Abstracts of Invited Speakers

Teaching mathematics in context as a social construct

H Barnes, University of Pretoria

Why is teaching in context a necessary option to consider in the teaching of mathematics? What does it mean to teach mathematics from and in contexts? And what are the possible challenges associated with this practice? The aim of this paper is to attempt to address these questions from literature and data from a developmental research project carried out using the theory of Realistic Mathematics Education. The philosophical shift that has occurred within the domain of mathematics has brought with it a wave of reform in mathematics education. The former absolutist paradigm that dominated undermined the social responsibility of mathematics in human affairs such as value, wealth and power (Ernest, 1991). The shift has challenged the infallibility of mathematics and acknowledged it as a product of human inventiveness (Davis and Hersh, 1980) and a human activity (Freudenthal, 1973), thus making it a social construct. While the reform propagating the teaching of mathematics as a social construct is a positive move, along with the introduction of mathematical literacy in many countries, how can we effectively implement this reform? This paper seeks to examine the area of teaching mathematics in context to support this reform and how we can consider implementing this in both school and tertiary education.

Mathematicians and Fisheries Management: a Marriage of Opportunism or Necessity?

Doug S Butterworth, MARAM (Marine Resource Assessment and Management Group), Department of Mathematics and Applied Mathematics, University of Cape Town

For perspective, first a brief summary is given on how the commercial fisheries of South Africa and Namibia contribute to fishing worldwide and to local economies. The fundamental (and conflicting) objectives of fisheries management are explained, together with the key role of mathematical analysis in quantifying the trade-offs between them. This includes a short account of the basic process of resource assessment (what data are typically used in an integrative mathematical process to estimate the size and productivity of a fish population), together with the control rules applied to such results to set catch limits in a feedback manner that aims to move the population to a desired size. The critical need to deal with difficulties arising because of uncertainties, related both to poor statistical precision and to model structure, is emphasised. An account is given of the developing Management Procedure simulation-testing approach (in which South Africa has played a pioneering role) that aims to provide robust attainment of objectives in these circumstances. The example of South Africas most important fishery (that for hake) is used to illustrate how this approach is being applied at the moment to select between options to achieve an economically essential improvement in catch rates. Reference is also made to aspects of analysis for the major local fishery for the small near-surface shoaling species, sardine and anchovy, which is complicated by the unavoidable catch of juveniles of the former in hauls targeted at the latter, and for abalone where estimation of the substantial level of poaching is an essential component. Results are shown of attempts to apply species interaction models to explain the trends of the main whale and seal species around Antarctica following their heavy exploitation and subsequent protection. Finally some comments on the current state of the marriage: improving with age, or headed towards the divorce courts?

TOTAL DOMINATION IN GRAPHS AND TRANSVERSALS IN HYPERGRAPHS

Michael Henning, School of Mathematical Sciences, University of KwaZulu-Natal

A set S of vertices in a graph G without isolated vertices is a total dominating set of G if every vertex of G is adjacent to a vertex in S. The total domination number of G is the minimum cardinality of a total dominating set in G. The transversal number of a hypergraph is the minimum number of vertices meeting every edge. We observe that total domination in graphs can be translated to the problem of finding transversals in hypergraphs. In this talk, we survey recent results on total domination in graphs and present new results using transversals in hypergraphs.

RELIABLE FINITE DIFFERENCE SCHEMES FOR IMPACT OSCILLATORS WITH ONE DEGREE OR TWO DEGREES OF FREEDOM

Jean M-S Lubuma, Department of Mathematics and Applied Mathematics, University of Pretoria

Vibration analysis is a major area of research in mechanics. Of particular and growing interest is the study of impact oscillators, which arise in a variety of engineering and real-life problems such as rattling gears, vibrating absorbers, car suspension and impact print hammers. These oscillators are nonsmooth mechanical systems with complex and even chaotic behaviors. Typically, at the time of the impact, which is supposed to be instantaneous, the velocity of the oscillator changes instantaneously to the extent that this may induce unwanted phenomena such as excessive noise, wear and fatigue. Thus, there is an essential need to study these systems properly in order to improve their performance.

We consider an oscillator system with one degree or two degrees of freedom whose motion is subject to convex constraints in the sense that the involved body can hit fixed stops and thus the position of the body is constrained to remain inside a convex set. Using a suitable percussion law that takes into account the loss of energy during each impact time, the problem is formulated as a differential inclusion or a variational inequality. As usual for problems of this nature, the solution cannot in general be found analytically. Hence, there is a crucial need of reliable numerical methods that permit to gain some useful information on the problem. To this end, we address the issues listed below.

- Existence and/or uniqueness results for the associated variational inequality.
- Description of some essential physical properties of the impact oscillator problems; this includes the principle of conservation of energy between two consecutive impact times.
- Design of non-standard finite difference schemes that replicate the essential physical properties of the exact solution; in the process, we explain along the lines of [1], [2] and [3] what non-standard schemes are and what are their advantages over the standard finite difference schemes.
- Extension of the study to multi-dimensional impact oscillators.

References

[1] R. Anguelov and J.M-S. Lubuma, Contributions to the mathematics of the nonstandard finite difference method and applications, *Numer. Methods Partial Differential Eq.* **17** (2001), 518-543.

[2] Y. Dumont and J.M.-S. Lubuma, Non-standard finite difference methods for vibro-impact problems, *Proc. R. Soc. London*, **461A** (2005), 1927-1950.

[3] R.E. Mickens, *Nonstandard finite difference models of differential equations,* World Scientific, Singapore, 1994.

ON THE UNIQUENESS OF THE REPRESENTATION OF TILED MATRIX RINGS

L van Wyk*, Department of Mathematics, University of Stellenbosch

S Dascalescu, Kuwait University

R Khazal, Kuwait University

In this talk we discuss the possible recovery, up to isomorphism, of some of the underlying structures in various kinds of matrix rings, for example, complete matrix rings, structural (or blocked triangular) matrix rings and tiled matrix rings. These underlying structures include modules, bimodules, ideals, rings and directed graphs.

Abstracts of Short Talks

Elements of Complete Symmetry Groups

K Andriopoulos, University of KwaZulu-Natal

In this talk we introduce the concept of complete symmetry groups in differential equations. Theorems are presented about the dimensions of the groups and their representations. The necessity for the introduction of nonlocal symmetries is highlighted.

Local Connectedness and the Wallman Compactification

Dharmanand Baboolal, University of KwaZulu-Natal

We study frames having a locally connected Wallman basis. For such frames we obtain several equivalent conditions for its Wallman compactification to be perfect and locally connected.

Modelling and Simulation with Application to Natural Gas Networks

MK Banda, School of Mathematical Sciences, University of KwaZulu-Natal

We introduce a model for gas flow in pipeline networks based on the isothermal Euler's equations. Intersections of pipes are discussed by considering solutions to Riemann problems. An intersection of multiple pipes is modelled by posing additional assumptions at the intersection in order to obtain unique solutions near the intersection. A method to obtain solutions to the gas network problem is given and numerical results on sample networks are presented.

Gravitational Collapse with decay of Vacuum Energy

A Beesham*, University of Zululand
Syamala Krishnannair, University of Zululand

The gravitational collapse of a spherically symmetric star consisting of radiation in a background with Λ is discussed, i.e., modified Vaidya collapse. In addition the decay of the vacuum energy, modelled by the Λ term, is considered. Implications for the cosmic censorship hypothesis are pointed out.

A categorical proof of Gödel's Completeness Theorem

Michael Ben-Yosef, Department of Mathematics and Applied Mathematics
University of Cape Town

We present the concept of a Boolean algebra with quantifiers, which has been known in categorical logic circles as a profitable alternative to cylindrical algebras in the algebraic approach to first-order logic. We explain how the usual notions of first-order theories and their models can be defined in terms of these structures. Gödel's Completeness Theorem is then obtained as an algebraic result using this framework.

NETWORK RELIABILITY AS A RESULT OF REDUNDANT CONNECTIVITY

F Binneman*, Department of Mathematical Sciences, University of Stellenbosch
JH Van Vuuren, Department of Mathematical Sciences, University of Stellenbosch

The vertex connectivity number of a connected graph G is the minimum number of vertices whose removal from G produces a disconnected subgraph of G. The problem of finding spanning subgraphs with the same vertex connectivity number as the original graph is known to be NP-hard. Algorithms for finding this type of spanning subgraph for a graph satisfying certain properties are presented in this talk.

The calculation of gravitational radiation

NT Bishop, University of South Africa

The Bondi-Sachs metric provides a natural formalism for describing a gravitationally radiating space-time. Within this formalism, and using the standard separation of variables technique, analytic linearized solutions of the Einstein equations have been found. We apply this method to obtain, in the linearized limit, the gravitational radiation emitted by a binary. As expected, we recover the quadrupole formula to lowest multipole order; but in addition the calculation yields the higher multipole components.

The generalised ADS property

GL Booth, Nelson Mandela Metropolitan University

It is well known that Kurosh-Amitsur radicals of associative rings have the ADS property, namely the radical of an ideal of a ring R is an ideal of R. Birkenmeier and Heatherly formulated a generalised ADS (GADS) property which any class of rings or other algebraic objects may possess. In this talk we consider radicals of associative rings, and involution rings whose semisimple classes possess the GADS property.

Concrete reflections and coreflections in quasi-uniform spaces.

Guillaume Brümmer*, University of Cape Town
Hans-Peter Künzi, University of Cape Town

Besides concreteness, we shall require the (co)reflections to preserve the first topology of every quasi-uniform space. We consider how these functors interact with the quasi-uniform bicompletion. Such work was done for the case of uniform spaces in [1] (Section 7). An impression of how things will look in the quasi-uniform case may be gained from [2] (Definition 1 and Proposition 2).

References

[1] G. C. L. Brümmer and A. W. Hager, Functorial uniformization of topological spaces, Topology Appl. 27 (1987), 113-127.
[2] G. C. L. Brümmer and H.-P. A. Künzi, Idempotency of extensions via the bicompletion, Preprint, May 2006.

Thoughts on having more topological structure

Guillaume Brümmer, University of Cape Town

We consider concrete categories over a fixed category X. In [1] (p. 36) there is the weird claim that a concrete category (A, T) is initially complete if and only if (A, T) can be fully and concretely embedded into every concrete category (C, V) which has more structure than (A, T). For the case X = SET one calls (A, T) a topological category if (A, T) is initially complete, has small fibres, and has singleton fibres over sets of at most one point. The message of the above weird claim is duly advertised in [1] (p. 28) by claiming that "this circumstance is one of the surprising distinctions between topology and algebra". In fact the latter claim is bolstered in [1] (p. 36) by a comparison between the results 5.4 and 5.6 of [1]. In the present talk we discuss to what extent such claims are justifiable. In the same vein we consider a few instances of applications of simple, small home-grown pieces of very elementary category theory which were designed for applications in general topology by the present author, jointly on occasion with Sergio Salbany, Rudolf-Eberhard Hoffmann, Buzz Fay, Keith Hardie, Bernhard Banaschewski, Tony Hager, Hans-Peter Künzi, Eraldo Giuli, Horst Herrlich or Mark Sioen. What we particularly had in mind were cases where having more topological structure is important, such as when dealing with uniform spaces and their underlying topologies, or many generalisations of these, especially the asymmetric spaces such as quasi-uniform spaces and their underlying T_0 topologies or bitopologies.

Reference

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Solution of the Heisenberg-Okubo Equations

Erwin Brüning, University of KwaZulu-Natal

It is known that the system of equations

$$\begin{cases} \phi(x) = \left(\dfrac{cm}{\hbar}\right)^2 \phi(x) \\ \left(i\dfrac{\hbar}{c}\gamma_\mu \dfrac{\partial}{\partial x_\mu} - \tilde{m}\right)\psi(x) = 2\gamma_\mu l^2 \psi(x)\phi(x)\dfrac{\partial \phi(x)}{\partial x_\mu} \end{cases} \tag{1}$$

called the Heisenberg-Okubo equation, is solvable in the sense of classical field theory. I report on the solution of the quantized version of these equations obtained recently in a collaboration with S. Nagamachi (see [2]). In the quantized version the fields ϕ and ψ are generalized functions on space-time (tempered ultra hyperfunctions) which take values in a certain $*-$algebra of unbounded operators. We use stochastic quantization and nonstandard analysis to calculate the sequence of correlation functions determining the fields ϕ and ψ and find a solution in our recently developed framework of *relativistic quantum field theory with a fundamental length* (see [1]).

References

[1] E. Brüning and S. Nagamachi. Relativistic quantum field theory with a fundamental length. *J. Math. Phys.* Vol. **45**, 2199–2231, 2004.

[2] S. Nagamachi and E. Brüning. Solution of the Heisenberg-Okubo equation as a relativistic quantum field theory with a fundamental length. Preprint. Department of Mathematics. The University of Tokushima, Japan, 2006.

ON THE PLACEMENT OF A NUMBER OF STRINGS IN A COLLECTION OF HATS

AP Burger*, Department of Mathematical Sciences, University of Stellenbosch

JH van Vuuren, Department of Mathematical Sciences, University of Stellenbosch

The following problem is considered in this talk: Suppose k strings of unit length are to be distributed amongst x hats. If cuts in the strings are allowed, how should the string (parts) be distributed amongst the hats so that, if the shortest string is removed from each hat, the remaining (combined) string length in the hat with the most string is as small as possible? This problem is related to the problem of colouring a complete multipartite graph with the smallest number of colours such that each colour induced subgraph has maximum degree at most some specified value.

Linear maps between vector lattices with asymmetric norms.

Jurie Conradie, University of Cape Town

The topology and the order on a real normed lattice E is determined by the asymmetric norm p defined by $p(x) = \|x^*\|$. A real linear functional f on E is continuous with respect to the topology induced by p and the topology on the real line induced by the asymmetric norm q defined by $q(t) = \max\{t, 0\}$ if and only if f is positive and continuous with respect to the norm of E. We investigate the possibility of extending this result to linear maps between normed lattices.

On the Method of Lines for Diffusion Equations in four Variables x, y, z and t

OA Daman*, Mathematics Department, University of Botswana

T Motsumi, Mathematics Department, University of Botswana

T Stys, Mathematics Department, University of Botswana

In this paper the diffusion equation $u_t = au_xx + u_yy + cu_zz + f(t, x, y, z, u)$ is considered. Applying the finite difference approximation with respect to the space variables x, y, z, it results into the system of the Method of Lines, that is , the system of ordinary differential equations $U_t = AU + F$, where A is the block matrix on $(n+2)^3$, where $n+2$ is the number of mesh points in each of x, y, z directions. It has been proven in [1] that the Maximum Principle given for the an ordinary differential operator is applicable to the system of ordinary differential equations. Using the maximum principle, $O(h^2)$ uniform convergence of the method of lines has been proved. An efficient algorithm for solving the diffusion equation based on the method of lines, on the absolutely stable implicit Eulerś rule combined with well convergent Gauss-Seidel iterative method is given. The method is tested by examples with use of enclosed Mathematica module solveDiffusion. Large systems of ordinary differential equations

of the method of lines have been solved with $n^3 = 100000$ equations and unknowns. The module solveDiffusion gives the solution $O(h^2)$ accurate in the form of a table and graphical form.

Distances in Trees

Peter Dankelmann, University of KwaZulu-Natal

Let T be a tree of order n and let k be a positive integer, $k \leq n-1$. If D_k denotes the number of unordered pairs of vertices at distance k in T, then the sequence $D = (D_1, D_2, \ldots, D_{n-1})$ is called the *distance distribution* of T. In this talk we present results on D.

A Finite difference scheme for Burgers' equation

J Djoko*, University of Pretoria
R Anguelov, University of Pretoria
J Lubuma, University of Pretoria

In this talk, a finite difference scheme that preserve essential properties of the solution related to the Burgers' equation is discussed. Using the nonlocal approximation of nonlinear term proposed by Micken's in 94, we derived a family of schemes which replicates the properties of the kinetic energy of the system. Representative numerical results which validate our approach is presented.

An example of a Problem-based Learning exercise for first year engineering students

Jeanetta du Preez, University of Pretoria

First year Engineering students are often reluctant to accept that Calculus is part and partial of their undergraduate studies. Introducing simple real life problems into their course helps to convince them that there is a need for Calculus. Furthermore, research shows that students do not integrate their knowledge gained in different mathematics modules, not to even mention other areas such as physics, electronics and mechanics. The example is designed to integrate knowledge from the modules Linear Algebra and Calculus. Students were briefly introduced to on-line research and had to report back in a group of at most four. The airship-exercise deals with group work, modeling, estimation, integration, and finally a valuation of students´results. The students involved were 120 Industrial, Mechanical and Mining Engineering students from the University of Pretoria. Although the assignment contributed only 5% towards students final mark, more than 95% of the students did more than what was expected from them and concluded that they found the assignment relevant, enjoyable and challenging.

ALGEBRAICALLY EMBEDDING SPACES IN FUNCTIONALLY COMPACT ONES

Themba Dube, Department of Mathematics, Unisa

By introducing a conservative notion of functional compactness to frames, we identify two classes of topological spaces that can be densely embedded in functionally compact ones.

Well-posedness of a model of strain gradient plasticity for plastically irrotationally materials

F Ebobisse*, University of Cape Town, South Africa
Daya Reddy, University of Cape Town, South Africa
Andrew Mcbride , University of Cape Town, South Africa

In this talk, I will present the initial boundary value problem corresponding to a model of strain gradient plasticity due to Gurtin and Anand (Gurtin, M., Anand L., 2005: A theory of strain gradient plasticity for isotropic, plastically irrotational materials. Part I: Small deformations. J. Mech. Phys. Solids 53, 1624-1649). In a joint work with Daya Reddy and Andrew Mcbride (UCT), we formulated and studied the model as a variational inequality in which the unknowns are the displacement, the plastic strain and the hardening parameter. The aim of this talk is to discuss the properties of the problem that give existence of a unique solution for the variational inequality and also to present the difficulties that one encounters in trying to extend the model to the case where softening is present.

Representation of $L^q(\mu) \overset{\wedge}{\otimes} X$ by vector valued function spaces.

Jan Fourie, North-West University (Potchefstroom Campus)

In this talk we use results from the theory of tensor products of Banach spaces to establish the isometry of the space of so called strongly p-integrable functions in a Banach space X to the space of integral X-valued operators on $L^p(\mu)$ and the complete projective tensor product of $L^p(\mu)$ with an arbitrary Banach space X. Although there are similar results in recent papers, our contribution in the present paper is to remove all restrictions attached to the Banach space X. Through similar techniques, the result is then also considered in the context of the complete projective tensor product of a Banach lattice X (satisfying some conditions) with an arbitrary Banach space Y.

ON RINGS GENERATING SUPERNILPOTENT AND SPECIAL ATOMS

H France-Jackson*, Department of Mathematics, Nelson Mandela Metropolitan University
NJ Groenewald, Department of Mathematics, Nelson Mandela Metropolitan University

A semiprime ring A is called a $*$-ring if every proper homomorphic image of A is prime radical. It is well known that every nonzero $*$-ring generates an atom of the lattice of all supernilpotent (respectively special) radicals. A big open question in radical theory asks whether the prime radical coincides with the upper radical generated by the essential cover of the class $*$ of all $*$-rings. In this talk we consider the class $**$ of all $**$-rings, that is prime rings A such that the smallest special class containing A is closed under semiprime homomorphic images of A. We will show that the class $**$ strictly contains the class $*$ and that the smallest supernilpotent (respectively special) radical generated by a nonzero $**$-ring is also an atom of the lattice of all supernilpotent (respectively special) radicals. We then ask the similar question concerning $**$-rings and note that a negative answer to our question implies a negative answer to the big question.

Remarks on dimension functions for frames

JL Frith, Department of Mathematics and Applied Mathematics, University of Cape Town

I will discuss some new results for dimension functions for frames (point free topologies). I makes use of techniques derived from rings of continuous functions in the point free setting. This is a continuation of results presented in the Aspects of Contemporary Topology workshop held in Antwerp in 2005, but the talk will be self contained.

NONSTANDARD FINITE DIFFERENCE SCHEMES FOR HAMILTON-JACOBI EQUATIONS

M Froduald, Department of Mathematics and Applied Mathematics, University of Pretoria

We consider the numerical solution of the Hamilton-Jacobi equation of the particular form using a finite element space discretization coupled with a nonstandard finite difference time discretization. One of the main advantages of the nonstandard finite difference method is that in addition to the usual properties of consistency, stability and hence convergence, it produces numerical solutions which also exhibit essentially physical properties of the exact solution. The schemes for Hamilton-Jacobi equations applying standard finite difference techniques are typically monotone only under some restriction on the time step size. One of the main advantage of the nonstandard approach proposed here is that the obtained numerical scheme is monotone without restriction on the time step size. The numerical solutions of the scheme converge to the viscosity solution of the Hamilton-Jacobi equation.

On Analysis of Rayleigh and Bishop Equations

Ying Gai*, Department of Mathematical Technology, Tshwane University of Technology
Igor Fedotov, Department of Mathematical Technology, Tshwane University of Technology
Michael Shatalov, Department of Mathematical Technology, Tshwane University of Technology

The traditional approach to the analysis of Tonpilz transducers is based on longitudinal vibrations of systems, composed of distributed bars and lumped masses, with implementation of equivalent electric circuits to predict the resonance frequency and level of transmission voltage response. In this approach, the lateral effects are neglected in comparison to the major longitudinal vibrations and the analysis is founded on elementary theory of bars, which is applicable in the case of sections having lengths much longer than a characteristic dimension of the cross-section. In the present paper we attempt to improve the prediction by applying a more advanced theory of vibration of distributed bars. Rayleigh and Bishops theories are employed, considering the effects of lateral inertia and shear stress. The equation of motion for the Rayleigh or Bishop bar with the boundary conditions is derived from the corresponding Lagrangian and variation principle. We note that the given boundary conditions have physical interpretation and can be automatically generalised for an N-stepped bar of an arbitrary complexity. The eigenvalues and eigenfunctions are found by constructing the characteristic equation. It is possible to prove that two types of orthogonality for the eigenfunctions exist. The analytic solution to the Rayleigh-Bishop problem is obtained in terms of Green function. Finally, one example is given for the Rayleigh-Bishop equation with constant coefficients.

Classifying spaces and Hochschild cohomology

J-B Gatsinzi, University of Botswana

In his paper [1], Gerstenhaber defined a Lie bracket on the cohomology algebra $H^*(A, A)$ where A is an associative algebra. More precisely, if $\alpha \in H^m(A, A)$ and $\beta \in H^n(A, A)$, the Lie bracket $[\alpha, \beta] \in H^{m+n-1}(A, A)$. With a slight modification, we define a Lie bracket on $H^*(L, L)$, where L is a graded Lie algebra. If one considers the coformal space of which $L = \pi_*(\Omega X) \otimes \mathbf{Q}$, then $H^*(L, L)$ computes the rational homotopy algebra of the classifying space $B\mathfrak{aut}_1(X)$, where $\mathfrak{aut}_1(X)$ is the space of mappings from X to X that are homotopic to the identity.

References

[1] M. Gerstenhaber, *The cohomology structure of an associative ring*, Annals of Math. 78 (1963), 267–288.

A Duality with Borel Spaces

D Baboolal, University of KwaZulu-Natal
PP Ghosh*, University of KwaZulu-Natal

In this short paper we shall develop a duality between Borel spaces and Boolean σ frames. The class of sober Borel spaces shall be defined and characterised as precisely those Borel spaces which separate points and on which the {0, 1} valued probabilities are concentrated at some point. All separated Borel spaces then follow to be sober and examples of non separated sober Borel spaces are also provided.

"Durban Category Seminar" Report

PP Ghosh, University of KwaZulu-Natal

This is a report on the topics that have been covered in the "Durban Category Seminar". In particular, I shall present the tripleability theorem due to Beck and different variations of the same (like CTT, VTT) and present applications of these methods to homology/homotopy.

Bernhard Banaschewski - the South African Connections

Christopher RA Gilmour, University of Cape Town

I will give an overview of the mathematical interactions of Bernhard Banaschewski with South African colleagues over the past 30 years. If there is time I will present some joint work with Jumani Clarke linking spectral sigma-spaces with Banaschewski's notion of sigma-spectral topological spaces.

Closed sums of subspaces of Banach function spaces

JJ Grobler, North-West University

If a Banach function space is defined on a probability space, we associate with each sub-sigma-algebra of the algebra of measurable sets in the probability space a subspaces of the Banach function space.

We consider their embedding into the original space and in particular the question of the closedness of sums of such spaces.

Edge-removal criticality of the lower irredundance number of a graph

P Grobler, Department of Mathematical Sciences, University of Stellenbosch

We denote the domination number, the independent domination number and the lower irredundance number of a graph G by $\gamma(G)$, $i(G)$ and $ir(G)$ respectively. For $\pi \in \{\gamma, i, ir\}$, G is π-ER-critical if $\pi(G - e) > \pi(G)$ for all $e \in E_G$, where E_G denote the edge-set of G. The γ-ER-critical graphs as well as the i-ER-critical graphs are precisely the disjoint unions of non-trivial stars. Such a simple characterization does not exist for ir-ER-critical graphs. In this talk we discuss necessary conditions for ir-ER-critical graphs as well as characterizations of ir-ER-critical graphs with small lower irredundance numbers.

An inversion formula of Gohberg-Heinig type for matrix valued integral operators involving Hankel operators

GJ Groenewald, Department of Mathematics and Applied Mathematics, North-West University

We discuss necessary and sufficient conditions for the invertibility of operators $I - K_2K_1$, where K_1 and K_2 are Hankel integral operators acting between vector-valued L_1-spaces over $[0, \infty]$. Also, a Gohberg-Heinig type inversion formula is given for these operators. This is joint work with M.A. Kaashoek, Vrije Universiteit, Amsterdam.

The Stabilization of a One-Dimensional Wave Equation by Boundary Feedback with Non-Collocated Observation

Bao-Zhu Guo*, School of Computational and Applied Mathematics, University of the Witwatersrand

Cheng-Zhong Xu, Universite Claude Bernard Lyon I

This note addresses the stabilization of a one-dimensional wave equation with control at one end and non-collocated observation at another end. A simple exponentially convergent observer is constructed. The dynamical stabilizing boundary output feedback is designed via the observed state. While the closed-loop system is non-dissipative, we show its exponential stability using Riesz basis approach.

Absolutely closed Tychonoff spaces

David Holgate, University of Stellenbosch

A Tychonoff space X is a *Conway space* if it is sequentially closed in βX, its Stone-Čech compactification. In a recent preprint ([1]) this notion is naturally extended to define C-*Conway* spaces as Tychonoff spaces which are C-closed in βX for C a closure operator on **Tych**. Closely related is the notion of an *absolutely* C-*closed* space, one which is C-closed in any other space in which it is embedded. The relationship between absolutely C-closed and C-Conway spaces is explored in [1]. In this talk we introduce and briefly study the above concepts and explore their relationship via the pullback closure operator as introduced in [2], first in **Tych** and then in more general categories.

References

[1] D. Dikranjan, E.M. Peinador, V. Tarieladze. Conwayś question: the chase for completeness. *Preprint, September 2006.*

[2] D. Holgate. The pullback closure operator and generalisations of perfectness. *Applied Categorical Structures* **4** (1996) 107-120.

Mathematics Teaching in the Eastern Cape Province

PS Jaca, Walter Sisulu University

This presentation is a report of part of an ambitious project that is currently taking place in the province of the Eastern Cape. The aim of this part of the project is to characterize mathematics teaching in secondary schools of the province. Eastern Cape districts have identified several teachers as good mathematics teachers in their respective districts. The districts were not asked what qualities they use to identify these teachers or what is their understanding in general of what is a good mathematics teacher. This project attempts to characterize the mathematics teaching of these teachers. Twenty-four mathematics teachers were identified by the districts with the added quality that these teachers should teaching in areas that were previously disadvantaged. Again the districts were not expected to report what they used as qualities that represent previously disadvantaged. Currently the author is busy visiting all these teachers in their respective schools collecting relevant data. In this presentation an analysis of the data collected from three teachers is presented.

Developing tutorial learning environments: A case study of a statistical analysis in an authentic task

Jacob J Jaftha, UCT

Ubiquitous applications such as Microsoft Excel are widely used in educational contexts, including many undergraduate university courses. These are typically used within introductory activities or as applications students are encouraged to use while working independently. Much less consideration has been given to how one could integrate such ubiquitous applications into tutorial learning environments by creating scaffolded activities or automating assessment. We describe our experience in customising Excel to support flexible tutorial designs over several years, focusing on a statistical analysis in an authentic task. In our contact university context, we see it as important to use tutors to assist students with their understanding of the content rather than administering hand-in or assessment processes that could be automated. The automated assessment additionally shortens the feedback cycle while we also automated submissions and copying of data to simplify tedious tasks otherwise performed by students. While many similar ideas have been applied before, we have integrated a rich set of functionality in this project and applied in a large class, allowing us to reflect on various design strategies and approaches. We found it very important to have robust components while allowing flexibility in making changes to the content, even while tutorials were in progress.

Topics in semi-abelian algebra

George Janelidze, Department of Mathematics and Applied Mathematics
University of Cape Town

Semi-abelian categories have been introduced in authors joint work with L. Márki and W. Tholen in 1999, although there is a long story of previous investigations initiated by S. Mac Lane more than fifty years ago. These categories are to groups, rings, and various kinds of algebras over rings as abelian categories are to abelian groups and modules. We discuss semi-abelian generalizations of constructions and results known for some of the aforementioned algebraic structures, which involve the following concepts: object actions, ideals and clots, semidirect and tensor products, crossed modules, extensions and cohomology, commutators, and radicals. Particular topics here were developed jointly with F. Borceux, D. Bourn, A. Carboni, G. M. Kelly, L. Márki, M. C. Pedicchio, W. Tholen, A. Ursini, and other co-authors.

On standard homological lemmas in general pointed categories

Tamar Janelidze, Department of Mathematics and Applied Mathematics
University of Cape Town

Various classical homological lemmas known for abelian categories have been recently extended to so-called homological categories by D. Bourn; see [F. Borceux and D. Bourn, Mal'cev, protomodular, homological and semi-abelian categories, Mathematics and its Applications, Kluwer, 2004], and references there. We investigate conditions on the exact sequences involved, under which the same lemmas can be proved in general pointed categories; in particular these results apply to the relative case considered in [T. Janelidze, Relative homological categories, Journal of Homotopy and Related Structures, To appear].

Characterizations of additive categories via subtractivity

Zurab Janelidze, Department of Mathematics and Applied Mathematics
University of Cape Town

The notion of a subtractive category, introduced in [Z. Janelidze, Subtractive categories, *Applied Categorical Structures* 13, 2005, 343-350], extends the notion of a subtractive variety of universal algebras, due to A. Ursini, to abstract pointed categories. As shown later in [Z. Janelidze, Closedness properties of internal relations I: A unified approach to Mal'tsev, unital and subtractive categories, *Theory and Applications of Categories* 16, 2006, 236-261], the process of obtaining the class of subtractive categories from that of subtractive varieties, is an instance of a more general process which allows to construct a class of categories from a class of varieties determined by a suitable term condition. As it was also shown there, Mal'tsev categories in the sense of A. Carboni, J. Lambek, and M. C. Pedicchio can be obtained in the same way from Mal'tsev varieties, and unital categories in the sense of D. Bourn - from Jónsson-Tarski varieties in the sense of J. D. H. Smith. It is known that a pointed category $\mathcal{C}$ with products is half-additive in the sense of P. Freyd and A. Scedrov, that is, it is enriched in commutative monoids, if and only if both $\mathcal{C}$ and its dual category are unital. We prove a similar result, replacing "unital" with "subtractive" and "half-additive" with "additive". Although any additive category is half-additive, a subtractive category need not be unital. However, many categories which resemble the category of groups are both unital and subtractive; e.g. any semi-abelian

category in the sense of G. Janelidze, L. Márki, and W. Tholen is such. The above result implies the known result that a category $\mathcal{C}$ is abelian if and only if both $\mathcal{C}$ and its dual are semi-abelian. This also follows from another known fact that a category is additive if and only if it is half-additive and subtractive. Thus, we have: "subtractive" + "unital" + "co-unital" = "subtractive" + "co-subtractive" = "additive".

Mathematics of Interferometry and Non-Invasive Imaging Applications

E Jonathan, Harare Institute of Technology

The focus of this paper is the mathematics of optical interferometry. A mathematical model for interferometry is developed. The mathematical formulations are then applied to the design and performance prediction of interferometer-based non-invasive imaging devices. Representative example medical and non-medical applications of such devices are discussed.

A note on the weakly connected domination subdivision number of a graph.

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E Jonck*, Department of Mathematics, University of Johannesburg
LR Marcus, Department of Mathematics, De Anza College, USA

A dominating set D is a weakly connected dominating set of a connected graph $G = (V, E)$ if $(V, E \cap (D \times V))$ is connected. The weakly connected domination number of G, denoted $\gamma_{wc}(G)$, is $\min\{|S| \mid S \text{ is a weakly connected dominating set of } G\}$. The weakly connected domination subdivision number $sd_{wc}(G)$ of a graph G is the minimum number of edges that must be subdivided (where each edge in G can be subdivided at most once) in order to increase the weakly connected domination number. We provide a general upper bound on the weakly connected domination subdivision number of a connected graph. We show that the weakly connected domination subdivision number of a tree is either one or two and so a tree can be classified as either Class 1 or Class 2 depending on whether its weakly connected domination subdivision number is one or two. We then provide a constructive characterization of all trees in Class 2. Moreover, all caterpillars in Class 2 are also characterized.

HOW CAN MATHEMATICS STUDENTS BENEFIT FROM E-LEARNING COURSES?

KE Junquiera, Central University of Technology

At the end of the 2000 academic year, an instruction from the top management of the Central University of Technology, Free State (CUT, FS) was given to lecturing staff to adjust as many courses as possible into a web-based format. This instruction was the result of a worldwide shift in higher education towards the presentation of courses by means of electronic teaching and learning methods. The exercise made me wonder if the presentation of mathematics by means of e-learning could be done successfully and if a blended approach would indeed add value to students' learning experiences in mathematics. A need to answer these questions led to this study.

The purpose of the study was to explore and understand possible weaknesses and strengths of e-learning as a medium of instruction for undergraduate mathematics, as well as to identify the

challenges and opportunities that accompany this approach to teaching and learning. The study furthermore endeavoured to exploit the possibilities of e-learning at the CUT, FS, by making recommendations for implementing electronic teaching and learning strategies for undergraduate mathematics that would contribute to the improvement of students' learning and performance in the subject.

This study made use of a combined research approach which involved quantitative and qualitative methods. A case study of the teaching and learning of undergraduate mathematics of two main groups of students at the CUT, FS was considered. These groups were the first-, second- and third-year mathematics students enrolled in the School of Mechanical Engineering and Applied Mathematics during 2005, as well as the third-year mathematics education students enrolled in the School of Teacher Education during the same year. The case study provided data regarding the extent to which the use of e-learning in the teaching of mathematics at the CUT, FS contributes to a better understanding of the mathematical content and the improvement of the academic performance of the students.

The following instruments were used to collect the data: Artefacts in the form of computerised class lists and mark sheets; unstructured interviews; and two different questionnaires combining open-ended and closed questions.

The study found that mathematics can be presented by means of e-learning, and that it can serve a very important purpose with regard to blended learning. The success of e-learning, however, will depend on many factors which include student capabilities and prior knowledge; lecturer attitudes and facilitation skills; technology usage; as well as the quality of the e-learning materials. Current mathematical material cannot just be put online and new e-learning material, even for courses that have been taught for many years, cannot be developed within days, but will take time to design. There is furthermore not only one application that can and should be used to design mathematics e-learning material, but a wide range of technologies that the prospective e-learning mathematics lecturer can choose from. Last, and most importantly, the study found that students enjoy using e-learning in their mathematics studies and that a significant and constant use of the e-learning materials contributes towards improved performance in the subject by individual students.

Flowing from the research, a model for the presentation of mathematics by implementing a blended approach, involving classroom teaching and learning as well as e-learning, was developed.

Exact solutions for static charged spheres

K Komathiraj*, University of KwaZulu-Natal
SD Maharaj, University of KwaZulu-Natal

We present solutions to the Einstein-Maxwell system of equations with a specified form of the electric field intensity by assuming that the hypersurface t = constant are spheroidal. The condition of pressure isotropy yields two linearly independent classes of solutions which are obtained by restricting the spheroidal parameter K and the electric field intensity parameter . Consequently it is possible to find exact solutions in terms of elementary functions, namely polynomials and algebraic functions. Our result contains models found previously including the neutron star model of Tikekar (1990) and Maharaj and Leach (1996).

Sobriety revisited

W Kotzé, Rhodes University, Emeritus Professor

Characterizations of sobriety of both crisp and fuzzy topological spaces are considered, particularly i.t.o. observative nets.

Black hole linear perturbation equations

A Kubeka, University of South Africa

We discuss some discrepancies we found for the Ricci tensors of non-stationary axisymmetric spacetimes. We then review linear perturbation equations for three types of black holes: Schwarzschild, Kerr and Reissner-Nordstrom.

OBSERVATIONS ON QUASI-UNIFORM PRODUCTS

HA Kunzi*, Department of Mathematics and Applied Mathematics, University of Cape Town
S Dost, Hacettepe University, Ankara, Turkey

We study the question whether and to what extent the following results from symmetric topology can be extended to some appropriate asymmetric setting: The product of any family of quotient maps in the category of uniform spaces and uniformly continuous maps is a quotient map. Any uniformly continuous map defined on an arbitrary product of uniform spaces into a metric space depends on countably many coordinates. Any product of proximally fine totally bounded uniformities is totally bounded and proximally fine.

Positive and negative resonances in the Painleve Analysis

PGL Leach, University of KwaZulu-Natal

The roles of positive and negative resonances in the Painleve analysis of ordinary differential equations are described in a manner which makes sense of results reported sporadically in the literature of the last decade or so.

On idempotents in Banach algebras

Dr L Lindeboom*, Unisa
Dr RM Brits, University of Johannesburg
Prof H Raubenheimer, University of Johannesburg

The collection of idempotents in a Banach algebra A will be denoted by the symbol $A^\bullet$. Note that $A^\bullet$ is always closed in A. If $p \in A^\bullet$ then we denote by K_p the connected component of $A^\bullet$ containing the element p. The sets K_p turn out to be intimately related to the connected component of A^{-1} containing the identity element 1: using the fact that $\exp(A)$ acts transitively on K_p, via the mapping $\nu \rightarrow \nu p \nu^{-1}$, J. Zemanek showed that

$$K_p = \{\nu p \nu^{-1} : \nu \in \exp(A)\}.$$

Let A be a semiprime Banach algebra. In this paper we are going to show that every element of a minimal left(right) ideal $L = Ap (R = pA)$ can be written as a linear combination of two idempotents, both of which belong to the same connected component of $A^\bullet$. We are also going to show

that all the nonzero elements of $L(R)$ can be obtained by multiplication of the idempotent p on the left(right) by the connected component of 1.

Best Approximation in Vector-valued Function Spaces

Sizwe Mabizela, Department of Mathematics, Rhodes University

Let $(X, \|\cdot\|)$ be a (real) normed linear space, T a compact Hausdorff space and $C(T, X)$ the Banach space of all continuous functions $f : T \to X$ equipped with the norm

$$\|f\|_\infty = \sup_{t \in T} \|f(t)\|.$$

Let $(T, \mathcal{S}, \mu)$ be a finite measure space. For $1 \leq p < \infty$ (*resp.* $p = \infty$), let $L_p(\mu, X)$ be the Banach space of X-valued Bochner p-integrable (*resp.* essentially bounded) functions on T. This talk will present some of what is known about best approximation in the function spaces $C(T, X)$ and $L_p(\mu, X)$. We shall also point out some of the problems that have not yet been investigated in this area.

Applications of Keychains

Z Mahlasela, University of FORT HARE and RHODES University

The idea of keychain is an extension of preferential fuzzy subsets of finite sets. It can be used in modeling uncertainty in economics, social sciences. and in many more other fields. Here we are going to look at the set such that $a_k \geq a_{k-1} \geq \cdots \geq a_1 \geq a_0$ is a **keychain** of length $k+1$ where $a_i \in [0, 1]$ for $i = 0, 1, 2, \ldots, n$. This paper shows how keychains can be used as a tool of modelling different fuzzy real life situations.

A DECOMPOSITION APPROACH TO PREY-PREDATOR MODEL WITH CONSTANT EFFORT PREY HARVESTING

OD Makinde, Department of Applied Mathematics, University of Limpopo

From the point of view of human needs, the exploitation of biological resources and harvesting of populations are commonly practiced in fishery, forestry, and wildlife management. There is a wide range of interest in the use of bio-economic models to gain insight into the scientific management of renewable resources like fisheries and forestries. This is related to the optimal management of renewable resources. In this paper, an algorithm based on Adomians decomposition method is developed to approximate the solution of the ratio-dependent predator-prey system with constant effort harvesting. The convergence of the decomposition series is enhanced using Pad approximation technique. The qualitative analysis of the model reveals that constant effort prey harvesting may contribute to mutual extinction as a possible outcome of predator-prey interaction. Some plots of the population of predator and prey versus time are presented to illustrate the performance and the accuracy of the algorithm.

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THERMAL STABILITY OF A REACTIVE VISCOUS FLOW THROUGH A CHANNEL FILLED WITH A POROUS MEDIUM

OD Makinde, Department of Applied Mathematics, University of Limpopo

Catalytic converter used in an automobileś exhaust system is made up of a finely divided platinum-iridium catalyst (i.e. forming a porous matrix) and provides a platform for exothermic chemical reaction where unburned hydrocarbons completely combust. This helps to reduce the emissions of toxic car pollutant such as carbon monoxide (CO) into the environment. However, in order to ignite, stabilize and operate under steady-state conditions, the thermal criticality of a burner based on combustion in inert porous media like catalytic converter must be determined. In this paper, the steady-state solutions of a strongly exothermic reaction of a viscous combustible fluid in a channel filled with a saturated porous medium under Arrhenius kinetics, neglecting reactant consumption, are presented. The Brinkman model is employed and analytical solutions are constructed for the governing nonlinear boundary-value problem using a perturbation technique together with a special type of Hermite-Pad approximants and important properties of the temperature field including bifurcations and thermal criticality are discussed.

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THE DOITCHINOV COMPLETION OF A QUASI-PSEUDOMETRIC SPACE REVISITED

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HA Künzi, Department of Mathematics and Applied Mathematics, University of Cape Town

Various completion theories have been developed for quasi-pseudometric spaces. In particular the bicompletion and the Yoneda completion are well known. While these two completion theories can be applied to an arbitrary quasi-pseudometric space, another - from the point of view of category theory - very satisfactory construction is due to Doitchinov, but unfortunately it only applies to balanced quasi-metric spaces. In our article we investigate whether the ideas underlying Doitchinov's extension theory can be amended in such a way that his theory becomes useful in a broader context. We note that related investigations were also conducted by J. Deak.

On the number of solutions a second order linear ODE can have

JM Manale, UNISA

Some authors believe that a second order linear ordinary differential equation can have only two solutions. Theoretically, the justification is usually drawn from the Existence and Uniqueness theorems of solutions. In practice, authors go against the evidence generated by their methods and settle for the two solutions. In this contribution, we show that other solutions are possible. Why they have remained hidden is largely due to lack of attention paid to a word contained in the theorems. The value of this word started fading away a little over a century ago. Today it has become a useless appendage that hardly ever gets acknowledged. Some texts do not even carry it. As an example, we demonstrate the significance of such solutions in turbulent fluid flow.

LINDENSTRAUSS' SHORT PROOF OF THE LYAPUNOV CONVEXITY THEOREM: REVISITED

P Maritz, Department of Mathematical Sciences, University of Stellenbosch

This talk deals with the 1966 proof ("the slickest proof to end all proofs" [HAL, p.156]) by J. Lindenstrauss [LIN] of the Convexity Theorem of A.A. Lyapunov. It is widely recognised that the Lindenstrauss proof revived interest in the subject of vector measures, mainly because it manoeuvred the Convexity Theorem out of its measure theoretic environment. The impact and applications of the short proof will be discussed.

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Group invariant solution for a free two-dimensional turbulent jet with eddy viscosity

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DL Hill, Department of Mathematics and Statistics, University of Western Australia

A free two-dimensional turbulent jet described by eddy viscosity is analysed. A Lie point symmetry generator of the two-dimensional boundary layer equations for the turbulent jet exists provided the eddy viscosity satisfies a first order quasi-linear partial differential equation. The general form of the group invariant solution for the fluid velocity and the eddy viscosity is derived. The hypothesis due to Prandtl that the eddy viscosity is constant across a turbulent jet is investigated. Using the group invariant solution a numerical solution is derived in which the eddy viscosity varies across the jet. The results are compared with the corresponding results for a laminar two-dimensional jet.

Bi-Alexandroff spaces (Lower and upper cozero sets of bitopological spaces)

P Matutu, Rhodes University

Bi-Alexandroff spaces are defined as extensions of Alexandroff spaces. Urysohnś lemma for bi-Alexandroff spaces is used to show that lower and upper cozero sets of bitopological spaces are bi-Alexandroff spaces. An adjunction between bi-Alexandroff spaces and pairwise completely regular bitopological spaces is established.

MHD JEFFERY-HAMEL FLOWS: ITS NATURE AND STABILITY

Mr PY Mhone*, Applied Maths. Dept., University of Limpopo
Prof OD Makinde, Applied Maths. Dept., University of Limpopo

In this paper, we study incompressible viscous conducting fluid in convergent/divergent channels under the influence of an externally applied transverse magnetic field for very small magnetic Reynolds number Rm. For steady flow, we employ the Jeffery-Hamel similarity transformation to reduce the governing equations to a nonlinear differential equation which we solve by a perturbation method. The validity of this basic flow solution is extended by a special type of Hermite-Pade approximation semi-numerical approach. This technique reveals the analytical structure of the solution function and the important properties of overall flow structure including flow reversal and bifurcations are obtained and discussed. The temporal development of small disturbances on the basic flow is investigated by employing a modified form of normal modes that satisfy the linearised governing equations asymptotically far downstream. The resulting fourth order eigenvalue problem which reduces to the well known Orr-Sommerfeld equation under some limiting cases is solved numerically by spectral collocation technique with expansions in Chebyshev polynomials. The results indicate that an increase in the magnetic field intensity has a strong stabilizing effect on the flow for both diverging and converging channel's geometry. Keywords: Jeffery-Hamel flows; Hermite-Pade approximation; Small disturbances stability; Spectral method; Orr-Sommerfeld equation.

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Lie group analysis of unsteady flow of a variable viscosity liquid film along an inclined heated plate

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OD Makinde, University of Limpopo

Lie transformation group approach is employed to investigate the unsteady flow of a variable viscosity liquid film along an inclined heated plate. The upper surface of the liquid film is considered free with uniform temperature. Classical Lie point and potential symmetry analysis results in a rich array

of admitted symmetry for the special cases of the temperature dependent viscosity. Some invariant solutions are constructed.

Mathematical thinking and citizenship

M Moodley, University of Kwazulu-Natal

My topic is on how children solve problems by reflecting on their thinking. I talk about construction and reconstruction of ideas, taking risks and gaining ownership. An analysis of teacher - pupil dialogues is very interesting and important to diagnose what really happens in the classroom. In this paper is no teacher involvement. This did not occur through any organised design. It was by chance largely due to political strike by educators in the country (South Africa). However the conversational analysis of the learner-learner dialogue gives rise to a pedagogy landscaped in a social and multicultural paradigm where learners learn to think with their own ideas. The South African classroom is diverse in culture and language. Meeting the challenges of a changing classroom is strenuous if an open mind system is lacking. A teacher needs to empower the learners to design their thinking to eventually take their rightful places in society. Implicit in this is empowerment of teachers to walk into this culturally diverse classroom with confidence, take risks, motivate and satisfy the enthusiasm of the learners. This paper illustrates that children when given the opportunity can add sound to their voices, discuss, argue, negotiate and add meaning to their learning . The nature of talk and what happens in the classroom impacts on society and prepares the learner for critical citizenship, hopefully. The role of the teacher in taking children through a guided tour without indoctrinating them but empowering them lies in his/her ability to listen to the dialogue of learners, attitude, belief and concern of "why educate?".

Completeness for symmetric spaces.

SP Moshokoa, Unisa

In the literature [[2],[3]] completeness for symmetric spaces is done through the classical Cauchy criterion for metric spaces. However, unlike the situation in metric spaces a convergent sequence in a symmetric space is not necessarily a Cauchy sequence. We introduce the notion of convergence completeness for symmetric spaces and characterize completeness for these spaces without appealing to the notion of a Cauchy sequence. The new notion is equivalent to completeness when restricted to the class of metric spaces.

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ON THE BOUNDARY SPECTRUM IN BANACH ALGEBRAS

S Mouton, Department of Mathematics, University of Stellenbosch

In this talk we consider the set $S_\partial(a) = \{\lambda \in \mathbf{C} : \lambda - a \in \partial S\}$ (which we call the boundary spectrum of a) where ∂S denotes the topological boundary of the set S of all non-invertible elements of a Banach algebra A, and where a is an element of A. If the particular Banach algebra A needs to be emphasized, the notation $S_\partial(a, A)$ will be used. The boundary spectrum has several interesting properties. In particular, the following will be investigated: certain mapping properties of S_∂, the relationship between $S_\partial(a, A)$ and $S_\partial(a, B)$ where B is a closed subalgebra of a Banach algebra A such that B contains the unit of A, the relationship between $S_\partial(a, A)$ and $S_\partial(Ta, B)$ where B is another Banach algebra and $T : A \to B$ a homomorphism, and some applications in ordered Banach algebras.

A Lagrangian formulation of a generalized Lane-Emden equation and double reduction

CM Khalique, Department of Mathematical Sciences, North West University
B Muatjetjeje*, Department of Mathematical Sciences, North West University
FM Mahomed, School of Computational and Applied Mathematics, University of the Witwatersrand

We classify the Noether point symmetries of the generalized Lane-Emden equation $y'' + \frac{n}{x}y' + f(y) = 0$ with respect to the standard Lagrangian $L = \frac{x^n}{2}y'^2 - x^n \int f(y)dy$ for various functions $f(y)$. We then obtain first integrals of the various cases which admit Noether point symmetry. Moreover, we obtain reduction to quadratures for the cases that admit Noether symmetries. Three new case arises. One of them is when $f(y) = \alpha y^r$, where $r \neq -1, 0, 1$. The case $r = 5$ was considered previously and only a one-parameter family of solutions was presented. Here we provide a complete integration not only for $r = 5$ but for other r values. The Noether point symmetry is utilised for double reduction.

A New Time-Valued Model for HIV Transmission

S Mukwembi*, University of KwaZulu-Natal
S Borgatti, Boston College

Since several years, models for disease transmission networks have been based on simple graphs either valued or non-valued. We propose a more superior time-valued HIV transmission network model based on multigraphs. We also develop some mathematical tools to analyze the network and construct variants of some known existing measures of centrality to generate measures that are meaningful to our model.

Enumerative Properties of Gap-Bounded Sequences

A Munagi, University of the Witwatersrand, Johannesburg, RSA

A sequence of positive integers is gap-bounded if the set of differences between adjacent elements is bounded from above. We compute the number of gap-bounded sequences of given length, and obtain applications to the multi-step Fibonacci numbers and compositions of numbers. We also obtain enumeration formulas for sets of integer partitions and set partitions consisting of gap-bounded

sequences. Our results include a new interpretation of the Bell number recurrence and a generalization of a partition theorem of P. A. MacMahon.

Ordered Partitions and finite preferential fuzzy sets

V Murali, Mathematics, Rhodes University

The kernel of a fuzzy subset of an n-element-set $X = X_n = \{x_1, x_2, \cdots, x_n\}$ generates an ordered partition on X, the blocks of which are formed by elements with same membership value and the ordering is that of membership values. Conversely ordered partitions give rise to fuzzy subsets on X when a set of real numbers from the unit interval $\mathbf{I}$, one for each block, are assigned in such a way that the fuzzy subset becomes an order reversing function from the blocks to the unit interval $\mathbf{I}$. But the correspondence is clearly not one-to-one. With a view to make this one-to-one we introduce an equivalence relation called *preferential equality* on fuzzy subsets which reduces to equality of crisp sets when membership values are 0 and 1. In this paper we discuss some aspects of this correspondence with respect to counting and their implications to generating functions $P(x,t) = e^{tx}/(2-e^x)$ for some integral values of t.

THE METHOD OF ADDITIONAL CONDITIONS TO FIND THE NONLINEAR NONSINGULAR SOLUTIONS OF MATHEMATICAL PHYSICS

IN Neygebauer, Department of Mathematics, University of Rwanda

The solutions of Laplaces equation were created by using the singular Newtons law of gravitation that gives infinite value by zero distance between particles. There could be taken different nonsingular laws which can create another nonlinear equations of mathematical physics. The solutions of this equations could be considered and compared with traditional solutions of potential theory. The solutions of Laplaces equation with boundary point singularities could be regarded as approximations to the nonlinear nonsingular solutions of different physical problems. It is possible to suggest a method to find these nonsingular solutions using additional condition that is fulfilled for any nonsingular solution. The similar problem exist in fracture mechanics. The Griffiths problem for the tension of a linear elastic plate has singular infinite solution for stresses near the tip of a crack. Using additional condition the zero J-Integral that is satisfied for all bounded elastic potentials one can get the stress concentration factor that corresponds to experiments of Inglis and Griffiths. They write about these experiments in the paper that is regarded as the start of new theory of fracture mechanics. This approach could be transformed to another problems of mathematical physics such as heat transform or electromagnetic field, Helmholz equation or Schroedinger equation.

Lattices, Trees and Preferential Fuzzy Subgroups

S Ngcibi*, Mathematics Department, Rhodes University
V Murali, Mathematics Department, Rhodes University

A preferential fuzzy subgroup is a subchain in the lattice of subgroups of a group G under inclusion. In this communication we develop an alternative to the subgroup lattice for a p-group G of rank two, in the form of a tree diagram of nested subgroups under inclusion. The reverse direction of constructing the lattice from a tree diagram is also discussed. We illustrate with a specific example

how the tree diagram facilitates counting preferential fuzzy subgroups.

ESTRUTURAO DE UM CURSO DE INTRODUO A ANLISE NUMRICA ASSISTIDA POR COMPUTADOR.

JC Pedro, Citmatel-Tecnologias de la Informacin y Servicios Telemticos
Francisco Nodarse

O presente trabalho consiste numa proposta de estruturao de um Curso de introduo a Anlise Numrica , com o objectivo fundamental de mostrar algumas vias que permitam uma compreenso bsica dos mtodos de soluo numrica de problemas com recurso ao Computador. A estruturao do curso apoiada por um TUTORIAL HIPERMEDIAL, desenhado e implementado para o efeito, permitindo assim ao usurio uma independncia na definio e assimilao de conceitos com base em elementos de navegao inteligente (INTELIGNCIA ARTFICIAL), aplicados a aprendizagem combinando a incidncia condutivista com a riqueza hipermedial. O trabalho constituido por trs partes fundamentais que comportam um levantamento de trabalhos de vrios autores no campo da inteligncia artificial aplicada a aprendizagem particularmente da matemtica, uma proposta de estrutura do Curso de introduo a Anlise Numrica e por fim o desenho e a implementao do TUTORIAL HIPERMEDIAL sobre o referido curso e sua utilizao combinada com o Assistente Matemtico DERIVE for Windows Verso 4.01.

Evaluating infinite range exponential integrals using decomposition method

PA Phiri*, University of Limpopo
OD Makinde, University of Limpopo

This paper presents a new efficient algorithm based on Adomain decomposition approach for evaluating infinite range exponentially decreasing integrals in the form

$$\int_0^\infty Q(x)e^{-ax^n}dx, a > 0, n > 0.$$

The algorithm has been implemented in the computer algebra system (MAPLE) to enhance the application of the method. A number of illustrative examples are presented to demonstrate the computation efficiency of the programme.

Iterative approaches to convex feasibility problems in Banach spaces

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Hong-Kun Xu, School of Mathematical Sciences, UKZN
JG O'Hara, School of Statistics and Actuarial Sciences, UKZN

The convex feasibility problem (CFP) of finding a point in the nonempty intersection $\cap_{i=1}^{N} C_i$ is considered, where $N \geq 1$ is an integer and each C_i is assumed to be the fixed point set of a nonexpansive mapping $T_i : X \to X$ with X a Banach space. It is shown that the iterative scheme $x_{n+1} = l_{n+1}y + (1 - l_{n+1})T_{n+1}x_n$ is strongly convergent to a solution of (CFP) provided the Banach space X either is uniformly smooth or is reflexive and has a weakly continuous duality map, and provided the sequence $\{l_n\}$ satisfies certain conditions. The limit of $\{x_n\}$ is located as $Q(y)$, where Q is the sunny nonexpansive retraction from X onto the common fixed point set of the T_i's.

Duality via Truth: Semantic frameworks for lattice-based logics

Ingrid Rewitzky*, University of Stellenbosch

Ewa Orlowska, Warsaw National Institute for Telecommunications

A type of duality, called Duality via Truth, is presented for showing that a class of algebras and a class of frames provide equivalent semantics of a formal language whose signature coincides with that of the algebras in question. Consequently, the algebras and frames may be seen as expressing equivalent notions of truth. The approach is illustrated for logics based on not necessarily distributive lattices.

ON THE BINARY CODES FROM THE STRONGLY REGULAR $(40, 12, 2, 4)$ GRAPHS

BG Rodrigues,School of Mathematical Sciences, University of KwaZulu-Natal

We examine some properties of the binary codes obtained from the adjacency matrices of the 28 non-isomorphic strongly regular $(40, 12, 2, 4)$ graphs and describe their properties and the nature of some classes of codewords as well as give a description of the structure of the automorphism groups of the graphs, designs and codes and explore their links. Some of the codes are optimal or near optimal for the given length and dimension. Some dual codes admit majority logic decoding.

Solving General Equations

EE Rosinger, University of Pretoria

A method based on order completion for solving general equations is presented. In particular, this method can be used in order to solve large classes of nonlinear systems of PDE's with possibly associated initial and/or boundary value problems.

THE FUNDAMENTAL THEOREM OF ASSET PRICING UNDER PROPORTIONAL TRANSACTION COSTS

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The fundamental theorem of asset pricing characterises models of financial markets without arbitrage or free lunch, i.e., the making of risk-free profit without initial investment. It is well known that a classical friction-free model containing a risky stock and a bank account admits no arbitrage if and only if there exists a probability measure on the model under which the stock price, discounted by the interest rate on the bank account, is a martingale. In turn, the collection of such probability measures play a fundamental role in the pricing of contingent claims and derivative securities. We extend the fundamental theorem to a model where the risky stock is subject to proportional transaction costs in the form of bid-ask spreads, so that a share can always be sold for the bid price and bought for the ask price, and the bank account has different interest rates for borrowing and lending. We show that such a model is free of arbitrage if and only if one can embed in it a friction-free model that is itself free of arbitrage, in the sense that there exists an artificial friction-free price for the stock between its bid and ask prices and an artificial interest rate between the borrowing and lending interest rates such that, if one discounts this stock price by this interest rate, then the resulting process is a martingale under some non-degenerate probability measure. Restricting ourselves to the simple case

of a finite number of time steps and a finite number of possible outcomes for the stock price, the proof follows by combining classical arguments based on finite-dimensional separation theorems with duality results from linear optimisation.

Minimal asymmetric graphs

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A graph G is called asymmetric if the identity map on $V(G)$ is the only automorphism of G. In the 1960's Erdös and Renyi proved that almost all graphs are asymmetric (in the sense that the fraction of graphs of order n having non-trivial automorphism group tends to zero as n goes to infinity). A graph is called minimal asymmetric if it is asymmetric but none of its proper non-trivial induced subgraphs is asymmetric. A conjecture of J. Nesetril dating from the 1980's states that - contrary to what one might expect (especially in the light of the result of Erdös and Renyi) - there are only finitely many minimal asymmetric graphs. Our result is that Nesetril's conjecture is true; in fact, there are surprisingly few minimal asymmetric graphs and they are quite small: Theorem: There are exactly 18 minimal asymmetric graphs; they are at the same time the minimal graphs not having any involution. All these graphs are of order $6, 7$ or 8.

Smallest compactifications of biframes

Anneliese Schauerte, University of Cape Town

The locally compact, Hausdorff topological spaces are precisely the ones which have one-point, that is, smallest, compactifications. The situation in pointfree topology is similarly well understood - the frames with smallest compactifications are the regular, continuous ones. However, in the asymmetric world of bispaces and biframes things are not so clear. We discuss a construction of smallest compactifications for biframes under the assumptions of weak local compactness and regularity.

Reference

A. Schauerte, Notions of local compactness and smallest compactifications of biframes. Applied Categorical Structures (2006) 14:259-272.
SAMS Subject Classifications: 2, 5 and 15

Levy Processes

IM Schoeman, NWU (Potchefstroom campus)

Levy processes have an advantage over Brownian motion in that they are able to reflect the non-continuous nature of the dynamics of most financial instruments. We will have a look at the motivation, properties, modelling and simulation of Levy processes.

A group of the form $2^7 : S_8$

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The group $S_8 \cong O_6^+(2)$ is a maximal subgroup of $SP_6(2)$ of order 40320 and index 36. It can be generated by two elements of order 2 and 7 respectively inside $SP_6(2)$. The group $2^7 : S_8$ is a maximal subgroup of full automorphism group $\overline{Fi_{22}}$ of the smallest Fischer sporadic simple group Fi_{22} of index 694980. The object in this paper is to compute the Fischer - Clifford matrices of $2^7 : S_8$ which can then be used together with the ordinary character tables of the inertia factors of S_8 to compute its full ordinary character table.

On Well-Posedness, Regularity and Exact Controllability for Problems of Transmission of Plate Equation with Variable Coefficients

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A system of transmission of Euler-Bernoulli plate equation with variable coefficients under Neumann control and collocated observation is studied. Using geometric multiplier method on Riemannian manifold, it is shown that the system is well-posed in the sense of D. Salamon. This concludes the equivalence between the exact controllability of open-loop system and the exponential stability of closed-loop system under the proportional output feedback control. The regularity of the system in the sense of G. Weiss is also proved, and the feedthrough operator is found to be zero. These properties make this PDE system parallel in many ways to the finite-dimensional ones. Finally, the exact controllability of open-loop system is also developed under a uniqueness assumption by establishing the observability inequality for the dual system.

ANISOTROPIC EFFECT ON THE SURFACE TENSION OF COMPACT STARS

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In an earlier treatment it was demonstrated that general relativity gives higher values of surface tension in strange stars with quark matter than neutron stars. We generate the modified Tolman-Oppenheimer-Volkoff equation to incorporate anisotropic matter and use this to show that pressure anisotropy provides for a wide range of behaviour in the surface tension than is the case with isotropic pressures. In particular it is possible that anisotropy drastically decreases the value of the surface tension.

Hall current effects on unsteady rotating disc MHD flow.

P Sibanda, University of KwaZulu-Natal

The present investigation is concerned with the effects of Hall current, the magnetic field effect, the viscosity variation and the porosity parameter on unsteady MHD laminar incompressible flow of an electrically conducting incompressible fluid on a porous rotating disc in the presence of a uniform magnetic field. The unsteady boundary layer equations are solved using perturbation and asymptotic techniques coupled with a shooting method for small and large times respectively. The solutions are obtained in terms of local radial skin friction, local tangential skin friction and local rate of heat transfer at the surface of the disc for different values of pertinent parameters.

A spectral Mapping Theorem for Measurable Operators

ID Tembo, University of Cape Town

A Borel functional calculus for unbounded self-adjoint operators has been widely studied. Let $\mathcal{M}$ be a semifinite von Neumann algebra on a Hilbert space $\mathcal{H}$, equipped with a semifinite faithful normal trace τ. Let S be a self adjoint operator in the algebra $\widetilde{\mathcal{M}}$ of τ-measurable operators. We investigate the class of Borel functions f for which $f(S)$ lies in $\widetilde{\mathcal{M}}$. For positive τ-compact operators, we use a characterisation of the point spectrum in terms of decreasing rearrangements to obtain a spectral mapping theorem in $\widetilde{\mathcal{M}}$ for positive increasing functions.

Mining bases for $\mathbf{M_{GK}}$-valid association rules

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Jean Diatta, University of La Reunion, France
Daniel R Feno, University of La Reunion, France

In this paper, in the light of very interesting properties of the so-called quality measure MGK, which has been shown to be the normalized quality measure associated to most of the quality measures used in the data mining literature, and which enables to handle both positive and negative association rules, we present three new algorithms for generating respectively the bases for negative MGK-exact rules, positive MGK-approximate rules and negative MGK-approximate rules. On the other hand, following the example construction of the well known basis of the association rules according to confidence, we characterize and generate a basis for MGK-valid association rules in terms of a closure operator induced by a Galois connection.

The Efficiency of Algorithms for the Simulation of Diffusions

Jack Urombo, School of Mathematical Sciences, Harare Institute of Technology

Numerical methods for the simulation of stochastic differential equations(SDEs) have been developed. The algorithms for SDEs simulation exhibit different levels of accuracy and efficiency. The accuracy depends on the discretisation or time stepping scheme. The discretisation can be deterministic and stochastic. Adaptive methods increase the efficiency of the simulation by optimally changing the time stepping when the SDE is stiff. In this work a hybrid algorithm is proposed. It is a combination of different schemes that involves finding weak and strong solution depending on whether the SDE is stiff, has jumps and is stable.

(Approach) frames: similarities and adjunctions

Christophe van Olmen, University of Antwerp

Approach frames were introduced by B. Banaschewski in 1999 to provide a means to define sobriety for approach spaces. They are defined as an abstraction of regular function frames, more specifically, an approach frame is a frame with two extra families of unary operations: for each $\alpha \in [0, \infty]$ we have an 'addition' by α and we have a 'truncated subtraction' by α and these operations need to satisfy all algebraic laws that hold on $[0, \infty]$ for addition, truncated subtraction and the frame operations. We will argument that this notion gives us a pointfree counterpart of approach spaces, similar to what frames are to topological spaces: both have the same categorical relationship with their spatial counterpart, namely they arise as Eilenberg-Moore algebras induced by the Hom-functor to the respective initial objects. Furthermore, a pointfree counterpart of the simultaneously bireflective and bicoreflective embedding of topological spaces into approach spaces will be described. We have an embedding of frames into approach frames with a left and right adjoint and as these functors 'complete the square' of categories we will also look at the commutations between the involved functors.

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ON THE ASYMPTOTIC GROWTH OF THE LOTTERY NUMBER

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In this talk it is shown that the lottery number $L(m, n, t; k)$ grows at a polynomial rate as $m \to \infty$ (for any constant values of $\{n, t\} \geq k$), where $L(m, n, t; k)$ denotes the smallest cardinality of a lottery playing set, consisting of a collection of n-sets drawn from a universal m-set, that guarantees the player a k-match with the randomly chosen winning t-set (also drawn from the universal m-set). In particular, the limit $\lim_{m\to\infty} L(m, n, t; k)/m^k$ is shown to exist. Finally, it is conjectured that the limiting constant above is such that

$$\lim_{m\to\infty} \frac{L(m, n, t; k)r}{\frac{t}{k}\binom{m}{t}} = 1,$$

where $r = \sum_{i=k}^{n} \binom{n}{i}\binom{m-n}{t-i}$, and some evidence is provided to support this conjecture. If this conjecture is true, then the optimal density of a lottery design is asymptotic to t/k.

The process followed at the Nelson Mandela Metropolitan University to identify the criteria to include in a Mathematics placement test

Shirley Wagner-Welsh, NMMU

In this paper the process that was followed to identify the criteria that are indicators of success in a Mathematics 1 course at the Nelson Mandela Metropolitan University is described. The identification of the criteria is the first phase in the development of a placement test for first year Mathematics students.

The Kink Phenomenon in Fejer and Clenshaw-Curtis Quadrature

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The Fejér and Clenshaw-Curtis rules for numerical integration exhibit a curious phenomenon when applied to certain analytic functions. When N (the number of points in the integration rule) increases, the error does not decay to zero evenly but does so in two distinct stages. For N less than a critical value, the error behaves like $O(\rho^{-2N})$, where ρ is a constant greater than 1. For these values of N the accuracy of both the Fejér and Clenshaw–Curtis rules is almost indistinguishable from that of the more celebrated Gauss–Legendre quadrature rule. For larger N, however, the error decreases at the rate $O(\rho^{-N})$, i.e., only half as fast as before. Convergence curves typically display a kink where the convergence rate cuts in half. In this talk we derive explicit as well as asymptotic error formulas that provide a complete description of this phenomenon.

On Topologizing Groups

Yevhen Zelenyuk, University of the Witwatersrand

We show that every infinite group admits a nondiscrete zero dimensional Hausdorff topology with continuous translations and inversion.