Idiosyncratic Risk and Security Returns*

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Abstract

The traditional CAPM approach argues that only market risk should be incorporated into asset prices and command a risk premium. This result may not hold, however, if some investors can not hold the market portfolio. For example, if one group of investors fails to hold the market portfolio for exogenous reasons, the remaining investors will also be unable to hold the market portfolio. Therefore, idiosyncratic risk could also be priced to compensate rational investors for an inability to hold the market portfolio. A variation of the CAPM model is derived to capture this observation as well as to draw testable implications. Under both the Fama and MacBeth (1973) and Fama and French (1992) testing frameworks, we find that idiosyncratic volatility is useful in explaining cross-sectional expected returns. We also discover that returns from constructed portfolios directly co-vary with idiosyncratic risk hedging portfolio returns.

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Abstract

The traditional CAPM approach argues that only market risk should be incorporated into asset prices and command a risk premium. This result may not hold, however, if some investors can not hold the market portfolio. For example, if one group of investors fails to hold the market portfolio for exogenous reasons, the remaining investors will also be unable to hold the market portfolio. Therefore, idiosyncratic risk could also be priced to compensate rational investors for an inability to hold the market portfolio. A variation of the CAPM model is derived to capture this observation as well as to draw testable implications. Under both the Fama and MacBeth (1973) and Fama and French (1992) testing frameworks, we find that idiosyncratic volatility is useful in explaining cross-sectional expected returns. We also discover that returns from constructed portfolios directly co-vary with idiosyncratic risk hedging portfolio returns.

Introduction

The usefulness of the well-celebrated CAPM theory of Sharpe, Lintner, and Black to predict cross-sectional security and portfolio returns has been challenged by researchers such as Fama and French (1992, 1993). It is still debatable, however, whether Fama and French's empirical approach has invalidated the CAPM (see, for example, Berk, 1995; Ferson and Harvey, 1999; Kothari, Shanken, and Sloan, 1995; Jagannathan and Wang, 1996; and Loughran, 1996). Moreover, as Roll (1977) has pointed out, it is difficult, if not impossible, to devise an adequate test of the theory. Nevertheless, financial economists have worked in several directions to improve the theory of asset pricing. The first route has involved relaxing the underlying assumptions of the model, including the introduction of a tax effect on dividends (e.g., Brennan (1970)), non-marketable assets (e.g., Mayers (1972)), as well as accounting for inflation and international assets (e.g., Stulz (1981)). A second route has been to extend the one period CAPM to an intertemporal setting (e.g., Merton, 1973; Lucas, 1978, Breeden, 1979, and Cox, Ingersoll and Ross, 1985). Ross[1] (1976) has taken a different route by assuming that the stochastic properties of asset returns are consistent with a factor structure, and our approach is in that tradition. We accept the CAPM as a reasonable first order approximation, but find that the model has different implications when we assume that not all investors are able to hold the market portfolio.

Starting from mean variance analysis,[2] the traditional CAPM theory predicts that only market risk should be priced in equilibrium; any role for idiosyncratic risk is completely excluded through diversifications. CAPM must surely hold if investors are alike and can hold a combination of the market portfolio and a risk-free asset as the theory prescribes. In reality, however, institutional investment managers will often deliberately structure their portfolios to accept considerable idiosyncratic risk in an attempt to obtain extraordinary returns. Even in the absence of unusual cases such as Enron and WorldCom, these investors fully appreciate the importance of idiosyncratic factors in affecting the risk to which they are exposed. Therefore, as Merton (1987) wrote in his AFA presidential address, "... financial models based on frictionless markets

[1]Also see, for example, Chamberlain and Rothschild, 1983; Chen, Roll, and Ross, 1986; Connor, 1984; Connor and Korajczyk, 1988; Dybvig, 1983; Lehmann and Modest, 1988; Shanken, 1982; and Shukla and Trzcinka 1990).

[2]This is consistent with expected utility maximization for any concave utility function (see Ross (1976)).

and complete information are often inadequate to capture the complexity of rationality in action." When one group of investors – who we call "constrained" investors – is unable to hold the market portfolio for various reasons, such as transactions costs, incomplete information, and institutional restrictions including limitations on short sales, taxes, liquidity constraints, imperfect divisibility of securities, or any other exogenous factors, then the remaining investors – labeled as "free" or "unconstrained" investors – will also be unable to hold the market portfolio. This is so because the *constrained* investors' holdings and the *free* investors' holdings together make up the whole market. An inability to hold the market portfolio will force investors to care about total risk not simply market risk. Since the relative per capita supply will be high for those stocks that the constrained investors do not hold or only hold in very limited amounts, the prices of these stocks must be relatively low. In other words, an idiosyncratic risk premium can be rationalized to compensate investors for the "over supply" or "unbalanced supply" of some assets.

Still another intuition can be gained in terms of diversification. Suppose the actual market portfolio consists of only tradable securities. In other words, the market portfolio is observable and measurable. Under our assumption that some investors are constrained from holding all securities, the "available market portfolio" that unconstrained investors can hold will be less diversified than the *actual* market portfolio. Therefore, its corresponding risk will be higher, and a larger risk premium will be required. When individual investors use the *available* market portfolio to price individual securities, the corresponding risk premia tend to be higher than those under the CAPM where all investors are able to hold the *actual* market portfolio. This is because some of the systematic risk would be considered as idiosyncratic risk relative to the *actual* market portfolio. Hence, idiosyncratic risk would be priced in the market. We will fully develop this idea below and investigate the empirical implications.

There are many reasons why individual investors might not be able to hold the market portfolio. First, transactions costs are likely to prevent individual investors from holding large numbers of individual stocks in their portfolios. In fact, Hirshleifer (1988) has predicted that trading costs limit the participation of some classes of traders in commodity futures markets and that idiosyncratic risk will be priced cross-sectionally. Furthermore, more than half of U.S. households have accounts with brokerage firms. Because of limited resources or their desires to exploit the unique characteristics of individual stocks, these investors normally only hold a handful of stocks.[3] In addition, as Hirshleifer (2001) has pointed out that "there is also experimental evidence that investors sometimes fail to form efficient portfolios and violate two-fund separation." Also, in order to provide financial incentives for their employees that are not charged against corporate income, many companies now grant stock options to their employees or match the employee contributions to 401K retirement plans with company stock. In general, such employees are constrained from liquidating their positions or to hedge the stocks of their own firms, hence they tend to hold very unbalanced portfolios.[4] Moreover, some stock traders and market makers hold large positions in individual stocks.

Finally, there are several thousand actively managed mutual funds and pension funds in existence. While these funds are able to hold a market portfolio, they typically do not do so.[5] Moreover, Day, Wang, and Xu (2000) have demonstrated that the portfolios of equity mutual funds are not even mean-variance efficient with respect to their holdings. These "active" portfolio managers are able to obtain large management fees because they claim to be able to find "undervalued" securities and hence offer investors the possibility of risk-adjusted returns superior to the market averages. While there is no evidence that they can achieve this goal even before expenses (see Jensen (1968) and Malkiel (1995)), they do affect the relative supply of stocks available for other investors. Equity mutual funds hold portfolios comprising almost one-third of the total capitalization of the U.S. stock market, and thus they have the potential to alter the supply of securities available to other investors in an important way. The fact that investors are willing to pay the high costs to invest in non-indexed mutual funds indicates that they do not choose to allocate their portfolios between a market portfolio and a risk-free asset as the CAPM theory assumes.

Behavioral finance provides additional insights that help explain why institutional

[3] One may argue that the idiosyncratic volatility of a portfolio is close to zero when there are more than 20 stocks. However, this conclusion is based on a random sampling. In reality, investors do not randomly select their stocks. In addition, Campbell, Lettau, Malkiel, and Xu (2000) have shown that a well-diversified portfolio must have 40 or more stocks in recent decades because idiosyncratic volatility has increased.

[4] This practice is particularly prevalent in high technology industries.

[5] Institutional investors are more likely to purchase index funds than are individual investors. A proximately 10 percent of the mutual funds held by individuals were indexed in 2002 while about one quarter of institutional funds were indexed. Thus, the vast majority of investors do not hold the market portfolio. According to a recent survey, about 15% of individual investors only held one stock in their account and an average investor only owned about three stocks.

investors may be sensitive to the idiosyncratic risk of individual securities even though such volatility can be diversified away. The prospect theory of Kahneman and Tversky (1979)makes clear that the major force influencing the decisions of investors is loss aversion. Mutual fund managers, pension fund managers and other institutional investors are usually required to report quarterly to their directors, trustees, etc., on their recent investment performance. That report typically includes a discussion of the best and worst performing stocks in their portfolios. Even if balanced by favorable performance in other parts of the portfolio, it is extraordinarily difficult to explain why the manager bought and held those stocks, that declined sharply in value. Trustees and directors are quite likely to ask the indelicate question of the manager, "How could you have held WorldCom or Enron as these common stocks had lost essentially all of their value?" Trustees and directors are unlikely to be sympathetic to arguments that idiosyncratic factors (accounting fraud, unexpected industry overcapacity, etc.) are valid excuses for such investment errors. It is not unreasonable, therefore, to believe that stocks that are sensitive to substantial idiosyncratic risks may be subject to additional risk premiums.

There is no doubt, therefore, that not every investor is willing or able to hold the market portfolio. Indeed, even index funds that attempt to replicate the very broad market indexes, such as the Wilshire 5000 and Russell 3000, do not hold all the stocks in the index in order to minimize transactions costs. To what extent this distortion will affect the CAPM is purely an empirical question. Our approach is not to conduct a direct investigation of the portfolio holdings of investors. Instead, we will take as given that investors are unable to hold the market portfolio. Starting from there, we investigate the consequences.

The role of idiosyncratic risk in asset pricing has been studied in the literature to some extent. Most theoretical works have been focused on the effect of idiosyncratic (or uninsurable) income risk on asset pricing (see for example, Heaton and Lucas (1996), Thaler (1994), Aiyagari (1994), Lucas (1994), Telmer (1993), Franke, Stapleton, and Subrahmanyam (1992), and Kahn (1990)). Based on assumptions similar to ours, Levy (1978) derived a modified CAPM that revealed possible bias in the beta estimator as well as a possible role for idiosyncratic risk. In contrast, our model demonstrates an explicit role of idiosyncratic risk in asset pricing. Furthermore, we show that the beta estimator will be unbiased if idiosyncratic risk is appropriately account for. Perhaps the most relevant study to this paper is Merton (1987). Starting from a single factor structure of returns, he assumes that investors can only invest in securities where they have exact information about the expected returns, beta loadings, and volatilities. This assumption seems to be too restrictive in today's investment environment. We therefore, use assumptions from the supply side instead. Although both the Merton model and ours yield simular pricing implications, our model is more general in two respects. First, we do not require that idiosyncratic returns are uncorrelated across individual stocks. If this condition is imposed, our model will reduce to that of Merton (1987). Second, we demonstrate that the price of idiosyncratic risk for an individual stock depends on its correlation with the aggregated undiversified idiosyncratic return. This motivates the construction of the return proxy for the idiosyncratic risk used in our time series study.

The CAPM is the most extensively tested model. Early empirical studies that rejected the model include Black, Jensen, and Scholes (1972), Douglas (1969), Lintner (1965), and Miller and Scholes (1972). In particular, Douglas (1969) concluded that residual variance was also priced based on a single cross-sectional regression using average returns.[6] Fama and MacBeth's (1973) important study both rejected the role of idiosyncratic risk in the CAPM and provided a more powerful cross-sectional test. More recently, Lehmann (1990) studied the significance of residual risk in the context of statistical testing methodology. Some indirect evidence regarding the role of idiosyncratic risk has also surfaced in recent years. Falkenstein (1996) found some evidence that the equity holdings of mutual fund managers appeared to be related to idiosyncratic volatility. Using Swedish government lottery bonds where the underlying risk is idiosyncratic by construction, Green and Rydqvist (1997), find prices of the bonds appear to reflect aversion to idiosyncratic risk. Bessembinder (1992) finds strong evidence that idiosyncratic risk was priced, looking at a cross-section of foreign currency and agricultural futures. In studying the volatility linkage between national stock markets, King, Sentana, and Wadhwani (1994) provided evidence that idiosyncratic economic shocks are priced and that the 'the price of risk' is different across stock markets.

In this paper, we provide a theory of idiosyncratic risk and test some of the implications of our model with constructed portfolio returns, individual stock returns, and equity mutual fund returns. Since most empirical evidence supporting the role of idiosyncratic risk from early studies in asset pricing was disregarded after the comprehensive

[6]Miller and Scholes (1972) suggested that several sources of bias may exist including omitting the risk free rate, errors in beta measures, and correlation between betas and residual variances. They claimed that the errors-in-variables issue is especially important.

study by Fama and MacBeth (1973), we start our empirical study by replicating the Fama and MacBeth study and extending it to different settings and sample periods. In addition, we also consider Fama and French's (1992) frameworks. The empirical results support our model by showing that (1) idiosyncratic volatility alone is important in explaining cross-sectional expected return differences; (2) its explanatory power does not seem to be taken away by other variables, such as the size, the book-to-market, and liquidity variables; and (3) the findings are robust to Japanese stock return data.

The paper is organized as follows: A simple CAPM type of model with some constrained investors is constructed in the first section. After studying the implications of the model, we discuss issues related to empirical testing and data construction in section 2. Section 3 presents cross-sectional evidence in the spirit of Fama and MacBeth (1973) and Fama and French (1992). Times series evidence concerning the role of idiosyncratic volatility is briefly discussed in Section 4. Section 5 presents concluding comments.

1 The basic model and its implications

The Capital Asset Pricing Model is an equilibrium model in which the demand for equity securities is determined under a mean-variance optimization framework. The market clearing condition then equates demand and the exogenous supply to achieve equilibrium. Since it is assumed that investors are homogenous and are able to hold every asset in the market portfolio, their holdings will be similar in equilibrium. As a result, investors' holdings of risky stocks will be comprised of shares held in proportion to the market portfolio, which is a value-weighted portfolio of all the securities available for investment. In other words, the market portfolio is always feasible and will be the only portfolio held in equilibrium. Such an available market portfolio will be altered, however, when a group of investors does not or cannot hold every stock for any of the reasons described in the previous section.

1.1 Asset returns in a traditional CAPM world

For ease of exposition, we assume that there are three risky stocks denoted a, b, and c that generate a return vector $\mathbf{R} = [R_a, R_b, R_c]'$ and one riskless bond that pays interest rate r. Not all of the stocks are necessarily on the traditional mean-variance efficient frontier. The final result will not depend on the number of stocks assumed since we use vector notations. The risk structure for the three stocks is represented by their variance-covariance matrix of returns, $\mathbf{V} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} & \sigma_{bc} \\ \sigma_{ab} & \sigma_b^2 & \sigma_{ca} \\ \sigma_{ca} & \sigma_{bc} & \sigma_c^2 \end{bmatrix}$. Each investor has the following utility function,

$$u(W) = E(W) - \frac{1}{2\tau} Var(W), \tag{1}$$

where W represents future wealth and τ is the coefficient of risk tolerance. This particular utility function is consistent with the family of exponential utility functions when future wealth has a normal distribution. If we denote $\mathbf{X}_j = [x_{a,j}, x_{b,j}, x_{c,j}]$ as investor j's dollar amount invested in the three stocks, the corresponding budget constraint can be written as,

$$W_j = W_{0,j}(1 + r) + \mathbf{X}_j'(\mathbf{R} - r\underline{\mathbf{1}}), \tag{2}$$

where $W_{0,j}$ is the initial endowment. Utility maximization of equation (1) subject to the budget constraint represented by equation (2) leads to the following demand function,

$$\mathbf{X}_j = \tau \mathbf{V}^{-1}(\boldsymbol{\mu} - r\underline{\mathbf{1}}), \tag{3}$$

where $\boldsymbol{\mu} = E(\mathbf{R})$ is the vector of expected returns of individual stocks. Note that demand is independent of investors' initial wealth because we do not constrain borrowing at the rate r on riskless bonds. Although the variances and covariances among stock returns are given exogenously in this framework, the expected returns should be determined in equilibrium by total supply. In other words, the equity market clears with the condition of $\sum_j^n \mathbf{X}_j = \mathbf{S}$, where $\mathbf{S} = [S_a, S_b, S_c]$ is the total supply of individual stocks and n is the total number of investors. The equilibrium expected returns in this unrestricted world can thus be written as,

$$\boldsymbol{\mu} - r\underline{\mathbf{1}} = \frac{1}{n\tau}\mathbf{VS}. \tag{4}$$

Following convention, we define the market portfolio as $\boldsymbol{\alpha} = \frac{1}{M}\mathbf{S}$, where $M = \mathbf{S}\underline{\mathbf{1}}' = S_a + S_b + S_c$. Under this notation, the expected market return and market volatility can be expressed as $\mu_m = \boldsymbol{\alpha}'\boldsymbol{\mu}$ and $\sigma_m^2 = \boldsymbol{\alpha}'\mathbf{V}\boldsymbol{\alpha}$. Equation (4) can thus be converted into the traditional CAPM,

$$\boldsymbol{\mu} - r\underline{\mathbf{1}} = \boldsymbol{\beta}(\mu_m - r), \tag{5}$$

where $\boldsymbol{\beta} = [\beta_a, \beta_b, \beta_c] = \frac{1}{\sigma_m^2}\mathbf{V}\boldsymbol{\alpha}$ is the conventional measure of systematic risk. Substituting equation (5) back into equation (3), it is clear that, in an economy where investors are homogenous and unconstrained, only a linear combination of the risk-free bond and the (efficient) market portfolio m generated from all the individual stocks will be held. What makes this single factor model a truly equilibrium model is the existence of an equilibrium market portfolio, which is determined by the aggregate supply. Since the variance-covariance structure and the total supply of stocks are common knowledge, this market portfolio can be constructed by an econometrician even when there are limited investment opportunities.[7]

[7]For illustrative purposes, if we assume that only the NYSE/AMEX/NASDAQ listed stocks form the entire universe of investment assets, the conventional market index portfolio, such as the value weighted NYSE/AMEX/NASDAQ index, is indeed the market portfolio and is observable to everyone. Since we will study the effect of limited investment opportunities under such a scenario where we know the market portfolio, Roll's (1977) critique is inapplicable in this context.

Equation (5) says that only systematic risk, represented by the scaled covariance between individual stock returns and the market return, matters for valuation. Idiosyncratic risk can be diversified away in this framework, and will not command a risk premium. Based on equation (5), we can express individual stock returns as,

$$R_{i,t} - r_{f,t} = \beta_i (R_{m,t} - r_{f,t}) + \epsilon_{i,t}, \tag{6}$$

where $\epsilon_{i,t}$ is the idiosyncratic return. In the discussion that follows, we will use idiosyncratic volatility to measure idiosyncratic risk. In light of equation (6), idiosyncratic volatility $\mathbf{V}_\epsilon$ can simply be calculated as,

$$\mathbf{V}_\epsilon = \mathbf{V} - \boldsymbol{\beta} \sigma_m^2 \boldsymbol{\beta}'. \tag{7}$$

1.2 Asset returns under an imperfect market portfolio

While the CAPM must prevail when all individuals are unconstrained in holding underlying assets, this assumption is frequently violated in practice. When some investors can not or do not hold every security for the reasons discussed in the previous section, the CAPM will fail to hold. For ease of exposition, we assume that there are three groups of investors. While the "free" investors in the second group have full investment opportunities and can hold all securities, the first and the third groups of investors are assumed to be constrained from holding the first and the third stocks, respectively. Following the same steps, we can derive demand equations similar to equation (3) for representative investors in each group as,

$$\begin{aligned}
\mathbf{X}_{(1)} &= \tau \begin{bmatrix} 0 & \underline{\mathbf{0}}' \\ \underline{\mathbf{0}} & \Sigma_{bc}^{-1} \end{bmatrix} (\boldsymbol{\mu} - r\underline{\mathbf{1}}), \\
\mathbf{X}_{(2)} &= \tau \mathbf{V}^{-1} (\boldsymbol{\mu} - r\underline{\mathbf{1}}), \\
\mathbf{X}_{(3)} &= \tau \begin{bmatrix} \Sigma_{ab}^{-1} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}}' & 0 \end{bmatrix} (\boldsymbol{\mu} - r\underline{\mathbf{1}}),
\end{aligned}$$

where

$$\Sigma_{ab} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix}, \quad and \quad \Sigma_{bc} = \begin{bmatrix} \sigma_b^2 & \sigma_{bc} \\ \sigma_{bc} & \sigma_c^2 \end{bmatrix}.$$

If there are n_1, n_2, and n_3 number of investors in the first, the second, and the third groups, respectively, the market clearing condition leads to the following,

$$\mathbf{S} = n_1 \mathbf{X}_{(1)} + n_2 \mathbf{X}_{(2)} + n_3 \mathbf{X}_{(3)} = n\tau[\eta_{1.3}(\mathbf{V}_*)^{-1} + \eta_2 \mathbf{V}^{-1}](\boldsymbol{\mu}^c - r\underline{\mathbf{1}}), \tag{8}$$

where $\boldsymbol{\mu}^c$ is the vector of expected equilibrium stock returns in this constrained world, and $\mathbf{V}_* = (\frac{n_1}{n_1+n_3}\begin{bmatrix} 0 & \underline{\mathbf{0}}' \\ \underline{\mathbf{0}} & \Sigma_{bc}^{-1} \end{bmatrix} + \frac{n_3}{n_1+n_3}\begin{bmatrix} \Sigma_{ab}^{-1} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}}' & 0 \end{bmatrix})^{-1}$ is the aggregate variance-covariance matrix perceived by constrained investors. $\eta_{1.3} = (n_1 + n_3)/n$ is the proportion of constrained investors and $\eta_2 = n_2/n = 1 - \eta_{1.3}$ is the proportion of "free" investors. The expected equilibrium return vector $\boldsymbol{\mu}^c$ is then determined by the following equation,

$$\boldsymbol{\mu}^c - r\underline{\mathbf{1}} = \frac{1}{n\tau}[\eta_{1.3}(\mathbf{V}_*)^{-1} + \eta_2\mathbf{V}^{-1}]^{-1}\mathbf{S}. \tag{9}$$

Equation (9) says that in a constrained market, investors apply an altered variance-covariance matrix $[\eta_{1.3}(\mathbf{V}_*)^{-1} + \eta_2\mathbf{V}^{-1}]^{-1}$ to price stocks instead of using the true variance-covariance matrix $\mathbf{V}$ of stock returns. In other words, a CAPM would prevail if the altered variance-covariance matrix represented the true risk structure. However, since the risk structure is given, we should rewrite equation (9) alternatively as,

$$\boldsymbol{\mu}^c - r\underline{\mathbf{1}} = \frac{1}{n\tau}\mathbf{V}[\eta_{1.3}(\mathbf{V}_*)^{-1}\mathbf{V} + \eta_2\mathbf{I}]^{-1}\mathbf{S} = \frac{1}{n\tau}\mathbf{V}\mathbf{S}_*. \tag{10}$$

where $\mathbf{S}_*$ is the effective supply. Therefore, equation (10) can be interpreted as if investors are subject to an altered market portfolio[8] $\boldsymbol{\alpha}_*(= \frac{\mathbf{S}_*}{\mathbf{S}_*'\underline{\mathbf{1}}})$. *In other words, a CAPM type of relationship continues to hold with regard to the altered market portfolio in equilibrium. But the CAPM relationship will not hold with respect to the actual total market portfolio.*

Equation (10) also suggests that $\mathbf{S}_*$ properly normalized is a tangency portfolio in the space of $(\boldsymbol{\mu}^c, V)$. However, since we (the econometricians) do not know the distribution of investors among different groups, it is difficult to test equation (10) directly. When investors determine individual stock returns with respect to the *altered* market portfolio available to them, the econometricians tend to find an imperfect CAPM. This is because econometricians can only use the market return ${R_m}^\dagger$ derived from the actual observable market portfolio weights $\boldsymbol{\alpha}$ constructed from all of the outstanding shares of stocks, that is ${R_m}^\dagger = \boldsymbol{\alpha}'\mathbf{R}$. The net effect is that we will perceive that idiosyncratic risk is a priced factor with respect to the actual *observed* market portfolio.

By recognizing the differences, perhaps it is possible to test the model indirectly. In order to illustrate the point, we rewrite equation (9) in the following way;

$$\boldsymbol{\mu}^c - r\underline{\mathbf{1}} \quad = \quad \frac{1}{n\tau}[\mathbf{V}^{-1} - \eta_{1.3}(\mathbf{V}^{-1} - \mathbf{V}_*^{-1})]^{-1}\mathbf{S}$$

[8]In the case of two risky stocks, it is the actual market portfolio less the holdings of the constrained investors.

$$
\begin{aligned}
&= \frac{1}{n\tau}\mathbf{VS} + \frac{\eta_{1.3}}{n\tau}\mathbf{V}[(\mathbf{I}-\mathbf{V}_*^{-1}\mathbf{V})^{-1} - \eta_{1.3}\mathbf{I}]^{-1}\mathbf{S} \\
&= \frac{1}{n\tau}\mathbf{VS} + \frac{\eta_{1.3}}{n\tau}\mathbf{V}\boldsymbol{\omega}, \qquad (11)
\end{aligned}
$$

where $\boldsymbol{\omega}$ is the supply adjustment. Equation (11) reveals that the equilibrium expected returns will adjust both to the actual total supply, which is prescribed by the traditional CAPM, and to the supply adjustment from constrained investors. When the number of constrained investors is relatively small, i.e., $\eta_{1.3} \approx 0$, the supply adjustment is trivial. Expected returns will not deviate much from those predicted by a traditional CAPM (equation (4)). However, if the aggregate demand from the constrained investors is large, as we suggest it is, substantial adjustments will be required. Next, we multiply both sides of equation (11) by the actual market portfolio weights $\boldsymbol{\alpha}$, that is,

$$
{\mu_m}^{\dagger} - r = \frac{M}{n\tau}\boldsymbol{\alpha}'\mathbf{V}\boldsymbol{\alpha} + \frac{M}{n\tau}\eta_{1.3}\boldsymbol{\alpha}'\mathbf{V}\boldsymbol{\omega}_* = \frac{M}{n\tau}\sigma_m^2 + \frac{M}{n\tau}\eta_{1.3}\sigma_m^2\boldsymbol{\beta}'\boldsymbol{\omega}_*, \qquad (12)
$$

where ${\mu_m}^{\dagger} = \boldsymbol{\alpha}'\boldsymbol{\mu}_c$ is the observed expected market return, and $\boldsymbol{\omega}_* = \frac{1}{M}\boldsymbol{\omega}$ is the relative supply adjustment. Substituting equation (12) back into equation (11) and applying equation (7), we have the following result,

$$
\begin{aligned}
\boldsymbol{\mu}^c - r\underline{\mathbf{1}} &= \boldsymbol{\beta}\frac{{\mu_m}^{\dagger} - r}{1+\eta_{1.3}\boldsymbol{\omega}_*'\boldsymbol{\beta}} + \frac{({\mu_m}^{\dagger} - r)/\sigma_m^2}{1+\eta_{1.3}\boldsymbol{\omega}_*'\boldsymbol{\beta}}\eta_{1.3}\mathbf{V}\boldsymbol{\omega}_* \\
&= \boldsymbol{\beta}({\mu_m}^{\dagger} - r) + \frac{({\mu_m}^{\dagger} - r)/\sigma_m^2}{1+\eta_{1.3}\boldsymbol{\omega}_*'\boldsymbol{\beta}}\eta_{1.3}[\mathbf{V}\boldsymbol{\omega}_* - \boldsymbol{\beta}\sigma_m^2\boldsymbol{\beta}'\boldsymbol{\omega}_*] \\
&= \boldsymbol{\beta}({\mu_m}^{\dagger} - r) + \kappa\delta_{SR}\mathbf{V}_\epsilon\boldsymbol{\omega}_*, \qquad (13)
\end{aligned}
$$

where $\mathbf{V}_\epsilon$ is the idiosyncratic volatility defined in equation (7), $\kappa = \frac{\eta_{1.3}}{1+\boldsymbol{\omega}_*'\boldsymbol{\beta}}$ and $\delta_{SR} = \frac{{\mu_m}^{\dagger}-r}{\sigma_m^2}$ are constant and the market Sharpe Ratio, respectively.

If we define the undiversified market wide idiosyncratic return with respect to equation (7) as $\epsilon_m^I = \boldsymbol{\epsilon}'\boldsymbol{\omega}_*$, equation (13) can be rewritten as,

$$
\mu_i^c - r = \beta_i({\mu_m}^{\dagger} - r) + \beta_{I,i}\mu_\epsilon, \qquad (14)
$$

where $\beta_{I,i} = \frac{Cov(R_i,\epsilon_m^I)}{Var(\epsilon_m^I)}$ represents the sensitivity coefficient of the market wide undiversified idiosyncratic risk factor in the spirit of an APT model, and $\mu_\epsilon = \kappa Var(\epsilon_m^I)\delta_{SR}$ is the market wide undiversified idiosyncratic risk premium that arises in our model because of the constrained investors. Similar to the implication of Shanken's (1987) model[9], what matters here is the covariance risk between individual stocks return and the market wide undiversified idiosyncratic risk.

Equation (14) says that, if investors cannot hold a market portfolio, the expected return for each individual stock will be related not only to the observed market expected return through the conventional beta measure, but will also include an extra risk premium because some undiversified idiosyncratic risk will be forced on investors by the constraints imposed. Of course, the portfolio $\boldsymbol{\omega}_*$ is unobservable to an econometrician, but we are able to construct an idiosyncratic risk hedging portfolio in the spirit of Fama and French (1993) to approximate portfolio $\boldsymbol{\omega}_*$ and will briefly discuss our empirical findings in section 4.

Similar to the CAPM model, our model offers cross-sectional implications that can be tested directly. Similar to the assumption used in Dybvig (1983) and Grinblatt and Titman (1983), we further assume that the idiosyncratic returns for individual stocks have very low pairwise correlations, i.e. $Cov(\epsilon_i, \epsilon_j) \approx 0$. Therefore, equation (13) can be further simplified as,

$$\mu_i^c - r \approx \beta_i({\mu_m}^\dagger - r) + \kappa\delta_{SR} w_{*,i}\sigma_{I,i}^2, \tag{15}$$

where $\sigma_{I,i}^2$ is the conventional measure of ith stock's idiosyncratic volatility. The appearance of the Sharpe Ratio, δ_{SR}, in addition to the idiosyncratic volatility makes perfect sense in this context. It translates the idiosyncratic risk into a comparable risk premium. Since there is no particular reason suggesting that stocks with large supply adjustments will have either large or small idiosyncratic volatilities, we can assume that $w_{*,i}$ and $\sigma_{I,i}^2$ are independent cross-sectionally.

$$\mu_i^c - r \approx \beta_i({\mu_m}^\dagger - r) + \kappa\delta_{SR}\bar{w}\sigma_{I,i}^2 + e_i, \tag{16}$$

where $e_i = \kappa\delta_{SR}(w_{*,i} - \bar{w})\sigma_{I,i}^2$. It is easy to see that e_i is independent of $\sigma_{I,i}^2$, which means that the cross-sectional regression coefficient on $\sigma_{I,i}^2$ is unbiased.

Equation (16) is useful in understanding the cross-sectional implications of the pricing of idiosyncratic risk. It suggests that the differences among individual stocks' expected returns will be related not only to their firms' systematic volatilities (β), but also to the firms' idiosyncratic volatilities. In other words, firms that are subject to large

[9] In contrast, as (the referee) points out, an asset's deviation from a misspecified pricing relation in Shanken's model, say the CAPM relation, is proportional to the covariance between the assets residual return and the correct benchmark.

idiosyncratic shocks will tend to have high expected returns. Although the assumption of zero residual correlation will not hold precisely in practice, with low levels of residual correlation, the qualitative implication from equation (16) should still obtain.[10] This is the testable implication studied from the cross-sectional perspective in section 3.

In study the APT model, Dybvig (1983) and Grinblatt and Titman (1983) have shown that the residual risk, although may be important, its influence on asset returns are generally quite small. This may seem to be contradict to what we suggest here. However, there are important difference in the model structure. First, they assume that the number of factors are correctly specified. Second, each factor is correctly represented. When we fail to correctly specify some of the factors, as is the case here, the effect of idiosyncratic risk relative to the specified factors can be large.

[10]The average absolute value of correlations among residuals of the 100 portfolios constructed in the next section is about 0.33, which can be considered relatively small. In other words, the qualitative conclusion should hold.

2 The data and idiosyncratic risk proxies

Three data sets are employed in this study. The first data set is the 2001 version of the COMPUSTAT tape which is primarily used to obtain the book values for individual stocks in the later part of the study. The second data set is from the 2000 version of the CRSP (Center for Research in Security Prices) tape, which includes NYSE, AMEX, and NASDAQ stock returns. Since so many papers have been written on testing the CAPM and most researchers rely on the CRSP tapes, there might be a data snooping concern (Leamer, 1983, and Lo and MacKinlay, 19990). In order to address this issue, we also examine the Japanese stock market for all stocks listed on the First and Second Sections of the Tokyo Stock Exchange (TSE). The monthly individual stock returns and annual financial statements (for book value of equity) data are from the PACAP Japan database.[11] The period covered in this study is from 1975 to 1999. The total number of stocks available varies from 1174 to 1607.

Our study covers both the Fama and MacBeth (1973) sample period from January 1935 to June 1968 and the extended Fama and French (1992) sample period from July 1963 to June 2000. Since the Fama and MacBeth (1973) study was influential in dismissing the role of idiosyncratic risk, we begin our investigation by replicating their study using their choice of sample of NYSE stocks only. Like Fama and MacBeth (1973), the whole sample period under consideration is divided into portfolio formation, estimation, and testing periods. In particular, there are 9 consecutive testing periods with four sample years each starting from 1935. The five-year estimation period proceeding to each testing period is used to estimate individual stocks' betas and residual volatilities from a market model using an equal weighted NYSE index. Since tests are performed on portfolios, these individual estimates are aggregated into the corresponding portfolio estimates using equal weights.[12] According to Fama and MacBeth (1973), 20 portfolios are constructed based on each stock's beta obtained from the seven-year portfolio formation period prior to the corresponding estimation period.[13] Portfolio be-

[11]We are grateful to Yasushi Hamao for providing the data.

[12]As Miller and Scholes (1972) pointed out, the residual variance will bias the coefficient estimate of the beta variable in an cross-sectional regression. Therefore, we only use the orthogonal part of the residual variance to the beta variable in the cross-sectional regressions of Tables 5 and 6. However, we still use the original residual variance in Table 4 in order to make our results comparable to the original Fama and MacBeth (1973) study.

[13]The first portfolio formulation period has only four years from 1926 to 1929. According to Fama and MacBeth (1973), a security available in the first month of a testing period must also have data for all five years of the preceding estimation period and for at least four years of the portfolio formation

tas and idiosyncratic volatilities are time varying due to delisting and annual updates in the same way as Fama and MacBeth (1973).

For ease of comparison, we show in Table 1 the number of stocks, portfolio betas, and portfolio idiosyncratic volatilities in the four selected portfolio estimation periods reported in Table 2 of Fama and MacBeth (1973). Note that we tend to select a few more stocks than that of Fama-MacBeth. For example, during the estimation period of 1934-38, there are 581 stocks selected in our sample versus 576 stocks in Fama-MacBeth. This could due to the fact that CRSP constantly updates their stock files over the years. Therefore, we do not have an exact replication of the Fama-MacBeth portfolios. Although our portfolio beta estimates in each of the estimation period are similar in magnitude to those of Fama and MacBeth, they do not increase monotonically over the 20 portfolios as was the case in the Fama and MacBeth study. However, the portfolio idiosyncratic volatility measures are extremely close to those of Fama and MacBeth.

Insert Table 1

The size variable is one of the most important variables studied in Fama and French (1992) and has been widely used in later research. We have also incorporate the size variable in the Fama and MacBeth (1973) framework in the following way. Stocks are first sorted into five size groups according to their market capitalization in the month prior to each testing period. There is no particular reason to choose five size groups except for insuring that the portfolios have sufficient numbers of stocks. Within each size group, stocks are then sorted into ten beta portfolios as in the original study.

Since volatilities, especially idiosyncratic volatilities, are unobservable, we need estimates in order to perform empirical tests of equation (15). Presumably these estimates can be obtained from the residuals of an asset pricing model. Empirically, however, it is very difficult to interpret the residuals from the CAPM or even a multi-factor model as solely reflecting idiosyncratic risk. One can always argue that these residuals simply represent omitted factors. Therefore, we can only assert that the residuals from a market model measure idiosyncratic risk in the context of that model. In other words, it would be legitimate to measure idiosyncratic volatility using the mean square of residuals as suggested in the model discussed in the previous section. In fact, this is the

period. In order to have comparable numbers of stocks selected for the first period, we require each stock have at least three years of data in the portfolio formation period.

approach used in Fama and MacBeth (1973). We also realize that the current literature has leaned toward a three-factor model of Fama and French (1993) such as shown in equation (17) below,

$$R_{i,t} = \beta_{m,i} R_{m,t} + \beta_{smb,i} R_{smb,t} + \beta_{hml,i} R_{hml,t} + \epsilon_{i,t}, \tag{17}$$

where $R_{m,t}$ is the market return, with R_{smb} and R_{hml} respectively representing the returns on portfolios formed to capture the size effect and the book-to-market equity effect.[14] Therefore, in this part of the investigation, we use idiosyncratic volatility estimates both from a market model and from the above Fama-French three-factor model.

Forming large portfolios not only reduces the errors-in-variables problem in the beta estimates, but also makes the residual variance estimates more accurate. At the same time, due to diversification effects, idiosyncratic volatilities do not have much variability across portfolios when there are too many stocks in each portfolio. In order to increase the power of our tests, we have also investigated a different specification with 50 portfolios in our empirical study. As a result the average standard error of the 50 portfolio betas will be in the order of one half to one-eleventh that of individual stocks.[15] Moreover, we extend the Fama and MacBeth study to the sample period from 1963 to 2000 for NYSE/AMEX stocks.

The essential characteristics for the 50 portfolios over the same sample period are shown in Table 2. We report the average monthly returns for each of the 50 portfolios sorted on both size and beta computed from a market model. In general, portfolio returns decrease with the portfolio sizes except for the portfolio with the lowest beta and smallest size. Portfolio returns also increase with portfolio betas. But this relation weakens when portfolio size increases and when betas are large. At the same time portfolio betas are monotonic over both the beta group and the size group. Although these betas range from 0.64 to 1.72, only one-quarter of the portfolios have betas less than one. It is also interesting to note that portfolio size does not vary much across beta groups. In contrast, portfolio idiosyncratic volatilities aggregated from the root mean squared residuals of individual stocks computed from a market model, vary considerably

[14]We are grateful to Eugene Fama for making these data available to us.

[15]Fama and MacBeth (1973) reported the average standard error of the twenty portfolio betas is of the order of one-third to one-seventh that of individual stocks. When 50 portfolios are used instead, the relative increase in the standard error is about $\sqrt{50/20} = 1.58$. In fact, the empirical results suggests that the significance of beta variable is not affected by this grouping approach.

both across the size groups and beta deciles. This suggests that the idiosyncratic volatility variable may be more useful in explaining the cross-sectional return difference than the size variable.

Insert Table 2

For the recent sample period from 1963 to 2000, as show in Table 3, the average log market capitalizations of the 50 portfolio have gone up between 40% to 60%. For example, the overall average portfolio size for Fama and MacBeth period is 3.4 while that for the current sample period is 5.2 (not shown in the table). The increase in the portfolio size are uniform across portfolios. However, portfolio returns seem to vary much less both across the size groups and beta deciles than those in the previous sample period. This suggests that cross-sectional test results might be weaker for the recent sample period. Similarly, variations in portfolio betas are also much smaller from 0.64 to 1.37 and are more symmetrical around 1. In contrast, variations in the portfolio aggregate idiosyncratic volatilities increase across beta deciles but decrease across size quintiles.

Insert Table 3

Perhaps a more powerful test is to run cross-sectional regressions on individuals stocks in the spirit of Fama and French (1992). Similar to their study, portfolio betas of the 100 portfolios are assigned to each individual stock within each portfolio in order to reduce errors-in-variables problems. In particular, since there are so many small NASDAQ stocks in terms of market capitalization, portfolio breakdowns are determined using only NYSE stocks to avoid the small size portfolios from being too small. Each year, all NYSE stocks on the CRSP tapes are sorted in groups according to their size. The ten NYSE size deciles are then used to split the whole sample. At the same time, the beta of each stock is estimated from a market model using the previous 24 to 60 months of sample returns. Within each size group for NYSE stocks only, stocks are sorted again by their betas into ten equal number groups. Similarly, the break points thus obtained are used to sort all the stocks in our sample. All portfolios are rebalanced on June each year. The 100 portfolios thus constructed are very close to those used in Fama and French (1992), except that we have extended the sample period to June

2000. Using the whole sample period returns, we then estimate individual portfolio betas from the sum of the beta coefficients of regressing individual portfolio returns on the market and the lagged markets (see Fama and French, 1992). These portfolio beta estimates are then assigned to individual stocks in the corresponding portfolios. Individual stocks used in this part of the study should also have book values identifiable from the COMPUSTAT tape. One average, we have 2537 stocks per month for the extended sample period. Finally, we use NYSE/AMEX/NASDAQ index returns as the market returns and the 3-month treasury-bill rates from Ibbotson Associates (2001) as the risk-free rates.

In this part of the investigation, we use two measures of idiosyncratic volatility. As mentioned in the last section, we can reasonably assume that the supply adjustment and the idiosyncratic volatility of individual stocks are uncorrelated with each other. Under this assumption, we can use the June's residual root mean square error of individual stocks estimated from either a market model or the Fama and French (1992) three-factor model using the previous 24 to 60 months of sample returns as the idiosyncratic volatility measure in the cross-sectional regressions. At the same time, if we consider the residual risk of a non randomly selected portfolio as the undiversified idiosyncratic risk, the size-beta sorted portfolio idiosyncratic volatility measures can also be used in the cross-sectional regression of individual stocks. This is legitimate since our model suggests that it is the undiversified idiosyncratic volatilities that affect individual stocks returns. Using portfolios' idiosyncratic volatilities also makes sense under the independence assumption between the supply adjustment and the idiosyncratic volatility. The supply adjustment is unlikely to vary much across portfolios. Finally, by assigning portfolio idiosyncratic volatility to each individual stock within each portfolio will also reduce errors-in-variables problems if individual betas can not be accurately estimated. We also recognize that, due to the diversification effect, a portfolio's idiosyncratic volatility will not be representative of individual stocks' undiversified idiosyncratic volatilities in the portfolio when there are too many stocks. Therefore, in order to balance the benefit of accurate estimates and the diversification effect, we use 200 portfolios' idiosyncratic volatilities to approximate those of the individual stocks within each portfolio. For robustness, we also use portfolio residuals from Fama-French three-factor model.

3 The cross-sectional evidence

Ever since doubts were raised about the CAPM model by Fama and French (1992), considerable attention has been devoted to risk measurement. For example, Jagannathan and Wang (1996) have argued that a conditional CAPM behaves well. Some have argued that the variables used by Fama and French are not robust (see for example Loughran (1996), and Kothari, Shanken, and Sloan (1995)). Others have suggested that a multifactor model in the spirit of the Merton's (1973) ICAPM model provides a better explanation of returns than a single factor CAPM model.[16] As noted by Fama and French (1992) and others, the most significant factors in "explaining" cross-sectional returns appear to be the market capitalization (size) and book to market ratios. These are largely empirical findings rather than equilibrium implications. Therefore, it is difficult to understand why these factors should matter in determining expected returns unless they are proxies for other (systematic) risk factors. Moreover, combining time series evidence of return predictability and cross-sectional testing in a conditional framework, Ferson and Harvey (1999) have rejected the three-factor model advocated by Fama and French (1993) as a conditional asset pricing model. Meanwhile, Malkiel and Xu (1997) have found that size and idiosyncratic volatility are highly correlated. Therefore, the so-called size effect may just as well be attributed to idiosyncratic risk. Guided by our theoretical model, we will study the empirical significance of idiosyncratic risk in addition to other factors from both cross-sectional and time series perspectives.

As suggested by our model (16), idiosyncratic volatilities for individual securities and their expected returns will be related. In other words, we need cross-sectional evidence to conclude that return differences among securities can be partially explained by differences in their idiosyncratic volatilities. In order to provide an overview of the relationship, we first plot the average monthly returns versus the average idiosyncratic volatility calculated from the residuals to the three-factor model for the ten decile portfolios in Figure 1. Clearly there is a positive association between idiosyncratic volatility and average returns. The significance of such a relationship is further demonstrated in the following cross-sectional tests.

Insert Figure 1

[16]This does not mean that the market factor is unimportant, only that other factors are important as well.

3.1 The Fama and MacBeth (1973) study revisited

The Fama and MacBeth (1973) study is an important one not only in terms of its testing methodology, which was widely used in later studies, but also in terms of its empirical results that support the CAPM model. In addition, this study also reversed earlier findings on the role of idiosyncratic risk. As a natural starting point in investigating the cross-sectional implications of idiosyncratic risk, we try to replicate Fama and MacBeth's (1973) Table 3 in our Table 4. In particular, we report the time series averages of the gamma estimates from cross-sectional regressions for each time t. For example, we calculate the time series average $\bar{\gamma}_x = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{x,t}$ for the cross-sectional estimates $\hat{\gamma}_{x,t}$. The corresponding t ratio in Fama and MacBeth (1973) and Fama and French (1992) is defined as $t_\gamma = \sqrt{T}\bar{\gamma}_x/std(\hat{\gamma}_{x,t})$. Since the betas used in the cross-sectional regression are themselves generated regressors, starting with Table 5[17], we use the Shanken (1992) correction factor $(1+\hat{\mu}_m^2/\hat{\sigma}_m^2)$. In addition, since there might be autocorrelation in the estimates $\hat{\gamma}_{x,t}$, we use Newey and West (1987) estimator,[18]

$$\sigma^2(\bar{\gamma}_x) = \sum_{j=-k}^{k}\left(\frac{k-|j|}{k}\right)\frac{1}{T}\sum_{t=1}^{T}(\hat{\gamma}_{x,t}\hat{\gamma}_{x,t-j}).$$

Because we are using monthly data, k is set to 6.

Insert Table 4

When the portfolio beta variable $\hat{\beta}_{p,t-1}$ is used alone in the cross-sectional regressions, the gamma estimates and the corresponding t-ratios closely match those of Fama and MacBeth over both the whole sample period from 1935 to June 1968 and the six five-year subsample periods. After introducing the additional variable of portfolio beta squared, $\hat{\beta}^2_{p,t-1}$, our estimates suggest that the beta variable is very significant compared with only marginal significance in Fama and MacBeth study. The main difference arises in the first two subsample period. When the additional variable introduced in the cross-sectional regressions is idiosyncratic volatility (residual standard deviation) $\bar{s}_{p,t-1}(\epsilon)$ instead, Fama and MacBeth's result continue to hold except that the gamma

[17]The purpose of Table 4 is to replicate Fama and MacBeth (1973). We do not adjust the t ratios in the table.

[18]In order to truthfully replicate Fama and MacBeth's (1973) results, we use $std(\hat{\gamma}_{x,t})$ to compute t ratios in Table 4.

estimate is much smaller and is statistically insignificant for the whole sample period. Examining the gamma estimates from each subsample period, we have a reasonable match except for the first subsample period.

Finally, we include both the $\hat{\beta}^2_{p,t-1}$ variable and the $\bar{s}_{p,t-1}(\epsilon)$ variable in addition to the beta variable in the cross-sectional regressions. In contrast to Fama and MacBeth's finding that only the beta variable is marginally significant, we find that both the beta variable and the beta-square variable are statistically significant at a 5% level for the whole sample period. Therefore, Fama and MacBeth would have concluded that the asset pricing relationship is non-linear if the current version of the CRSP tape is used. It is also interesting to note that, despite the fact that the idiosyncratic risk variable is still insignificant, it has made the squared beta variable significant. This suggests that the two variables might be correlated with each other. In Panel A of Table 5, we find that each of the three variables is very significant when used alone from the cross-sectional regressions over the whole sample period even adjusted for bias and possible autocorrelations. Therefore, we cannot totally ignore the idiosyncratic risk factor.

Insert Table 5

One caveat is that the significance of idiosyncratic volatility variable could be due to a bias created when beta and the residual variable are correlated, as has been pointed out by Miller and Scholes (1972). Since the Fama and MacBeth procedure is more powerful than the simple cross-sectional regression using average returns that was popular in the early studies and the beta variable itself is not significant when used alone except for the case of early sample period, we do not believe that this is the case here. Nevertheless, we only use the orthogonal part of the idiosyncratic volatility to the corresponding beta variable in the cross-sectional regressions in the rest of the paper.

It is also possible that idiosyncratic volatility does not have sufficient variability when grouped into 20 portfolios and is swamped by the beta variable. Since there is no particular reason for Fama and MacBeth to choose 20 portfolios except for combating the errors-in-variables problem, we have increased the number of portfolios to 50 in Panel B of Table 5 using the same portfolio grouping procedure.[19] Surprisingly, not only do all the three individual variables continue to be statistically significant at a

[19]As mentioned earlier, this may only have moderate impact on estimation errors (see footnote 11). Similar results hold when grouping into 30 or 40 portfolios.

1% level when used alone in cross-sectional regressions, but also all the variables are now statistically significant in the multiple cross-sectional regressions with all the three variables as well. This result weakens a little when the value weighted index is used to estimate the market model as shown in the second block of Panel B of Table 5. The idiosyncratic volatility variable is now only significant at a 5% level when used alone and at a 1% level in the multiple cross-sectional regression. In contrast, both beta variables are insignificant when used alone. Therefore, we conclude that both beta and idiosyncratic volatility appear to be important in explaining the cross-sectional return differences for the early sample period. This evidence supports our model implications.

The Fama and French (1992) study and the Fama and MacBeth (1973) study conducted under a different framework, came to different conclusions regarding the importance of the beta variable. For consistency, it is important to study the same issue within the same framework. Since AMEX stocks were introduced into CRSP tape after July 1962, we examine NYSE/AMEX stocks in this part of the study over the extended Fama and French (1992) sample period from 1963 to 2000 using both 20 and 50 portfolio groupings. When an equally weighted index is used in the market model, only the idiosyncratic volatility variable is significant at a 10% level when used alone as shown in Panel C of Table 5. None of the three variables are statistically significant at a 5% level in the multiple cross-sectional regressions. However, when a value-weighted index is used in the market model that estimates both beta and idiosyncratic volatility, the idiosyncratic volatility variable is again statistically significant at a 1% level while the beta squared variable is significant at a 5% level. Although, the significance of the beta variable is not robust in terms of number of grouped portfolios as shown in Panel D of Table 5, the idiosyncratic volatility variable becomes even more significant. Therefore, the difference in the significance of beta found in the two studies is largely due to differences in sample periods and in portfolio groupings. In contrast, the idiosyncratic volatility variable is significant in both sample periods. One may wonder why idiosyncratic volatility variable is significant in the multivariate regression when none of the variable is significant individually as in the left block of Panel D in Table 5.[20] One explanation is that there is substantial and correlated noise in all the three variables. If the true beta variable does not have much explanatory power to begin with, noise in the beta variable may help to cancel out noise in the idiosyncratic volatility variable

[20]The beta variable and the idiosyncratic volatility variable are orthogonal to each other at the individual stock level.

when used jointly.

3.2 The role of the size and the idiosyncratic volatility variables in the framework of Fama and MacBeth (1973)

Differences in the two studies mentioned above could also have contributed to different results with respect to the size variable. In addition, as suggested by Malkiel and Xu (1997), portfolio size and idiosyncratic volatility are highly correlated, therefore, one could argue that the significance of the idiosyncratic volatility simply captures the well-documented size effect. It is necessary, therefore, to consider the size variable simultaneously. Thus, we extend the basic Fama-MacBeth sorting procedure by first sorting stocks into five size group.[21] Stocks in each size group are then sorted into ten beta portfolio using exact procedure used in Fama and MacBeth (1973). The details of this procedure was described in the previous section. For robustness, we apply both the market model and the Fama-French three-factor model to estimate the idiosyncratic volatilities used in estimation. The cross-sectional regression results for the 50 size-beta sorted portfolios are reported in Table 6.

Insert Table 6

For the early sample period of Fama and MacBeth (1973), using a market model to obtain beta and idiosyncratic volatility, results in the first block of Panel A in Table 6 show that all the four individual variables ($\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, $log(ME_{p,t-1})$, and $\bar{s}_{p,t-1}(\epsilon)$) are statistically significant at a 1% level with correct signs when used alone in cross-sectional regressions. Under the original specification of Fama-MacBeth, the three variables–$\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$ are simultaneously significant at a 1% level, which not only confirms our finding from Table 5 using a different sorting approach but also supports our model suggesting a role for idiosyncratic risk. If we replace the idiosyncratic volatility measure with the size measure of $log(ME_{p,t-1})$, a very similar result holds. This also suggests that idiosyncratic volatility and size are likely to be highly correlated. Therefore, we examine the cross-sectional regressions including both the size and the idiosyncratic volatility variables. The result shows that the idiosyncratic

[21] The reason for five size groups instead of ten groups as in Fama and French (1992) is to have 50 portfolios that are consistent with Table 5 and to allow for enough stocks in each portfolio for the early sample period to reduce the errors-in-variables problem.

volatility is still significant at a 5% level while the size variable is insignificant. When all the variables are used simultaneously, all the variables, except the size variable, continue to be significant. This means that the size variable will not replace the role of idiosyncratic risk in asset pricing. Moreover, the significance of the beta and beta squared variables is consistent with Table 5.

Results are little different when idiosyncratic volatilities are estimated from the residuals of the Fama-French three-factor model. From the second block of Panel A in Table 6 we see that the significance of the idiosyncratic volatility variable is virtually unchanged. When both the size variable and the idiosyncratic volatility variable are used in the cross-sectional regression, only the latter variable is statistically significant at a 5% level. In other words, the idiosyncratic volatility variable take away all the explanatory power of the size variable. When all the variables are included in the last equation of the right block of Panel A, the size variable is again statistically insignificant while idiosyncratic volatility variable continues to be very significant. Therefore, the results are robust to different specifications of idiosyncratic risk estimates.

For the most recent sample period of 1963-2000, results are not as strong as those of the previous sample period in general. This is partly due to the fact that portfolio returns are not as variable as before. Although the size variable and the idiosyncratic variable are statistically significant at 5% and 7% levels, respectively when used alone, only the idiosyncratic volatility variable continue to be significant at a 6% level in the multiple cross-sectional regression while the size variable is insignificant (see the left block of Panel B in Table 6). Although this specification is different from that of the Fama and French (1992), which was based on individual stocks, the insignificance of the beta variable is confirmed here. It is also interesting to see in a multiple cross-sectional regression without the size variable that the idiosyncratic volatility variable is significant at a 3% level. Therefore, our hypothesis that the size variable is a proxy for the idiosyncratic volatility variable, is again confirmed from evidence for the most recent sample period. When idiosyncratic volatility is estimated from the Fama-French three-factor model, results do not change very much.

We, therefore, conclude that (1) beta estimated from a market model is important in explaining return differences for the early sample period but its role has substantially weakened in the recent sample period; (2) the idiosyncratic volatility variable is very important especially in the previous sample period no matter how it is measured; (3)

the size variable is useful when used alone; and (4) the size effect is swamped by the idiosyncratic risk factor in both sample periods.

3.3 A further look at the role of idiosyncratic volatility and other variables in the framework of Fama and French (1992)

The fundamental difference between Fama and French (1992) and Fama and MacBeth (1973) studies are that (1) the cross-sectional regressions are run for individual stocks; (2) new variables such as size are considered; and (3) portfolio betas (β_p) are assigned to individual stocks within the portfolio, in the former study. We extend the Fama and French study by extending their sample period to June 2000 and introducing the idiosyncratic volatility variable in order to show its importance in explaining the cross-sectional return difference. Note that we have dropped the beta-squared variable (β_p^2) introduced in Fama and MacBeth's (1973) study since it is insignificant for current sample period. While the log size variable ($ME_{i,t-1}$) can be specified accurately, idiosyncratic volatility is unobservable and has to be estimated. Just like beta estimates for individual stocks that suffer from errors-in-variables problems, idiosyncratic volatility estimates for individual stocks face the same challenge since they are estimated from the same models. In order to mitigate the problems, we apply the same approach as Fama and French (1992) by assigning the portfolio idiosyncratic volatility (s_p) to each stock within the portfolio. As pointed out earlier, we face the diversification challenge when using portfolio idiosyncratic volatility for that of individual stocks. Therefore, we increase the number of portfolios in computing the idiosyncratic volatilities to 200. Using portfolios' idiosyncratic volatilities is also justified by the fact that it is the undiversified idiosyncratic risk that matters in determining the asset prices from our model. For a robustness check, we also include individual stocks' June idiosyncratic volatilities ($s_{i,t-1}$) in the cross-sectional regression. Similar to Fama and French (1992), we use the book value from the last fiscal year to compute the log book-to-market value ($B/M_{i,t-1}$). The results reported in Table 7 are for all the NYSE/AMEX/NASDAQ stocks over the sample period from July 1963 to June 2000.

Insert Table 7

Comparable to Fama and French's (1992) results, the first equation in Panel A of Table 2 shows a statistically insignificant estimate for the beta variable when used alone although it is a little stronger now. This result provides supplemental evidence that beta does not appear to be useful in explaining the cross-sectional stock return differences. However, as noted by Fama and French (1992), the size variable does appear to explain the cross sectional variability of returns as shown in the third equation with an estimate of $-.0019$ and a $t-$value of -3.50. This is much stronger than Fama and French's (1992) estimates using a shorter sample period. The size variable continues to be very important when both the beta variable β_p and the size variable $ME_{i,t-1}$ are added in the cross-sectional regression. In addition, the book-to-market variable the similar estimate to that of Fama and French no matter whether it is used alone or with other variables. As predicted by equation (15), high idiosyncratic risk, measured by idiosyncratic volatility, is associated with high returns on average. This is exactly the case as shown in the fourth equation for s_p and in the fifth equation for s_i of Panel A. The significance of the book-to-market, the size, and the idiosyncratic volatility variables remain when the beta variable is also added in the cross-sectional regression. The first idiosyncratic volatility measure (s_p) again takes away the explanatory power of the size variable. However, the result is reversed when s_i is used instead. This could indicate that s_i is noisy proxy for the undiversified idiosyncratic volatility.

More importantly, including the B/M_i variable does not alter the gamma estimate of the idiosyncratic volatility variable as shown in equations 13 and 14. Therefore, controlling for the book-to-market variable does not have a large impact on the significance of the idiosyncratic volatility variable. In other words, our conclusion with regard to the role of idiosyncratic volatility appears also to be robust whether or not the additional variable B/M_i is accounted for. Finally, when all the four variables are used, both the size and the idiosyncratic volatility variables continue to be statistically significant, while the size variable is insignificant in the regression with the s_p variable.

3.4 The Robustness of Our Results

When both idiosyncratic volatility measures s_p and s_i are computed from the residuals of a three factor model, the fourth and the fifth equation from Panel B shows that the positive relationship between return and idiosyncratic risk continues to be significant at the 1% and the 5% levels, respectively. In fact, all the results for either of the idiosyncratic volatility measures used with other variables only change slightly relative to those using the CAPM residuals. Therefore, using the more restrictive definition for idiosyncratic volatility does not affect our result.

As noted above, portfolio size is strongly related to idiosyncratic volatility. Stocks of smaller size tend to have larger idiosyncratic risks than stocks of larger size. Thus, the size effect in Fama and French and our idiosyncratic volatility effect may be substitutes. Although Berk (1995) and Cochrane (2001) have forcefully argued that the characteristic based variables, such as the size variable, can not be a risk factor, the size variable can effectively serve as a proxy for the idiosyncratic risk. As Loughran (1996) has found that the size effect may have concentrated on small stocks, we need to study the robustness of the idiosyncratic risk factor relative to small stocks. NASDAQ stocks were added to the CRSP tape in July, 1973 and the additions mostly consisted of small stocks. A natural empirical design is to exclude these NASDAQ stocks from our study. Results for NYSE/AMEX stocks only are reported in Table 8.

Insert Table 8

Comparing estimates with those shown in Table 7, the overall results are a little weaker for any single variable. For example, the size variable is now significant at a 5% level. Despite the fact that individual stocks' idiosyncratic volatility s_i is now only significant at a 7% level variable, the s_p measure is still very significant at a 1% level when estimated from the CAPM residual shown in Panel A of Table 8. It is interesting to note that when the s_i variable is used with the beta variable, it is again significant at a 5% level.[22] both idiosyncratic volatility measures continue to be significant at a conventional level even controlling for the book-to-market effect as shown in equations (13) and (14) of Panel A. The size variable is insignificant when all the variables are used in the cross-sectional regression. However, the s_p variable continue to be significant at a 1% level. The insignificance of the s_i in the last equation could simply due to the multicollinearity problem with the size variable.

When idiosyncratic volatilities are estimated from the Fama and French three-factor model, results are again very similar as shown in Panel B of Table 8. Therefore, we conclude that the usefulness of idiosyncratic volatility variables is also robust when the sample includes only large stocks. At the same time, the difference in the significance of the two idiosyncratic volatility measure, s_p versus s_i, suggests that the undiversified

[22] s_i is orthogonal to individual stocks' beta measure. It is not necessarily orthogonal to the portfolio beta used in the cross-sectional regression.

idiosyncratic risk is more relevant in explaining the cross-sectional return difference of individual stocks.

The evidence continues to support our hypothesis that idiosyncratic volatility helps to explain the cross-sectional variability in average returns in an important way. It is also reasonable to conclude that the idiosyncratic risk factor is more robust than the size variable in explaining the cross-sectional difference of asset returns over the different sample periods considered here.

3.5 The Liquidity Effect

Liquidity may play an important role in affecting asset prices. Although different researchers have offered different definitions, it is generally believed that liquidity measures how easy to trade a large amount of shares without altering the share prices. There are many theoretical papers including Amihud and Mendelson (1986), Constantinides (1986), Heaton and Lucas (1996), and Huang (2002), that tie liquidity to asset prices. Intuitively, assets that are "difficult to trade" should have lower prices, other things being equal, in order to compensate investors for the inability to trade or for increasing the cost of trading. Indeed, many empirical studies, such as Amihud and Mendelson (1986), Brennan, Chordia, and Subrahmanyam (1998), and Datar, Naik, and Radcliffe (1998) have generally found a negative relationship between liquidity and expected stock returns. Alternatively, using market wide liquidity as a state variable (or factor), Pastor and Stambaugh (2002) and Jones (2002) find stocks with high covariance with the market liquidity generally offers high expected return. This is reasonable since those stocks will face severe liquidity problems when the market liquidity is low.

If liquidity is indeed priced, residuals from any asset pricing model that excludes liquidity factor will reflect it. In other words, the idiosyncratic volatility measure constructed from residuals could potentially capture some of the liquidity effect. In this section, we will control for it. The bid-ask spread is often used in the literature as a measure of liquidity. Due to data availability, we use trading volume in this study as the liquidity measure since it is the most important determinant of the bid-ask spread (see Stoll, 1978). Unlike Brennan, Chordia, and Subrahmanyam (1998), we use relative volume defined as the ratio between share volume and share outstanding instead of log dollar volume. When both log size and log dollar volume are used in the cross-sectional regressions, they share log price as the common component. Therefore, we believe it is better to use the relative volume variable here. In particular, we use last month's relative volume in the cross-sectional regression of current month. In this part of the study, we use the s_p constructed from the CAPM residuals as the idiosyncratic risk measure. The results are reported in Table 9.

Insert Table 9

For all stocks, the volume variable is statistically significant at a 1% level when used alone. Surprisingly, however, the sign is wrong.[23] Potentially, the size variable could be correlated with the volume variable. One could argue that large stocks are more actively traded than small stocks. Therefore, the positive sign on the relative volume variable could simply due to the size effect. This is confirmed in the multiple regression with the size variable as shown in the fifth equation of Panel A of Table 9. The size variable virtually takes away all the explanatory power of the volume variable. When the idiosyncratic volatility and the volume variable are use simultaneously, the significance of the idiosyncratic volatility is unchanged. This suggests that our idiosyncratic volatility measure is not contaminated by liquidity. More importantly, the volume variable now has the right sign although it is still insignificant. From previous results, we understand that this is due to the fact that the idiosyncratic volatility has taken away some of the "size effect" in the volume variable. Equation (7) is similar to Brennan, Chordia, and Subrahmanyam's (1998) specification except for the lagged return variables. The liquidity proxy is again statistically insignificant with the right sign. We do not want to attach too much significance to this result since we did not use the lagged cumulative returns in the regression. When all the variables are used in the cross-sectional regression as shown in the last equation, both the magnitude and the significance of the idiosyncratic volatility variable is virtually unchanged. Very similar results continue to hold when only the NYSE/AMEX stocks are used in the cross-sectional regression shown in the Panel B of Table 9. Therefore, we conclude that liquidity variable will not have much impact to the significance of the idiosyncratic volatility variable.

[23] We obtain similar results when the log dollar volume is used instead. Since Brennan, Chordia, and Subrahmanyam (1998) did not report univariate regression result, we do not know if similar results would occur in their framework.

3.6 Idiosyncratic Risk in a Different Market

Considerable recent attention has been paid to data snooping problems in empirical studies. Since the CAPM debate is mostly based on the same data source, i.e., the CRSP tape for US stock data, such a problem is inevitable at least conceptually. In order to avoid the problem, we also study Japanese stock returns. The available data set runs from 1975 to 1999, and we use the first five years of data to do the presorting in constructing our portfolios. Therefore, the actual sample used in the study runs from 1980 to 1999. During this particular episode, the Japanese stock market went through dramatic changes–from the pre bubble period (1980-1984) to the bubble period (1985-1989), and followed by a decade of post-crash market slump (1990-1999). Therefore, we also study the two subsample period: 1980-1989 and 1990-1999. The average numbers of stocks in each sub-period are 1321 and 1564, respectively. We followed the exact procedures employed with U.S. data to construct portfolios and perform cross-sectional regressions. The results are shown in Table 10.

Insert Table 10

First, note that the beta variable is insignificant no matter whether it is used alone or with other variables in the cross-sectional regressions. The same results hold for different subsample periods. This reconfirms our results from the US data. In contrast to the US experience, however, the size variable is insignificant in all of the cross-sectional regressions. The book-to-market variable is significant for the whole sample period both when used alone and together with other variables. It is interesting to note, however, that the significance mostly arises from the effect in the post crash period. As for the idiosyncratic volatility variable, it is significant at a 6% level for the whole sample period when used alone. It is very significant during the first subsample period but not in the second subsample period when used alone. What is most interesting is that the idiosyncratic volatility variable is always very significant when used with the size measure, as shown in the last equation of all the three blocks of Table 10. This again provides evidence that the idiosyncratic volatility variable is much stronger than the size variable. Moreover, the size variable helps reduce the measurement error in the idiosyncratic volatility variable. Therefore, Japanese stock market data also support the important role of idiosyncratic volatility in explaining cross-sectional returns.

While there are some (usually short) periods in which beta measures do appear to play some role in explaining the cross-sectional pattern of returns in accordance with the CAPM, the general conclusion is that book value/market value and especially idiosyncratic volatility are the only variables that show a consistently strong relationship with returns. Of course, it is possible to interpret our measures of idiosyncratic volatility as simply an approximation for size as well as for some omitted systematic risk factor(s). However, at the very least, our model seems to offer a consistent and empirically supported explanation for some of the deficiencies of popular asset pricing models. It is also true that beta does not perform as well in the framework of Fama and French (1992) as in the framework of Fama and MacBeth (1973). As pointed out by Xu (2001), this is partly due to the procedure used by Fama and French to estimate portfolio betas. Their procedure is subject to a particularly large errors-in-variables problem because of the high correlations between alphas and betas estimated from the market model. In any case, what is clear is that the beta of the capital asset pricing model appears to be a very imperfect measure of the risk that is related to securities returns.

4 Preliminary Time Series Evidence on Idiosyncratic Risk

Generally speaking, two approaches have been applied in testing the CAPM. Cross-sectional studies of the return and risk implications of the CAPM have received considerable recent attention. This is what we have undertaken in the previous section by showing that idiosyncratic volatility appears to be an independent pricing factor. As our model predicts, it is also important to examine an alternative time-series approach that utilizes the constraint on the intercept of a market model. Given the emphasis of this paper, however, we will only present some preliminary results on the explanatory power of idiosyncratic volatility from an *ex post* perspective.

We study the time series behavior for the same 100 portfolios constructed in the cross-sectional studies. In order to perform time series tests on equation (14), we need a proxy for the market wide undiversified idiosyncratic risk factor. The proxy $R_{I,t}$ that we propose to use here is the idiosyncratic risk hedging portfolio. In order to motivate it, we rewrite the time-series CAPM and our model (equation (14)) as the followings,

$$\begin{aligned} I: \quad \tilde{R}_{i,t} - R_{f,t} &= \beta_i(R_{M,t} - R_{f,t}) + \epsilon_{i,t}, \\ II: \quad \tilde{R}_{i,t} - R_{f,t} &= \beta_i(R_{M,t} - R_{f,t}) + \beta_{I,i}R_{I,t} + \xi_{i,t} \end{aligned} \tag{18}$$

where $\tilde{R}_i$, $R_{f,t}$, and $\tilde{R}_M$ are the returns for security i, the risk free rate, and the market portfolio respectively. If model II is indeed true, the part of $\beta_{I,i}R_{I,t}+\xi_{i,t}$ will be specified as $\epsilon_{i,t}$ in the CAPM specification of model I. Therefore, firms that are more sensitive to $R_{I,t}$ will likely to have large $\sigma(\epsilon_{i,t})$. Since $R_{M,t}$ and $R_{I,t}$ are independent, the two portfolios constructed by sorting stocks according to their $\sigma(\epsilon_{i,t})$ will have similar beta sensitivities to $R_{M,t}$ and very different beta sensitivities to $R_{I,t}$.

In implementation, we follow the logic of Fama and French (1993) by constructing six size-idiosyncratic volatility sorted portfolios. We split the sample into two size groups. Within each size group, stocks are sorted again by their idiosyncratic volatilities into three groups of equal numbers of securities.[24] The idiosyncratic volatility measure for each stock is estimated using the mean squared residuals from the CAPM model over the previous 24 to 60 month period in order to be consistent with our model. We denote B as *big* size, S as *small* size, H as *high* idiosyncratic volatility, M as *median* idiosyncratic volatility, and L as *low* idiosyncratic volatility. The six portfolios can be characterized

[24]The actual breakdowns are based on NYSE stocks only for the same reason discussed previously.

as the B/L portfolio, the B/M portfolio, the B/H portfolio, the S/L portfolio, the S/M portfolio, and the S/H portfolio. The return proxy for the market wide idiosyncratic risk factor is then calculated as the difference between the average returns for portfolios B/L and S/L and the average returns for portfolios B/H and S/H. Therefore, our method of constructing the idiosyncratic risk hedging portfolio is consistent with our model.

Our empirical results in this section are based on the portfolio returns of the 100 size-beta sorted portfolios obtained by equally weighting each security's returns in the portfolio. As a first step, we report the average and the standard deviation of coefficient estimates for individual portfolios in Table 11. For comparison, we report estimates for both the Fama and MacBeth sample period (1935.1-1968.6) and the extended Fama and French sample period (1963.7-2000.6). Although the average R^2s across all the portfolios are relatively high with 69.9% and 67.10% for each of the sample periods respectively, the CAPM has failed to hold because most of the individual regressions have significant intercept α In general, adding additional variables to the CAPM specification does not reduce either the magnitude or the significance of alpha. However, the idiosyncratic risk factor is very significant in both sample periods. In particular, the distribution of the beta estimates on $R_{I,t}$ is on the positive side and far from zero. On average, each percentage increase in the idiosyncratic hedging proxy $R_{I,t}$ is associated with .57% increase in return, over the early sample period and .31% in the recent sample period. For individual portfolios, the idiosyncratic hedging proxy is significant at a 5% level for over 80% of these portfolios in both sample periods. At the same time, the average adjusted R^2 across all the portfolios jumps to 77.1% for the early sample period and 73.3% for the recent sample period. Therefore, the preliminary results support our model in equation (14).

Insert Table 11

Fama and French (1993) have argued that a size proxy and a book-to-market proxy are also very important in explaining return fluctuations over time. Their three-factor model is reconfirmed in the third equation of Table 11. As expected, the average adjusted R^2 has been improved dramatically to 79.5% and 81.1% for the previous and recent sample periods, respectively. At the same time, as we suggested in discussing the cross-sectional evidence, the size factor may very well be measuring the idiosyncratic risk. In this case, the idiosyncratic risk factor will be more appropriate, which is represented in the fourth equations of Table 11. For both sample periods, we see that the corresponding coefficient estimates of both the idiosyncratic hedging factor and the book-to-market factor are distributed far from zero. The number of individual portfolios showing statistical significant estimates on the two factors at a 5% level also resembles that of the three factor model in the recent sample period. Similar results hold for the early sample period. Therefore, idiosyncratic volatility also appears to capture the size effect in the time series framework.

One might argue that our idiosyncratic risk proxies do nothing more than to capture the size and book-to-market factors. Thus, it is important to put our model in perspective relative to these two additional factors. In the last equation, we control for both the size and book-to-market effect. It is evident that, for the Fama-MacBeth sample period, the majority of the portfolios have statistically significant estimates on the idiosyncratic risk factor. For example, after controlling for the size and the book-to-market factors, there are still 54% of the portfolios showing statistically significant estimates at the 5% level on the idiosyncratic risk factor. In other words, there is a strong connection between idiosyncratic risk and individual portfolio returns, even in the presence of size and book-to-market effects. For the Fama and French sample period, the percentage of portfolios with statistically significant estimates on the idiosyncratic risk factor increases to 77%.

While a more detailed time series analysis is required for more conclusive results, the preliminary evidence from individual portfolios supports our theoretical prediction that idiosyncratic volatility appears to be important in explaining asset return fluctuations over time. Note also that the significant beta estimates from the time series regressions do not contradict Fama and Frenchs (1992) findings that were discussed in our cross-sectional study.

One may argue that the role we ascribe to idiosyncratic volatility may result from our inability to define the market portfolio. Perhaps idiosyncratic risk is just an artifact of approximation error. If so, we view our argument here as deepening the critique of Roll. In this paper, we have argued, however, that even if investors are shortsighted and only pay attention to tradable financial assets, in which case Roll's critique is irrelevant, we may still find an imperfect CAPM from an econometrician's point of view. The essence of our argument is that such an imperfection can be ameliorated by including an idiosyncratic risk measure.

5 Concluding comments

In this paper, we have made some progress in understanding the role of idiosyncratic risk in asset pricing both theoretically and empirically. Other things being equal, idiosyncratic risk will affect asset returns when not every investor is able to hold the market portfolio. Even after controlling for factors such as size, book-to-market, and liquidity, evidence from both individual US stocks and a sample of Japanese equities supports the predictions of our model. Most importantly, the cross-sectional results demonstrate that idiosyncratic volatility variable is more powerful than either beta or size measures in explaining the cross section of returns. We also suggested that beta performs relatively well in a non-linear way for the early sample period and poorly in the recent sample period. At the very least, our results provide a unique perspective in understanding the possible role of idiosyncratic risk in asset pricing.

References

[1] Aiyagari, Rao S. (1994), 'Uninsured Idiosyncratic Risk and Aggregate Saving,' *Quarterly Journal of Economics,* Vol. 109, No. 3, pp. 659-84, August.

[2] Amihud, Y., and H. Mendelson (1986), 'Asset Pricing and the Bid-ask Spread,' *Journal of Financial Economics,* Vol. 17, pp. 223-249.

[3] Berk, J. (1995), 'A Critique of Size-Related Anomalies,' *Review of Financial Studies,* Vol. 8, pp. 275-286.

[4] Bessembinder, Hendrik (1992), 'Systematic Risk, Hedging Pressure, and Risk Premiums in Future Markets,' *Review of Financial Studies,* Vol. 5, pp. 637-67.

[5] Breeden, D. T. (1979), 'In Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities,' *Journal of Financial Economics,* Vol. 7, pp. 265-96.

[6] Brennan, M. (1970), 'Taxes, Market Valuation and Corporate Financial Policy,' *National Tax Journal,* Vol. 23, pp. 417-27.

[7] Brennan, Michael J., Tarun Chordia, and Avanindhar Subrahmanyam (1998), 'Alternative Factor Specifications, Security Characteristics, and the Cross-section of Expected Stock Returns,' *Journal of Financial Economics,* Vol. 49, pp. 345-373.

[8] Campbell, John Y., Martin Lettau, Burton G. Malkiel, and Yexiao Xu (2000), 'Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk,' *The Journal of Finance,* Vol. 56, pp. 1-43.

[9] Campbell, John, Sanford J. Grossman, and Jiang Wang (1993), 'Trading Volume and Serial Correlation in Stock Returns,' *Quarterly Journal of Economics,* Vol. 108, pp. 905-939.

[10] Chamberlain, G., and M. Rothschild (1983), 'Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets,' *Econometrica,* Vol. 51, pp. 1281-1304.

[11] Chen, N., R. Roll, and S. A. Ross (1986), 'Economic Forces and the Stock Markets,' *Journal of Business,* Vol. 59, pp. 383-403.

[12] Cochrane, J. H. (2001), *Asset Pricing,* Princeton University Press: Princeton.

[13] Connor, G. (1984), 'A Unified Beta Pricing Theory,' *Journal of Economic Theory,* Vol. 34, pp. 13-31.

[14] Connor, G. and R. Korajczyk (1988), 'Risk and Return in an Equilibrium APT,' *Journal of Financial Economics,* Vol. 21, pp. 255-289.

[15] Constantinides, George (1986), 'Capital Market Equilibrium with Transaction Costs,' *Journal of Political Economy,* Vol. 94, pp. 842-862.

[16] Cox, J. C., J. E. Ingersoll, and S. A. Ross (1985), 'An Intertemporal General Equilibrium Model of Asset Prices,' *Econometrica,* Vol. 53, pp. 385-407.

[17] Datar, Vinay T., Narayan Y. Naik, and Robert Radcliffe (1998), 'Liquidity and Asset Returns: An Alternative Test,' *Journal of Financial Markets,* Vol. 1, pp. 203-219.

[18] Douglas, G. W. (1969), 'Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency,' *Yale Economic Essays,* Vol. 9, pp. 3-45.

[19] Dybvig, P. H. (1983), 'An Explicit Bound on Individual Assets' Deviations from APT Pricing in a Finite Economy,' *Journal of Financial Economics,* Vol. 12, pp. 483-96.

[20] Falkenstein, Eric G. (1996), 'Preferences for Stock Characteristics as Revealed by Mutual Fund Portfolio Holdings,' *Journal of Finance,* Vol. 51, No. 1, pp. 111-35, March.

[21] Fama, E. and J. MacBeth (1973), 'Risk, Return, and Equilibrium: Empirical Tests,' *Journal of Political Economy,* Vol. 81, pp. 607-636.

[22] Fama, Eugene F. and French, Kenneth R. (1996), 'Multifactor Explanations of Asset Pricing Anomalies,' *Journal of Finance,* Vol. 51, pp. 55-84.

[23] Fama, Eugene F., and Kenneth R. French (1993), 'Common Risk Factors in the Returns on Stock and Bonds,' *Journal of Financial Economics,* Vol.33, pp.3-56.

[24] Fama, Eugene F., and Kenneth R. French (1992), 'The Cross-Section of Expected Stock Returns,' *Journal of Finance,* Vol.47, pp.427-465.

[25] Fama, Eugene F. and French, Kenneth R. (1988), 'Permanent and Temporary Components of Stock Prices,' *Journal of Political Economy,* Vol. 96, pp. 246-73.

[26] Ferson, W. and C. Harvey (1991), 'The Variation of Economic Risk Premiums,' *Journal of Political Economy,* Vol. 99, pp. 385-415.

[27] Ferson, Wayne E. and Campbell R. Harvey (1999), 'Conditioning Variables and the Cross Section of Stock Returns,' *Journal of Finance,* Vol. 54, pp. 1325-1360.

[28] Ferson, Wayne E. and Campbell R. Harvey (1991), 'The Variation of Economic Risk Premiums,' *Journal of Political Economy,* Vol. 99, pp. 385-415.

[29] Ferson, Wayne E., Sergei Sarkissian, and Timothy Simin (1998),'The Alpha Factor Asset Pricing Model: A Parable,' *Journal of Financial Markets,* Vol. 2, 49-68.

[30] Franke, Gunter, Richard G. Stapleton, Marti G. Subrahmanyam (1992), 'Idiosyncratic Risk, Sharing Rules and the Theory of Risk Bearing,' *INSEAD Working Paper,* No. 93/02/FIN.

[31] French, Kenneth R., G. William Schwert, and Robert F. Stambaugh (1987), 'Expected Stock Returns and Volatility,' *Journal of Financial Economics,* Vol. 19, pp. 3-29.

[32] Green, Richard, Rydqvist, Kristian (1997), 'The Valuation of Nonsystematic Risks and the Pricing of Swedish Lottery Bonds,' *Review of Financial Studies,* Vol. 10, pp. 447-80.

[33] Hansen, Lars Peter (1982), 'Large Sample Properties of Generalized Method of Moments Estimators,' *Econometrica,* Vol. 50, pp. 1029-54, November.

[34] Heaton, John, and Deborah J. Lucas (1996), 'Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing,' *Journal of Political Economy,* Vol.104, No. 3, pp. 443-87, June.

[35] Hirshleifer, David (1988), 'Residual Risk, Trading Costs, and Commodity Future Risk Premia,' *Review of Financial Studies,* Vol. 1, pp. 173-193.

[36] Hirshleifer, David (2001), 'Investor Psychology and Asset Pricing,' *Journal of Finance,* Vol. 56, pp. 1533-1597.

[37] Huang, Ming (2002), 'Liquidity Shocks and Equilibrium Liquidity Premia,' *Journal of Economic Theory,* forthcoming.

[38] Ibbotson Associates. (2001), *Stocks, Bonds, Bills, and Inflation: 2001 Yearbook,* Chicago.

[39] Jagannathan, Ravi, and Zhenyu Wang (1996), 'The Conditional CAPM and the Cross-Section of Expected Returns,' *Journal of Finance,* Vol. 51, pp. 3-53.

[40] Jensen, Michael C. (1968), 'The Performance of Mutual Funds in the Period 1945-1964,' *Journal of Finance,* Vol. 23, pp. 389-416.

[41] Jones, Charles M. (2002), 'A Century of Stock Market Liquidity and Trading Costs,' Working Paper, Columbia University.

[42] Kahn, James A. (1990), 'Moral Hazard, Imperfect Risk-Sharing, and the Behavior of Asset Returns,' *Journal of Monetary Economics,* Vol. 26, No. 1, pp. 27-44, August.

[43] Kahneman, Daniel, and Amos Tversky. (1979), 'Prospect Theory: An Analysis of Decision Under Risk,' *Econometrica,* Vol. 47, No. 2, pp. 263-291.

[44] King, Mervyn, Enrique Sentana, and Sushil Wadhwani (1994), 'Volatility and Links Between National Stock Markets,' *Econometrica,* Vol. 62, No. 4, pp. 901-33, July.

[45] Kothari, S.P., Jay Shanken, and Richard G. Sloan (1995), 'Another Look at the Cross-Section of Expected Stock Returns,' *Journal of Finance,* Vol. 50, pp. 185-244.

[46] Leamer, E. (1983), 'Let's Take the Con Out of Econometrics,' *American Economic Review,* Vol. 73, pp. 31-43.

[47] Lehmann, Bruce N. (1990), 'Residual Risk Revisited,' *Journal of Econometrics,* Vol. 45, pp. 71-97.

[48] Lehmann, B.N., and D. M. Modest (1988), 'The Empirical Foundations of the Arbitrage Pricing Theory,' *Journal of Financial Economics,* Vol. 21, pp. 213-254.

[49] Lo, Andrew and Craig A. MacKinlay (1990), 'Data Snooping Biases in Tests of Financial Asset Pricing Models,' *Review of Financial Studies,* Vol. 3, pp. 431-467.

[50] Loughran, Tim (1996), 'Book-to-Market Across Firm Size, Exchange, and Seasonality: Is There an Effect?' Working Paper, University of Iowa.

[51] Lintner, John (1965), 'The Valuation of Risky Assets and the Selection of Risky Investment in Stock Portfolio and Capital Budgets,'*Review of Economics and Statistics,* Vol. 47, pp. 13-37.

[52] Lucas, R. E., Jr. (1978), 'Asset Prices in an Exchange Economy,' *Econometrica,* Vol. 46, No. 6, pp. 1429-45.

[53] Lucas, Deborah (1994), 'Asset Pricing with Undiversifiable Income Risk and Short Sales Constraints: Deepening the Equity Premium Puzzle,' *Journal of Monetary Economics,* Vol. 34, No. 3, pp. 325-41, December.

[54] Malkiel, Burton G. (1995), 'Returns from Investing in Equity Mutual Funds 1971 to 1991' *Journal of Finance*, Vol. 50, No.2, pp. 549-572.

[55] Malkiel, Burton G. and Xu, Yexiao (1997), 'Risk and Return Revisited' *Journal of Portfolio Management*, Vol. 23, No.3, pp. 9-14.

[56] Mayers, D. (1972), 'Non-marketable Assets and Capital Market Equilibrium Under Uncertainty,' In *Studies in the Theory of Capital Markets,* ed. M. Jensen, New York: Praeger.

[57] Merton, R. C. (1973), 'An Intertemporal Capital Asset Pricing Model,' *Econometrica,* Vol. 41, pp. 867-887.

[58] Merton, R. C. (1987), 'A Simple Model of Capital Market Equilibrium with Incomplete Information,' *Journal of Finance,* Vol. 42, pp. 483-510.

[59] Merville, L. and Y. Xu (2002), 'The Changing Factor Structure of Equity Returns,' Working Paper, The University of Texas at Dallas.

[60] Miller, M. H. and M. Scholes (1972), 'Rates and Return in Relation to Risk: A Re-examination of Some Recent Findings,' in Michael C. Jensen, ed.: *Studies in the Theory of Capital Markets,* Praeger: New York, pp. 47-78.

[61] Newey, W. K., and K. D. West (1987), 'A Simple, Positive Semi-definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix,' *Econometrica,* Vol. 55, pp. 703-708.

[62] Pastor, L. and R. F. Stambaugh (2001), 'Liquidity Risk and Expected Stock Returns,' NBER Working Paper No. 8462.

[63] Roll, R. (1977), 'A Critique of the Asset Pricing Theory's Tests; Part 1: On Past and Potential Testability of the Theory,' *Journal of Financial Economics,* Vol. 4, pp. 129-176.

[64] Ross, Stephen A. (1976), 'Arbitrage Theory of Capital Asset Pricing,' *Journal of Economic Theory,* Vol. 13, pp. 341-360.

[65] Schwert, G. William (1991), 'Market Volatility: Review,' *Journal of Portfolio Management,* Vol. 17, No. 4, pp. 74-78.

[66] Schwert, G. William, and Paul J. Seguin (1990), 'Heteroscedasticity in Stock Returns,' *Journal of Finance,* Vol.45, pp. 1129-55.

[67] Shanken, J. A. (1982), 'The Arbitrage Pricing Theory: Is it Testable?' *Journal of Finance,* Vol. 37, pp. 1129-1140.

[68] Shanken, J. A. (1992), 'On the Estimation of Beta-pricing Models' *Review of Financial Studies,* Vol. 5, pp. 1-33.

[69] Sharpe, William F. (1964), 'Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk,' *Journal of Finance* Vol. 19, pp. 425-442.

[70] Shukla, R. and C. Trzcinka (1990), 'Sequential Tests of the Arbitrage Pricing Theory: A Comparison of Principal Components and Maximum Likelihood Factors,' *Journal of Finance,* Vol. 45, pp. 1541-1564.

[71] Stoll, H. (1978), 'The Pricing of Dealer Services: An Empirical Study of NASDAQ Stocks,' *Journal of Finance,* Vol. 33, pp. 1152-1173.

[72] Telmer, Chris I. (1993), 'Asset Pricing Puzzles and Incomplete Markets,' *Journal of Finance,* Vol.48, No.5, pp. 1803-32, December.

[73] Xu, Yexiao (2001), 'Extracting Factors with Maximum Explanatory Power,' Working Paper, The University of Texas at Dallas.

Table 1: **Fama-MacBeth Portfolio Characteristics in Estimation Periods**

This table reports the portfolio beta and idiosyncratic volatility for the 20 Fama-MacBeth portfolios in four selected estimation periods. It is meant to replicate Table 2 of Fama-MacBeth (1973). In general, stocks from the NYSE are selected into one of the 20 portfolios according to the selection period (seven years) betas estimated from a market model utilizing an equal-weighted index. Beta estimates from the estimation period (next five years) are used in cross-sectional regressions using data from the following four years. Details are available in Fama-MacBeth (1973). $\hat{\beta}_{p,t-1}$ and $\bar{s}_{p,t-1}(\epsilon)$ denote portfolio beta and portfolio residual standard deviation, respectively, which are equal-weighted averages of individual stocks' betas and residual standard deviations.

Pfl.	Period:1934-38 N=581		Period:1942-46 N=708		Period:1950-54 N=812		Period:1958-62 N=869	
No.	$\hat{\beta}_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\hat{\beta}_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\hat{\beta}_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\hat{\beta}_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$
1	0.288	0.077	0.465	0.054	0.446	0.040	0.648	0.050
2	0.690	0.084	0.532	0.053	0.609	0.043	0.622	0.050
3	0.485	0.069	0.616	0.064	0.658	0.046	0.708	0.054
4	0.702	0.090	0.607	0.054	0.830	0.050	0.794	0.059
5	0.792	0.094	0.709	0.056	0.730	0.049	0.814	0.064
6	0.684	0.077	0.759	0.065	0.800	0.052	0.863	0.060
7	0.922	0.115	0.768	0.062	0.942	0.052	0.977	0.067
8	0.955	0.105	0.827	0.062	0.950	0.053	0.910	0.068
9	0.970	0.111	0.810	0.065	1.007	0.056	1.000	0.067
10	1.053	0.095	0.859	0.067	0.987	0.053	0.924	0.065
11	1.013	0.096	0.940	0.070	1.101	0.058	1.003	0.069
12	1.153	0.130	1.048	0.079	1.131	0.062	1.018	0.066
13	1.110	0.119	0.955	0.070	1.131	0.059	1.047	0.065
14	1.236	0.124	1.075	0.088	1.227	0.063	1.056	0.068
15	1.186	0.118	1.416	0.108	1.118	0.066	1.047	0.061
16	1.318	0.131	1.254	0.091	1.275	0.063	1.080	0.067
17	1.299	0.124	1.322	0.081	1.239	0.063	1.100	0.077
18	1.371	0.158	1.344	0.088	1.281	0.068	1.112	0.071
19	1.415	0.138	1.658	0.120	1.447	0.071	1.217	0.067
20	1.496	0.175	1.610	0.114	1.531	0.088	1.414	0.079

Table 2: **Size-Beta Portfolio Characteristics (Sample Period: January 1935-June 1968)**

This table reports the average return, average beta, average log size, and average residual standard deviation for the 50 size-beta sorted portfolios over the Fama and MacBeth (1973) sample period of January 1935 to June 1968. Stocks from the NYSE/AMEX are first sorted into five size groups according their market capitalization in December of the last year of the estimation period. Each group of stocks is then sorted again into decile portfolios according to the selection period betas. Detailed information on portfolio selection, estimation, and testing periods are available in Fama and MacBeth (1973). Both the reported betas and the idiosyncratic volatilities as well as betas used in sorting are estimated from a market model with a value-weighted index.

	Low-β	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	High-β
	Average Monthly Returns (in Percent)									
Small-ME	1.08	1.89	1.83	2.00	1.82	1.96	2.03	2.38	2.50	1.89
ME-2	2.14	1.18	1.23	1.55	1.69	1.52	1.87	1.72	1.81	1.87
ME-3	1.73	1.30	1.39	1.22	1.27	1.46	1.41	1.51	1.36	1.52
ME-4	1.32	.889	1.08	1.04	1.24	1.34	1.27	1.25	1.21	1.22
Large-ME	1.44	.806	.896	1.03	1.01	1.16	1.27	1.16	1.02	1.12
	Average portfolio beta									
Small-ME	.990	1.11	1.24	1.41	1.31	1.31	1.45	1.53	1.71	1.72
ME-2	.806	.901	1.07	1.22	1.31	1.39	1.49	1.50	1.59	1.68
ME-3	.719	.909	1.06	1.17	1.26	1.35	1.35	1.44	1.54	1.69
ME-4	.644	.841	.991	1.10	1.11	1.20	1.27	1.31	1.52	1.57
Large-ME	.643	.723	.845	0.911	1.00	1.11	1.15	1.19	1.28	1.41
	Average portfolio log size									
Small-ME	1.66	1.68	1.61	1.67	1.61	1.61	1.64	1.62	1.61	1.63
ME-2	2.59	2.49	2.51	2.56	2.52	2.60	2.58	2.58	2.56	2.54
ME-3	3.40	3.36	3.33	3.31	3.38	3.39	3.37	3.34	3.34	3.28
ME-4	4.28	4.28	4.26	4.32	4.26	4.27	4.24	4.21	4.25	4.25
Large-ME	5.85	5.82	5.82	6.00	5.80	5.96	5.76	5.83	5.60	5.58
	Average portfolio idiosyncratic volatility (in Percent)									
Small-ME	12.2	11.8	13.2	13.1	12.3	13.2	14.2	13.7	15.0	14.7
ME-2	7.76	8.12	9.13	9.52	9.80	10.3	10.5	10.2	10.6	11.8
ME-3	6.77	7.09	7.64	8.11	7.86	8.24	8.46	8.10	8.80	9.36
ME-4	5.64	6.29	6.46	7.02	7.06	7.07	7.33	7.31	8.97	8.43
Large-ME	4.70	4.78	5.17	5.04	5.36	5.73	6.05	5.73	6.39	6.53

Table 3: **Size-Beta Portfolio Characteristics (Sample Period: 1963-2000)**

This table reports the average return, average beta, average log size, and average residual standard deviation for the 50 size-beta sorted portfolios over the extended Fama and French (1992) sample period of 1963-2000. Stocks from the NYSE/AMEX are first sorted into five size groups according their market capitalization in December of the last year of the estimation period. Each group of stocks is then sorted again into decile portfolios according to the selection period betas. Detailed information on portfolio selection, estimation, and testing periods are available in Fama and MacBeth (1973). Both the reported betas and the idiosyncratic volatilities as well as betas used in sorting are estimated from a market model with a value-weighted index.

	Low-β	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	High-β
	Average Monthly Returns (in Percent)									
Small-ME	1.04	1.52	1.31	1.45	1.58	1.22	1.63	1.69	1.75	1.67
ME-2	1.78	1.15	1.16	1.35	1.33	1.48	1.44	1.25	1.32	1.30
ME-3	1.43	1.04	1.12	1.28	1.16	1.43	1.26	1.27	1.19	1.29
ME-4	1.43	1.01	1.13	1.09	1.15	1.17	1.36	1.17	1.20	1.20
Large-ME	1.09	1.03	1.09	1.09	1.13	1.22	1.03	1.10	1.10	.969
	Average portfolio beta									
Small-ME	.783	.932	.939	1.04	1.05	1.10	1.19	1.18	1.28	1.37
ME-2	.576	.798	.928	1.00	1.10	1.19	1.21	1.26	1.37	1.44
ME-3	.635	.805	.945	1.02	1.10	1.15	1.20	1.28	1.36	1.51
ME-4	.628	.735	.934	1.00	1.05	1.09	1.14	1.18	1.30	1.45
Large-ME	.643	.786	.860	.908	.999	1.03	1.06	1.07	1.16	1.25
	Average portfolio log size									
Small-ME	2.63	2.75	2.78	2.70	2.80	2.81	2.73	2.80	2.67	2.77
ME-2	4.10	4.08	4.11	4.14	4.13	4.17	4.17	4.22	4.18	4.10
ME-3	5.19	5.25	5.25	5.23	5.28	5.22	5.21	5.17	5.23	5.27
ME-4	6.27	6.33	6.32	6.38	6.35	6.34	6.41	6.32	6.35	6.32
Large-ME	7.65	7.94	7.88	8.06	7.82	7.88	7.83	7.90	7.64	7.55
	Average portfolio idiosyncratic volatility (in Percent)									
Small-ME	10.2	11.5	11.3	12.1	11.9	12.4	13.1	12.8	14.2	14.6
ME-2	6.41	7.54	8.51	8.99	9.11	9.93	10.2	10.2	10.8	11.6
ME-3	6.32	6.58	7.38	7.64	8.00	8.14	8.60	8.89	9.73	10.1
ME-4	5.38	5.80	6.50	6.56	6.92	7.01	7.17	7.74	8.10	8.94
Large-ME	5.11	5.26	5.71	5.71	5.84	5.93	6.09	6.18	6.63	7.31

Table 4: **Replicating Fama-MacBeth Cross-sectional Regression on Portfolios**
This table is intended to replicate the cross-sectional regression results reported in Fama-MacBeth (1973) Table 3. In general, stocks from the NYSE are selected into one of 20 portfolios according to the selection period (seven years) with betas estimated from a market model. Details are available in Fama-MacBeth (1973). Both the beta estimates and idiosyncratic volatility estimates used in cross-sectional regressions of the testing periods are from the estimation period (next five years). In particular, the independent variables $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$ are equal-weighted averages of individual stocks' beta, beta squared, and residual standard deviation, respectively, estimated using a market model with an equal weighted index.

	Replicating results					Fama-MacBeth Table 3				
	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$
1935-6/68	.0061	.0087			.30	.0061	.0085			.29
t Ratio	3.26	2.69				3.24	2.57			
1935-40	.0012	.0125			.22	.0024	.0109			.23
t Ratio	.155	.950				.320	.790			
1941-45	.0071	.0212			.36	.0056	.0229			.37
t Ratio	1.72	2.46				1.27	2.55			
1946-50	.0048	.0031			.39	.0050	.0029			.39
t Ratio	1.22	.511				1.27	.480			
1951-55	.0122	.0026			.24	.0123	.0024			.24
t Ratio	4.98	0.576				5.06	.530			
1956-60	.0148	-.0058			.23	.0148	-.0059			.22
t Ratio	5.62	-1.34				5.68	-1.37			
1961-6/68	.0013	.0141			.33	.0001	.0143			.32
t Ratio	.351	2.73				.030	2.81			
1935-6/68	.0027	.0165	-.0037		.33	.0049	.0105	-.0008		.32
t Ratio	1.04	2.72	-1.46			1.92	1.79	-.290		
1935-40	-.0056	.0302	-.0091		.23	.0013	.0141	-.0017		.24
t Ratio	-.669	1.44	-1.21			.160	.75	.190		
1941-45	.0094	.0166	.0018		.38	.0148	.0004	.0108		.39
t Ratio	1.63	1.72	.277			2.28	.030	1.15		
1946-50	-.0004	.0143	-.0047		.45	-.0008	.0152	-.0051		.44
t Ratio	-.092	1.09	-1.14			-.180	1.14	-1.24		
1951-55	.0004	.0280	-.0120		.28	.0004	.0281	-.0122		.28
t Ratio	.089	2.53	-2.63			.100	2.55	-2.72		
1956-60	.0124	-.0003	-.0025		.26	.0128	-.0015	-.0020		.25
t Ratio	3.18	-.028	-.676			3.38	-.160	-.540		
1961-6/68	.0030	.0101	.0021		.35	.0029	.0077	.0034		.34
t Ratio	.429	.665	.299			.420	.530	.510		

Table 2–**Continued**

	Replicating Results					Fama-MacBeth Table 3				
	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$
1935-6/68	.0058	.0083		.0037	.33	.0054	.0072		.0198	.32
t Ratio	2.45	2.36		.085		2.10	2.20		.460	
1935-40	.0027	.0152		-.0450	.25	.0036	.0119		-.0170	.25
t Ratio	.324	1.09		-.653		.370	.970		-.190	
1941-45	.0021	.0102		.1610	.39	-.0006	.0085		.2053	.41
t Ratio	.309	1.65		1.29		-.080	1.25		1.46	
1946-50	.0076	.0095		-.1230	.44	.0069	.0081		-.0920	.42
t Ratio	1.46	1.07		-1.33		1.56	.950		-1.41	
1951-55	.0149	.0072		-.1230	.28	.0150	.0069		-.1185	.27
t Ratio	3.95	1.29		-1.33		4.05	1.24		-1.31	
1956-60	.0122	-.0086		.0925	.26	.0127	-.0081		.0728	.26
t Ratio	2.54	-1.41		.569		2.68	-1.40		.480	
1961-6/68	.0000	.0123		.0483	.34	-.0014	.0122		.0570	.33
t Ratio	.001	1.97		.523		-.320	2.12		.640	
1935-6/68	-.0011	.0175	-.0061	.0630	.34	.0020	.0114	-.0026	.0516	.34
t Ratio	-.315	2.85	-2.21	1.32		-.550	1.85	-.860	1.11	
1935-40	-.0083	.0320	-.0117	.0224	.26	.0009	.0156	-.0029	.0025	.26
t Ratio	-.760	1.51	-1.35	.300		.070	.780	-.290	.030	
1941-45	-.0076	.0235	-.009	.2370	.40	.0015	.0073	.014	.1767	.43
t Ratio	-.684	2.46	-1.43	1.63		.120	.520	.150	1.16	
1946-50	.0032	.0136	-.0029	-.0719	.44	.0011	.0141	-.0040	-.0313	.44
t Ratio	.576	1.05	-.749	-.984		.180	1.03	-.730	-.410	
1951-55	.0034	.0275	-.0104	-.0773	.30	.0023	.0277	-.0112	-.0443	.29
t Ratio	.691	2.50	-2.33	-.860		.480	2.53	-2.54	-.530	
1956-60	.0087	-.0042	-.0028	.1370	.28	.0103	-.0047	-.0020	.0979	.28
t Ratio	1.21	-.430	-.649	.717		1.63	-.470	-.490	.590	
1961-6/68	-.0025	.0120	-.0008	.1130	.37	-.0017	.0088	.0013	.0957	.35
t Ratio	-.310	.767	-.110	1.12		-.210	.580	.190	1.02	

Table 5: **Fama-MacBeth Cross-sectional Regression on Portfolios**

This table reports the cross-sectional regression results for portfolios over both the Fama-MacBeth (1973) sample period of January 1935 to June 1968 and the extended Fama-French (1992) sample period of January 1963 to December 2000. In general, stocks from the NYSE/AMEX are selected into one of 20 portfolios according to the selection period (seven years) betas estimated from a market model. Details are available in Fama-MacBeth (1973). Both the beta estimates and idiosyncratic volatility estimates used in the cross-sectional regressions of the testing periods are from the estimation period (next five years). In particular, the independent variables $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$ are equal-weighted averages of individual stock's beta, beta squared, and residual standard deviation, respectively, estimated using a market model.

		Equally-Weighted Index				Value-Weighted Index			
Eq. #		Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$
		Panel A: 1935-1968/6 with 20 portfolios							
1	$\bar{\gamma}$	.0061	.0087			.0073	.0057		
	$t(\bar{\gamma})$	2.75	2.34			3.12	1.91		
2	$\bar{\gamma}$	.0101		.0039		.0106		.0020	
	$t(\bar{\gamma})$	4.07		2.32		4.26		1.73	
3	$\bar{\gamma}$	-.0075			.0769	-.0022			.0574
	$t(\bar{\gamma})$	-1.10			2.27	-0.34			1.77
4	$\bar{\gamma}$	-.0079	.0161	-.0057	.0459	.0014	.0187	-.0056	-.0070
	$t(\bar{\gamma})$	-1.17	2.40	-2.11	1.50	0.21	2.85	-2.52	-0.26
		Panel B: 1935-1968/6 with 50 portfolios							
1	$\bar{\gamma}$	.0067	.0080			.0076	.0054		
	$t(\bar{\gamma})$	3.15	2.24			3.41	1.86		
2	$\bar{\gamma}$	.0105		.0036		.0110		.0018	
	$t(\bar{\gamma})$	4.11		2.33		4.33		1.64	
3	$\bar{\gamma}$	-.0055			.0701	-.0019			.0560
	$t(\bar{\gamma})$	-0.94			2.31	-0.34			1.93
4	$\bar{\gamma}$	-.0075	.0123	-.0045	.0507	-.0084	.0136	-.0049	.0475
	$t(\bar{\gamma})$	-1.44	1.92	-1.99	2.36	-1.55	2.74	-2.94	2.17
		Panel C: 1963-2000 with 20 portfolios							
1	$\bar{\gamma}$	.0081	.0045			.0079	.0043		
	$t(\bar{\gamma})$	3.84	1.42			3.51	1.43		
2	$\bar{\gamma}$	.0103		.0020		.0101		.0018	
	$t(\bar{\gamma})$	4.99		1.20		5.08		1.32	
3	$\bar{\gamma}$	.0006			.0460	.0018			.0387
	$t(\bar{\gamma})$	0.01			1.66	0.32			1.47
4	$\bar{\gamma}$	.0019	.0091	-.0047	.0227	-.0100	.0107	-.0069	.0666
	$t(\bar{\gamma})$	0.26	1.26	-1.44	0.71	-1.41	1.36	-1.97	2.48
		Panel D: 1963-2000 with 50 portfolios							
1	$\bar{\gamma}$	.0084	.0043			.0082	.0040		
	$t(\bar{\gamma})$	4.00	1.39			3.73	1.37		
2	$\bar{\gamma}$	.0105		.0018		.0104		.0016	
	$t(\bar{\gamma})$	5.09		1.12		5.15		1.24	
3	$\bar{\gamma}$	.0011			.0417	.0028			.0348
	$t(\bar{\gamma})$	0.20			1.60	0.53			1.44
4	$\bar{\gamma}$	.0025	.0083	-.0035	.0187	-.0043	.0057	-.0035	.0529
	$t(\bar{\gamma})$	0.51	1.24	-1.22	0.92	-.791	0.96	-1.36	2.59

Table 6: **Cross-sectional Regression for Size-Beta Portfolios**

This table reports the cross-sectional regression results on the 50 size-beta sorted portfolios over both the Fama-MacBeth (1973) sample period of January 1935 to June 1968 and the extended Fama-French (1992) sample period of January 1963 to December 2000. Stocks from the NYSE/AMEX are first sorted into five size groups according their market capitalization in December of the last year of the estimation period. Each group of stocks is then sorted again into decile portfolios according to the selection period betas. Detailed information on the portfolio selection, estimation, and testing periods are available in Fama and MacBeth (1973). Under the column 'Based on CAPM', both the reported betas and the idiosyncratic volatilities as well as betas used in sorting are estimated from a market model with a value-weighted index. Similarly, under the column 'Based on FF3 Model', the idiosyncratic volatilities are estimated from the Fama and French (1993) three-factor model instead. Both the beta estimates and idiosyncratic volatility estimates used in cross-sectional regressions of the testing periods are from the estimation period. In particular, the independent variables $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, $log(ME_{p,t-1})$, and $\bar{s}_{p,t-1}(\epsilon)$ are equal-weighted averages of individual stock's beta, beta squared, log size, and residual standard deviation, respectively.

Eq.		Based on CAPM				Based on FF3 Model			
#		$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$log(ME_{p,t-1})$	$\bar{s}_{p,t-1}(\epsilon)$	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$log(ME_{p,t-1})$	$\bar{s}_{p,t-1}(\epsilon)$
		Panel A: For sample period of 1935-1968/6							
1	$\bar{\gamma}$	.0077				.0077			
	$t(\bar{\gamma})$	2.24				2024			
2	$\bar{\gamma}$		.0027				.0027		
	$t(\bar{\gamma})$		2.13				2.13		
3	$\bar{\gamma}$			-.0022				-.0022	
	$t(\bar{\gamma})$			-2.74				-2.74	
4	$\bar{\gamma}$				0.070				0.077
	$t(\bar{\gamma})$				2.61				2.61
5	$\bar{\gamma}$	.0141	-.0045	-.0018		.0141	-.0045	-.0018	
	$t(\bar{\gamma})$	2.46	-2.29	-2.73		2.46	2.29	-2.73	
6	$\bar{\gamma}$	.0134	-.0055		0.073	.0124	-.0049		0.077
	$t(\bar{\gamma})$	2.55	-3.10		2.69	2.36	-2.76		2.70
7	$\bar{\gamma}$			-.0004	0.062			-.0004	0.067
	$t(\bar{\gamma})$			-0.77	2.03			-0.76	2.03
8	$\bar{\gamma}$	.0163	-.0063	-.0004	0.053	.0157	-.0058	-.0005	0.051
	$t(\bar{\gamma})$	2.95	-3.36	-0.96	2.16	2.83	-3.12	-1.13	2.09
		Panel B: For sample period of 1963-2000							
1	$\bar{\gamma}$	.0044				.0044			
	$t(\bar{\gamma})$	1.57				1.57			
2	$\bar{\gamma}$		.0018				.0018		
	$t(\bar{\gamma})$		1.48				1.48		
3	$\bar{\gamma}$			-.0009				-.0009	
	$t(\bar{\gamma})$			-1.94				-1.94	
4	$\bar{\gamma}$				0.043				0.048
	$t(\bar{\gamma})$				1.81				1.88
5	$\bar{\gamma}$	.0089	-.0029	-.0009		.0089	-.0029	-.0009	
	$t(\bar{\gamma})$	1.30	-1.13	-2.05		1.30	-1.13	-2.05	
6	$\bar{\gamma}$	.0059	-.0033		0.054	.0055	-.0031		0.059
	$t(\bar{\gamma})$	0.97	-1.33		2.16	0.92	-1.26		2.19
7	$\bar{\gamma}$			-.0003	0.034			-.0003	0.036
	$t(\bar{\gamma})$			-0.69	1.36			-0.83	1.36
8	$\bar{\gamma}$	.0072	-.0036	-.0003	0.041	.0071	-.0035	-.0003	0.040
	$t(\bar{\gamma})$	1.08	-1.39	-0.69	1.86	1.06	-1.37	-0.82	1.76

Table 7: **Cross-sectional Fama and French Regression for Individual NYSE/AMEX/NASDAQ Stocks (1963-2000)**

This table reports the cross-sectional regression results for all NYSE/AMEX/NASDAQ individual stocks over the extended Fama-French (1992) sample period of July 1963 to June 2000. The beta estimates (β_p) used in the cross-sectional regressions are estimated from the 100 size-beta sorted portfolios and then assigned to individual stocks. Portfolio breakdowns in each year are determined by first sorting the NYSE stocks only into 10 size groups according their market capitalization in June of each year. Each group of the NYSE stocks is then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model (with lagged market returns) of previous 24 to 60 monthly returns. Similarly, the idiosyncratic volatility measure $s_p(\epsilon)$ is the root mean square error of either the market model or the Fama-French (1993) three-factor model for each of the 20(size) $\times$ 10(beta) portfolios and then assigned to individual stocks. In June of each year, we also compute $s_{i,t-1}(\epsilon)$ as the root mean square error of either a market model or the three-factor model for stock i based on the previous 24 to 60 monthly returns. This measure is used in the cross-sectional regression of the next 12 months. $ME_{i,t-1}$ and $B/M_{i,t-1}$ are June's log market capitalization and last fiscal year's log book to market measure, respectively. In Panel A, both idiosyncratic volatility measures are estimated from the market model. While in Panel B, both idiosyncratic volatility measures are estimated from the Fama and French (1993) three-factor model.

		Panel A: Based on CAPM					Panel B: Based on FF3 Model				
Eq#		β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$s_{i,t-1}(\epsilon)$	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$s_{i,t-1}(\epsilon)$
1	$\bar{\gamma}$	.0047					.0047				
	$t(\bar{\gamma})$	1.37					1.37				
2	$\bar{\gamma}$		.0042					.0042			
	$t(\bar{\gamma})$		5.43					5.43			
3	$\bar{\gamma}$			-.0019					-.0019		
	$t(\bar{\gamma})$			-3.50					-3.50		
4	$\bar{\gamma}$				0.315					0.318	
	$t(\bar{\gamma})$				4.75					3.59	
5	$\bar{\gamma}$					.0534					.0515
	$t(\bar{\gamma})$					2.53					2.43
6	$\bar{\gamma}$	.0056	.0041				.0056	.0041			
	$t(\bar{\gamma})$	1.66	5.53				1.66	5.53			
7	$\bar{\gamma}$	-.0003		-.0020			-.0003		-.0020		
	$t(\bar{\gamma})$	-0.11		-4.34			-0.11		-4.34		
8	$\bar{\gamma}$	.0054			0.327		-.0090			0.460	
	$t(\bar{\gamma})$	1.54			4.83		-3.06			5.16	
9	$\bar{\gamma}$	.0035				.0487	.0036				.0464
	$t(\bar{\gamma})$	1.10				2.58	1.12				2.44
10	$\bar{\gamma}$			-.0007	0.234				-.0005	0.265	
	$t(\bar{\gamma})$			-1.09	3.62				-1.46	3.00	
11	$\bar{\gamma}$			-.0017		.0194			-.0017		.0165
	$t(\bar{\gamma})$			-3.66		1.27			-3.67		1.08
12	$\bar{\gamma}$	.0016	.0030	-.0016			.0016	.0030	-.0016		
	$t(\bar{\gamma})$	0.58	4.08	-3.23			0.58	4.08	-3.23		
13	$\bar{\gamma}$	.0061	.0030		0.280		-.0066	.0030		0.405	
	$t(\bar{\gamma})$	1.77	4.15		4.05		-2.51	4.22		4.51	
14	$\bar{\gamma}$	.0042	.0044			.0556	.0043	.0044			.0546
	$t(\bar{\gamma})$	1.35	5.84			2.90	1.37	5.85			2.82
15	$\bar{\gamma}$	.0081	.0031	.0006	0.360		-.0070	.0030	.0003	0.446	
	$t(\bar{\gamma})$	2.44	4.16	1.65	4.87		-2.62	4.09	0.74	5.43	
16	$\bar{\gamma}$	.0019	.0035	-.0010		.0380	.0019	.0035	-.0010		.0357
	$t(\bar{\gamma})$	0.66	5.29	-2.57		2.40	0.64	5.29	-2.67		2.30

Table 8: **Cross-sectional Fama and French Regression for Individual NYSE/AMEX Stocks (1963-2000)**

This table reports the cross-sectional regression results for NYSE/AMEX individual stocks only over the extended Fama-French (1992) sample period of July 1963 to June 2000. The beta estimates (β_p) used in the cross-sectional regressions are estimated from the 100 size-beta sorted portfolios and then assigned to individual stocks. Portfolio breakdowns in each year are determined by first sorting NYSE stocks only into 10 size groups according their market capitalization in June of each year. Each group of NYSE stocks is then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model (with lagged market returns) of previous 24 to 60 monthly returns. Similarly, the idiosyncratic volatility measure $s_p(\epsilon)$ is the root mean square error of either the market model or the Fama-French (1993) three-factor model for each of the 20(size) $\times$ 10(beta) portfolios and then assigned to individual stocks. In June of each year, we also compute $s_{i,t-1}(\epsilon)$ as the root mean square error of either a market model or the three-factor model for stock i based on the previous 24 to 60 monthly returns. This measure is used in the cross-sectional regression of the next 12 months. $ME_{i,t-1}$ and $B/M_{i,t-1}$ are June's log market capitalization and last fiscal year's log book to market measure, respectively. In Panel A, both idiosyncratic volatility measures are estimated from the market model. While in Panel B, both idiosyncratic volatility measures are estimated from the Fama and French (1993) three-factor model.

		Panel A: Based on CAPM					Panel B: Based on FF3 Model				
Eq#		β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$s_{i,t-1}(\epsilon)$	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$s_{i,t-1}(\epsilon)$
1	$\bar{\gamma}$	.0039					.0039				
	$t(\bar{\gamma})$	1.19					1.19				
2	$\bar{\gamma}$		.0039					.0039			
	$t(\bar{\gamma})$		5.22					5.22			
3	$\bar{\gamma}$			-.0013					-.0013		
	$t(\bar{\gamma})$			-2.47					-2.47		
4	$\bar{\gamma}$				0.199					0.244	
	$t(\bar{\gamma})$				3.53					2.89	
5	$\bar{\gamma}$					.0404					.0379
	$t(\bar{\gamma})$					1.86					1.71
6	$\bar{\gamma}$	.0035	.0036				.0035	.0036			
	$t(\bar{\gamma})$	1.10	5.10				1.10	5.10			
7	$\bar{\gamma}$	-.0002		-.0014			-.0002		-.0014		
	$t(\bar{\gamma})$	-0.09		-3.19			-0.09		-3.19		
8	$\bar{\gamma}$	.0043			0.213		-.0030			0.293	
	$t(\bar{\gamma})$	1.29			3.61		-1.13			3.99	
9	$\bar{\gamma}$	.0031				.0372	.0032				.0341
	$t(\bar{\gamma})$	1.03				1.95	1.05				1.75
10	$\bar{\gamma}$			-.0007	0.119				.0003	0.299	
	$t(\bar{\gamma})$			-1.11	1.95				0.88	3.59	
11	$\bar{\gamma}$			-.0012		.0112			-.0012		.0069
	$t(\bar{\gamma})$			-2.47		0.72			-2.49		0.44
12	$\bar{\gamma}$	.0010	.0028	-.0009			.0010	.0028	-.0009		
	$t(\bar{\gamma})$	0.38	4.35	-2.01			0.38	4.35	-2.01		
13	$\bar{\gamma}$	.0040	.0028		0.156		-.0017	.0027		0.227	
	$t(\bar{\gamma})$	1.19	4.49		2.73		-0.66	4.26		3.18	
14	$\bar{\gamma}$	.0027	.0036			.0367	.0028	.0037			.0356
	$t(\bar{\gamma})$	0.92	5.13			1.90	0.93	5.13			1.81
15	$\bar{\gamma}$	.0050	.0028	.0003	0.195		-.0021	.0028	.0008	0.342	
	$t(\bar{\gamma})$	1.72	4.36	0.73	3.85		-0.83	4.30	1.94	4.83	
16	$\bar{\gamma}$	.0011	.0030	-.0006		.0243	.0011	.0030	-.0006		.0216
	$t(\bar{\gamma})$	0.40	4.96	-1.48		1.44	0.39	4.97	-1.56		1.28

Table 9: **Investigating the Liquidity Effect for Individual Stocks (1963-2000)**
This table reports the cross-sectional regression results for individual stocks over the extended Fama-French (1992) sample period of July 1963 to June 2000. The beta estimates (β_p) used in the cross-sectional regressions are estimated from the 100 size-beta sorted portfolios and then assigned to individual stocks. Portfolio breakdowns in each year are determined by first sorting NYSE stocks only into 10 size groups according their market capitalization in June of each year. Each group of NYSE stocks are then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model (with lagged market returns) of the previous 24 to 60 monthly returns. Similarly, the idiosyncratic volatility measure $s_p(\epsilon)$ is the root mean square error of the market model for each of the 20(size) $\times$ 10(beta) portfolios and then assigned to individual stocks. $Vlm_{i,t-1}$ is the relative volume of the previous month. $ME_{i,t-1}$ and $B/M_{i,t-1}$ are June's log market capitalization and the last fiscal year's log book to market measure, respectively. In Panel A, all the NYSE/AMEX/NASDAQ individual stocks are used in the cross-sectional regression. In Panel B, only NYSE/AMEX stocks are used in estimation.

		Panel A: NYSE/AMEX/NASDAQ Stocks					Panel B: NYSE/AMEX Stocks				
Eq#		β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$Vlm_{i,t-1}$	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$Vlm_{i,t-1}$
1	$\bar{\gamma}$				0.315					0.199	
	$t(\bar{\gamma})$				4.75					3.53	
2	$\bar{\gamma}$					.0065					.0066
	$t(\bar{\gamma})$					2.68					2.80
3	$\bar{\gamma}$	.0048				.0068	.0037				.0066
	$t(\bar{\gamma})$	1.39				2.92	1.16				3.04
4	$\bar{\gamma}$		.0042			.0039		.0038			.0031
	$t(\bar{\gamma})$		5.37			1.72		5.16			1.48
5	$\bar{\gamma}$			-.0019		.0001			-.0012		.0013
	$t(\bar{\gamma})$			-3.40		0.04			-2.31		0.78
6	$\bar{\gamma}$				0.317	-.0015				0.194	-.0011
	$t(\bar{\gamma})$				4.79	-0.85				3.57	-0.79
7	$\bar{\gamma}$	.0013	.0030	-.0016		-.0018	.0009	.0028	-.0009		-.0007
	$t(\bar{\gamma})$	0.49	4.08	-3.32		-1.35	0.33	4.36	-2.04		-0.69
8	$\bar{\gamma}$	.0060	.0030		0.284	-.0016	.0039	.0028		0.157	-.0004
	$t(\bar{\gamma})$	1.72	4.16		4.15	-1.14	1.18	4.49		2.81	-0.37
9	$\bar{\gamma}$	.0057		.0008	0.352	-.0021	.0036		-.0002	0.199	-.0005
	$t(\bar{\gamma})$	1.60		0.21	4.76	-1.59	1.23		-0.60	3.93	-0.50
10	$\bar{\gamma}$	.0078	.0031	.0006	0.360	-.0015	.0048	.0028	.0003	0.195	-.0011
	$t(\bar{\gamma})$	2.37	4.16	1.55	4.86	-1.19	1.67	4.37	0.67	3.87	-1.03

Table 10: **Cross-sectional Regressions for Individual Japanese Stocks (Jan. 1980- Dec. 1999)**

Both the beta estimates and idiosyncratic volatility estimates used in cross-sectional regressions are estimated from 100 size-beta sorted portfolios and then assigned to individual stocks. In each year, stocks are first sorted into 10 size groups according their market capitalization in December of the previous year. Each group of stocks are then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model of previous 24 to 60 monthly returns. We use all the listed stocks that also have book-to-market information over the whole sample period. The idiosyncratic volatilities are estimated from the Fama and French (1993) three-factor model. In particular, $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, $log(ME_{i,t-1})$, $(BE/ME)_{i,t-1}$ and $\bar{s}_{p,t-1}(\epsilon)$ are the independent variables of beta, beta squared, log size, book-to-market equity, and residual standard deviation, respectively.

	Const.	β_p	BE/ME	$log(ME_{i,t-1})$	$s_p(\epsilon)$	Const.	β_p	BE/ME	$log(ME_{i,t-1})$	$s_p(\epsilon)$
	Panel A: The whole sample									
$\bar{\gamma}$	-.00367	.0121								
$t(\bar{\gamma})$	-0.555	1.362								
$\bar{\gamma}$	0.00523		.0081							
$t(\bar{\gamma})$	1.163		2.569							
$\bar{\gamma}$	0.02518			-.0016						
$t(\bar{\gamma})$	1.651			-1.470						
$\bar{\gamma}$	0.00332				1.358					
$t(\bar{\gamma})$	0.935				1.867					
$\bar{\gamma}$	0.01445	.0052	.0070	-.0013						
$t(\bar{\gamma})$	0.991	0.707	2.375	-1.285						
$\bar{\gamma}$	0.00076	.0003	.0071		1.306					
$t(\bar{\gamma})$	0.123	0.037	2.312		1.871					
$\bar{\gamma}$	-.00632	-.0003	.0073	.0006	1.686					
$t(\bar{\gamma})$	-0.457	-0.041	2.466	0.701	3.354					
	Panel B: The first subsample					Panel C: The second subsample				
$\bar{\gamma}$	0.00843	.0132				-.01580	.0110			
$t(\bar{\gamma})$	0.893	1.206				-1.697	0.773			
$\bar{\gamma}$	0.01922		.0077			-.00877		.0085		
$t(\bar{\gamma})$	3.268		1.614			-1.273		2.028		
$\bar{\gamma}$	0.04748			-.0024		0.00289			-.0007	
$t(\bar{\gamma})$	2.499			-1.773		0.124			-0.455	
$\bar{\gamma}$	0.01470				1.835	-.00805				0.881
$t(\bar{\gamma})$	3.083				1.990	-1.501				0.801
$\bar{\gamma}$	0.04366	.0020	.0060	-.0024		-.01476	.0084	.0080	-.0002	
$t(\bar{\gamma})$	1.861	0.186	1.343	-1.647		-0.891	0.782	2.017	-0.131	
$\bar{\gamma}$	0.01603	-.0038	.0065		2.031	-.01451	.0043	.0078		0.581
$t(\bar{\gamma})$	1.673	-0.344	1.386		1.937	-1.847	0.419	1.891		0.654
$\bar{\gamma}$	0.02501	-.0034	.0062	-.0006	1.625	-.03765	.0028	.0085	.0019	1.747
$t(\bar{\gamma})$	1.152	-0.329	1.389	-0.510	2.435	-2.262	0.270	2.100	1.600	2.322

Table 11: **Summary Statistics for Individual Times Series Regressions of 100 Portfolios**

This table reports the average and standard deviation of regression coefficients for different time-series models. $R_{VW,t}$ and $R_{i,t}$ denote returns from a value-weighted index and the $i-th$ portfolio respectively; $R_{I,t}$ denotes the return proxies for idiosyncratic volatility from a hedging portfolio; $R_{SMB,t}$ and $R_{HML,t}$ denote proxies for the size and book-to-market factors respectively. The dependent variable is $R_{i,t}$. "Ind." counts the percentage of individual regressions for each portfolio with significant coefficients at 5% level.

	α	$R_{VW,t}$	$R_{SML,t}$	$R_{HML,t}$	$R_{I,t}$	$R^2(\%)$
	Panel A: 1935.1-1968.6					
Mean	0.105	1.279				69.94
St.D	(.195)	(.088)				(13.48)
Ind.(%)	26.00	100.0				
Mean	0.166	0.957			0.574	77.09
St.D	(.166)	(.082)			(.141)	(7.66)
Ind.(%)	39.00	97.00			82.00	
Mean	-0.002	1.021	0.679	0.347		79.46
St.D	(.147)	(.061)	(.116)	(.123)		(7.10)
Ind.(%)	34.00	100.0	87.00	71.00		
Mean	0.106	0.961		0.225	0.462	77.62
	(.163)	(.080)		(.110)	(.132)	(7.47)
Ind.(%)	30.00	98.00		56.00	73.00	
Mean	0.021	0.997	0.609	0.303	0.094	80.20
St.D	(.155)	(.079)	(.125)	(.105)	(.160)	(6.52)
Ind.(%)	27.00	99.00	91.00	77.00	54.00	
	Panel B: 1963.7-2000.6					
Mean	0.079	1.057				67.10
St.D	(.161)	(.051)				(11.96)
Ind.(%)	14.00	100.0				
Mean	0.087	0.889			0.314	73.25
St.D	(.137)	(.052)			(.073)	(9.76)
Ind.(%)	16.00	100.0			80.00	
Mean	-0.091	1.019	0.577	0.311		81.13
St.D	(.117)	(.037)	(.055)	(.066)		(7.08)
Ind.(%)	35.00	100.0	91.00	85.00		
Mean	-0.064	0.920		0.369	0.439	77.26
St.D	(.130)	(.049)		(.076)	(.055)	(7.45)
Ind.(%)	27.00	100.0		88.00	87.00	
Mean	-0.091	1.019	0.576	0.311	0.000	82.19
St.D	(.111)	(.044)	(.081)	(.064)	(.073)	(6.46)
Ind.(%)	37.00	100.0	92.00	88.00	77.00	

Figure 1 The Relationship between Average Return and Idiosyncratic Volatility