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The “optical lever” intracavity readout scheme for gravitational-wave antennae

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Abstract

An improved version of the “optical bar” intracavity readout scheme for gravitational-wave antennae is considered. It can be called “optical lever” because it can provide significant gain in the signal displacement of the local mirror similar to the gain which can be obtained using ordinary mechanical lever with unequal arms. In this scheme displacement of the local mirror can be close to the signal displacement of the end mirrors of hypothetical gravitational-wave antenna with arm lengths equal to the half-wavelength of the gravitational wave. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

All contemporary large-scale gravitational-wave antennae are based on common principle: they convert phase shift of the optical pumping field into the intensity modulation of the output light beam being registered by photodetector [1]. This principle allows to obtain sensitivity necessary to detect gravitational waves from astrophysical sources. However, its use in the next generations of gravitational-wave antennae where substantially higher sensitivity is required, encounters serious problems.

An very high value of optical pumping power which also depends sharply on the required sensitivity, is likely to be the most important one. For example, at the stage II of the LIGO project the light power circulating in the interferometer arms will be increased

to about 1 MW [2]. In particular, so high values of the optical power can produce undesirable non-linear effects in the large-scale Fabry–Perot cavities [3].

This dependence of pumping power on the sensitivity can be explained easily using the Heisenberg uncertainty relation. Really, in order to detect displacement Δx of test mass M it is necessary to provide perturbation of its momentum $\Delta p \geq \hbar/2\Delta x$. The only source of this perturbation in the interferometric gravitational-wave antennae is the uncertainty of the optical pumping energy $\Delta \mathcal{E}$. Hence, the following conditions have to be fulfilled: $\Delta \mathcal{E} \propto (\Delta x)^{-1}$. If pumping field is in the coherent quantum state then $\Delta \mathcal{E} \propto \sqrt{\mathcal{E}}$, and therefore $\mathcal{E} \propto (\Delta x)^{-2}$.

Rigorous analysis (see [4]) shows that pumping energy stored in the interferometer have to be larger than

$$\mathcal{E} = \frac{ML^2\Omega^2\Delta\Omega}{4\omega_p\xi^2}, \quad (1)$$

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where Ω is the signal frequency, $\Delta\Omega < \Omega$ is the bandwidth where necessary sensitivity is provided, ω_p is the pumping frequency, L is the length of the interferometer arms, $\xi < 1$ is the ratio of the amplitude of the signal which can be detected to the amplitude corresponding to the standard quantum limit.

This problem can be alleviated by using optical pumping field in squeezed quantum state [5], but cannot be solved completely, because only modest values of squeezing factor have been obtained experimentally yet. Estimates show that usage of squeezed states allows to decrease ξ by the factor of $\simeq 3$ for the same value of the pumping energy (see [6]), and the energy still remains proportional to ξ^{-2} .

In the article [7] the new principle of *intracavity* readout scheme for gravitational-wave antennae has been considered. It has been proposed to register directly redistribution of the optical field *inside* the optical cavities using quantum non-demolition (QND) measurement instead of monitoring output light beam.

The main advantage of such a measurement is that in this case a non-classical optical field is created by the measurement process automatically. Therefore, sensitivity of these schemes does not depend directly on the circulating power and can be improved by increasing the precision of the intracavity measurement device. The only fundamental limitation in this case is the condition $\Delta\mathcal{E} \leq \mathcal{E}$, which leads to the inequality

$$\frac{\Delta x}{L} \gtrsim \frac{\Omega}{\omega_p N}, \quad (2)$$

where N is the number of optical quanta in the antenna (see also Eqs. (1)–(3) of the paper [9]).

In the articles [8,9] two possible realizations of this principle have been proposed and analyzed. Both of them are based on the pondermotive QND measurement of the optical energy proposed in the article [10]. In these schemes displacement of the end mirrors of the gravitational-wave antenna caused by the gravitational wave produces redistribution of the optical energy between the two arms of the interferometer. This redistribution, in its turn, produces variation of the electromagnetic pressure on some additional local mirror (or mirrors). This variation can be detected by measurement device which monitors position of the local mirror(s) relative to reference mass placed outside the optical pumping field (for example, a small-

scale optical interferometric meter can be used as such a meter).

The optical pumping field works here as a passive medium which transfers the signal displacement of the end mirrors to the displacement of the local one(s) and, at the same time, transfers perturbation of the local mirror(s) due to measurement back to the end mirrors.

In this Letter an improved version of the “optical bar” scheme considered in the article [8] is proposed. This scheme can be called “optical lever” because it can provide a gain in displacement of the local mirror similar to the gain which can be obtained using ordinary mechanical lever with unequal arms. This scheme is discussed in Section 2.

In Section 3 the instability which can exist in both “optical bar” and “optical lever” schemes (namely, in so-called *X*-topologies of these schemes) and which was not mentioned in the article [8] is analyzed.

It is supposed in this article for simplicity that all optical elements of the scheme are ideal. It means that reflectivities of the end mirrors are equal to unity, and all internal elements have no losses. We presume that optical energy have been pumped into the interferometer using very small transparency of some of the end mirrors, and at the time scale of the gravitation-wave signal duration the scheme operates as a conservative one.

It has been shown in the article [8] that losses in the optical elements limited the sensitivity at the level

$$\xi \gtrsim \frac{1}{\sqrt{\Omega \tau_{\text{opt}}^*}}, \quad (3)$$

where τ_{opt}^* is the optical relaxation time. Taking into account that value of τ_{opt}^* can be as high as $\gtrsim 1$ s, one can conclude that the optical losses do not affect the sensitivity if $\xi \gtrsim 10^{-1}$.

2. The optical lever

One of the possible “optical lever” scheme topologies (*L*-topology) is presented in Fig. 1 (another variant—*X*-topology—is considered in the next section). It differs from the *L*-topology of the “optical bar” scheme [8] by two additional mirrors *A'* and *B'* only. These mirrors together with the end mirrors *A* and *B* form two Fabry–Perot cavities with the same

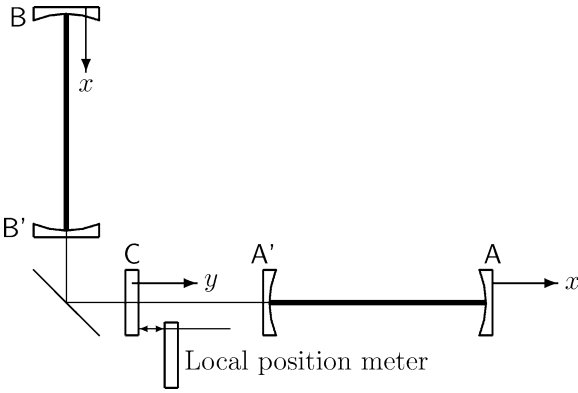


Fig. 1. L-topology of the “optical lever” scheme.

initial lengths L coupled by means of the central mirror C with small transmittance T_C . Exactly as in the case of the “optical bar” scheme, due to this coupling eigen modes of such a system form the set of doublets with frequencies separated by the value of Ω_B which is proportional to the T_C and can be made close to the signal frequency Ω .

Let distances between the mirrors to be adjusted in such a way that Fabry–Perot cavities AA' and BB' are tuned in resonance with the upper frequency of one of the doublets, and additional Fabry–Perot cavities A'C and B'C are tuned in antiresonance with this frequency.

It is supposed here that (i) distances l between the mirrors A', B' and the coupling mirror C are small enough to neglect values of the order of magnitude close to $\Omega l/c$, and that (ii) it is possible to neglect the mirrors A', B' motion. For example, they can be attached rigidly to the platform where the local position meter is situated.

Let only the upper frequency of this doublet to be excited initially. In this case most of the optical energy is concentrated in the cavities AA' and BB' and distributed evenly between them. Small differential change x in the cavities optical lengths will redistribute the optical energy between the arms and hence will create difference in pondermotive forces acting on the mirrors. In other words, an optical pondermotive rigidity exists in such a scheme.

In the article [8] the analogy with two “optical springs”, one of which was situated between the mirrors A and C and another one (L-shaped) between the mirrors B and C, has been used. It has been shown in that article that if the optical energy exceeded the

threshold value of

$$\mathcal{E} \sim \frac{ML^2\Omega^3}{\omega_p}, \tag{4}$$

then these springs became rigid enough to transfer displacement x of the mirrors A, B to the same displacement y of the mirror C.

It is rather evident that if additional mirrors A', B' are present then displacement x of the end mirrors A, B is equal to about \mathcal{F} times greater displacement x of the mirrors C, where \mathcal{F} is finesse of the Fabry–Perot cavities AA' and BB' (one can imagine, for simplicity, that these Fabry–Perot cavities are replaced by delay lines). Therefore, one can expect that scheme presented in Fig. 1 provides gain in the mirror C displacement relative to its displacement in the original “optical bars” scheme, and this gain has to be close to \mathcal{F} .

The analysis shows that it is true. The mechanical degrees of freedom equations of motion in spectral representation have the form (we omit here some very lengthy but rather straightforward calculations devoted to excluding variables for electromagnetic degrees of freedom and reducing the full equations set for the system to mechanical equations only):

$$[-2M_x\Omega^2 + K_{xx}(\Omega)]x(\Omega) = K_{xy}(\Omega)y(\Omega) + F_{\text{grav}}(\Omega), \tag{5a}$$

$$[-M_y\Omega^2 + K_{yy}(\Omega)]y(\Omega) = K_{xy}(\Omega)x(\Omega) + F_{\text{fluct}}(\Omega), \tag{5b}$$

here M_x is the mass of the mirrors A, B, M_y is the mass of the mirror C, x is the displacement of the mirrors A, B, y is the displacement of the mirror C, $F_{\text{grav}}(\Omega)$ is the signal force acting on the mirrors A, B, and F_{fluct} is fluctuational back action force produced by the device which monitors variable y . We consider here only differential motion of the mirrors A, B, when their displacements have the same absolute value and the directions shown in Fig. 1. This motion corresponds to the gravitational wave with optimal polarization. It has been shown in the article [8] that the symmetric motion of the mirrors A, B (when both mirrors approach to or move from the mirror C) did not coupled with the degrees of freedom x and y and could be excluded from the consideration.

Factors K_{xx} , K_{yy} , K_{xy} , K_{yx} which form the matrix of the pondermotive rigidities are equal to

$$K_{xx}(\Omega) = \frac{2\omega_p \mathcal{E}}{c^2 \tau \cos^2 \Omega \tau} \frac{\tan \Omega_B \tau}{\tan^2 \Omega_B \tau - \tan^2 \Omega \tau}, \quad (6a)$$

$$K_{yy}(\Omega) = \frac{2\omega_p \mathcal{E}}{c^2 \tau F^2} \frac{\tan \Omega_B \tau}{\tan^2 \Omega_B \tau - \tan^2 \Omega \tau}, \quad (6b)$$

$$K_{xy}(\Omega) = \frac{2\omega_p \mathcal{E}}{c^2 \tau F \cos \Omega \tau} \frac{\tan \Omega_B \tau}{\tan^2 \Omega_B \tau - \tan^2 \Omega \tau}, \quad (6c)$$

where

$$\tau = \frac{L}{c}, \quad (7)$$

$$F = \frac{1+R}{1-R} \approx \frac{2}{\pi} \mathcal{F}, \quad (8)$$

$$\Omega_B = \frac{1}{\tau} \arctan\left(\frac{\tan \phi}{F}\right), \quad (9)$$

$\phi = \arcsin T_C$ and R is the reflectivity of the mirrors A' , B' . It have to be noted that these rigidities exactly satisfy the condition

$$K_{xx}(\Omega)K_{yy}(\Omega) - K_{xy}^2(\Omega) = 0. \quad (10)$$

Suppose then $\Omega \tau \ll 1$ (in the case of the contemporary terrestrial gravitational-wave antennae $\tau \lesssim 10^{-5}$ s and $\Omega \lesssim 10^3$ s $^{-1}$, so $\Omega \tau \lesssim 10^{-2}$). In this case we obtain that

$$K_{xx}(\Omega) = \frac{2\omega_p \mathcal{E}}{L^2} \frac{\Omega_B}{\Omega_B^2 - \Omega^2}, \quad (11)$$

and

$$K_{xx}(\Omega) = F K_{xy} = F^2 K_{yy}. \quad (12)$$

There is a very simple mechanical model which also can be described by the Eqs. (5), (12), putting aside for a while particular spectral dependence (11). This is an ordinary mechanical lever with arm lengths ratio F (see Fig. 2). Rigidities K_{xx} , K_{yy} and K_{xy} in this case are proportional to the bending rigidity of the lever bar. It is evident that if the motion is sufficiently slow and therefore it is possible to neglect bending then the y -arm tip will be F times greater than the x -arm tip displacement. Consequently, if the observation frequency Ω is sufficiently small then in the “optical lever” scheme presented in Fig. 1 the mirror C motion will repeat the end mirrors A, B motion with the gain factor F .

In all other aspects this scheme is similar to the “optical bars” scheme. As it follows from the symmetry

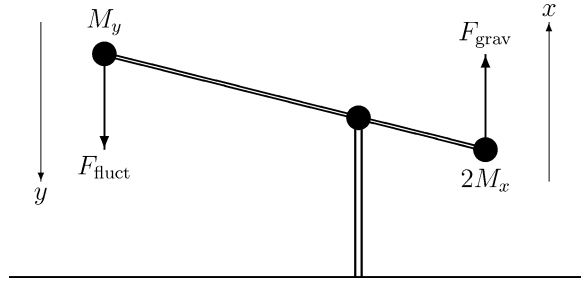


Fig. 2. Mechanical model of the “optical lever” scheme.

conditions (12) if one replaces in the Eqs. (5) y by y/F , F_{fluct} by $F \times F_{\text{fluct}}$, M_y by $F^2 \times M_y$ and then replaces all rigidities by their values corresponding to “optical bars” scheme (with $F = 1$) then these equations still remain valid.

It means that if in the “optical bars” scheme one (i) replaces the mass M_y by F^2 smaller one; (ii) decreases back action noise of the meter by the factor of F and increases proportionally its measurement noise (for example, by decreasing pumping power in the interferometric position meter by the factor of F^2); and (iii) inserts the additional mirrors A' , B' with reflectivity defined by the Eq. (8) then signal-to-noise ratio (relative to the local meter noises) and dynamical properties of the scheme will remain unchanged, with the only evident replacement of y by Fy .

Two characteristic regimes of the “optical lever” scheme are similar to the quasistatic and resonant regimes of the “optical bars” scheme described in the article [8], and therefore here we consider them in brief only.

Characteristic equation of the equations set (5) is the following:

$$\Omega^2 \left(\Omega^4 - \Omega_B^2 \Omega^2 + \frac{2\omega_p \mathcal{E} \Omega_B}{M_{\text{eff}} L^2} \right) = 0, \quad (13)$$

where

$$M_{\text{eff}} = \left(\frac{1}{2M_x} + \frac{1}{F^2 M_y} \right)^{-1}. \quad (14)$$

Root $\Omega = 0$ of this equation corresponds to the quasistatic regime. If pumping energy is sufficiently high:

$$K_{xx}(\Omega) = \frac{2\omega_p \mathcal{E}}{L^2} \frac{\Omega_B}{\Omega_B^2 - \Omega^2} \gg M_x \Omega^2,$$

$$K_{yy}(\Omega) = \frac{2\omega_p \mathcal{E}}{F^2 L^2} \frac{\Omega_B}{\Omega_B^2 - \Omega^2} \gg M_y \Omega^2, \tag{15}$$

then the equations set (5) solution for this regime can be presented as

$$y(\Omega) = Fx(\Omega) = -\frac{F F_{\text{grav}}(\Omega) + F^2 F_{\text{fluct}}(\Omega)}{\Omega^2(2M_x + F^2 M_y)}. \tag{16}$$

It is evident that maximal value of the signal response can be obtained here if

$$F = \sqrt{\frac{2M_x}{M_y}}, \tag{17}$$

and in this case we will get

$$y(\Omega) = Fx(\Omega) = -\frac{1}{2\Omega^2} \left(\frac{F_{\text{grav}}(\Omega)}{\sqrt{2M_x M_y}} + \frac{F_{\text{fluct}}(\Omega)}{M_y} \right). \tag{18}$$

Taking into account, that gravitational-wave signal force is proportional to the mass of the end mirrors: $F_{\text{grav}} \propto M_x$, we can conclude that in the gravitational-wave experiments this regime can provide a wide-band gain in signal displacement proportional to $\sqrt{M_x/M_y}$.

It is necessary to note that this gain by itself does not allow to overcome the standard quantum limit because the value of the standard quantum limit for the test mass M_y rises exactly in the same proportion. But it does allow to use less sensitive local position meter and it does increase the signal-to-noise ratio for miscellaneous noises of non-quantum origin and therefore makes it easier to overcome the standard quantum limit using, for example, variational measurement in the local position meter [11,12].

Another two roots of Eq. (13)

$$\Omega_{1,2}^2 = \frac{\Omega_B^2}{2} \pm \sqrt{\frac{\Omega_B^2}{2} - \frac{4\omega_p \mathcal{E} \Omega_B}{M_{\text{eff}} L^2}}, \tag{19}$$

correspond to the more sophisticated resonant regime of the scheme. Placing these two roots evenly in the spectral band of the signal it is possible to obtain sensitivity a few times better than the standard quantum limit for a free mass in relatively wide band, as it has been proposed in the article [13]. Using the value of pumping power

$$\mathcal{E} = \frac{M_{\text{eff}} L^2 \Omega_B^3}{8\omega_p}, \tag{20}$$

it is possible to implement the second-order-pole test object and obtain the sensitivity substantially better than the standard quantum limit for both free mass and harmonic oscillator in narrow band near the frequency $\Omega_B/\sqrt{2}$ [14]. In both cases “optical lever” allows to increase the signal displacement of the local mirrors and therefore makes it easier implementation of the local position meter.

3. X-topologies of the “optical bars” and the “optical lever” schemes

In the article [8] two possible topologies of the “optical bars” scheme have been considered, the *L*-topology discussed in the previous section and the *X*-topology similar to the Michelson interferometer topology. The latter one also can be converted to the “optical lever” scheme using additional mirrors *A'* and *B'* as it is shown in Fig. 3. Here *C* is the coupling mirror with transmittance $T_C = \sin \phi$. In this topology one optical “spring” exists between the mirrors *A* and *A''*, and another one between the mirrors *B* and *B''*.

In the article [8] *L*- and *X*-topologies were considered as identical ones with the only difference that in the case of *X*-topology the value of Ω_B was about two times greater,

$$\Omega_B = \frac{1}{\tau} \arctan\left(\frac{\tan 2\phi}{F}\right). \tag{21}$$

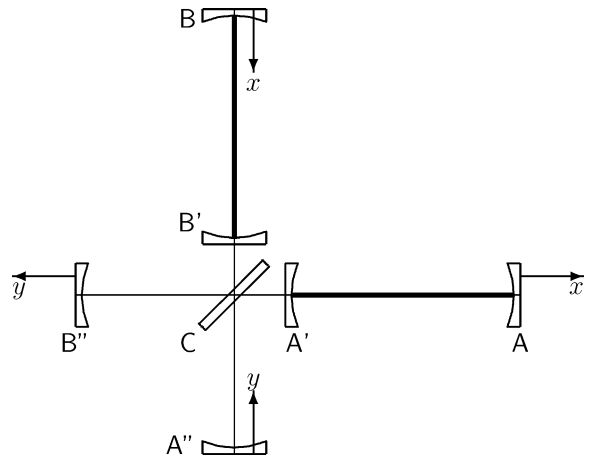


Fig. 3. X-topology of the “optical lever” scheme.

More rigorous analysis shows, however, that this is not the case. Really, the rigidities which appear in Eqs. (5) for the case of the X -topology are equal to

$$K_{xx}(\Omega) = \frac{2\omega_p \mathcal{E}}{c^2 \tau \cos^2 \Omega \tau} \frac{\tan \Omega_B \tau}{\tan^2 \Omega_B \tau - \tan^2 \Omega \tau}, \quad (22a)$$

$$K_{yy}(\Omega) = \frac{2\omega_p \mathcal{E}(1 + F^2 \tan^2 \Omega \tau)}{c^2 \tau F^2} \times \frac{\tan \Omega_B \tau}{\tan^2 \Omega_B \tau - \tan^2 \Omega \tau}, \quad (22b)$$

$$K_{xy}(\Omega) = \frac{2\omega_p \mathcal{E}}{c^2 \tau F \cos \Omega \tau \cos 2\phi} \times \frac{\tan \Omega_B \tau}{\tan^2 \Omega_B \tau - \tan^2 \Omega \tau}, \quad (22c)$$

and

$$K_{xx}(\Omega)K_{yy}(\Omega) - K_{xy}^2(\Omega) = -\left(\frac{2\omega_p \mathcal{E}}{c^2 \tau \cos \Omega \tau}\right)^2 \frac{\tan^2 \Omega_B \tau}{\tan^2 \Omega_B \tau - \tan^2 \Omega \tau} \neq 0. \quad (23)$$

It means that the low-frequency mechanical mode which in the article [8] has been considered as a free mass mode (see factor p^2 in the equation (C.3) in the above-mentioned article) and which does represent free mass in the case of L -topology, has a non-zero rigidity in the case of X -topology. Moreover, if $\Omega < \Omega_B$ then this rigidity is negative, and therefore asynchronous instability exists in the system.¹

Characteristic time for this instability is equal to

$$\tau_{\text{instab}} = \left(\frac{2\omega_p \mathcal{E} \Omega_B}{L^2(2M_x/F^2 + M_y)}\right)^{-1/2}. \quad (24)$$

Taking into account condition (15) an supposing that $\Omega \sim \Omega_B$, one can obtain that

$$\tau_{\text{instab}} \Omega \approx \frac{1}{F \Omega \tau}. \quad (25)$$

This value is rather large ($\sim 10^2$ if, say, $\Omega \sim 10^3 \text{ s}^{-1}$ and $\tau \sim 10^{-5} \text{ s}$) in the case of pure “optical bars” scheme ($F = 1$). Therefore, this instability can be easily dumped by the feed-back system in this case.

On the other hand, in the case of the “optical lever” scheme can be $\tau_{\text{instab}} \sim \Omega^{-1}$, if one attempts to use too large value of F .

In the article [15] it has been shown, however, that even such a strong instability can be dumped in principle by feed-back scheme without any loss in the signal-to-noise ratio.

4. Conclusion

Properties of the “optical bars” intracavity scheme [8] can be substantially improved by converting arms of the antenna into Fabry–Perot cavities similar to ones used in traditional topologies of gravitational-wave antennae with extracavity measurement. This new “optical lever” scheme allows to obtain the gain in signal displacement of local mirror approximately equal to finesse of the Fabry–Perot cavities.

This gain by itself does not allow to overcome the standard quantum limit in wide-band regime. But it allows to use less sensitive local position meter and increases the signal-to-noise ratio for miscellaneous noises of non-quantum origin, making it easier to overcome the standard quantum limit using, for example, variational measurement in the local position meter.

The value of this gain is limited, in principle, by the formula (9) only. As it follows from this formula F cannot exceed value $(\Omega_B \tau)^{-1}$. If $\Omega_B \sim \Omega \sim 10^3 \text{ s}^{-1}$ and $\tau \sim 10^{-5} \text{ s}$ (which corresponds to arms length of LIGO and VIRGO antennae) then $F \lesssim 10^2$. If $\tau \sim 10^{-6} \text{ s}$ (GEO-600 and TAMA) then this value can be about one order of magnitude larger, $F \lesssim 10^3$. It is interesting to note that if F is close to its limiting value $(\Omega_B \tau)^{-1} \sim (\Omega \tau)^{-1}$ then signal displacement of the local mirror is close to the signal displacement of the end mirrors of hypothetical gravitational-wave antenna with arm lengths equal to the half-wavelength of the gravitational wave.

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¹ In the article [8] another instability has been considered which has different origin and depends on the optical relaxation time. Instability considered here exists even if this relaxation time is equal to infinity.

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