $\|\tilde{p}\|_1 < \|p^\dagger\|_1$. The same reasoning applies to $p^\ddagger$. Thus, $p^*, p^\dagger$ and $p^\ddagger$ are solutions to the same system of linear equations. By Perron-Frobenius theory, we know that a positive solution of this system of linear equations is unique, and hence if $p^\dagger$ and $p^\ddagger$ exist so also does p^*, and conversely. ■

C. Log-convexity of the minimum total power

By the previous discussion, we know that if $\rho(q) < 1$ is true, $P_{\rm m}(q) = \|p^*\|_1 < +\infty$ exists and is given by

$$P_{\rm m}(q) = \sigma^2 1^T (I - \Gamma V)^{-1} \Gamma C 1 \,. \tag{24}$$

Recall that $\Gamma = \Gamma(q)$ depends on q. It is worth pointing out that in contrast to channels without power constraints, there is no need that ΓV is primitive.

Now we address the problem of convexity of the minimum total power $P_{\rm m} : \mathbb{F}_\gamma \to \mathbb{R}_+$ defined by (17). The following result shows that if $\gamma : \mathbb{G} \to \mathbb{R}_+$ is log-convex, then $P_{\rm m}$ is a log-convex function on $\mathbb{F}_\gamma$.

Theorem 4: Suppose that $\gamma : \mathbb{G} \to \mathbb{R}_+$ is log-convex. Then, $P_{\rm m} : \mathbb{F}_\gamma \to \mathbb{R}_+$ is log-convex so that

$$P_{\rm m}(q) \le P_{\rm m}(\hat{q})^{1-\alpha} P_{\rm m}(\check{q})^{\alpha} \tag{25}$$

for all $\hat{q}, \check{q} \in \mathbb{F}_\gamma$ and $\alpha \in [0,1]$.

Proof: The theorem was proven in [18]. For completeness, we included the proof in Appendix VII-B. ■

Just as in case of Theorem 1, Theorem 4 requires that γ defined by (2) be log-convex. Consequently, the minimum total power $P_{\rm m}$ is a log-convex function on the feasible log-SIR region, the feasible log-EB region and the feasible ESG region (see Section III-B). Figure 2 depicts $P_{\rm m}(q(\alpha))$ as a function of $\alpha \in [0,1]$ for these three examples of the function γ.

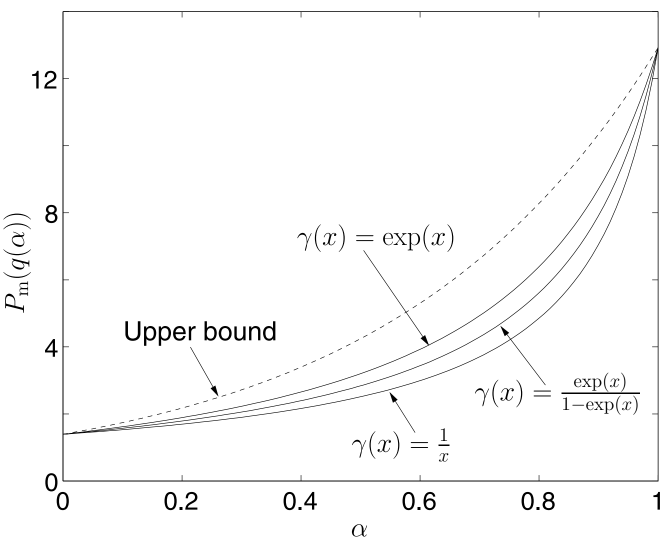

Fig. 2. The minimum total power $P_{\rm m}(q(\alpha))$ as a function of $\alpha \in [0,1]$ for some given $\hat{q}$ and $\check{q}$. We have $V_{k,l} > 0, 1 \le k,l \le K, k \ne l$.

In [18], we strengthened some of these results. In particular, we showed that $P_{\rm m}$ is strictly log-convex if $\gamma(x) = 1/x, x > 0$, and all the non-diagonal elements of V are positive. Furthermore, it was shown that the minimum total power is strictly log-convex if $\gamma(x) = \exp(x), x > 0$. Note that Figure 2 confirms these analytical results.

As an immediate consequence of Theorem 4, we have the following corollary

Corollary 2: Let V and P_{tot} be given. If $\gamma : \mathbb{G} \to \mathbb{R}_+$ is log-convex, then $\mathbb{F}_\gamma(P_{tot})$ is a convex set.

Proof: As the right-hand side of (25) is convex for all $\hat{q}, \check{q} \in \mathbb{F}_\gamma$, one obtains

$$P_{\rm m}(q(\alpha)) \le \max\{P_{\rm m}(\hat{q}), P_{\rm m}(\check{q})\} \le P_{tot}$$

for all $\alpha \in [0,1]$. ■

Since $P_{\rm m}$ is strictly log-convex if $\gamma(x) = \exp(x), x > 0$, the feasible log-SIR region is a strictly log-convex set. The same applies to the feasible ESG region if $V_{k,l} > 0, 1 \le k,l \le K, l \ne k$. Figure 3 shows the feasible log-SIR regions for different total power constraints.

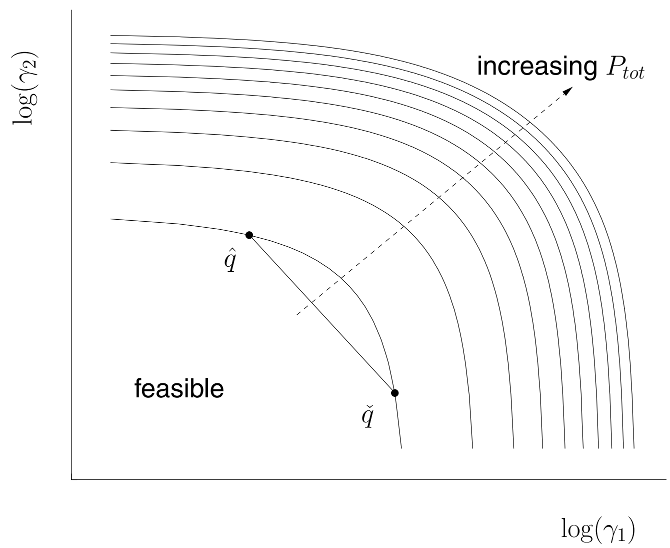

Fig. 3. The feasible log-SIR region for a 2-user CDMA channel with different total power constraints P_{tot}

D. Behavior of the minimum total power in a neighborhood of the boundary

Now we aim at investigating the behavior of the minimum total power $P_{\rm m}$ in a neighborhood of the boundary of $\mathbb{F}_\gamma$. To be more specific, we are interested in the order of divergence of $P_{\rm m}(\xi q)$ when ξq tends to the boundary of $\mathbb{F}_\gamma$. Indeed, it is easy to see that if V is irreducible, then we have

$$P_{\rm m}(\xi q) \to +\infty \quad \text{as} \quad \xi \to \frac{1}{\rho(q)}.$$

The problem of how fast the minimum total power tends to infinity may be relevant for some practical applications. Once the problem is solved, it is possible to estimate the cost (in terms of additional power) for improving QoS of the users in the vicinity of the boundary of $\mathbb{F}_\gamma$. This additional knowledge can be exploited when designing wireless communication networks, where power consumption is not an issue and the main objective is to guarantee the best possible performance for each user. Clearly, such networks must operate in a neighborhood of the boundary of $\mathbb{F}_\gamma$.

Assume that V is irreducible and note that $\mathbb{F}_\gamma$ is an open set. This immediately follows from the fact that Γ depends continuously on the QoS requirements and the spectral radius is a continuous function of the matrix elements. Thus, since the concatenation of continuous functions is continuous, the feasibility region must be an open set. Let $\overline{\mathbb{F}}_\gamma = \partial\mathbb{F}_\gamma \cup \mathbb{F}_\gamma$ be the closure of $\mathbb{F}_\gamma$ so that $\partial\mathbb{F}_\gamma$ denotes the boundary of $\mathbb{F}_\gamma$:

$$\partial\mathbb{F}_\gamma := \{q \in \mathbb{G}^K : C(q) = 1\},$$

where, for any fixed V,

$$C(q) := \frac{1}{\rho(q)}. \tag{26}$$

The quantity $C(q)$ is nothing else but the supremum in (6). Furthermore, since V is irreducible,

$$\begin{aligned} C(q) &= \sup_{p \in \mathbb{R}_+^K} \min_{1\le k\le K} \frac{\mathrm{SIR}_k(p)}{\gamma(q_k)} \\ &= \max_{\sum_k p_k = 1} \min_{1\le k\le K} \frac{p_k}{\gamma_k \sum_{l\ne k} p_l V_{k,l}}, \end{aligned}$$

where, without loss of generality, we assumed that $\sum_k p_k = 1$. This simply follows from the fact that $\mathrm{SIR}_k(\mu p) = \mathrm{SIR}_k(p)$ for any $\mu > 0$ in noiseless channels ($\sigma^2 = 0$). $C(q)$ is important for system analysis revealing which QoS requirements are feasible in channels without power constraints. By Section III-A, we know that if there are no power constraints, q is feasible if and only if $C(q) > 1$. Finally, let

$$\sigma(q) = \{\lambda_1(q), \lambda_2(q), \ldots, \lambda_p(q)\}$$

be the spectrum of the matrix $\Gamma V = \Gamma(q)V$. Without loss of generality, assume that $|\lambda_1(q)| = \rho(q)$ is equal to the spectral radius of ΓV. Thus, since ΓV is irreducible, $\lambda_1(q)$ is a simple positive real eigenvalue. First consider the following lemma.

Lemma 2: Suppose that V is a given irreducible matrix. Let $q \in \mathbb{F}_\gamma$ be arbitrary and fixed. Then, there exists a matrix $R(q) := R(\Gamma(q)V)$ so that

$$\begin{aligned} P_\mathrm{m}(q) = \sigma^2 &\frac{C(q)}{C(q)-1} \cdot 1^T x(q)\, y(q)^T \Gamma C 1 \\ &+ \sigma^2 1^T R(q) \Gamma C 1, \end{aligned} \tag{27}$$

where $x(q)$ and $y(q)$ are the right and left eigenvector of ΓV.

Proof: By [21], we have

$$\begin{aligned} (I - \Gamma V)^{-1} &= \frac{1}{1-\rho(q)} Z_{1,1} \\ &+ \underbrace{\sum_{k=2}^{p} \sum_{j=1}^{m_k} \frac{(j-1)!}{(1-\lambda_k(q))^j} Z_{k,j}}_{R(q)}, \end{aligned}$$

where the matrices $Z_{k,j}, j = 1, \ldots, m_k$ and $k = 1, \ldots, p$ are known as the principal component matrices. Furthermore, one has [21]

$$Z_{1,1} = x(q)\, y(q)^T,$$

from which the lemma follows. ■

The next lemma shows that the second sum term on the right-hand side of (27) is bounded.

Lemma 3: Let V be fixed. Suppose that $\epsilon > 0$ is chosen so that

$$U_\epsilon = \{q \in \mathbb{F}_\gamma : \gamma(q_k) \ge \epsilon, 1 \le k \le K\}$$

is not an empty set. Then, for all $q \in U_\epsilon$, there exists $\delta > 0$ so that

$$\lambda_2(q), \ldots \lambda_p(q) \notin \{|z| \le 1 : |z - 1| < \delta\}.$$

Proof: See Appendix VII-C ■

Now we are in a position to prove the limit of the product $P_\mathrm{m}(q) \cdot (C(q) - 1)$ as $q \to \hat{q}$ with $q \in L(\hat{q})$, where $\hat{q} \in \partial\mathbb{F}_\gamma$ is arbitrary and $L(\hat{q})$ is any curve in U_ϵ that ends at the point $\hat{q}$. The situation is illustrated in Figure 4 for a channel with 2 users.

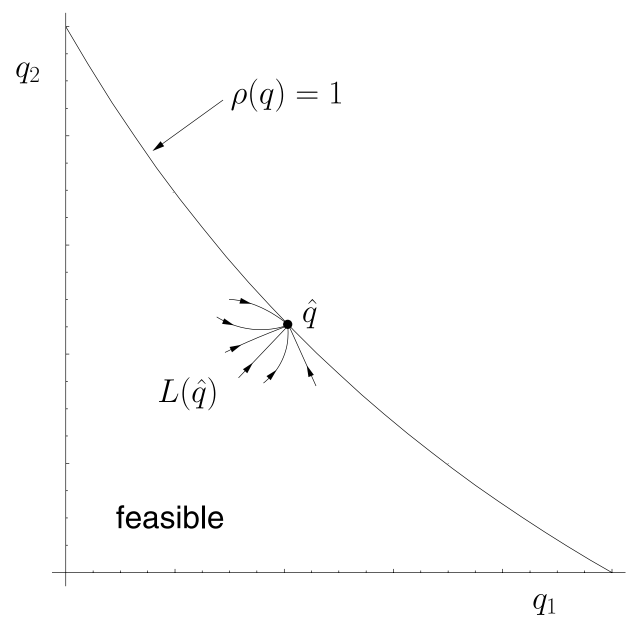

Fig. 4. Examples of different curves $L(\hat{q})$ that end at $\hat{q} \in \partial\mathbb{F}_\gamma$

.

Theorem 5: Let $\hat{q}$ and $L(\hat{q})$ be as above. Then, we have

$$\lim_{\substack{q\to\hat{q} \\ q \in L(\hat{q})}} P_\mathrm{m}(q) \cdot (C(q) - 1) = \sigma^2 1^T x(\hat{q}) \cdot y(\hat{q})^T \hat{\Gamma} C 1,$$

where $x(\hat{q})$ and $y(\hat{q})$ are the right and left eigenvectors of $\hat{\Gamma} V$, respectively, and $\hat{\Gamma} = \Gamma(\hat{q})$.

Proof: By Lemma 3, the matrix norm of $R(q)$ is uniformly bounded for any $q \in L(\hat{q})$ sufficiently close to $\hat{q}$. Thus, with $\lim_{q\to\hat{q}} C(q) = 1$, we have

$$\lim_{\substack{q\to\hat{q} \\ q \in L(\hat{q})}} \sigma^2 (C(q) - 1) 1^T R(q) \Gamma(q) C 1 = 0$$

The theorem follows from the fact that $x(q)$ and $y(q)$ are continuous functions of q. ■

In other words, Theorem 5 implies that if q is sufficiently close to $\hat{q} \in \partial\mathbb{F}_\gamma$, then

$$P_{\mathrm{m}}(q) \approx \frac{\sigma^2}{C(q)-1} 1^T x(\hat{q}) \cdot y(\hat{q})^T \hat{\Gamma} C 1 \,.$$

V. The 2-user case

In case of a CDMA system with 2 users, the minimum total power P_{m} can be computed explicitly as

$$\begin{aligned} P_{\mathrm{m}}(q) &= \sigma^2 \frac{\gamma(q_1)+\gamma(q_2)+2\varrho\gamma(q_1)\gamma(q_2)}{1-\varrho^2\gamma(q_1)\gamma(q_2)} \\ &= \sigma^2 \frac{\gamma_1+\gamma_2+2\varrho\gamma_1\gamma_2}{1-\varrho^2\gamma_1\gamma_2}\,, \end{aligned}$$

where, without loss of generality, we assumed that $\varrho = V_{1,2} = V_{2,1}$ and $C_1 = C_2 = 1$. The characteristic polynomial for the matrix ΓV is $p(\lambda) = \lambda^2 - \varrho^2\gamma_1\gamma_2$. Hence, P_{m} is bounded if and only if $\rho(q) = \varrho\sqrt{\gamma_1\gamma_2} < 1$.

In this simple case of 2 users, the non-feasible SIR region (denoted by $\mathbb{F}^c$) is a convex set (see the discussion at the end of Section III-B). To see this, it is sufficient to show that $P_{\mathrm{m}}(q) = P_{tot}$ or, equivalently,

$$\gamma_2(\gamma_1) = \frac{\mathrm{SNR}-\gamma_1}{1+2\varrho\gamma_1+\varrho^2\gamma_1\mathrm{SNR}}$$

is a convex function, where $\mathrm{SNR} = \frac{P_{tot}}{\sigma^2}$. It may be easily verified that

$$\gamma_2'(\gamma_1) = \frac{-(1+\varrho\mathrm{SNR})^2}{(1+\varrho(2+\varrho\mathrm{SNR})\gamma_1)^2}.$$

Thus, since the numerator is independent of γ_1 and the denominator is increasing in $\gamma_1 > 0$, we have $\gamma_2''(\gamma_1) \geq 0$ for every $\gamma_1 > 0$. From this, it follows that γ_2 is a convex function of γ_1, and hence $\mathbb{F}^c$ is a convex set. Furthermore, if $\varrho > 0$, then $\gamma_2''(\gamma_1) > 0$ for every $\gamma_1 > 0$ in which case $\mathbb{F}^c$ becomes a strictly convex set. Assuming $\mathrm{SNR} = 1$ and $\varrho = 1/2$, Figure 5 shows the resulting feasible SIR region.

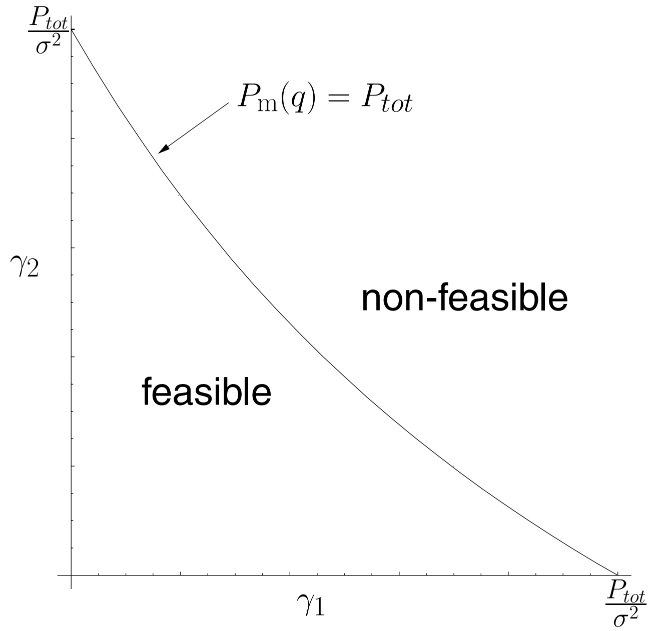

Fig. 5. The feasible SIR region ($\gamma(x) = x, x > 0$) for the 2-user case ($\mathrm{SNR} = 1, \varrho = 1/2$)

VI. Conclusions

The paper investigated a CDMA system with a linear receiver structure. We considered a network-centric problem where a set of transmission powers (called a power allocation) must be found so that each user satisfies its QoS requirement. We assumed that there is a one-to-one monotonic relationship between a QoS parameter of interest (data rate, service delay, etc.) and the signal-to-interference ratio (SIR) at the output of the linear receiver. For instance, under certain conditions, the data rate achieved by a user is a strictly increasing function of its SIR [1], [2]. We say that QoS requirements are feasible if there exists a power allocation for which each user meets its QoS requirement with all users being active concurrently. A set of feasible QoS requirements is called the feasibility region.

The structure of the feasibility region is of great interest since it shows whether and when scheduling strategies can improve network performance. In particular, if the feasibility region is a convex set, then concurrent transmission strategies are optimal and the optimal power allocation can be obtained efficiently via convex optimization techniques. The main results of this paper show that if SIR of each user is a log-convex function of its QoS parameter, then the feasibility region is a convex set regardless whether there are power constraints or not. In Section III-B, we presented some applications of these results. In particular, when QoS parameters depend logarithmically on SIR, the feasibility region (the so-called feasible log-SIR region) is a convex set. Note that at large values of SIR, the data rate achieved by a user can be often modeled as a logarithmic function of its SIR. Consequently, our results imply that concurrent transmission strategies should be used at large values of SIR when throughput performance of the users is considered.

In contrast, at small values of SIR, the data rate usually depends linearly on SIR. In this case, the feasibility region (the so-called feasible SIR region) is not convex, and hence scheduling strategy should involve timesharing over subsets of simultaneous transmitting users. We conjecture that the non-feasible SIR region is a convex set. In Section V, this was shown to be true for the 2-user case. If this conjecture was true in general, then timesharing between single-user transmissions would be an optimal scheduling strategy.

In Section III-C, we derived an upper bound on feasible QoS requirements. This bound, which turned out to be tight in the 2-user case, indicates the optimal tradeoff between QoS requirements. Note that the necessary and sufficient conditions in (6) and (9) do not reveal the dependence between QoS requirements under the optimal power allocation. For instance, QoS requirements go into (9) through the spectral radius of a certain non-negative matrix. Thus, by means of (9), it is quite difficult to say how they depend on each other.

In Section IV, we considered a CDMA system whose inputs are subject to a total power constraint. Interestingly, it turns out that it is sufficient to consider a channel with a total power constraint even if there are individual power constraints on each user. Finally, we investigated the behavior of the minimum total power when QoS requirements approach the boundary of the feasibility region for channels without power constraints. Clearly, when the coupling matrix is irreducible, then the minimum total power tends to infinity. The paper shows how fast the minimum total power tends to infinity when QoS requirements are sufficiently close to the boundary.

VII. Appendix

A. Proof of Theorem 1

Recall that $\gamma_k = \gamma(q_k), 1 \le l \le K$. By [19], we have

$$\log\rho(q) = \sup_{A\in\mathcal{S}}\Big(\sum_{\substack{k,l=1\\k\neq l}}^{K} u_k a_{k,l}\log V_{k,l} + \sum_{\substack{k,l=1\\k\neq l}}^{K} u_k a_{k,l}\log\gamma_l - \sum_{\substack{k,l=1\\k\neq l}}^{K} u_k a_{k,l}\log a_{k,l}\Big) \quad (28)$$

where $\mathcal{S} := \mathcal{S}(V)$ is a set of all doubly stochastic matrices so that $a_{k,l} = 0 \Leftrightarrow V_{k,l} = 0$ for each $A \in \mathcal{S}$ and $u = (u_1, \ldots, u_K)$ is the left Perron eigenvector of A with $\sum_{k=1}^{K} u_k = 1$. Notice that $\mathcal{S}$ depends on the choice of V. Combining (28) with (10) yields

$$\begin{aligned}
\log\rho(q(\alpha) &\le \sup_{A\in\mathcal{S}}\Big[\sum_k\sum_{l\neq k} u_k a_{k,l}\log V_{k,l} \\
&+ (1-\alpha)\sum_k\sum_{l\neq k} u_k a_{k,l}\log\hat{\gamma}_l \\
&+ \alpha\sum_k\sum_{l\neq k} u_k a_{k,l}\log\check{\gamma}_l - \sum_k\sum_{l\neq k} u_k a_{k,l}\log a_{k,l}\Big] \\
&= \sup_{A\in\mathcal{S}}\Big[(1-\alpha)\Big(\sum_k\sum_{l\neq k} u_k a_{k,l}\log(V_{k,l}\hat{\gamma}_l) \\
&- \sum_k\sum_{l\neq k} u_k a_{k,l}\log a_{k,l}\Big) \\
&+ \alpha\Big(\sum_k\sum_{l\neq k} u_k a_{k,l}\log(V_{k,l}\check{\gamma}_l) - \sum_k\sum_{l\neq k} u_k a_{k,l}\log a_{k,l}\Big)\Big] \\
&\le (1-\alpha)\sup_{A\in\mathcal{S}}\Big(\sum_k\sum_{l\neq k} u_k a_{k,l}\log(V_{k,l}\hat{\gamma}_l) \\
&- \sum_k\sum_{l\neq k} u_k a_{k,l}\log a_{k,l}\Big) \\
&+ \alpha\sup_{A\in\mathcal{S}}\Big(\sum_k\sum_{l\neq k} u_k a_{k,l}\log(V_{k,l}\check{\gamma}_l) \\
&- \sum_k\sum_{l\neq k} u_k a_{k,l}\log a_{k,l}\Big) \\
&= (1-\alpha)\log\rho(\hat{q}) + \alpha\log\rho(\check{q}),
\end{aligned}$$

where we used $\hat{\gamma}_l = \gamma(\hat{q}_l)$ and $\check{\gamma}_l = \gamma(\check{q}_l)$.

B. Proof of Theorem 4

Let $\Gamma(\alpha) = \Gamma(q(\alpha))$. By assumption, we have $\hat{q}, \check{q} \in \mathbb{F}_\gamma$. Thus, by Theorem 1, $P_{\rm m}(q(\alpha)) < +\infty$ exists for every $\alpha \in [0,1]$ and we can expand $(I - \Gamma(\alpha)V)^{-1}$ into a Neumann series to obtain

$$(I - \Gamma(\alpha)V)^{-1} = \sum_{l=0}^{\infty}\big(\Gamma(\alpha)V\big)^l .$$

From this it follows that

$$\begin{aligned}
P_{\rm m}(q(\alpha)) &= \sigma^2 1^T\sum_{l=0}^{\infty}(\Gamma(\alpha)V)^l\Gamma(\alpha)C1 \\
&= \sigma^2\sum_{l=0}^{\infty} 1^T\big[(\Gamma(\alpha)V)^l\Gamma(\alpha)C\big]1 \\
&= \sigma^2\sum_{l=0}^{\infty} g_l\big(\Gamma(\alpha)\big) .
\end{aligned}$$

Examining the matrix $\Gamma(\alpha)V$ reveals that its entries are of the form

$$(\Gamma(\alpha)V)_{k,l} = \gamma\big[(1-\alpha)\hat{q}_k + \alpha\check{q}_k\big]V_{k,l},\ 1 \le k,l \le K .$$

Consequently, if $V_{k,l} = 0$, one has $(\Gamma(\alpha)V)_{k,l} = 0$. Otherwise, if $V_{k,l} > 0$, then the entries are log-convex functions since $\gamma : \mathbb{G} \to \mathbb{R}_+$, by assumption, is log-convex. Now consider the following properties of log-convex functions:

(i) If two positive functions f and g are log-convex, then $f + g$ and $f \cdot g$ are log-convex

(ii) If f is log-convex and $n > 0$, then f^n is log-convex.

(iii) For any convergent sequence f_n of log-convex functions, the limit $f = \lim_{n\to\infty} f_n$ is log-convex provided that the limit is strictly positive.

Due to (ii), $g_l : \mathbb{G} \to \mathbb{R}_+$ is log-convex for each $l \ge 0$. By (i), $\sum_{l=0}^{M} g_l$ is log-convex for any $M > 0$. Furthermore, since $\sum_{l=0}^{M} g_l$ is a strictly increasing function, it must converge to a positive limit as $M \to +\infty$, and hence, by (iii), $P_{\rm m}$ is log-convex.

C. Proof of Lemma 3

Assume that the lemma is not true. Then, there exists $1 \le k_0 \le p$ and a sequence $\{q(n)\}_{n\in\mathbb{N}}$ with $q(n) = (q_1(n), \ldots, q_K(n)) \in U_\epsilon$ so that

$$|1 - \lambda_{k_0}(q(n))| \le 1/n,\ n \in \mathbb{N} .$$

Since $q_k(n)$ is bounded independent of $1 \le k \le K$ and $n \in \mathbb{N}$, there exists a subsequence $\{q(n_k)\}_{k\in\mathbb{N}}$ of $\{q(n)\}_{n\in\mathbb{N}}$ such that $\{q(n_k)\}_{k\in\mathbb{N}}$ uniformly converges to some $q^* \in U_\epsilon$. Because of the uniform convergence, one obtains

$$\lim_{k\to\infty}\sup|1 - \lambda_{k_0}(q(n_k))| = 0 ,$$

and hence

$$\lim_{k\to\infty}\lambda_{k_0}(q(n_k)) = \lambda_{k_0}(q^*) = 1.$$

Now since $|\lambda_{k_0}(q^*)| \le |\lambda_1(q^*)|, q \in U_\epsilon$, we have

$$\lim_{k\to\infty}\lambda_{k_0}(q(n_k)) = \lim_{k\to\infty}\lambda_1(q(n_k)) = 1.$$

This, however, contradicts the fact that $\lambda_1(q^*)$ is a simple eigenvalue since ΓV is irreducible.

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