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A Study on Particle Trajectory of Particle Swarm Optimization

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Abstract. For studying the sensitivity of PSO to control parameter choices, this paper proposes a special model of PSO theoretically. This model divides the position sequence of particle into the odd and even sub-sequences. The theorem demonstrates the position sequence of particle is affected by the parameter choices, the initialized position and velocity. Simulations for benchmark functions illustrate the validity of the odd-even property of particle trajectory.

Introduction

Particle swarm optimization (PSO) is an evolutionary computation algorithm which is motivated by the preying behavior of bird flocking [1]. Due to the simple but efficient characteristics, the PSO has been successfully applied to design and control of neural networks [2], data mining [3], dynamic system [4], and optimization of communication networks [5], etc..

It has been done to study the convergence performance of PSO [6-8]. From these studies it has been concluded that the PSO is sensitive to control parameter choices, specifically the inertia weight and acceleration coefficients. Wrong initialization of these parameters may lead to divergent.

To further understand the behavior of particle swarm, some theoretical studies have been done to analyze the trajectory of a single particle in PSO. Ozcan and Mohan [9-10] concluded that the trajectory of a particle in a simple PSO system was a sinusoidal wave where the initial conditions and parameter choices determined its amplitude and frequency. Van Den Bergh [11] developed a model of PSO considering the influence of the inertia weight. Clerk and Kennedy [12] provided a theoretical analysis of particle behavior in which a set of coefficients to control the system's convergence tendencies were analyzed.

In this paper, the PSO is analyzed theoretically and the special model of PSO is proposed. This model divides the position sequence of particle into the odd and even sub-sequences. The theorem demonstrates the position sequence of particle is affected by the parameter choices, the initialized position and velocity.

Overview of the PSO

PSO is a population-based stochastic optimization technique. In the PSO algorithm, an individual particle i is composed of three vectors: its position X_i, the best position found by itself p_i and its velocity V_i. Particles are originally initialized in a uniform random manner throughout the search space. Then, their positions are changed according to their own experience and that of the entire swarm.

The velocity and position are defined by the following rules:

$$V_i(t+1)=\omega\, V_i(t)+ \varphi_1\, (p_i(t)-X_i(t))+ \varphi_2\, (g(t) -X_i(t))) \quad (1)$$

$$X_i(t+1)= X_i(t) + V_i(t+1) \quad (2)$$

where $\varphi_1 = c_1 r_1$, $\varphi_2 = c_2 r_2$; r_1 and r_2 are uniformly distributed pseudorandom numbers in the range of 0-1; $p_i(t)$ is the personal best position, and $g(t)$ is the best position found by the swarm at the t-th iteration. The constriction factor ω is defined by Clerc and Kennedy [12].

The Special Model of the PSO

The particle position equation can by derived by the following transform:

substituting $V_i(t+1)$ into Eq. 2 :

$$X_i(t+1)= X_i(t) +\omega V_i(t)+ \varphi_1 (p_i (t)-X_i(t))+ \varphi_2 (g(t) -X_i(t))) \quad (3)$$

substituting $V_i(t)= X_i(t)-X_i(t-1)$ into Eq. 3:

$$X_i(t+1)= X_i(t) +\omega(X_i(t)-X_i(t-1))+ \varphi_1 (p_i (t)-X_i(t))+ \varphi_2 (g(t) -X_i(t)) \quad (4)$$

By substitution, the position of the i-th particle is updated by the following second-order non-homogeneous linear differential equation:

$$X_i(t+1)= (1+\omega- \varphi_1 - \varphi_2)X_i(t) -\omega X_i(t-1)+ \varphi_1 p_i (t)+ \varphi_2 g(t) \quad (5)$$

Let

$$P_i(t)= \varphi_1 p_i (t)+ \varphi_2 g(t) \quad (6)$$

we have

$$X_i(t+1)= (1+\omega - \varphi_1 - \varphi_2)X_i(t)- \omega X_i(t-1)+ P_i(t) \quad (7)$$

From Eq. 7, the constraint relationship among the parameters ω, φ_1 and φ_2 can be obtained thought studying the coefficients of $X_i(t)$ and $X_i(t-1)$. To analyze the parameters of PSO, there are two kinds of noticeable parameter settings: one is $\omega=0$, another is $1+\omega- \varphi_1 - \varphi_2 =0$.

When $\omega=0$, the previous velocity will not influence the new velocity and the memory of the previous flight direction will be erased. Eq. 7 can be simplified as shown below:

$$X_i(t+1)= (1 - \varphi_1 - \varphi_2)X_i(t) + P_i(t) \quad (8)$$

When $1+\omega- \varphi_1 - \varphi_2 =0$, $\omega= \varphi_1 + \varphi_2 -1$, $-\omega =1- \varphi_1 - \varphi_2$, Eq. 7 can be simplified as a special mode:

$$X_i(t+1)= -\omega X_i(t-1)+ P_i(t)=(1- \varphi_1 - \varphi_2) X_i(t-1)+ P_i(t) \quad (9)$$

By recurrence, Eq. 9 can be derived:

$X_i(2) = (1- \varphi_1 - \varphi_2) X_i(0)+ P_i(1)$

From Eq. 3, $X_i(1)$ can be obtained:

$X_i(1) = X_i(0) + \omega V_i(0)+ \varphi_1 (p_i (0)-X_i(0))+ \varphi_2 (g(0) -X_i(0))$

$= (1- \varphi_1 - \varphi_2) (X_i(0) - V_i(0))+ \varphi_1 p_i (0)+ \varphi_2 g(0) = (1- \varphi_1 - \varphi_2) (X_i(0) - V_i(0))+ P_i(0)$

$X_i(3) = (1- \varphi_1 - \varphi_2) X_i(1)+ P_i(2) = (1- \varphi_1 - \varphi_2)^2(X_i(0) - V_i(0))+ (1- \varphi_1 - \varphi_2)P_i(0) + P_i(2)$

$X_i(4) = (1- \varphi_1 - \varphi_2) X_i(2)+ P_i(3) = (1- \varphi_1 - \varphi_2)^2 X_i(0)+ (1- \varphi_1 - \varphi_2) P_i(1) + P_i(3)$

$X_i(5) = (1- \varphi_1 - \varphi_2)X_i(3)+ P_i(4)$

$= (1- \varphi_1 - \varphi_2)^3(X_i(0)-V_i(0))+(1- \varphi_1 - \varphi_2)^2P_i(0)+(1- \varphi_1 - \varphi_2)P_i(2)+ P_i(4)$

$X_i(6) = (1- \varphi_1 - \varphi_2)X_i(4)+ P_i(5) = (1- \varphi_1 - \varphi_2)^3X_i(0) + (1- \varphi_1 - \varphi_2)^2P_i(1) + (1- \varphi_1 - \varphi_2) P_i(3) + P_i(5)$

……

$X_i(2n-1) = (1- \varphi_1 - \varphi_2)^n(X_i(0) - V_i(0))+ (1- \varphi_1 - \varphi_2)^{n-1}P_i(0) + (1- \varphi_1 - \varphi_2)^{n-2} P_i(2) + (1- \varphi_1 - \varphi_2)^{n-3}P_i(4) +\ldots+ (1- \varphi_1 - \varphi_2)^1 P_i(2n-4) + (1- \varphi_1 - \varphi_2)^0P_i(2n-2)$

$$= (1- \varphi_1 - \varphi_2)^n(X_i(0) - V_i(0))+\sum_{r=1}^{n}(1 - \varphi_1 - \varphi_2)^{n-r}P_i(2r-2)$$

$X_i(2n) = (1- \varphi_1 - \varphi_2)^n X_i(0) + (1- \varphi_1 - \varphi_2)^{n-1}P_i(1) + (1- \varphi_1 - \varphi_2)^{n-2} P_i(3) + (1- \varphi_1 - \varphi_2)^{n-3}P_i(5)+\ldots+ (1- \varphi_1 - \varphi_2)^1 P_i(2n-3) + (1- \varphi_1 - \varphi_2)^0P_i(2n-1) = (1- \varphi_1 - \varphi_2)^n X_i(0) +\sum_{r=1}^{n}(1 - \varphi_1 - \varphi_2)^{n-r}P_i(2r-1)$

So the positions of the i-th particle form two sequences, $\{ X_i(2n)\}_{n=0}^{\infty}$ and $\{ X_i(2n-1)\}_{n=1}^{\infty}$. The sequence $\{ X_i(2n)\}_{n=0}^{\infty}$ is related to $X_i(0)$ and $P_i(n)$ at odd iteration steps; the sequence $\{ X_i(2n-1)\}_{n=1}^{\infty}$ is affected to $(X_i(0) - V_i(0))$ and $P_i(n)$ at even iteration steps.

THEOREM. If $1+\omega-\varphi_1-\varphi_2=0$, for all positive integrate n,

$$X_i(2n-1) = (1-\varphi_1-\varphi_2)^n(X_i(0) - V_i(0))+ \sum_{r=1}^{n}(1-\varphi_1-\varphi_2)^{n-r}P_i(2r-2) \tag{10}$$

$$X_i(2n) = (1-\varphi_1-\varphi_2)^n X_i(0) + \sum_{r=1}^{n}(1-\varphi_1-\varphi_2)^{n-r}P_i(2r-1) \tag{11}$$

Proof:

1) If n=1, we trivially have:

$$X_i(2\times 1-1) = (1-\varphi_1-\varphi_2)^1(X_i(0) - V_i(0))+ \sum_{r=1}^{1}(1-\varphi_1-\varphi_2)^{1-r}P_i(2r-2)$$

$$= (1-\varphi_1-\varphi_2)(X_i(0) - V_i(0))+P_i(0)= X_i(1)$$

$$X_i(2\times 1) = (1-\varphi_1-\varphi_2)^1 X_i(0) + \sum_{r=1}^{1}(1-\varphi_1-\varphi_2)^{1-r}P_i(2r-1)$$

$$= (1-\varphi_1-\varphi_2)X_i(0) +P_i(1) = X_i(2)$$

2) Assume that the equation is true for n=k, that is:

$$X_i(2k-1) = (1-\varphi_1-\varphi_2)^k(X_i(0) - V_i(0))+\sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{k-r}P_i(2r-2)$$

$$X_i(2k) = (1-\varphi_1-\varphi_2)^k X_i(0) +\sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{k-r}P_i(2r-1)$$

Then prove true for $n = k + 1$, that is,

$$X_i(2(k+1)-1) = (1-\varphi_1-\varphi_2)^{k+1}(X_i(0) - V_i(0))+ \sum_{r=1}^{k+1}(1-\varphi_1-\varphi_2)^{(k+1)-r}P_i(2r-2)$$

$$X_i(2(k+1)) = (1-\varphi_1-\varphi_2)^{k+1}X_i(0) + \sum_{r=1}^{k+1}(1-\varphi_1-\varphi_2)^{(k+1)-r}P_i(2r-1)$$

From Eq. 11, $X_i(2(k+1)-1)$ can be obtained:

$$X_i(2(k+1)-1) = X_i(2k+1)= (1-\varphi_1-\varphi_2)X_i(2k-1)+ P_i(2k)$$

$$= (1-\varphi_1-\varphi_2)((1-\varphi_1-\varphi_2)^k(X_i(0) - V_i(0))+\sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{k-r}P_i(2r-2)) + P_i(2k)$$

$$= (1-\varphi_1-\varphi_2)^{k+1}(X_i(0) - V_i(0))+ \sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{k-r+1}P_i(2r-2) + P_i(2k)$$

$$= (1-\varphi_1-\varphi_2)^{k+1}(X_i(0) - V_i(0))+ \sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{(k+1)-r}P_i(2r-2) + (1-\varphi_1-\varphi_2)^{(k+1)-(k+1)}P_i(2(k+1)-2)$$

$$= (1-\varphi_1-\varphi_2)^{k+1}(X_i(0) - V_i(0))+ \sum_{r=1}^{k+1}(1-\varphi_1-\varphi_2)^{(k+1)-r}P_i(2r-2)$$

From Eq. 11, $X_i(2(k+1))$ can be obtained:

$$X_i(2(k+1))= X_i(2k+2) = (1-\varphi_1-\varphi_2)X_i(2k)+ P_i(2k+1)$$

$$= (1-\varphi_1-\varphi_2)((1-\varphi_1-\varphi_2)^kX_i(0) +\sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{k-r}P_i(2r-1)) + P_i(2k+1)$$

$$= (1-\varphi_1-\varphi_2)^{k+1}X_i(0) + \sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{k-r+1}P_i(2r-1) + P_i(2k+1)$$

$$= (1-\varphi_1-\varphi_2)^{k+1}X_i(0) + \sum_{r=1}^{k}(1-\varphi_1-\varphi_2)^{(k+1)-r}P_i(2r-1) + (1-\varphi_1-\varphi_2)^{(k+1)-(k+1)}P_i(2(k+1)-1)$$

$$= (1-\varphi_1-\varphi_2)^{k+1}X_i(0) + \sum_{r=1}^{k+1}(1-\varphi_1-\varphi_2)^{(k+1)-r}P_i(2r-1)$$

So, if n=k+1, Eqs. 12 and 13 are obviously true.

Therefore the statement is true for all positive integers n.

Experiment Results

In order to illustrate the validity of the theorem, extensively adopted benchmark functions are used in the experiments, as listed in Table 1. A detailed description of these functions can be found in [13]. The population size is four particles in the experiments.

Table 1. Benchmark Functions

FUNCTION	MATHEMATICAL ESENTATION
Sphere	$f_1 = \sum_{i=1}^{D} x_i^2$
Generalized Rastrigin	$f_2 = \sum_{i=1}^{D} \{x_i^2 - 10\cos(2\pi x_i) + 10\}$
Ackley	$f_3 = -20\exp\{-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\} - \exp\{\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\} + 20 + e$

The parameters φ_1, φ_2 are uniformly distributed pseudorandom numbers. ω is initialed through the following equation: $1+\omega-\varphi_1-\varphi_2=0$.

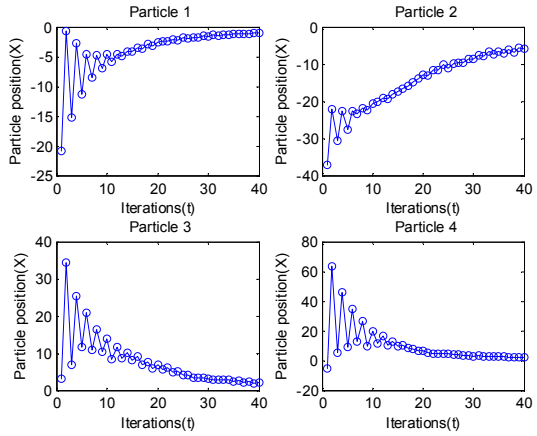

Figure 1 Sphere Function

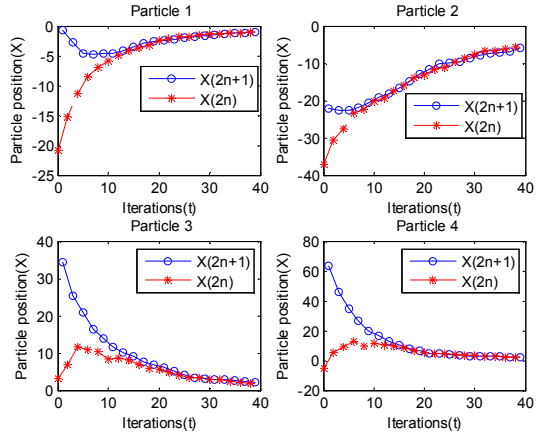

Figure 2 Sphere Function

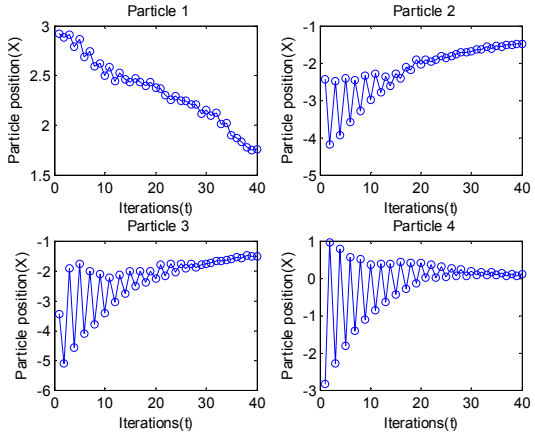

Figure 3 Generalized Rastrigin Function

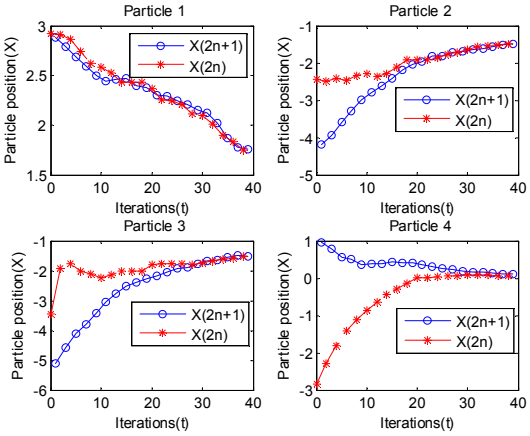

Figure 4 Generalized Rastrigin Function

In Fig. 1 and Fig. 3, the overall trajectories of the particles are shown; the odd and even trajectories of them are shown in Fig. 2 and Fig. 4.

The experiments show that the position sequence $\{X_i(n)\}_{n=0}^{\infty}$ of particle into the odd and even sub-sequences: $\{X_i(2n-1)\}_{n=1}^{\infty}$ and $\{X_i(2n)\}_{n=0}^{\infty}$. The sequence $\{X_i(2n)\}_{n=0}^{\infty}$ is related to $X_i(0)$ at odd iteration steps; the sequence $\{X_i(2n-1)\}_{n=1}^{\infty}$ is affected to $(X_i(0) - V_i(0))$ at even iteration steps.

Conclusions

For studying the sensitivity of PSO to control parameter choices, this paper proposes a special model of PSO. By analysis theoretically, when the algorithm parameters are selected and satisfy $1+\omega-\varphi_1-\varphi_2=0$, this model divides the position sequence $\{X_i(n)\}_{n=0}^{\infty}$ of particle into the odd and even sub-sequences: $\{X_i(2n-1)\}_{n=1}^{\infty}$ and $\{X_i(2n)\}_{n=0}^{\infty}$. The sequence $\{X_i(2n)\}_{n=0}^{\infty}$ is related to $X_i(0)$ at odd iteration steps; the sequence $\{X_i(2n-1)\}_{n=1}^{\infty}$ is affected to $(X_i(0) - V_i(0))$ at even iteration steps. These results provide a valuable guideline for selecting control parameters.

Acknowledgements

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