A New Method for Estimating Flow Duration Curves : an Application to the Burdekin River Catchment, North Queensland, Australia

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Abstract : This paper presents a new method of representing flow duration curves (FDC) using a logarithmic transformation. The FDC has been defined using two parameters – the 'cease to flow' point, and the slope of the FDC. This method for defining the FDC has been applied to 23 sub-catchments of the Burdekin River in North Queensland, Australia. The two parameters defining the FDC have been related to the area, mean annual precipitation, drainage density and total stream length of the catchments under consideration. Finally, a regionalisation procedure has been developed whereby the FDC for an ungauged catchment can be predicted based on the attributes of that catchment. Significant landuse change such as dam construction can have a significant impact on the FDC, implying that the FDC may be used as an indicator of landuse change in a catchment. Finally, the mirror-image nature of the FDC may imply that it is possible to predict high flows in a particular catchment based on analysis of low flows in that catchment.

Keywords : Flow duration curves, Burdekin catchment, tropical hydrology.

1. INTRODUCTION

The flow duration curve provides information about the percentage of time that a particular streamflow was exceeded over some historical period. Generally it is represented on a log-normal scale with exceedence probability on the x-axis and discharge (or some function of discharge) on the y-axis.

Flow duration curves have long been used as means of summarising catchment hydrologic response. More recently, they been used to validate the outputs of hydrologic models and/or compare observed and modelled hydrologic response. See for example Hansen *et al.* [1996] and Ye *et al.* [1997].

Recently however, it has also been suggested that flow duration curves may provide a hydrologic 'signature' of a catchment. This hydrologic signature has been used in top-down modelling approaches to determine the appropriate level of complexity required in a rainfall-runoff model [Jothityangkoon *et al.*, 2001]. It is hypothesised that this signature may reflect properties of the catchment, and may therefore be of potential use in regionalisation studies. For example, Yu *et al.* [2002] applied a regionalisation procedure to predict the low flow proportion of the flow duration curve at ungauged sites in southern Taiwan.

In relatively homogeneous areas, it may be possible to estimate the flow duration curve very simply, based on a relationship with catchment area. Yu and Yang [2000] demonstrated this approach in the homogeneous Gao-Ping Creek area in southern Taiwan. However, in order to be able to regionalise the flow duration curve in more heterogeneous areas (such as the Burdekin catchment in North Queensland, Australia), we need to be able to estimate it in a relatively simple yet robust way, such that it can be related to characteristics of the catchment which it represents.

Many alternative parameterisations of the flow duration curve have been presented in the literature. For example, Cigizoglu and Bayazit [2000] utilise a 5-parameter harmonic model, which they found could be simplified to a 2 or 3 parameter model in many cases. Best *et al.* [2003] examined the use of three different parameterisations of the flow duration curve, and found that the 5-parameter model was the only one that provided an adequate fit to be able to examine the impact of land use change on hydrologic response as defined by the flow duration curve.

In this paper, I will present an alternative method of defining the flow duration curve which requires just two parameters. This simple model is sufficient to adequately define the flow duration curve in a variety of catchments. It is therefore compatible with the 'top-down' approach where additional levels of complexity are added to a model only if demanded by the data [Sivapalan *et al.*, 2003].

2. DEFINING THE FLOW DURATION CURVE

Following the procedure defined by Best *et al.* [2003], streamflow (in cumecs) was normalised by dividing it by the median discharge of the non-zero flow days. This ensures that the log of the normalised streamflow crosses the axis at the median streamflow of the non-zero flow days (since *log*(1) = 0). It is impossible to normalise by the median streamflow over all days because some of the ephemeral catchments in the Burdekin flow for less than 50% of the time (making the median streamflow zero).

The log of normalised streamflow for Burdekin sub-catchment 120002 versus percent exceedence is shown by the dashed line in Figure 1.

Based on the shape of this curve, a logarithmic function appears to be most appropriate on to choose to represent it. The chosen function is shown in (1).

$$y = ln((a/x)-1)/b \tag{1}$$

where :

y = log(normalised streamflow)

x = percent exceedence

and a and b are variables as follows :

a = percent exceedence at the cease to flow value (for example, if a river flows for 90% of the time, a = 90).

b = a constant controlling the slope of the FDC

The fit to the observed data is shown by the solid line in Figure 1.

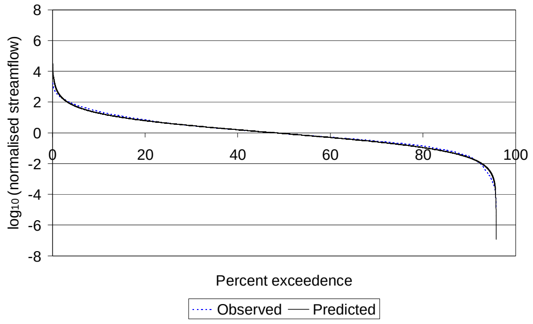

Figure 1 : Observed and predicted FDC for Catchment 120002

Here, a = 95.8 and b = 1.688. r^2 = 0.998.

3. APPLICATION TO BURDEKIN SUB-CATCHMENTS

The Burdekin River Catchment is a large (140,000 km^2) catchment in the dry tropics rangelands of North Queensland, Australia.

Table 1 shows the values of a and b for each of the 23 Burdekin sub-catchments. It also shows which region each of the catchments is in (the Burdekin can be divided into three physio-climatic regions as described in Post and Croke, [2002]). The catchments in each of these regions exhibit a different hydrologic behaviour.

Table 1 : FDC parameters for the 23 Burdekin sub-catchments

Region	Catchment	a	b	r^2
Upper	120002	95.8	1.69	.991
Suttor	120006*	95.7	1.66	.992
Suttor	120008*	93.4	1.74	.987
Suttor	120014	18.0	1.63	.964
Suttor	120015*	99.0	1.98	.986
Upper	120102	61.9	1.63	.983
Upper	120106	70.0	1.66	.974
Upper	120107	99.5	2.15	.978
Upper	120110	99.4	1.99	.980
Upper	120112	68.3	1.57	.991
Upper	120120	83.9	2.01	.997
Upper	120121	74.1	1.72	.994
Bowen	120205	98.9	2.06	.998
Bowen	120207^{+}	99.7	2.60	.995
Bowen	120209	99.6	2.09	.993
Bowen	120216^{+}	96.5	2.60	.998
Bowen	120299	89.0	2.24	.986
Suttor	120301	61.7	1.38	.979
Suttor	120302	52.9	1.14	.944
Suttor	120303	54.3	1.38	.963
Suttor	120304^{+}	38.0	1.31	.973
Suttor	120307	27.3	1.55	.969
Suttor	120309	38.6	1.37	.971

* Below the dam. Data only examined to July 1987 (pre-dam).

$^{+}$ Data only available until December1998.

3.1 Goodness of fit

It can be seen from Table 1 that in general, the 2-parameter logarithmic FDC model produces a very good fit to the observed flow duration curve, with all r-squared values exceeding 0.94. However, an examination of the goodness of fit of the models is illuminating. The poorest 6 fits are those obtained for catchments in the Suttor region. This is the area of the Burdekin catchment that is characterised by low rainfall, and infrequent discharge events. As a result, these catchments also have the lowest values of *a*.

Figure 2 shows the relationship between the cease to flow point and the goodness of fit (measured by r^2). It will be seen that more perennial catchments (flowing more than 80% of the time) all have r^2 values greater than 0.974. However, more ephemeral catchments (flowing less than 80% of the time) have r^2 values ranging from 0.944 to 0.994, with most of them having poorer fits than the more perennial catchments. This indicates that the procedure described here for using the FDC to define the hydrologic response of a catchment are more likely to work in humid, perennial catchments than in arid ephemeral catchments. As many of the sub-catchments of the Burdekin examined in this study are in the dry tropics, it suggests that this technique will be more applicable in the more humid parts of Australia (and the world), and possibly the wet tropics.

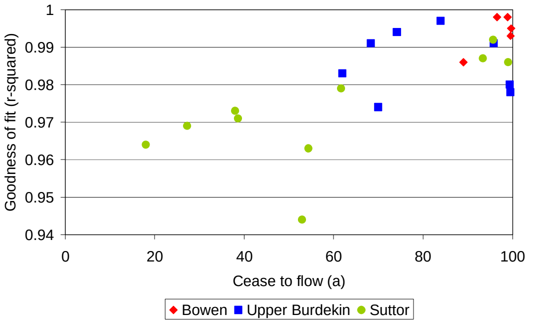

Figure 2 : Cease to flow versus goodness of fit for the 23 Burdekin sub-catchments

3.2 Cease to flow (*a* parameter)

The value of *a* in Equation 1 represents the 'cease to flow' point. That is, a perennial river would have a value of 100, while a river that flows for 80% of the time would have a value of 80.

One would expect that the value of *a* would be dependent on mean annual precipitation (MAP), with wetter catchments flowing for more of the time. This is true in part, as can be seen by Figure 3 showing MAP versus *a*. Catchments with high MAP (> 1000 mm) all have cease to flow values above 60%. However, drier catchments (500-1000 mm) can have cease to flow values anywhere between 20% and 100% depending on which region they are in (catchments in the Suttor in general flow for less of the time than catchments in the upper Burdekin with similar MAP). However, it will also be seen that the three regions have different patterns – catchments in the Suttor show little relationship to MAP, whereas catchments in the Bowen all have cease to flow values close to 100%. Wetter catchments in the upper Burdekin actually seem to have lower cease to flow values than drier catchments. Within each region therefore, MAP appears to have little impact on cease to flow for a particular catchment.

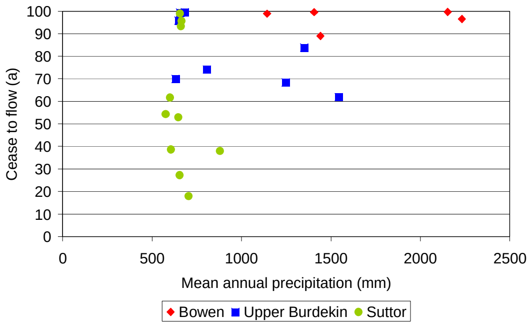

Figure 3 : Mean annual precipitation versus cease to flow for the 23 Burdekin sub-catchments

The second major control on cease to flow is catchment area (see Figure 4). Larger catchments tend to flow for longer because of their baseflow contributions and because it is likely that it will rain somewhere in them more often than it will rain somewhere in a smaller catchment. For example, the three gauging stations downstream of the dam (the three points in the upper right of Figure 4) flow for nearly all the time because there will be a contribution coming from one of the catchments upstream at pretty much any time of the year (the data are from before the dam, so it does not have an influence). A relationship can be seen for the Suttor, and perhaps the upper Burdekin, but little can be seen for the Bowen.

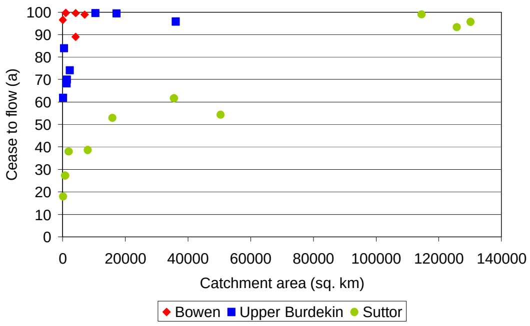

Figure 4 : Catchment area versus cease to flow for the 23 Burdekin sub-catchments

3.3 Slope of the FDC (*b* parameter)

In general, b shows the same relationship with mean annual precipitation and catchment area as a. As a result, these parameters are related to each other, with the slope of the FDC increasing with increasing values of cease to flow (ie. approaching 100). This correlation could mean that these parameters are inter-dependent, but there is insufficient information to be able to exclude one of them from the defining equation.

4. REGIONALISATION OF FDC PARAMETERS

To be able to derive flow duration curves for ungauged catchments, we need a way of deriving the values of the two parameters, a and b, as well as the median discharge. To determine the median discharge, one would need to monitor discharge for at least 50% of the year, although this could be done during the dry season when measurements are easier to take.

In the Burdekin catchment, the values of a and b depend not only on the following factors :

MAP	Mean annual precipitation (mm)
CA	Catchment area (km^2)
DD	Drainage density (km/km^2)
SL	Total stream length (km)

The values of a and b also depend on which region the catchment is located in – the Bowen, Upper Burdekin, or Suttor, with different relationships found in each region.

4.1 Suttor

The value of a can be predicted in the Suttor using (2).

$$a_S = 5.08x10^{-4}\, CA - 26.63\, DD + 51.24 \quad r^2=0.95 \tag{2}$$

The relationship with catchment area indicates that larger catchments flow for more of the time, while that with drainage density indicates that catchments with higher drainage density (where water will find its way into a stream more quickly) will flow for less time than catchments with longer sub-surface travel times.

The lack of a relationship with rainfall reflects the fact that MAP varies very little across the Suttor, from 576 mm to 880 mm (see Figure 3).

The value of b can be predicted in the Suttor using (3).

$$b_S = 0.547\, DD + 4.54x10^{-6}\, SL + 1.00 \quad r^2=0.73 \tag{3}$$

While the physical meaning of the b parameter is still not understood, as it controls the slope of the FDC, it also influences the range of meaningful values of log(streamflow). Thus catchments with smaller b values would have a greater range of streamflow. It seems reasonable that this should be related to the drainage system of a catchment.

4.2 Upper Burdekin

The value of a can be predicted in the Upper Burdekin using (4).

$$a_U = 8.52x10^{-4}\, CA + 74.22 \quad r^2=0.51 \tag{4}$$

Similarly to the Suttor, the relationship with catchment area indicates that larger catchments flow for more of the time. It can be seen from Figure 4 that the relationship between a and catchment area is not linear, and thus a transformation would possibly lead to a better fit. The addition of drainage density into this relationship only increased r^2 from 0.51 to 0.53 and is thus not justified.

No statistically significant relationship could be derived to define the value of b in the upper Burdekin. This may be related to its lack of physical meaning. However, learning from the other systems, it appears that catchment area, drainage density, and mean annual precipitation are vital controls on the shape of the FDC. Employing these three parameters in a linear regression leads to (5).

$$b_U = 0.31\, DD + 2.97x10^{-4}\, MAP + 4.0x10^{-4}\, CA \quad r^2=0.57 \tag{5}$$

4.3 Bowen

No statistically significant relationship could be found to derive the value of a in the Bowen. This is probably related to the fact that the range of catchment areas in the Bowen is very small compared to the overall range of catchment areas in this study (Figure 4). Also, as a only varies from 89.0 to 99.7, setting to it to 100 would be a suitable approximation.

The value of b can be predicted in the Bowen using (6).

$$b_B = 7.29x10^{-4}\, MAP + 1.40 \quad r^2=0.95 \tag{6}$$

Similar to the comments in the Suttor, the physical meaning of b is not completely understood, but a relationship with mean annual precipitation is not unreasonable.

5. IMPACT OF THE BURDEKIN FALLS DAM ON THE FDC

Figure 5 shows the flow duration curve for a catchment (120006) downstream of the Burdekin Falls Dam for the period before the dam was constructed. The modelled FDC fits the observed data very well (r^2=0.992). By comparison, Figure 6 shows the FDC for the period after construction of the dam. It will be seen that for the observed data, high flows are unaffected by the construction of the dam (comparison of the dashed lines in Figures 5 and 6). This is to be expected as the largest discharges pass over the dam spillway relatively unaffected by the dam. However, low flows are considerably changed after the construction of the dam. The very lowest flows have all but disappeared, and whereas the Burdekin River used to flow for only 95% of the time, it now flows all year round, and at a considerably higher level during low flows. Also, the model fit is very poor indeed, significantly underestimating both the high flows and the low flows.

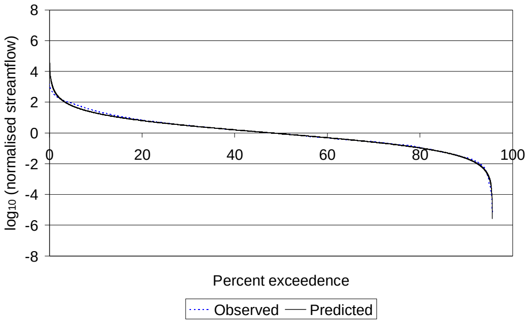

Figure 5 : Observed and predicted FDC for catchment 120006 (pre-dam)

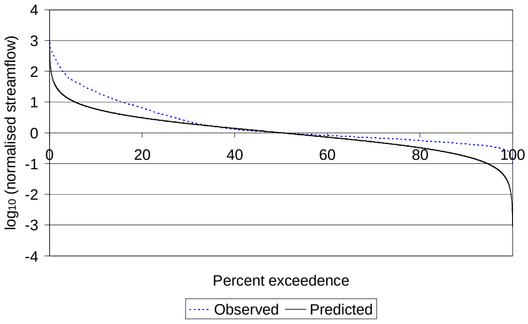

Figure 6 : Observed and predicted FDC for catchment 120006 (post-dam)

It is interesting that during pre-dam natural flow conditions, the derived model fits the observational data very well, but during the post-dam altered hydrologic regime, the model fits the observed data very poorly indeed. The form of the equation representing the FDC means that the FDC is reflected in the x-axis around 0 and the y-axis around the median non-zero discharge. Thus the two tails of the modelled FDC are mirror images of each other. This means that in rivers with unaltered flow in the Burdekin River Catchment, the log of the maximum discharges look very much like the log of the minimum discharges (Figure 5). When the natural hydrograph is altered (by construction of a dam for example) this relationship no longer holds (see for example Figure 6).

6. DISCUSSION

It was demonstrated in the previous section that the flow duration curve is a mirror image about y=0 and x=*a*/2. One question that immediately springs to mind is : why is this true? Why should it be that the log of normalised high flows looks very much like the log of normalised low flows? It could be that this relationship between high flows and low flows represents an important and useful property of natural rivers. Certainly this relationship holds true for catchments in this study. If it holds true for other catchments, we may have a way of estimating low flow response from measurements of high flows, or more importantly be able to estimate flood return intervals from measurements of baseflow. This supposition is unproven, but may represent a useful advance in the regionalisation of hydrologic response.

For example, Figure 7 shows a prediction of high flows for Catchment 120112 made by inverting the flow duration curve of the observed low flows. It will be seen that while there are some differences, in general the shape of the high flow FDC is reproduced using just data from the low flows. It is unlikely that the very highest flows will be reproduced accurately since they are predicted from the very lowest flows. These flows will depend on the accuracy of the monitoring equipment being used to measure low flows and thus may not necessarily represent a property of the catchment itself.

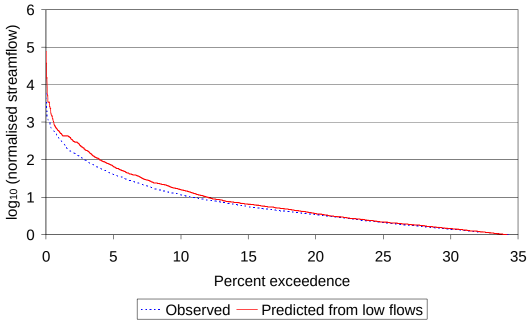

Figure 7 : Observed and predicted high flows for catchment 120112

Secondly, to produce these predictions, one needs to know the number of days of zero flow, so that the curve is 'mirrored' in the right spot (at x=$a/2$). Monitoring a stream only during the period of low flow is likely to yield this number. Finally, to convert the numbers on the FDC back into discharges, one needs to know the median discharge of the stream during periods of flow. This is more problematic, but may be obtained if the stream is monitored for at least 50% of the time.

Whether or not high flows can be predicted form low flows, this property of the FDC may provide a means of checking the accuracy of discharge data or the hydrologic impacts of landuse change. Deviations from the predicted FDC may reflect human-induced changes in either high flows or (more likely) low flows. Further research may also prove illuminating in terms of determining why it is that the FDC has this property, and whether this is true for all catchments, or just a sub-set.

7. CONCLUSIONS

This paper has presented a new way of representing the flow duration curve using a logarithmic function. This representation appears to reproduce the observed FDC in the 23 sub-catchments of the Burdekin River examined here to a reasonable degree of accuracy. Using it, the FDC can be reproduced through just two parameters – the cease to flow point and the slope of the FDC. The values of these two parameters can be predicted using catchment area, drainage density, stream length, and mean annual precipitation in each of the three regions of the Burdekin River, and thus may represent a way of regionalising flow duration curves into areas where we have little or no hydrologic data.

The mirror-image nature of the FDC presented here indicates that there may be some physical relationship between the nature of high flows, and that of low flows in a particular catchment. The cause of this relationship is not known, however it presents intriguing possibilities of predicting high flows in a partially gauged catchment through making use of the low flow record. It may also provide a way of checking the accuracy of hydrologic data, or assessing the extent of human interference on the hydrologic response of a catchment.

8. ACKNOWLEDGMENTS

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