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Modeling and simulation of water distribution systems using timed hybrid Petri nets

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Abstract

This paper deals with modeling and simulation of water distribution systems using Hybrid Petri Nets. A methodology for building a Timed Hybrid Petri Net (THPN) model is proposed; the model is derived from a set of equations describing conservation of mass and energy that specify the behaviour of a water distribution network. A simulation technique for the execution of a THPN in which the transitions firing velocities are established from the linearised conservation of energy equations is presented; it shows both transient and steady state for the analysis of hydraulic networks. Based on this technique, a simulator in Matlab has been developed.

Keywords

water distribution networks, hybrid models, timed hybrid petri nets, simulation

1. Introduction

Providing potable water is one of the most important services given by public administration. Performance in water distribution systems allows to the population to cover a basic issue in their life quality. Performance analysis plays an important role in the operation, maintenance and design of new water distribution systems. This analysis can be done either by formal techniques or by simulation; the last one is often used in systems analysis in order to verify ''what if'' scenarios; in both approaches the quality of results depends on the accuracy and correctness of the used model.

The analysis of hydraulic networks consists in determining pipes volume and verifying if water sources and pumps are appropriate, under certain physical and functional characteristics. There are several approaches for analysing water distribution networks in which each flow is determined by using iterative procedures and energy static conditions. For analysing actual water distribution systems it is necessary to solve a set of nonlinear equations that are solved using iterative methods. In literature there are two kinds of iterative methods: those based on Gauss-Seidel and Jacobi methods,[1,2] and those based on the solution of linear equations, namely Newton-Raphson.[3,4]

The difference between these two methods is that, in the first case, the solution process implies a simplification that yields equations having just one unknown variable; in the second case there is a linearization of the equations that are solved for all unknown variables.

Based on these methods some simulators have been developed; they allow friendly interfaces for data introduction and results presentation; EPANET[5,6] is one of these simulators. It implements the gradient method. In spite of its simplicity, analysis accuracy is low and its capabilities for hydraulic network configuration are limited. Also it does not perform transient analysis during the pipeline filling.

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Modeling and simulation of hydraulic networks have been previously addressed using Hybrid Petri Nets (HPN).[7] In[8] a closed loop hybrid control scheme is modelled using separately bond graphs and HPN for comparing purposes; the authors enhanced the advantages of HPN when hydraulic systems are dealt with.

In[9] the model of a basic potable water distribution system is presented; it consists of a tank with two input flows and one output flow; the tank is modelled by a continuous place and the flows by continuous transitions. One of the input flows is due to a pump whose source is a non-exhaustive natural resource. The transition that models this flow has a constant firing velocity; the other input flow is water from rain. The output flow represents the water consumption; the firing velocities for the transitions that model these flows have an associated piece-wise constant functions.

Other work addresses the modeling of the delay flow in a pipeline;[10] it is done by using a hybrid path formed by a continuous place, a discrete transition, and a continuous place. The first continuous place models water at the input extreme of the pipe, the discrete transition models the existent delay in the water passing through the pipe and the second continuous place models the water arriving at the end of the pipe.

In[11] two kinds of models for water canals are proposed: one of them is discrete and the other one is continuous. A discrete model uses discrete units; velocities are modelled as delays, due to discrete transitions. A continuous model uses a THPN extension using arcs (labelled with Δ) for representing the flow delay through each canal reach.

All the related works regarding hydraulic systems using THPN are very inaccurate, because they only take into account conservation of mass laws of the system; none of them consider system energy relationships. Besides they only propose modeling. In our approach, conservation of mass and energy laws of the system are considered; also a simulation technique is presented for the THPN model.

In this paper a methodology for modeling and simulation of potable water distribution systems is presented. It is based on the use of models expressed with Timed Hybrid Petri Nets (THPN),[10] which allows capturing both continuous and discrete behaviour.

The synthesis of the THPN model is done following a simple bottom-up technique: each hydraulic element corresponds to a THPN module; then, mass conservation equations help to integrate each THPN module to form a global model that describes the flow of water. A set of conservation of energy equations are used for obtaining parameters used during the execution of THPN model; these parameters are valid during a system operation time interval, so they must be recalculated several times during the simulation. This strategy allows simulating both the transient and steady states of hydraulic networks. This technique has been implemented as a THPN simulator using Matlab.

The remainder of paper is structured as follows. In Section 2 basic concepts on Petri Nets and hydraulics networks are overviewed. In Section 3 the proposed modeling methodology is presented. In Section 4 the developed simulator is described. Finally, in Section 5 two study cases are presented.

2. Background

2.1. Timed Hybrid Petri Nets

The concepts summarised herein can be found in.[10] A Discrete Petri Net (DPN) can be used to represent the state of a machine or the number of parts processed by a machine. In these cases the number of reachable markings may be large, thus some certain properties can be difficult to analyse. In order to cope with this problem, a new model called Continuous Petri Nets (CPN) is presented.

CPN are particularly used to model fluids or continuous production machines. This flow can be interrupted by a valve closure or a machine failure; these situations can be modelled by a hybrid net containing discrete and continuous elements.

A marked Hybrid Petri Net (HPN) is a sextuple $R = < P, T, Pre, Post, m_0, h>$, where: P is a finite and non empty set of places; T is finite and non empty set of transitions; $P \cap T = \emptyset$; $h : P \cup T \rightarrow \{D, C\}$, it is called hybrid function, which determines whether a node is continuous (Set P^C and T^C or C-Place or C-Transition) or discrete (Set P^D and T^D or D-Place or D-Transition); Pre: $P \times T \rightarrow Q^+$ or N, is the input incidence function; Post: $P \times T \rightarrow Q^+$ or N, is the output incidence function; $m_0 : P \rightarrow \Re^+$ or N, is the initial marking.

Functions Pre and Post must observe the following constraint: if P_i y T_j are such that $P_i \in P^D$ and $T_j \in T^C$, then $Pre(P_i,T_j)=Post(P_i,T_j)$.

A discrete transition in a HPN is enabled if each input place P_i to T_j holds the following condition: $m(P_i) \geq Pre(P_i,T_j)$.

A continuous transition T_j in a HPN is enabled if each input place P_i holds the following conditions: a) $m(P_i) \geq Pre(P_i,T_j)$, if P_i is a D-Place; b) $m(P_i) > 0$, if P_i is a C-Place, where $m(P_i)$ is the marking for the place P_i.

The fundamental equation for computing the marking evolution is the same that for a discrete model: $\mathbf{m_{i+1}} = \mathbf{m_i} + \mathbf{W} \cdot s$; $\mathbf{m_i}$. is the vector for the ith-marking; $\mathbf{W}$ is the incidence matrix, $\mathbf{W} = Post - Pre$; s is the firing vector, where each element is the quantity of firing of a transition. The vectors $\mathbf{m_{i+1}}$, $\mathbf{m_i}$ and s can be real or

positive integer numbers, and **W** can have rational or positive integers numbers (according to the related nodes).

A Timed HPN (THPN) is a pair <R, Tempo> such that: R is a marked HPN and Tempo is a function that maps the transitions set T to positive rational numbers or zero:

- If $T_j \in T^D$, $d_j = \text{tempo}(T_j)$ Timing associated with T_j.
- If $T_j \in T^C$, $U_j = \frac{1}{\text{tempo}(T_j)}$ Average flow associated with T_j.

The equation for marking evolution in a THPN is:

$$\mathbf{m(t_2)} - \mathbf{m(t_1)} + \mathbf{W}\left(\mathbf{n(t_2)} - \mathbf{n(t_1)} + \int_{t_1}^{t_2} \mathbf{v(u)}\,\mathbf{du}\right)$$

$\mathbf{m(t_i)}$ is the marking vector in time $\mathbf{t_i}$; $\mathbf{n(t_i)}$ is the fire's number since time 0 until time $\mathbf{t_i}$ for discrete transitions and $\mathbf{v(u)}$ is the firing velocity vector for continuous transitions.

There are two types of enabling to calculate the firing velocity of each continuous transition ($v_j(t)$):

- ***Strongly enabled.*** A T_j transition is strongly enabled if $\mathbf{m_i} > 0$ for all the places $P_i \in^{\circ} T_j$. Then its firing velocity is:

$$v_j(t) = U_j$$

- ***Weakly enabled.*** A transition T_j is weakly enabled if $\mathbf{m_i} = 0$ for at least one place $P_i \in^{\circ} T_j$ and there is a $T_k \in^{\circ} P_i$ such that $v_k > 0$, in other words P_i is feeding by T_k Let $Q_j(t)$ be the subset of places P_i in $^{\circ}T_j$ such that $\mathbf{m_j(t)} = 0$. If T_j is not involved in an actual conflict related to a place in $Q_j(t)$ (or if takes priority over the other transitions involved in the conflict), the instantaneous firing speed of T_j is:

$$v_j(t) = \min_{P_i \in Q_j(t)} \left\{ \frac{1}{pre(P_i,\ T_k)} \sum_{T_k \in oP_i} Post(P_i,\ T_k) v_k(t), U_j \right\}$$

An actual conflict in a PN is defined as the structure $k = < P_c,\ \{T_a,\ T_b\} >$ and a marking **m**, such that the set $\{T_a, T_b\}$ is enabled by **m** and the marking of P_c is not enough to fire both transitions. For a THPN there are four cases:

- ***Case 1:*** $T_a,\ T_b \in T^D$. There is an actual conflict when both transitions are fireable at the same time; it is solved by giving priority to one transition. For example: Transition T_a has priority over T_b (denoted as $T_a < T_b$).
- ***Case 2:*** $T_a,\ T_b \in T^C$ and $P_c \in P^C$. There is an actual conflict when the marking of a continuous place is zero, but the sum of the firing velocity for input transitions to P_c is less than the sum of firing velocity for $\{T_a, T_b\}$; this is solved by assigning a priority or by sharing input flow to P_c between transitions T_a and T_b.If the conflict is solved by sharing, it is expressed as $[\alpha T_a,\ \beta T_b]$, where:

$$\text{Flow rate to } T_a = \frac{\alpha}{\alpha + \beta} \quad \text{Flow rate to } T_b = \frac{\beta}{\alpha + \beta}$$

α and β must be positive numbers.

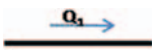
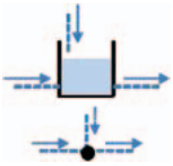
- ***Case 3:*** $T_a \in T^D$ and $T_b \in T^C$, or vice versa. It is solved by giving priority to the discrete transition.
- ***Case 4:*** $T_a,\ T_b \in T^C$ and $P_c \in P^D$. It is solved by sharing the marking of P_c between both transitions.

In this work only the cases 2 and 3 are dealt with.

2.2. Basic concepts on hydraulics

Most of the concepts overviewed in this section can be found in,[12,13] unless there is another reference. A potable water distribution network is a set of hydraulic elements used to distribute water in a specific zone. The most common elements are shown in Table 1.

Table 1. Hydraulic network elements

Element	Icon	Main purpose
Pipe		Conveys water from one node to another.
Node		Removes (demand) or adds (inflow) water from/to the system.
Pump		Raises system energy to overcome elevation differences and friction losses.
Valve		Controls flow in the system based on specified criteria.
Tank		Stores excess water within the system and releases it at times of high usage.

In networks of interconnected hydraulic elements, every element is influenced by each of its neighbours; the entire system is interrelated in such a way that the condition of one element must be consistent with the condition of all other elements. Two concepts define these interconnections: conservation of mass and conservation of energy.

(a) *Conservation of mass*

The principle of conservation of mass states that the fluid mass entering any pipe is equal to the mass leaving the pipe. In network modeling, all outflows are lumped at nodes or junctions.

$$\sum \mathrm{Q_i} - U = 0$$

where $\mathrm{Q_i}$ is the Inflow/Outflow to node in i-th pipe ($\mathrm{l^3/t}$), and U is the fluid used at node ($\mathrm{l^3/t}$), where l and t mean length and time units respectively.

The conservation of mass equation is applied to all nodes; an equation for each node is formulated.

(b) *Conservation of energy*

The principle of conservation of energy states that the difference in energy between two points must be the same regardless the path that is taken.

$$z_1 + \sum \mathbf{h_p} = z_2 + \sum \mathbf{h_L} + \sum \mathbf{h_m}$$

where $\mathrm{z_i}$ is the elevation height at point $_\mathrm{i}$, $\mathrm{h_p}$ is the head added by the pumps, $\mathrm{h_L}$ is the head loss in pipes, and $\mathrm{h_m}$ denotes minor head loses. Thus the difference in energy at any two points connected in a network is equal to the energy gained from pumps and the energy lost in pipes and fittings that occur in the path between them. The sum of head losses around a pipe loop is equal to zero.

Energy losses, also called head losses, are generally the result of both frictions along pipe walls (head losses due to friction), and turbulences due to changes in fluid lines through fittings (minor losses). One of the most used expressions to compute head losses due to friction is given below

$$h_L = k_P Q^z$$

where $\mathrm{k_P}$ is the resistance coefficient of the pipe ($\mathrm{t^Z/l^{3Z-1}}$), Q is the pipeline flow rate ($\mathrm{l^3/t}$), and Z is the exponent on the flow term. In the present work minor losses are not considered.

The velocity of a fluid through a pipe is not constant. The velocity of a particle fluid depends on its localization with respect to the pipe wall. In most cases, hydraulic models deal with average velocity in the transversal section of the pipe, which can be calculated with the following expression:

$$V = \frac{Q}{A}$$

where A is Pipe's transversal sectional area ($\mathrm{l^2}$).

For determining $\mathrm{k_p}$, it is used Darcy-Weisbach equation:

$$k_P = f\frac{L}{2gA^z D}$$

where f is Darcy-Weisbach factor, L is Pipe length(l), D is pipe diameter and z = 2.

To determine f in the previous equation it is necessary to know the fluid flow regime. Reynolds identifies and determinates a relationship to know the fluid flow regime; it is included in Table 2.

$$Re = \frac{VD}{\upsilon}$$

where Re is Reynold's number and v is kinematic viscocity ($\mathrm{l^2/t}$). For water v = 1.23–e05$\mathrm{ft^2/s}$.

The flow of water through urban water distribution systems is almost always turbulent, except in the periphery where water demand is low and intermittent; so the flow regime may be laminar. For this reason in this work it is only considered turbulent flow regime, then an approximation is used for computing f.[6]

$$f\frac{0.25}{\left[log_{10}\left(\frac{e}{3.7d} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

where e is pipe's roughness coefficient (l); this value depends on the age and the material of pipe.

In most of cases it is necessary to add energy to a hydraulic system in order to overcome elevations, losses due to friction, and minor losses. A pump is a device for providing mechanical energy to the water as total head. The head added is called *pump head;* it is a function of the flow rate through the pump.

$$h_P = h_0 - cQ_p^m$$

Table 2. Reynolds number for each flow regime

Flow regime	Reynolds number
Laminar	<2000
Transitional	2000–4000
Turbulent	>4000

where h_0 is cutoffhead (pump head at zero flow)(l), Q_p is pump discharge(l^3/t), and c,m are coefficients describing pump curve shape.

3. A methodology for modeling hydraulic networks

In this section a methodology to obtain a THPN from a hydraulic network is presented. First the general approach for modeling and simulation is outlined, and then the modeling procedure is described.

3.1. Modeling and simulation approach

The general scheme is described in Figure 1 in which the numbers reference components and steps of the general approach.[14] From the hydraulic system diagram (1) two sets of equations are obtained: the conservation of mass equations (3), and the conservation of energy equations (2).

The structure of the THPN model (4) is obtained from the mass equations. According to the initial marking, and based on the energy equations which are first linearised (5) and then solved (6), both the transition firing velocities and the flow rates for solving conflicts (7) are obtained.

These parameters are valid for the initial marking and can be used while the marking evolves within a predefined small range; thus during the model execution (8), when the marking of continuous or discrete places change, the blocks (6) and (7) must be iteratively performed for updating the firing velocities and flow rates. Transitions firing velocities and marking are recorded and graphically displayed at the end of the simulation process (9).

An initial flow transition represents a pipe or a pump; the input place to this transition represents tank and its direction flow is always from the tank to the end of the pipe or pump.

3.2. Construction of the THPN structure

The THPN structure is straightforward obtained from the hydraulic diagram by transforming the components of the system: places, transitions, and arcs, according to Table 3, where:

- If T_1 is an initial flow transition, Q_1 depends on equations of conservation of energy.

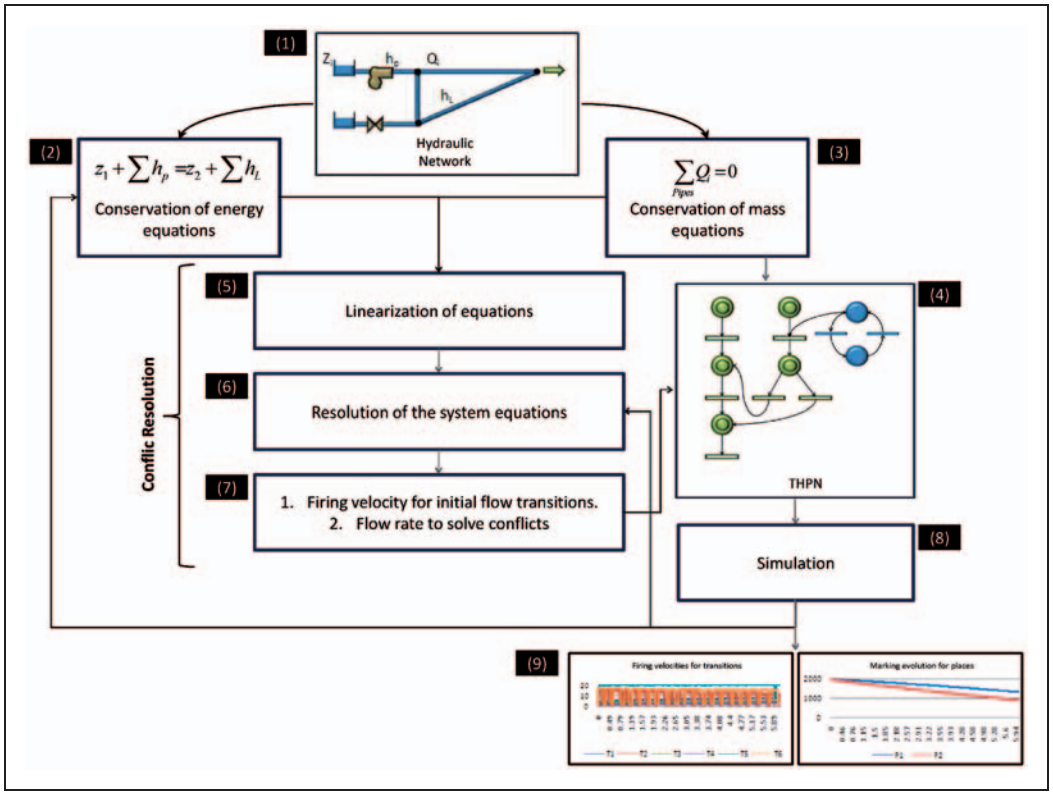
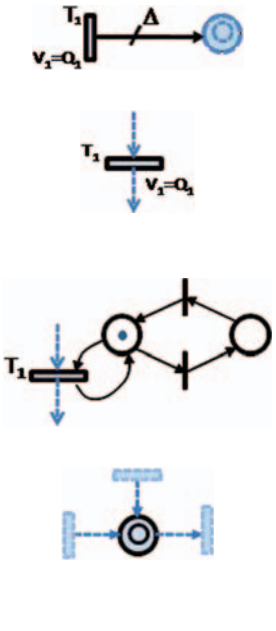


Figure 1. General scheme for modeling and simulation.

Table 3. THPN components

Hydraulic Element	THPN Structure
	

- If T_1 models consumption then its velocity is equal to the consumption. Otherwise $Q_1 = \infty$.
- The arc labelled with Δ introduces a delay d, similarly as proposed in;[11] d will change during the simulation time because it depends of v_1. The firing of T_1 modifies the marking of P_1 d time units later.

$$d = \frac{Pipe's\ Volume}{\upsilon_1}$$

- Conflicts are solved by priority when in the conflict there is a transition that represents consumption; it will have the highest priority in the conflict.
- Any other conflict is solved by sharing, where the flow rate depends on the energy equations.

The valves used in the hydraulic system are limited to pipes or pumps of which the source is a tank. Systems where valves are in the middle of a pipe are not dealt in this work because the change of state of valve modifies the network structure yielding two models for each valve. Afterwards the THPN structures are joined based on the conservation of mass equations. For valves, a place P_1 is joined to the system as an input and output place to the transition that represents the flow with the valve.

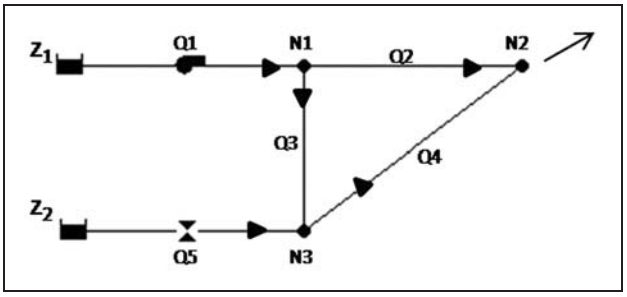


Figure 2. A simple hydraulic Network.

Table 4. Equations

Mass equations	Energy equations
$Q_1 = Q_2 + Q_3$ $Q_3 = Q_5 = Q_4$ $Q_2 = Q_4 = 20$	$z_1 + 133.33 - 8.33Q_1^3 - k_3Q_3^2 = z_2 - k_5Q_5^2$ $k_2Q_2^2 = k_3Q_3^2 + k_4Q_4^2$

Table 5. Pipes' dimensions

Pipe	Length(ft)	Diameter (ft)
Q_2	100	1
Q_3	200	1
Q_4	100	2.5
Q_5	200	2

Example 1. Consider the hydraulic system diagram showed in Figure 2; it includes two tanks that have 200 ft of height and their areas are 400 and 500 ft^2. The system also has one pump on flow Q_1, a valve on flow Q_5, a pipe system of flows Q_2–Q_4 and a consumption of 20 ft^3/s (N_2).

This hydraulic system is described by the equations in Table 4.

Table 5 shows information about pipes' dimensions. Roughness Coefficient for each pipe is 0.1 ft long (This value will be the same for the two examples presented in Section 5).

First, the transformation process from the hydraulic diagram to the THPN structure is performed. Each hydraulic element is translated to a THPN structure according to Table 6; then each element is joined according to conservation of mass equations. Finally, the structure showed in Figure 3 is obtained. Table 6 shows the correspondence of each element.

3.3. Conflict resolution

Figure 4 shows a general scheme for a conflict resolution procedure, which determines the maximal firing

Table 6. Element correspondence

Hydraulic element	THPN's nodes
Tank 1,2	P_1,P_2
Nodes 1–3	P_3, P_4, P_5
Pump	T_1
Pipes 2–5	T_2,T_3, T_4, T_6
Consumption	T_5
Valve	P_6,T_7, P_7,T_8

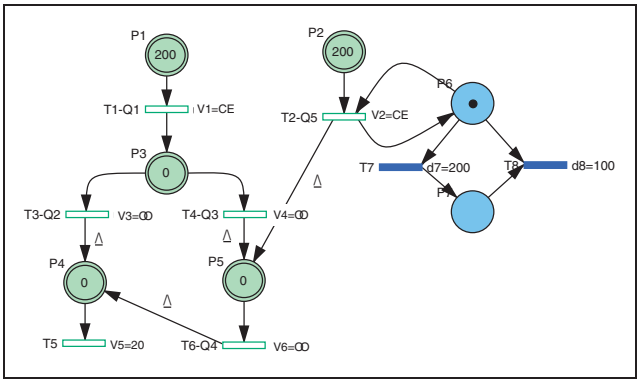


Figure 3. THPN model for the network.

velocity of initial flow transitions and the flow rate in structures where there is a conflict whose solution is by sharing.

First the linearisation of the conservation of energy equations using Taylor's series is done. Later a solution for conservation of mass equations is obtained. This solution will be the first operational set of linearised equations; then a substitution on the linearised equations is made (calculated flows and water levels for the tanks). So, a linear equations system is obtained and it is solved to determine the new flows.

At this point it is necessary to compare the obtained flows with those previously obtained; if they are equal then the procedure stops and the firing velocities for the required transitions and the flow rate to solve conflicts are obtained; if they are different, then the new flows are going to be the new operational set for the linearised equations and the procedure is stared again.

Example 2. Following the example 1, a conflict resolution procedure is executed. Conservation of energy equations are linearised; then it is obtained the set of equations that depend on operational points (Po_1–Po_5), tanks levels (z_1 and z_2) and the flows (Q_1–Q_5). The obtained equations are given below.

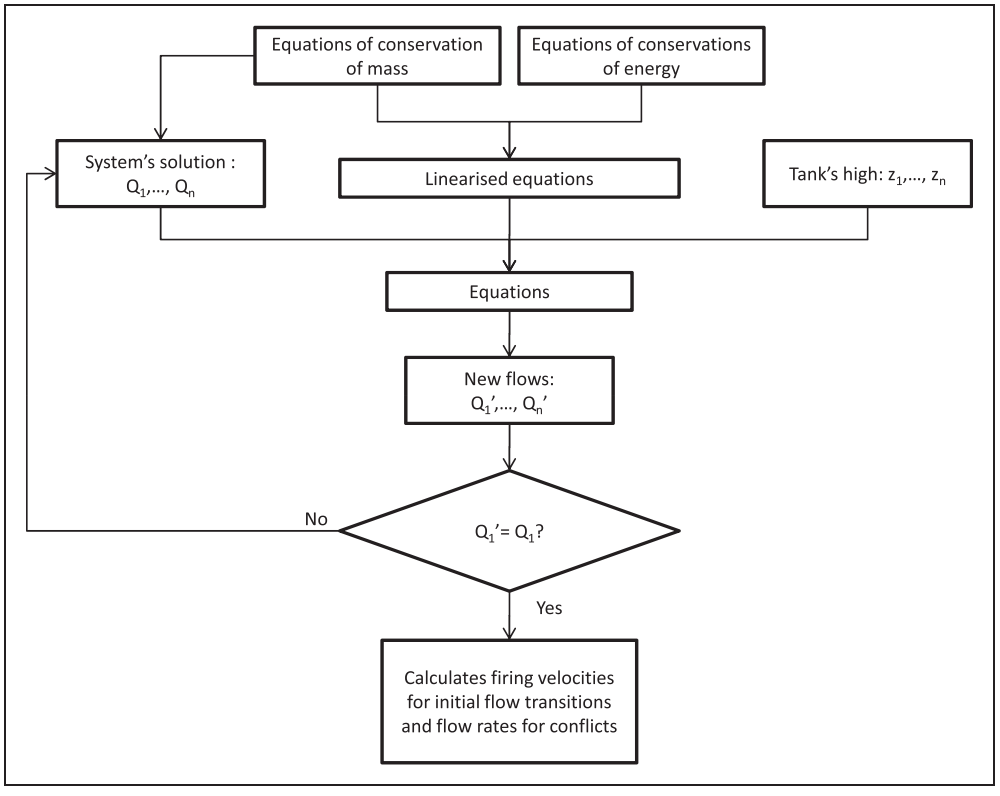


Figure 4. Conflict resolution procedure.

$$
\begin{aligned}
&\bullet\ -17Po_1(Q_1-Po_1)+(-.21e-1/\log(.14e-2+.18e\\
&-3/Po_3^{(9/10)})^3 Po_3^{(1/10)}/(.14e-2+.18e-3/Po_3^{(9/10)})\\
&.13e3/\log(.14e-2+.18e-3/Po_3^{(9/10)})^2 Po_3)(Q_3-Po_3)\\
&+(.12e-2/\log(.68e-3+.33e-3/Po_5^{(9/10)})^3\\
&Po_5^{(1/10)}/(.68e-3+.33e-3/Po_5^{(9/10)})\\
&+4.2/\log((.68e-3+.33e-3/Po_5^{(9/10)})^2 Po_5)\\
&(Q_5-Po_5)-8.3Po_1^2-67/\log(.14e-2+.18e-3/Po_3^{(9/10)})^2\\
&Po_3^2+2.1/\log(.68e-3+.33e-3/Po_5^{(9/10)})^2 Po_5^2\\
&+z_1-z_2+.13e3\\
&\bullet\ (.11e-1/\log(.14e-2+.18e-3/Po_2^{(9/10)})^3\\
&Po_2^{(1/10)}/(.14e-2+.18e-3/Po_2^{(9/10)})\\
&+67./\log(.14e-2+.18e-3/Po_2^{(9/10)})^2 Po_2)(Q_2-Po_2)\\
&+(-.21e-1/\log(.14e-2+.18e-3/Po_3^{(9/10)})^3\\
&Po_3^{(1/10)}/(.14e-2+.18e-3/Po_3^{(9/10)})\\
&-.13e3/\log(.14e-2+.18e-3/Po_3^{(9/10)})^2 Po_3)(Q_3-Po_3)\\
&+(-.25e-3/\log(.54e-3+.40e-3/Po_4^{(9/10)})^3\\
&Po_4^{(1/10)}/(.54e-3+.40e-3/Po_4^{(9/10)})\\
&-.68/\log(.54e-3+.40e-3/Po_4^{(9/10)})^2 Po_4)(Q_4-Po_4)\\
&+33/\log(.14e-2+.18e-3/Po_2^{(9/10)})^2\\
&Po_2^2-67./\log(.14e-2+.18e-3/Po_3^{(9/10)})^2\\
&Po_3^2-.34/\log(.54e-3+.40e-3/Po_4^{(9/10)})^2 Po_4^2
\end{aligned}
$$

Then a solution for mass conservation is obtained:

$$Q_1 = 12;\ Q_2 = 8;\ Q_3 = 4;\ Q_4 = 12;\ Q_5 = 8$$

These values (as operational points) and the water level of the tanks ($z_1 = z_2 = 200$) are substituted on the linearised equations yielding the following system equations:

$$
\begin{gathered}
-199.9Q_1 + 1340 - 4.1Q_3 + 1.804Q_5 = 0\\
4.099Q_2 - 7.954 - 4.1Q_3 - 0.4001Q_4 = 0\\
Q_1 = Q_2 + Q_3\\
Q_3 + Q_5 = Q_4\\
Q_2 + Q_4 = 20
\end{gathered}
$$

The system is solved and the new flows are:

$$Q_1 = 6.69;\ Q_2 = 4.39;\ Q_3 = 2.29;\ Q_4 = 15.61;\ Q_5 = 13.31$$

The previous solution and the new one are not equal, so the procedure is executed again; in this example four iterations are needed to obtain the final solution:

$$Q_1 = 4.22;\ Q_2 = 2.64;\ Q_3 = 1.58;\ Q_4 = 17.36;\ Q_5 = 15.78$$

In this example there are two initial flow transitions T_1 and T_2 representing flows Q_1 and Q_5 respectively; then it is established $V_1 = 4.22$ and $V_2 = 15.78$. For the conflict $< P_3, \{T_3, T_4\} >$, T_3 represents the flow Q_2, T_4 represents the flow Q_5, and the input flow to P_3 is given by transition T_2 (Q_1); then the conflict resolution is $[T_3, 0.6T_4]$.

4. Hydraulic networks simulation

4.1. Simulation engine

The simulator is a software tool that executes a THPN model of a hydraulic network as stated in the modeling methodology. The model structure, the firing velocities, and the energy equations are provided to the tool which has been developed in Matlab.[15] Additional procedures for conflict resolution and energy equations updating are used during the model execution. Figure 5 illustrates the main stages of the simulator which are briefly described below.

First, the conflict resolution procedure is executed for determining complementary data to the THPN model. Then, the model is executed considering alternatively the discrete part and the continuous part. First the discrete events are treated, and the simulation of the continuous part is performed. When a discrete event arises, an interruption occurs on the continuous simulation and the discrete marking is updated.

In the model a discrete event represents the switching of a valve state, which modifies physical configuration of the hydraulic net; so it is necessary to modify conservation of energy equations. When there is a modification on the equations, the conflict resolution procedure is executed again. When there are new values: velocities and flow rate for sharing, it is determined the next discrete event to occur; then it is simulated THPN continuous part again.

If water levels have changed in a predefined threshold, then the conflict resolution procedure is done, but in this situation it is not necessary to execute the complete procedure; it is enough to replace the new values (water levels) on the linearised equations.

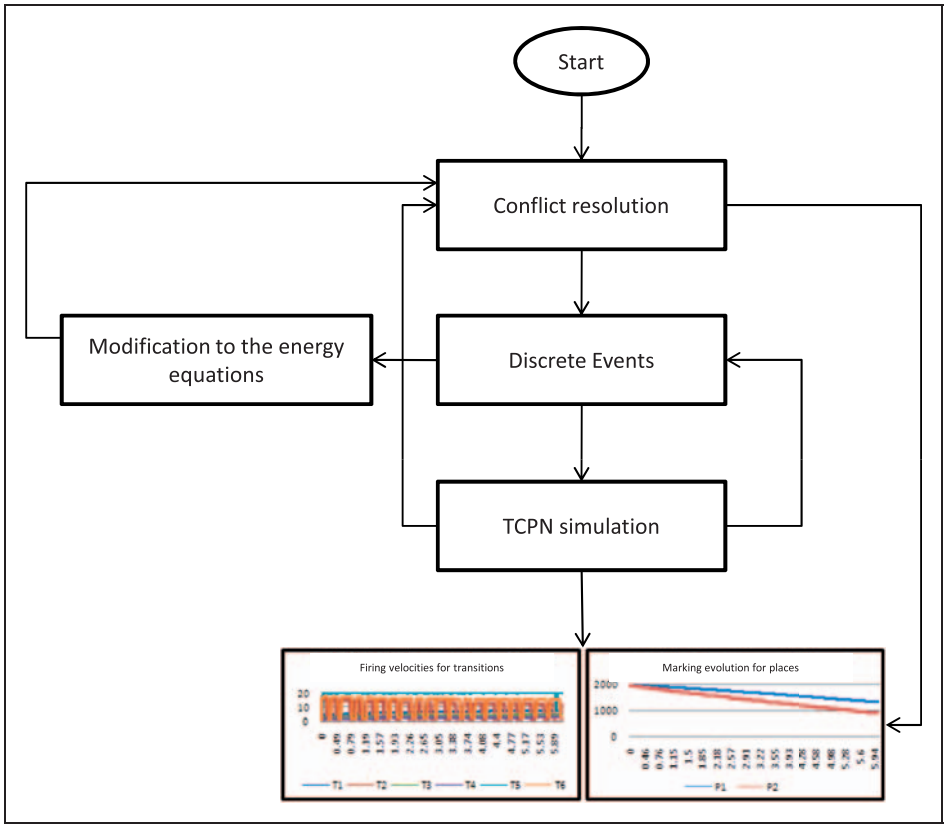


Figure 5. Simulation procedure.

The simulation stops when one of three situations are met:

- A flow becomes negative. This is because in a THPN model a pipe is represented as a transition with a defined direction flow.
- There are no more discrete or continuous events.
- Simulation interval has arrived to a pre-established simulation time limit.

A simulation interval is delimited by a situation in which the discrete marking and firing velocities for continuous transitions remains constant. When the simulator stops, the information about the relevant parameters (having a physical meaning in the hydraulic system) for the evolution of the model can be obtained.

4.2. Accuracy of the simulations

Simulations for several cases studies of diverse complexity have been made; the results have been compared with those obtained using the hydraulic nets simulator EPANET 2,[5,6] which has been recognised as accurate and efficient; this software operates using an iterative gradient method to solve non linear system equations describing the hydraulic system. The simulator determines the flows for each pipe given static conditions of energy; the procedure is executed periodically according to a parameter value given by the user.

The THPN based simulator presented herein updates the flows according to the system state values, rather than at predefined time periods, allowing that the velocities in the developed simulator be more consistent with current energy conditions of the hydraulic system.

The results delivered by both simulators differ when the modelled system evolves on changing energy conditions, but the results are the same when the simulators run on systems exhibiting static conditions of energy. The THPN based simulator provides more accurate evaluation during the transitory regime of flows.

5. A THPN based simulator

The presented simulation tool allows analysing the behaviour of a given network in order to verify its design. An experimented user may take advantages of this support during the design of a hydraulic system. In this section two cases study are presented; the first one illustrates the complete simulation method, and the

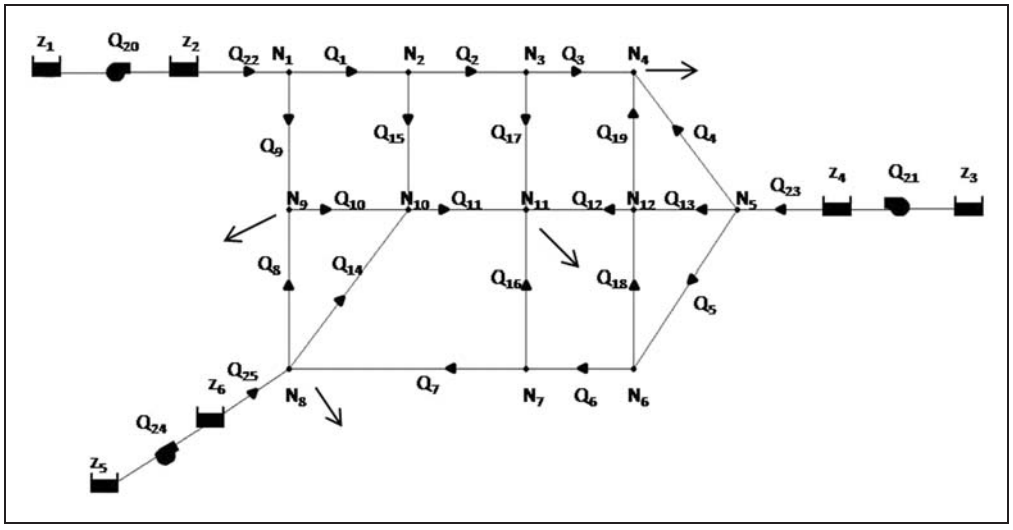


Figure 6. Hydraulic network.

Table 7. Pipes' dimensions

Pipe	Length (ft)	Diameter (ft)	Pipe	Length (ft)	Diameter (ft)
Q_1	1500	1	Q_{12}	1100	.83
Q_2	1000	.58	Q_{13}	1000	.5
Q_3	1200	.67	Q_{14}	1800	.66
Q_4	2000	1	Q_{15}	1100	2
Q_5	2800	2	Q_{16}	1800	.5
Q_6	1800	2	Q_{17}	1200	.66
Q_7	1000	.67	Q_{18}	1800	.5
Q_8	2500	.83	Q_{19}	1300	.5
Q_9	800	.58	Q_{22}	500	1
Q_{10}	1300	1	Q_{23}	500	1
Q_{11}	1000	.5	Q_{25}	800	1

Table 8. Tanks' dimensions

Tank	Height (z_i)(ft)	Base area (ft^2)	Tank	Height (z_i)(ft)	Base area (ft^2)
1	10	30	4	500	20
2	400	25	5	20	30
3	20	20	6	400	35

Table 9. Delays for the pumps

Pump	Delay on/off (sec)
1	1000
2	250
3	1100

Table 10. Flow Rates for the consumptions

Consumption	Flow rate (ft^3/sec)
N_4	20
N_8	5
N_9	3
N_{11}	10

second one is used for comparing with other standing simulator.

5.1. An illustrative case study

This case study aims to show all the process of modeling and simulation through four sceneries; in this example, some physical characteristics are modified to observe the behaviour of the water level in the tanks and to verify if the consumptions can be satisfied.

Consider the hydraulic network depicted in the scheme of figure 6. It consist of six sources (z_1 to z_6); three of them z_1, z_3 and z_5 model water reservoirs, in which water level is constant (they are filled by a natural source); the other three sources z_2, z_4 and z_6 model tanks with variable water level; z_1, z_3 and z_5 are used to maintain within a range the water level for the tanks z_2, z_4 and z_6 by means of three pumps that turn on and off periodically. Also four water consumption points in nodes N_4, N_8, N_9, N_{11} are included. The physical dimensions for pipes are shown in table 7 and tank's dimensions in table 8.

Table 9 presents time delays for the pumps on/off.

Table 10 presents the flow rates for the consumption points.

The following equations describe the system.

Figure 7 shows the THPN for the hydraulic system. Places P_{13}, P_{15} and P_{17} represent tanks 1, 3 and 5

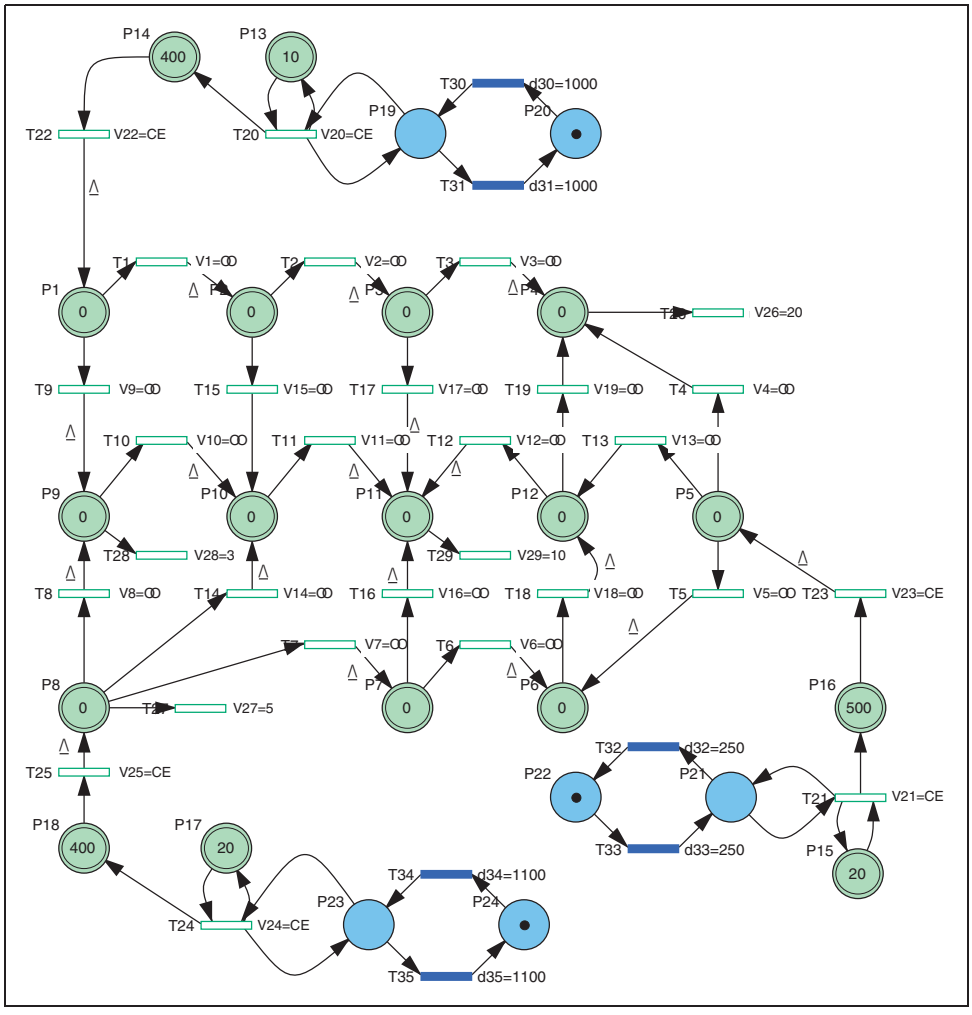


Figure 7. THPN model.

Table 11. Equations

Conservation of mass	Conservation of energy
$Q_{22} = Q_1 + Q_9$	$k_1 Q_1^2 + k_{15} Q_{15}^2 - k_9 Q_9^2 - k_{10} Q_{10}^2 = 0$
$Q_1 = Q_2 + Q_{15}$	$-k_{15} Q_{15}^2 + k_{17} Q_{17}^2 - k_{11} Q_{11}^2 + k_2 Q_2^2 = 0$
$Q_2 = Q_3 + Q_{17}$	$k_3 Q_3^2 - k_{17} Q_{17}^2 + k_{12} Q_{12}^2 - k_{19} Q_{19}^2 = 0$
$Q_3 = Q_4 + Q_{19} = 20$	$k_{19} Q_{19}^2 - k_4 Q_4^2 + k_{13} Q_{13}^2 = 0$
$Q_{23} = Q_4 + Q_5 + Q_{13}$	$k_5 Q_5^2 + k_{18} Q_{18}^2 - k_{13} Q_{13}^2 = 0$
$Q_5 = Q_6 + Q_{18}$	$-k_{12} Q_{12}^2 - k_{18} Q_{18}^2 + k_{16} Q_{16}^2 - k_6 Q_6^2 = 0$
$Q_7 = Q_{16} + Q_6$	$k_{11} Q_{11}^2 - k_{16} Q_{16}^2 + k_{14} Q_{14}^2 - k_7 Q_7^2 = 0$
$Q_{25} = Q_7 + Q_8 + Q_{14} + 5$	$k_{10} Q_{10}^2 - k_{14} Q_{14}^2 + k_8 Q_8^2 = 0$
$Q_8 = Q_9 + Q_{10} + 3$	$z_2 - z_6 - k_9 Q_9^2 + k_{25} Q_{25}^2 - k_{22} Q_{22}^2 + k_8 Q_8^2 = 0$
$Q_{10} = Q_{14} + Q_{15} = Q_{11}$	$z_4 - z_6 - k_5 Q_5^2 + k_6 Q_6^2 + k_7 Q_7^2 + k_{25} Q_{25}^2 - k_{23} Q_{23}^2 = 0$
$Q_{11} = Q_{12} + Q_{16} + Q_{17} = 10$	$z_1 - 4.63 Q_{20}^2 + 2366.68 = 0$ (1)
$Q_{13} = Q_{18} + Q_{12} + Q_{19}$	$z_3 - 4.63 Q_{21}^2 + 2366.68 = 0$ (2)
	$z_5 - 4.63 Q_{24}^2 + 2366.68 = 0$ (3)

respectively; they are input and output places (for maintaining a constant water level all the time) to transitions that represent the pumps.

The conflict solution is $[\alpha T_1, \beta T_9]$, $[\gamma T_2, \delta T_{15}]$, $[\varepsilon T_3, \zeta T_{17}]$, $[\eta T_4, \theta T_5, \iota T_{13}]$, $[\kappa T_6, \lambda T_{16}]$, $[\mu T_7, \nu T_8, \xi T_{14}, oT_{27}]$, $T_{10} < T_{28}$, $[\pi T_{12}, \rho T_{19}]$, $T_{31} < T_{20}$, $T_{33} < T_{21}$, $T_{35} < T_{24}$; where α, β, γ, δ, ε, ζ, η, θ, ι, κ, λ, μ, ν, ξ, o, π and ρ vary according to the water levels of the tanks change and when a pump turn on or turn off.

Table 12 shows the correspondence among THPN nodes and hydraulic system elements.

Scenario 1. In a first simulation (Figure 8) the evolution marking graph shows that the pump chosen as the input to tank 4 (P_{16}) is not the right one, because the water level is getting down and it is necessary that its level varies between a range.

Table 12. Elements correspondence

Hydraulic element	THPN nodes
Tanks 1,2,3,4,5,6	P_{13}–P_{18}
Nodes 1–12	P_1–P_{12}
Pipes 1–19 and 22-24	T_1–T_{19} y T_{22}–T_{24}
Consumptions 1,2,3,4	T_{26}–T_{29}
Pumps 1,2,3	T_{20},T_{21}, T_{24}
Valves 1,2,3	P_{19}, T_{30}, P_{20}, T_{31}; P_{21}, T_{32}, P_{22}, T_{33}; P_{23}, T_{34}, P_{24}, T_{35}

Scenario 2. In order to correct this, it is necessary to modify the energy equation for input flow velocity to tank 4 and the delay times for pump turning on/off.

Equation (2) (Table 11) is replaced with the following one:

Conservation of energy
$z_3 - 1.56Q_{21}^2 + 2266.68 = 0$

In table 13 are shown the new time delays for the pump:

After these modifications, the marking evolution graph showed in figure 9 is obtained. This simulation

Table 13. Updated pump timing

Transition	Delay (sec)
T_{32}, T_{33}	$d_{32} = d_{33} = 200$

Table 14. Updated pump timing

Transition	Delay (sec)
T_{30}, T_{31}	$d_{30} = d_{31} = 400$
T_{34}, T_{35}	$d_{34} = d_{35} = 250$

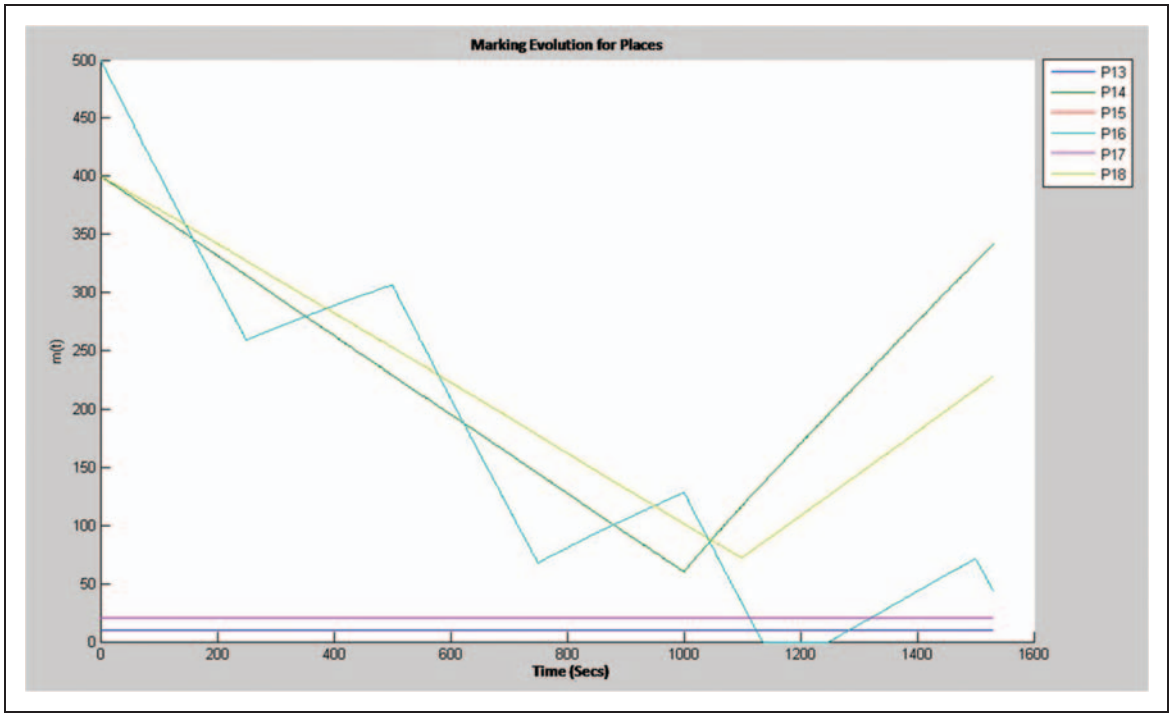


Figure 8. Marking evolution (scenario1).

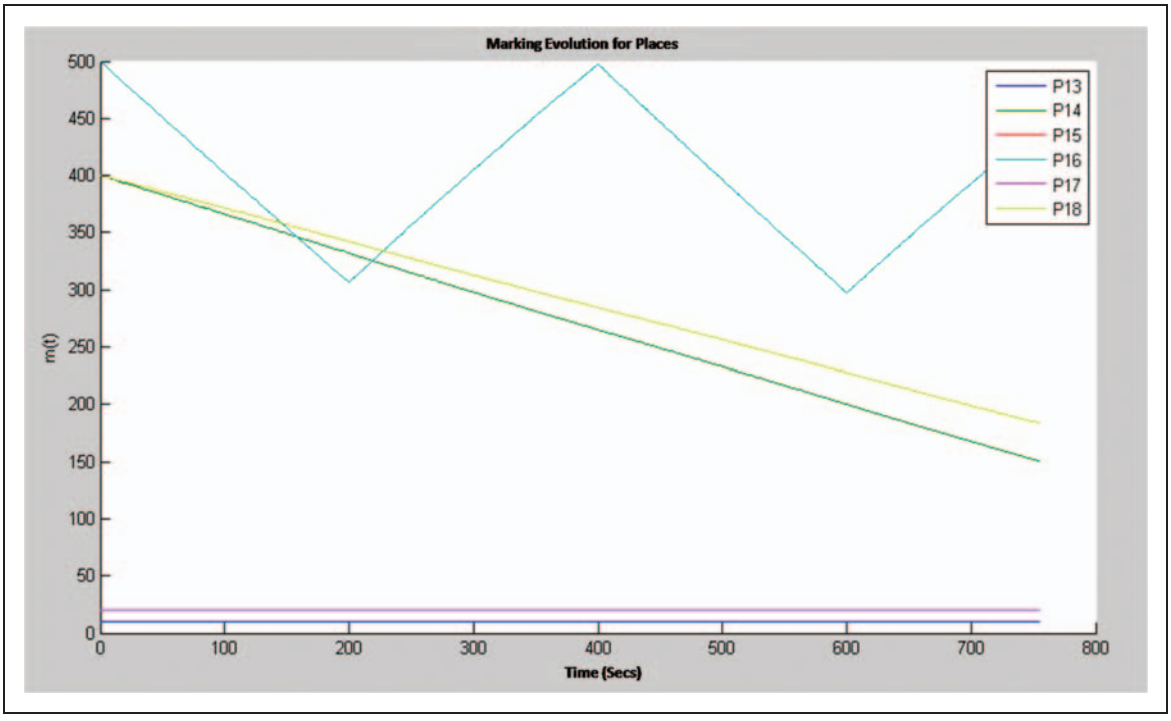


Figure 9. Marking evolution (scenario 2).

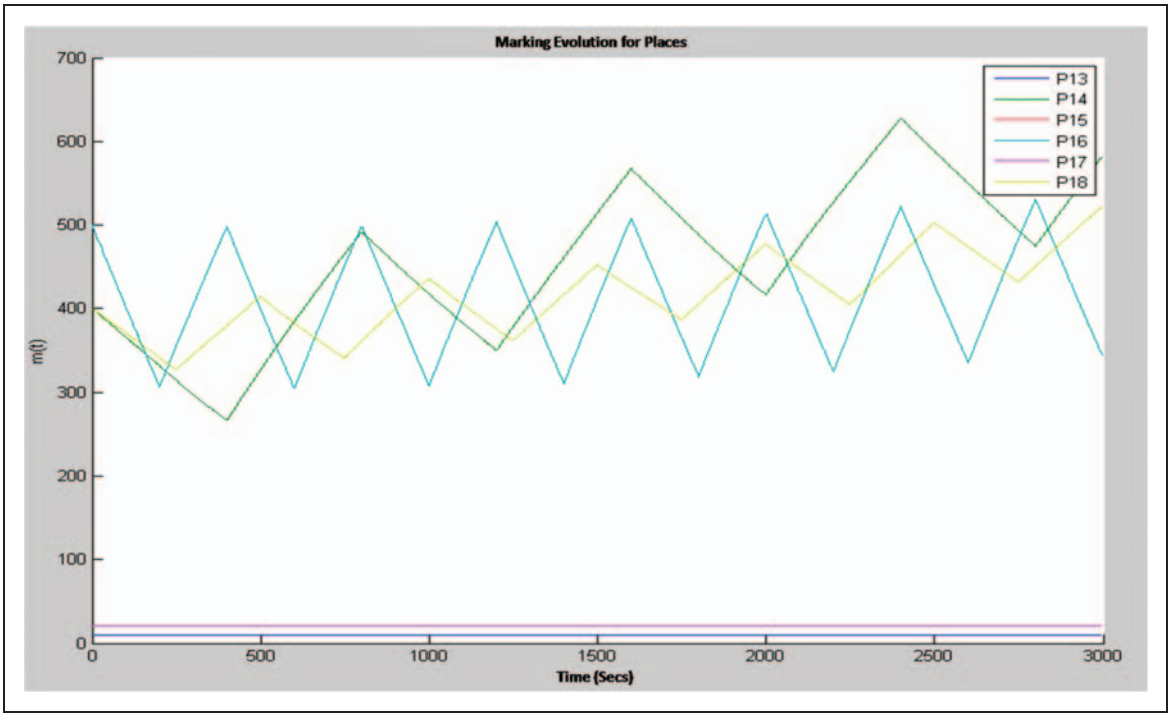


Figure 10. Marking evolution (scenario 3).

stops due to change flow direction (transition T_6). This occurs because water levels of tanks 2 and 6 are too low (P_{14} and P_{18}), then a shorter time delay is necessary to transitions that models pumps turning on/off.

Table 14 shows the time delay modifications for the discrete transitions.

Scenario 3. In a third simulation with new time delays, figure 10 shows the evolution of the water

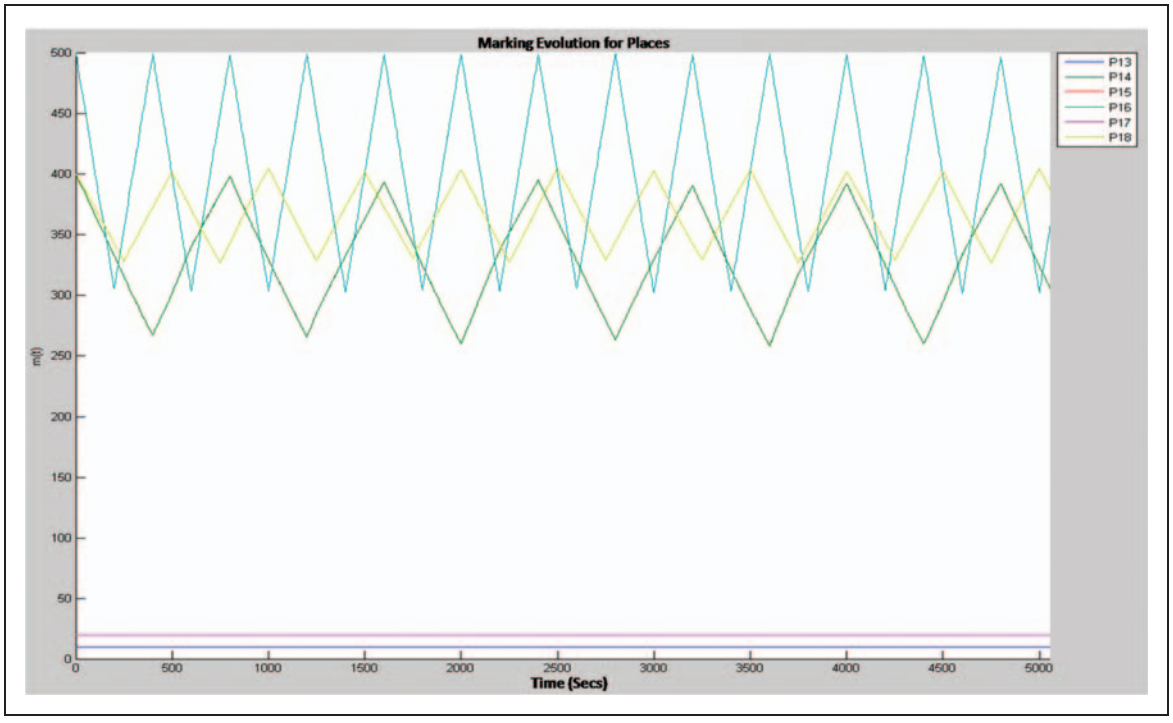


Figure 11. Marking evolution (scenario 4).

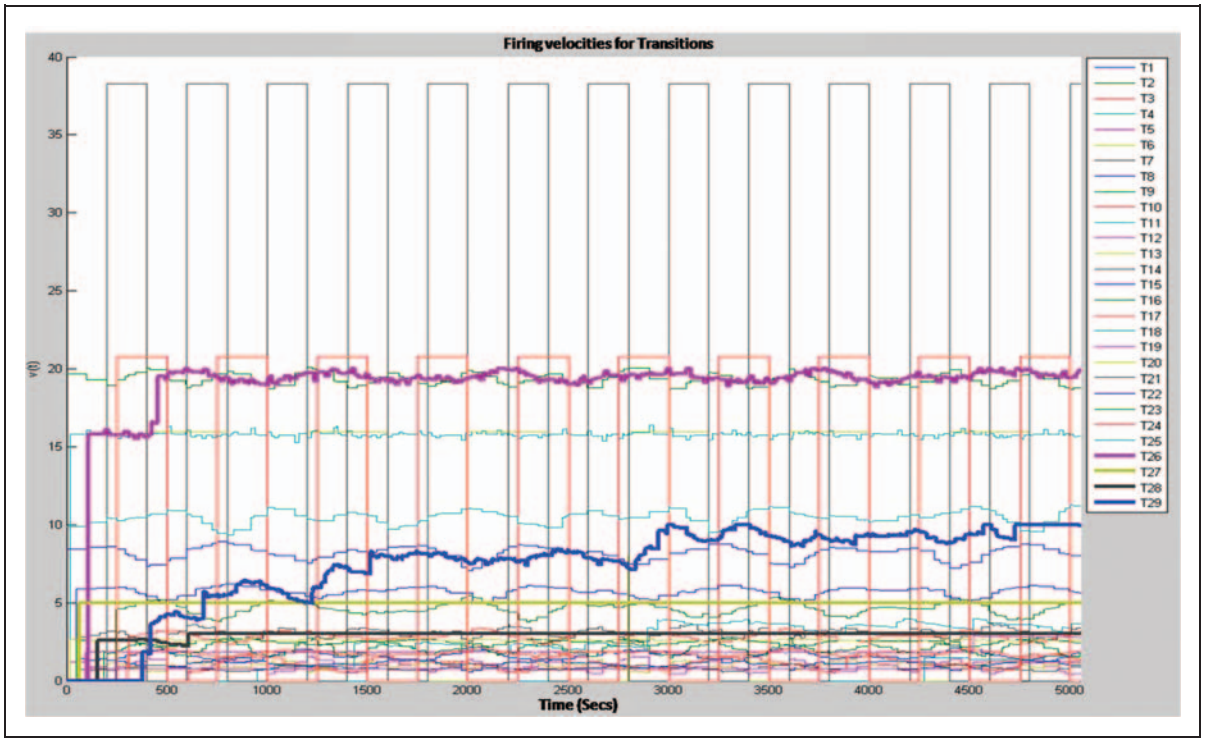


Figure 12. Firing velocities.

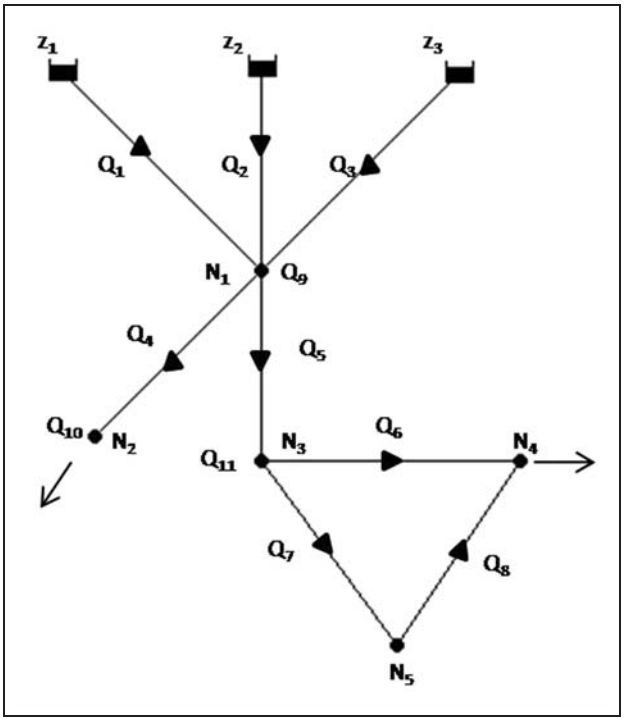


Figure 13. Hydraulic network.

Table 15. Firing velocity for transitions

Transition	Firing velocity (ft^3/sec)
T_{26}	$v_{26} = 19.7$
T_{27}	$v_{27} = 5$
T_{28}	$v_{28} = 3$
T_{29}	$v_{28} = 9.7$

Table 16. Pipes' dimensions

Pipe	Length (ft)	Diameter (ft)	Pipe	Length (ft)	Diameter (ft)
Q_1	100	2	Q_5	100	1
Q_2	100	1	Q_6	100	1
Q_3	200	1	Q_7	100	1
Q_4	100	1	Q_8	100	1

Table 17. Tanks' dimensions

Tank	Height (z_i)(ft)	Base area (ft^2)
1	200	20
2	200	25
3	200	25

level of the tanks. The pumps chosen as input flows to tanks 2 and 6, have not been the correct ones, because their water levels are increasing. The simulation stops again because a flow changed its direction (T_8).

Scenario 4. In order to correct this problem it is necessary to modify the conservation of energy equations that describe input flows to tanks 2 and 6. The equations (1) and (3) (Table 11) are replaced by the following ones:

Conservation of Energy equations
$z_1 - 8.9Q_{20}^2 + 2266.68 = 0$
$z_5 - 5.3Q_{24}^2 + 2266.68 = 0$

With these modifications and doing the simulation, the obtained graphics are shown in figures 11 and 12. Figure 11 shows how water levels for the tanks vary between the following ranks: Tank 2, 400 and 260 ft., Tank 4, 500 and 310 ft., and Tank 6, 400 and 325 ft.

In figure 12 is shown how water demands are satisfied (Transitions T_{26}, T_{27}, T_{28}, T_{29}), it occurs

Table 18. Flow rate for the consumptions

Consumption	Flow rate (ft^3/sec)
N_2	2
N_4	12

Table 19. Equations

Conservation of mass	Conservation of energy
$Q_1 + Q_2 + Q_3 = Q_4 + Q_5$	$z_1 - k_1Q_1^2 = Q_9$
$Q_4 = 2$	$z_2 - k_2Q_2^2 = Q_9$
$Q_5 = Q_6 + Q_7$	$z_3 - k_3Q_3^2 = Q_9$
$Q_7 = Q_8$	$Q_9 = Q_{10} + k_4Q_4^2$
$Q_6 + Q_8 = 12$	$Q_9 = Q_{11} + k_5Q_5^2$
	$k_6Q_6^2 = k_7Q_7^2 + k_8Q_8^2$

Table 20. Element correspondence

Hydraulic element	THPN nodes
Tanks 1,2,3	P_1,P_2, P_3
Nodes 1–5	P_4, P_5, P_6, P_7, P_8
Pipes 4–8	T_1,T_2, T_3, T_4, T_5, T_6, T_7, T_8
Consumptions 1 y 2	T_9, T_{10}

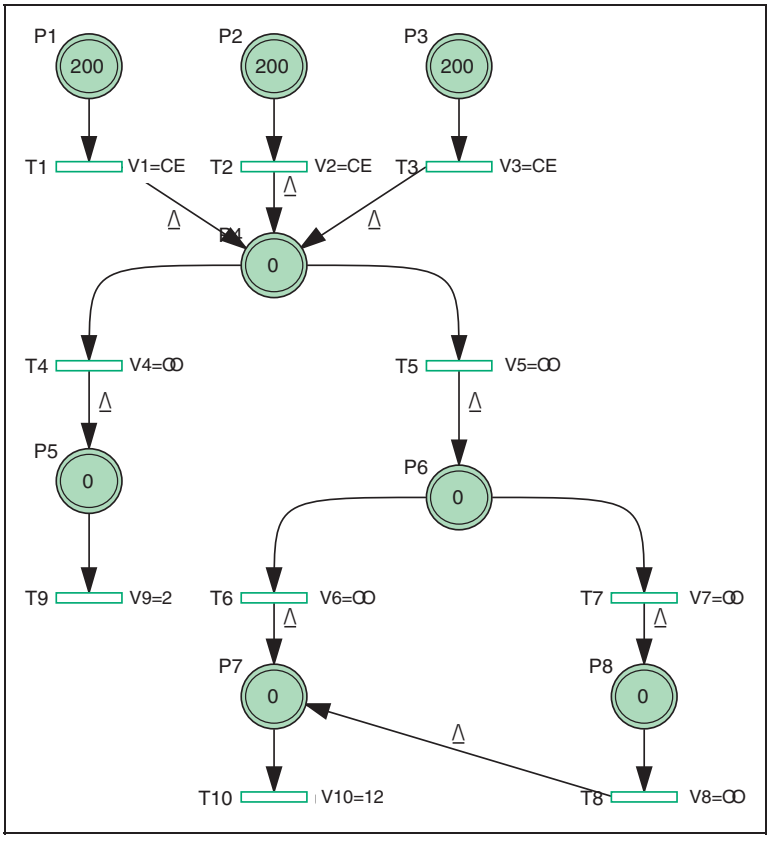


Figure 14. Hydraulic network.

Figure 15. Evolution of the water level of the tanks (EPANET).

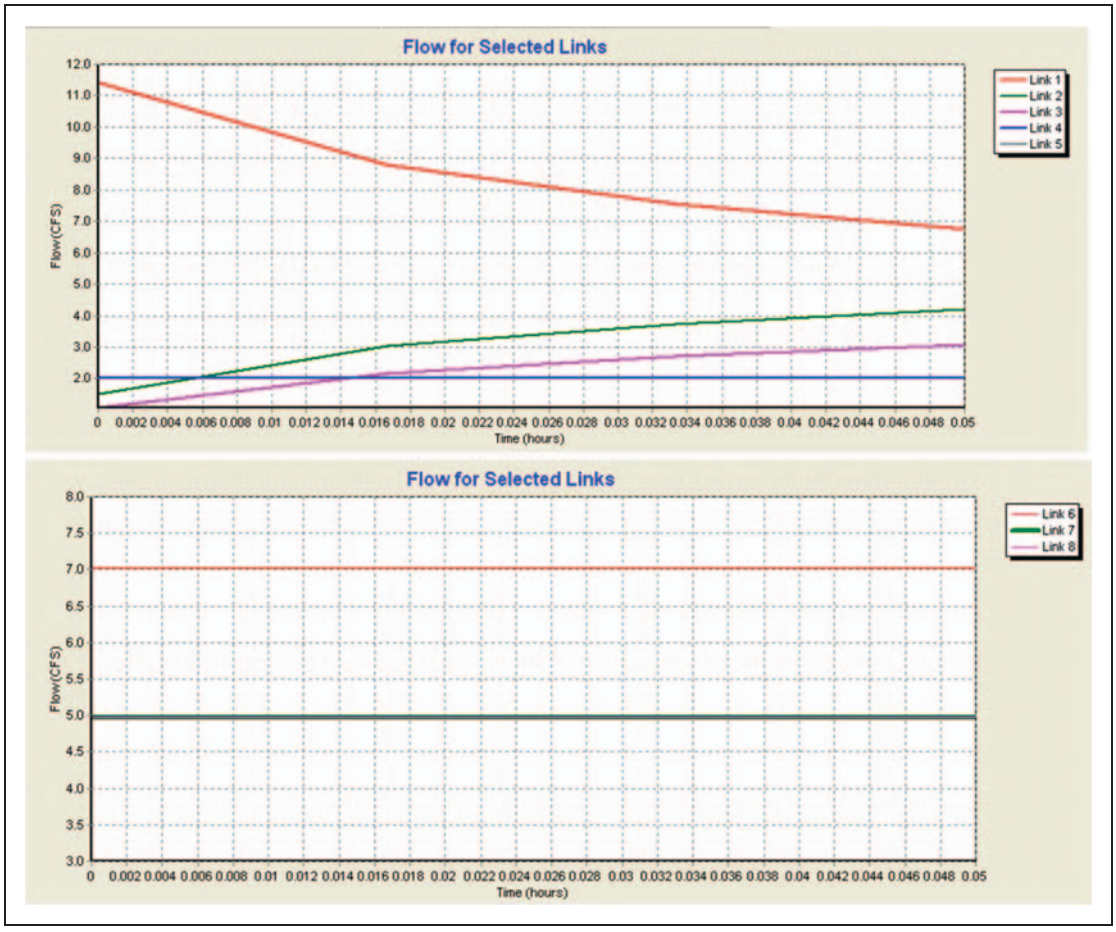


Figure 16. Evolution of flows rate (EPANET).

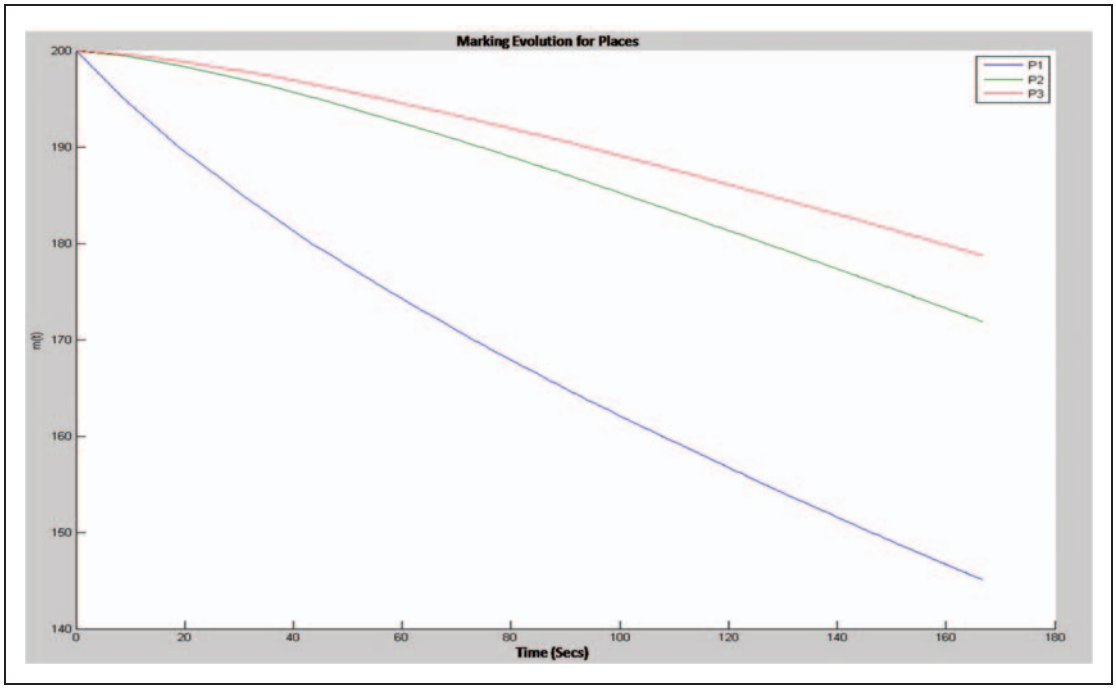


Figure 17. Evolution of the water level of the tanks (THPN Simulator).

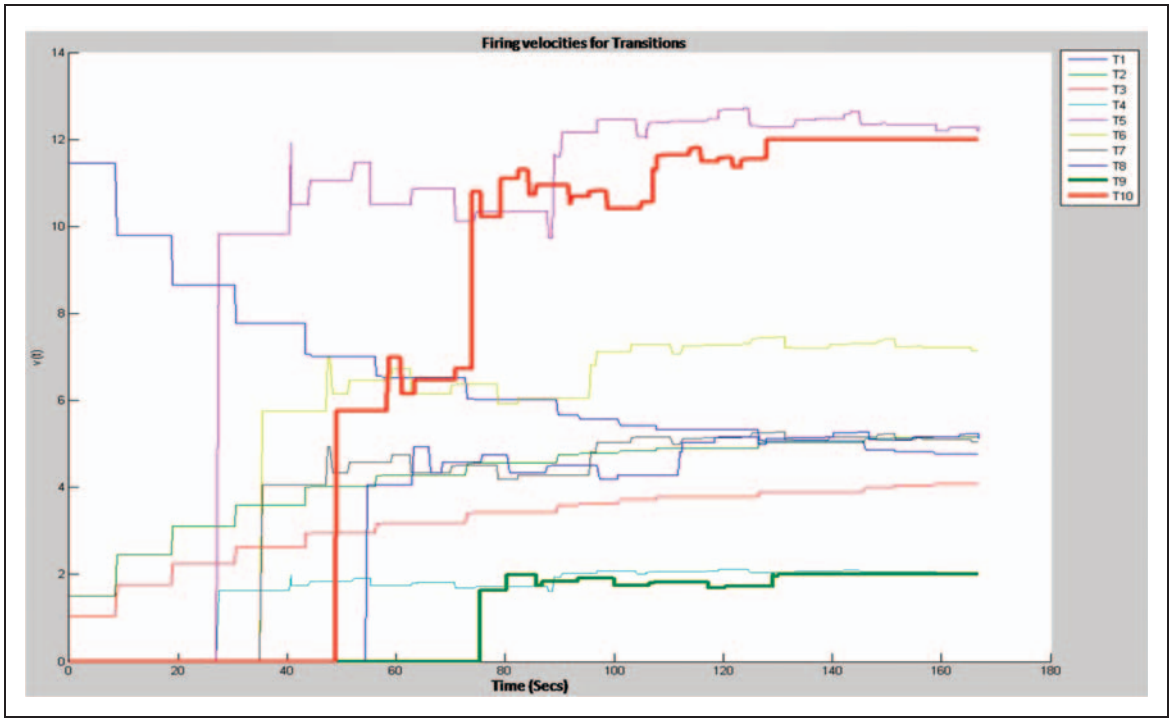


Figure 18. Evolution of flows rate (THPN Simulator).

approximately after 2965 seconds from the beginning. Water demands vary due to:

- Changes in energy levels for the tanks.
- Pumps on/off.
- Time delay for the two previous events.

The flow rate of transitions that represents consumptions (when the pipe's system is full) is shown in table 15.

5.2. A comparative case study

In this section is shown a case to appreciate the differences between the results obtained with EPANET and the THPN simulator described in Section 4. Consider the case depicted by figure 13, this hydraulic system has three tanks z_1-z_3 and two consumptions N_2 and N_4, its physical characteristics are presented in tables 16–19. Table 20 shows the relations between the hydraulic elements and the THPN nodes. Figure 14 shows the THPN for this hydraulic system.

Figures 15 and 16 show the results given by EPANET; it was configured to recalculate the flows every minute. There are two graphics because EPANET does not allow displaying more than five nodes in one graphic.

Figure 17 and 18 presents the results given by the THPN simulator; it was configured to recalculate the flows when the high levels of the tanks change in 5 ft.

It can be noticed that figure 17 shows a smoother evolution of water level; this is because of the re-computation based on the changes in energy conditions, an evolution more realistic (closer to the actual energy conditions) that the graphics displayed by EPANET in figure 15. Furthermore, in figures 16 and 18 there are a significant difference; EPANET does not show the evolution of pipeline filling at the beginning, it only computes every flow according to the energy conditions every minute. This difference is due because of the modelled water delay time through pipes of our proposal; the simulator computes and displays the transitory period at the beginning of the simulation.

6. Conclusion

This work presented an alternative solution for simulation of water distribution networks based on hybrid models; a modeling methodology and a simulation technique were proposed. The hybrid models, represented by a THPN, are obtained from conservation of mass and energy equations of the hydraulic system, and then for the execution of the model the transition firing velocities are dynamically determined from the linearised conservation of energy equations. Based on this technique a simulator in Matlab has been developed.

One of the key features of this approach is the determination of firing velocities of the transitions in the THPN model which represent the flow in the hydraulic

network. Since conservation of energy equations are non linear, the influence of each hydraulic element in the system, and the THPN semantics, it has been necessary to define and implement an external procedure including the existent energy relationships in the system; this allows approximating the variations on the flow during the system evolution. Thus conservation of energy equations complement the THPN model and play an important role within the simulation procedure yielding a more accurate computation of water flows and levels during transitory functioning. This feature is not provided by software tools such as EPANET.

The proposed simulation framework can be easily adapted for dealing with hydraulic networks that include valves placed ahead the sources. Since valves inside the system implies a change on the quantity of the network loops, a set of conservation of energy equations has to be defined for every situation resulting from the switching of valves.

Other interesting problems to be addressed are: a) the inclusion of other types of valves (such as pressure reducing, pressure sustaining, pressure breaker, etc.), and b) coping with the change of flow direction in pipes during the simulation.

Acknowledgement

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