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Generalized synchronization of chaos in RCL-shunted Josephson junction with uncertain parameters

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Abstract: This paper investigates the generalized synchronization of chaotic dynamics in resistive capacitive inductance (RCL)-shunted Josephson junctions with uncertain parameters.Based on Lyapunove stability theory and adaptive control method, unified nonlinear feedback controller and the parameter update laws are pesented .Numerical simulation illustrate that the system can realize generalized synchronization by different scaling factors .

1.Introduction

As a highly nonlinear device, there are complex chaotic behavior in Josephson junction and its associated circuits. The device of Josephson Junction itself have unique advantages such as low noise, low power dissipation and high operating frequency ,so the Josephson junction and its related circuit have attracted considerable research interest in the last few years[1-3]. Jopeson junction (JJ) has been represented by different models[4] ,one of which named RCLSJ model has been extensively investigated in recent years for the RCLSJ model has been found to be more useful in high-frequency applications[5-9].However, most works on the control and synchronization of the RCLSJ chaotic model are derived on the basis of the known parameters. In practice, determining some system parameters in advance may be difficult. Furthermore, most parametrical values are characterized by uncertainties related to physical conditions that can destroy or even break the control. These problems can be tackled to a certain extent through adaptive synchronization[10-12,15].The generalized projective synchronization , which includes complete synchronization, anti-synchronization and projective synchronization as its special item, is a novel type of chaos synchronization,and has attracted much attention recently [13-15].

The objective of this Letter is to design nonlinear feedback controller and the parameter update laws to realize generalized projective synchronization of the chaotic RCLSJ model with identical structure,and estimate the uncertain parameters simultaneously.

2.The RCLSJ model of Josephson junction

The standard form of the RCLSJ model is described by the following differential equations:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{\beta_C}[i - g(x_2)x_2 - \sin(x_1) - x_3] \\
\dot{x}_3 &= \frac{1}{\beta_L}[x_2 - x_3]
\end{aligned}
\tag{1}
$$

where x_1, x_2, x_3 represent the phase difference, junction voltage, and the inductive current respectively. β_C and β_L are constants that represent the capacitive and inductive values respectively. In this system, parameter i is an external current consisting of a dc component only. The nonlinear damping term $g(x_2)$ is approximated with a current–voltage relation between the two junction resistances and is defined by

$$g(x_2)=\begin{cases}0.366 & \text{if } |x_2|>2.9\\ 0.061 & \text{if } |x_2|\le 2.9\end{cases} \tag{2}$$

This dissipative model has been shown to exhibit chaotic or periodics dynamics for some values of its parameters.The chaotic attractor is shown in Fig. 1 with initial conditions are $(x_1(0), x_2(0), x_3(0)) =$（0.1,0.5,0）

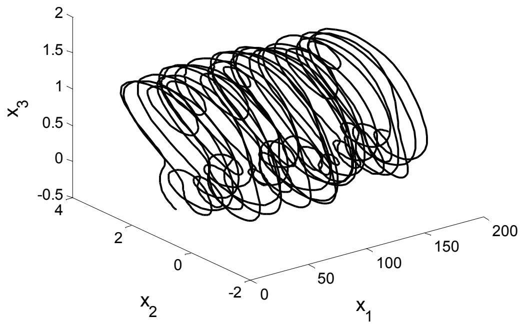


Fig 1. Chaotic attractors for $i=1.15$ and initial conditions $x_1(0), x_2(0), x_3(0) = (0.3, 0.5, 0)$

3. Unified adaptive controller design

We let $\dfrac{1}{\beta_C}=\alpha, \dfrac{1}{\beta_L}=\beta$ and add time dependent control input function $u(t)$ to system (1) to obtain

$$\begin{aligned}\dot{y}_1 &= y_2 + u_1(t)\\ \dot{y}_2 &= \alpha[i - g(y_2)y_2 - \sin(y_1) - y_3] + u_2(t)\\ \dot{y}_3 &= \beta[y_2 - y_3] + u_3(t)\end{aligned} \tag{3}$$

where α, β are parameters of the controlled system which need to be estimated,and $u_i(t)(i=1,2,3)$ are the nonlinear feedback controller to be designed . We define the error states between the system (3) and the system (1) as

$$e_1 = y_1 - k_1x_1, e_2 = y_2 - k_2x_2, e_3 = y_3 - k_3x_3 \tag{4}$$

where $k_i(i=1,2,3)$ are the scaling factors. The error dynamical system is

$$\begin{aligned}\dot{e}_1 &= y_2 + u_1 - k_1\dot{x}_1\\ \dot{e}_2 &= \alpha[i - g(y_2)y_2 - \sin(y_1) - y_3] + u_2 - k_2\dot{x}_2\\ \dot{e}_3 &= \beta[y_2 - y_3] + u_3 - k_3\dot{x}_3\end{aligned} \tag{5}$$

Hence the problem of generalized projective synchronization of chaos in RCLSJ becomes the stability of error dynamics(5). Based on Lyapunove stability theory and active control method we propose following controller for the controlled system (3)

$$\begin{aligned} u_1 &= -y_2 - d_1 y_1 + d_1 k_1 x_1(t) + k_1 \dot{x}_1(t) \\ u_2 &= -\alpha'[i - g(y_2) y_2 - \sin(y_1) - y_3] - d_2 y_2 + d_2 k_2 x_2(t) + k_2 \dot{x}_2(t) \\ u_3 &= -\beta'[y_2 - y_3] - d_3 y_3 + d_3 k_3 x_3(t) + k_3 \dot{x}_3(t) \end{aligned} \tag{6}$$

where $d_i (i = 1,2,3\ d_i > 0)$ are adjustable acceleration factors.we can adjust synchronization speed flexibility by choosing appropriate accelerate factors. α', β' are the estimate values of the system parameters α, β.The update laws for the two uncertain parameters are

$$\begin{aligned} \dot{\alpha}' &= e_2(i - g(y_2) y_2 - \sin(y_1) - y_3) \\ \dot{\beta}' &= e_3(y_2 - y_3) \end{aligned} \tag{7}$$

Theorem 1. For the given scaling factors $k_1 (i = 1,2,3)$, adjustable acceleration factors $d_i (i = 1,2,3$ $d_i > 0)$,controlled system(3) can achieve generalized projective synchronization with system (1) by the controller (6) and the parameter update laws (7).

Proof. Choose the following Lyapunov function

$$V = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 + \frac{1}{2} e_3^2 + \tilde{\alpha}^2 + \tilde{\beta}^2 \tag{8}$$

where $\tilde{\alpha} = \alpha' - \alpha$, $\tilde{\beta} = \beta' - \beta$ .The time derivative of V along the trajectory of error system (5) is

$$\begin{aligned} \dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + \tilde{\alpha} \dot{\tilde{\alpha}} + \tilde{\beta} \dot{\tilde{\beta}} = e_1(y_2 + u_1 - k_1 \dot{x}_1) + e_2[\alpha(i - g(y_2) y_2 - \sin(y_1) - y_3) + \\ u_2 - k_2 \dot{x}_2] + e_3[\beta(y_2 - y_3) + u_3 - k_3 \dot{x}_3] + \tilde{\alpha} \dot{\tilde{\alpha}} + \tilde{\beta} \dot{\tilde{\beta}} \end{aligned} \tag{9}$$

Inserting (6) and (7) into above equation yields the following:

$$\begin{aligned} \dot{V} = e_1(y_2 - y_2 - d_1 y_1 + d_1 k_1 x_1 + k_1 \dot{x}_1 - k_1 \dot{x}_1) + e_2[(\alpha - \alpha')(i - g(y_2) y_2 - \sin(y_1) - y_3) - \\ d_2 y_2 + d_2 k_2 x_2 + k_2 \dot{x}_2 - k_2 \dot{x}_2] + e_3[(\beta - \beta')(y_2 - y_3) - d_3 y_3 + d_3 k_3 x_3 + k_3 \dot{x}_3 - k_3 \dot{x}_3] + \\ \tilde{\alpha} e_2(i - g(y_2) y_2 - \sin(y_1) - y_3) + \tilde{\beta} e_3(y_2 - y_3) \end{aligned} \tag{10}$$

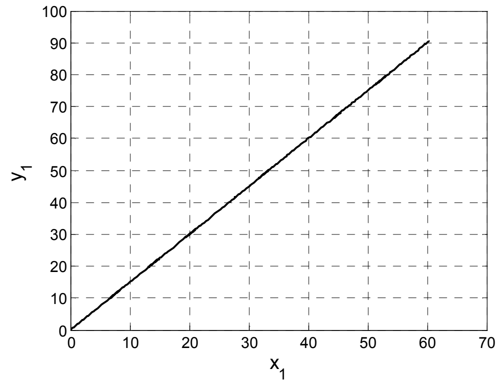
It is

$$\begin{aligned} \dot{V} &= e_1[-d_1(y_1 - k_1 x_1)] + e_2[-\tilde{\alpha}(i - g(y_2) y_2 - \sin(y_1) - y_3 - d_2(y_2 - k_2 x_2)] + \\ &= e_3[-\tilde{\beta}(y_2 - y_3) - d_3(y_3 - k_3 x_3)] + \tilde{\alpha} e_2(i - g(y_2) y_2 - \sin(y_1) - y_3) + \tilde{\beta} e_3(y_2 - y_3) \\ &= -d_1 e_1^2 - d_2 e_2^2 - d_3 e_3^2 \le 0 \end{aligned} \tag{11}$$

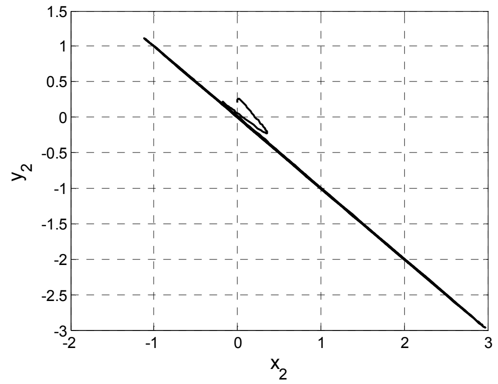
Then $\dot{V}$ is negative definite and according to LaSalle–Yoshizawa's theorem, the error dynamics will converge to zero and remains globally asymptotically stable. This completes the proof. Furthermore, according to the above formula ,we can solve for $e_i(t) = e(0)\exp(-d_i t)$ $(i = 1,2,3)$. It indicates that the speed of synchronization was increased with the increasing of accelerate factors d_i ,and the convergence of the errors $e_i(t)$ also were accelerated.Therefore, we can adjust the synchronous speed flexibility by choosing appropriate accelerate factors $d_i (i = 1,2,3\ \ d_i > 0.)$

4. Numerical simulations

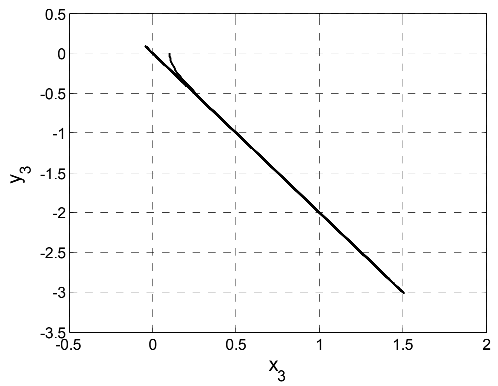
In this simulication , fourth order Runge–Kutta method is used to verify the effectiveness of the proposed controller with time step size 0.001.The parameters are choosen to be $\beta_C = 0.707$, $\beta_L = 2.6$, $i = 1.15$ so that the dynamics in RCL-shunted Josephson junctions exhibits chaotic behaviour if no control is applied.The initial conditions of the system (1) and system(3) are $(0.1, 0.5, 0)$ and $(0,0,0)$ respectively. The initial values of the estimated parameter are chosen as $\alpha' = 0, \beta' = 0$. The scaling factors are $k_1 = 1.5, k_2 = -1, k_3 = -2$ and the adjustable acceleration factors are choosen as $d_1 = d_2 = d_3 = 3$. The simulation results are illustrated in Fig.2. Fig.2 (a)-(d) demonstrate the generalized projective synchronization errors e_1, e_2, e_3 quickly tends to zero with time evolution. When the time $t \geq 5\text{s}$, there are $y_1 = 1.5x_1$, $y_2 = -x_2$, $y_3 = -2x_3$ .The evolution of the estimated two parameters are shown in Fig.3(e) .These results show that the generalized projective synchronization to identical RCLSJ system and parameters identification have been achieved with the nonlinear controller (6) and the parameter update laws (7).



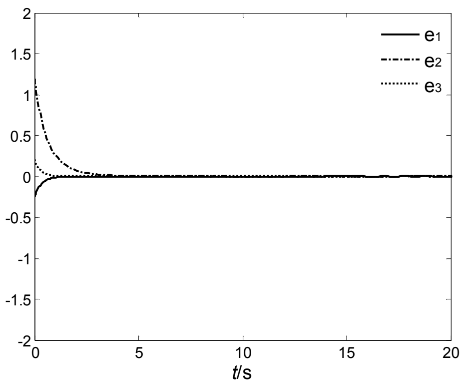
(a) Phase plot of $x_1 - r_1$



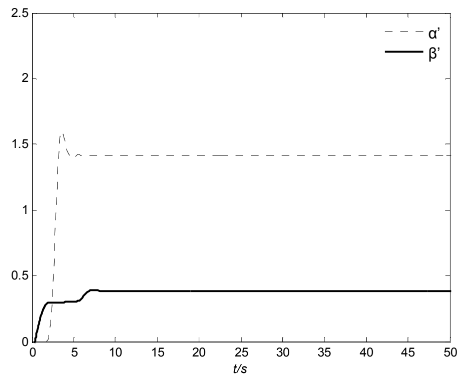
(b) Phase plot of $x_2 - r_2$



(c) Phase plot of $x_3 - r_3$



(d) $t - e_1, e_2, e_3$



(e) $t - \alpha', \beta'$

Fig.2.The simulation results of the generalized projective synchronization

5.Conclusions

In this paper, a unified nonlinear feedback controller and the parameter update laws are presented for the generalized projective synchronization of chaos in RCLSJ models with uncertain parameters. Based on the parameters modulation and the adaptive control techniques, the dynamical evolution of the two RCLSJ models can be synchronized according to desired scaling factors. This method takes into account the external disturbances on the system parameters, so it has a certain robustness. The corresponding numerical simulations demonstrate the effectiveness of the proposed scheme.

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