

# MOVING TARGETS TRAJECTORY PARAMETERS ESTIMATION USING THE SIGNATURE CURVATURE INFORMATION<sup>1</sup>

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## ABSTRACT

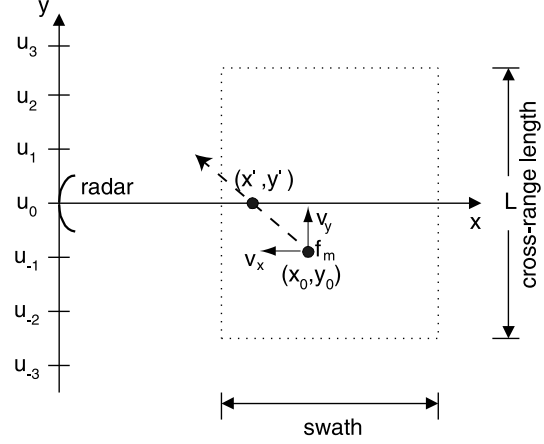
This paper presents a novel technique to estimate all the trajectory parameters of fast moving targets using a single SAR sensor without increasing the PRF. The basic reasoning is that, although the returned echoes may be aliased in the azimuth direction, their phase and amplitude are informative with respect to the moving target trajectory parameters. The proposed method samples the data in the spatial domain and extracts the data along the signature curve that depends of the moving target trajectory parameters. The resulting algorithm is a highly efficient (from the computational point of view) 1D matched filter. The effectiveness of the proposed algorithm is illustrated with simulated and real data.

## 1 INTRODUCTION

In SAR imaging it is well known that a moving target induces a Doppler-shift and a Doppler-spread on the returned signal in the slow-time frequency domain [1]. Most techniques proposed in recent literature to focus moving targets and obtain their velocity parameters take advantage of this knowledge [2], [3]. The azimuth velocity of a moving target is responsible for the spread in the slow-time frequency domain, whereas the slant-range velocity is responsible for the Doppler shift. Given a PRF, the Doppler-shift,  $f_D = 2v_x/\lambda$ , is confined to  $f_D < |\text{PRF}/2|$ , where  $v_x$  is the slant-range velocity of the moving target and  $\lambda$  is the carrier wavelength. If the induced Doppler-shift exceeds PRF/2 (the Nyquist limit) it has been mostly accepted that the true moving target slant-range velocity cannot be uniquely determined using a single antenna and a single pulse scheduling [4].

In [2] we have already presented an accurate method to detect and estimate all the moving target parameters using a single SAR sensor. Herein we present a novel technique that is much lighter, from the computational point of view, to estimate the same parameters. Basically, the technique corresponds to a 1D matched filter, that uses the signature acquired in the spatial domain along a curve that depends of the moving target trajectory parameters. By using both the phase and the amplitude information of the signature, the technique is able to retrieve all the moving target trajectory parameters using a single SAR sensor.

We start by writing the received signal, after pulse compression, in the spatial domain. We then write explicitly the moving target signature parameterized by the trajectory parameters. By assuming that the moving target signature is immersed in white noise, we derive a maximum likelihood



**Figure 1:** SAR slant-plane. At slow-time  $u_0$  the radar is broadside to the moving target.

estimator for the moving target trajectory parameters. At the end of the paper we present results using simulated and real data from the MSTAR public release data set.

## 2 PROPOSED APPROACH

Let us consider a moving target with slant-range and cross-range coordinates  $(x_0, y_0)$ , when the platform is at position  $u = 0$ , and velocity  $(v_x, v_y) = (\mu v_R, b v_R)$  defined in the slant-plane  $(x, y)$ ; symbol  $v_R$  denotes the platform speed and  $(\mu, b)$  is the target relative velocity with respect to the radar. The moving target coordinates are  $(x' = x_0 - \mu u_0, y' = u_0)$ , when the radar platform is broadside to the target, as illustrated in Fig. 1.

When the radar is positioned at coordinate  $y = u$ , the corresponding received echo from a moving target, after quadrature demodulation and pulse compression, is given by

$$s(x, u) = a(y_0 - \nu u) p_c(x - r(u)) f_m e^{-j2k_0 r(u)}, \quad (1)$$

where  $a(\cdot)$  is the two-way antenna radiation pattern,  $\nu \equiv 1 + b$ ,  $p_c(x)$  is the compressed transmitted pulse in the slant-range direction,  $k_0 \equiv 2\pi/\lambda$  is the wavenumber at wavelength  $\lambda$ , and  $r(u)$  is the distance between the platform and the moving target, given by

$$r(u) = \sqrt{(x_0 - \mu u)^2 + (y_0 - \nu u)^2}. \quad (2)$$

It is assumed that the dependency of  $a(\cdot)$  on  $x$  is negligible.

Let us define  $u' = u - u_0$  such that  $u' = 0$  corresponds to the position of the radar when the moving target is broadside to it. Therefore, defining  $r'(u') \equiv r(u' + u_0)$ , we obtain

$$r'(u') = \sqrt{(x' - \mu u')^2 + (-\nu u')^2}. \quad (3)$$

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Approximating  $r'(u')$  by a series expansion about  $u' = 0$  and retaining only the terms through the quadratic, results

$$r'(u') \approx x' - \underbrace{\mu u' + \frac{(\mu^2 + \nu^2)}{2x'} u'^2}_{\psi(u')}, \quad (4)$$

valid for  $|u'| \ll x'$ . If the range migration  $\psi(u')$  is known, then we can form a vector using data collected at coordinates  $s'(x, u') \equiv s[x = x' + \psi(u'), u' + u_0]$ , leading to

$$s'(x, u') = a(-\nu u') p_c(0) f_m e^{-j2k_0 r'(u')}. \quad (5)$$

Since  $p_c(\zeta)$  exhibits high resolution about  $\zeta = 0$ , then  $s'(x, u')$  becomes clustered about  $x = x' + \psi(u')$ , for all  $u'$  in the support of  $a(\cdot)$ . Therefore, we can form a vector containing the signature samples echoed by the moving target.

Let us assume white clutter in the spatial domain, and that  $p_c(0) = 1$ . Let  $z(x, u') = s'(x, u') + n(x, u')$  be the signal plus noise returns. Define vectors

$$\begin{aligned} \mathbf{z} &\equiv [z_{-N}, \dots, z_0, \dots, z_N]^T \\ \mathbf{s} &\equiv [s_{-N}, \dots, s_0, \dots, s_N]^T, \end{aligned}$$

where, for  $i = -N, \dots, N$ ,

$$\begin{aligned} z_i &= z(x' + \psi(u'_i), u'_i) \\ s_i &= a(-\nu u'_i) e^{-j2k_0 r'(u'_i)}, \end{aligned}$$

with  $u'_i = u'_s i$ , where  $u'_s$  is the sampling space in the cross-range direction chosen such that the clutter samples exhibit low correlation.

Notice that the observation vector  $\mathbf{z}$  should be formed with data sampled along the curve described by  $\psi$  and, in practical situations, data is available only at discrete values of  $(x, u')$ . For a fixed  $u'_i$ , the corresponding slant-range coordinate is computed by  $x' + \psi(u'_i)$ , and, in general, this coordinate does not correspond to a multiple of the sampling space in the slant-range direction. Therefore, a resampling or data interpolation should be done in the slant-range dimension. Nevertheless, as illustrated in the Results section, a simple nearest neighbor interpolation method may lead to satisfactory results.

Let us assume that the moving target parameters  $\boldsymbol{\theta} \equiv (\mu, \nu, x', y', u_0)$ , and the reflectivity  $f_m$  are known. In this case only the noise term is random. Therefore, the density of vector  $\mathbf{z}$  conditioned to  $\boldsymbol{\theta}$  and  $f_m$  is Gaussian given by

$$p(\mathbf{z} | f_m, \boldsymbol{\theta}) \sim \mathcal{N}(\boldsymbol{\mu}_z, \mathbf{C}_z), \quad (6)$$

where the mean is  $\boldsymbol{\mu}_z = f_m \mathbf{s}(\boldsymbol{\theta})$ , the covariance is  $\mathbf{C}_z = \sigma_n^2 \mathbf{I}$ , and the clutter cross-section is  $\sigma_n^2$ . Notice that  $z_i = z_i(\boldsymbol{\theta})$ . Consequently, the density in (6) is correct only for the true  $\boldsymbol{\theta}$ . However, for  $\boldsymbol{\theta} \simeq \boldsymbol{\theta}_0$ , the range migration is known and the data used to form vector  $\mathbf{s}$  is taken along the correct curve thus leading to  $z(\boldsymbol{\theta}) \simeq z(\boldsymbol{\theta}_0)$ . Thus, we still use (6) to compute the maximum likelihood (ML) estimate of  $\boldsymbol{\theta}$ . After some algebra, we obtain

$$\hat{\boldsymbol{\theta}}_{ML} = \arg \max_{\boldsymbol{\theta}} \frac{|\sum_i z_i s_i^*(\boldsymbol{\theta})|}{\sum_i |s_i(\boldsymbol{\theta})|^2}. \quad (7)$$

We note that  $u_0$  is not exactly known. However, assuming that  $u_0$  is a multiple of  $u_s$ , i.e.,  $u_0 = u_s i_0$ , then an error on  $u_0$  means an integer shift,  $i_0$ , on the sequence  $z_i$ . For this reason, we replace (7) by the correlation

$$(\hat{\mu}, \hat{\nu}, \hat{i}_0)_{ML} = \arg \max_{(\mu, \nu)} \left\{ \max_{i_0} \frac{|\sum_i z_i s_{i-i_0}^*(\boldsymbol{\theta})|}{\sum_i |s_i(\boldsymbol{\theta})|^2} \right\}. \quad (8)$$

Parameters  $u_0$ ,  $x_0$ , and  $y_0$  are then given as follows:

$$\hat{u}_0 = u_0 + \hat{i}_0 u_s, \quad (9)$$

$$\hat{x}_0 = x' + \hat{\mu} \hat{u}_0, \quad (10)$$

$$\hat{y}_0 = (\hat{\nu} - 1) \hat{u}_0. \quad (11)$$

The proposed methodology can be summarized as follows:

1. Obtain an image focused with static ground parameters. The ground will be focused and the moving targets will appear defocused and at wrong positions.
2. Detect the moving targets and estimate their approximate positions using methods such as those proposed in [5].
3. Isolate the defocused moving targets and re-synthesize their signatures back to the  $(x, u)$  domain as proposed in [6]. This procedure increases the signal to noise ratio, although it introduces some correlation due to the spatial windowing.
4. For each re-synthesized signature proceed as follows:

- Using the re-synthesized signature mass center, determine a rough estimate of the target coordinates  $(\hat{x}', u_0)$ ;
- Change the cross-range coordinate reference according to  $u' = u - u_0$ ;
- For all  $(\mu, \nu)$  pairs of interest, form the observation vector  $z$  using data extracted along the curve given by (4) and compute the correlation (8). Estimate as  $(\mu, \nu)$  the velocity pair for which the maximum absolute value occurs and store its coordinate  $i_0$ .
- Estimate  $(x_0, y_0)$  using equations (9), (10), and (11).

### 3 RESULTS

The scheme developed in the previous section to estimate the moving targets parameters is now evaluated using synthetic and real data.

#### A. Synthetic Data

We use herein three types of simulated targets: point-like targets, extended homogeneous targets, and extended targets with predominant scatterers. All the targets have initial coordinates  $(x_0, y_0) = (64, 209)\text{m}$ , velocity  $(v_x, v_y) = (-7.959, 8)\text{m/s}$  ( $v_x$  induces a Doppler-shift 3 times greater than the Nyquist limit). The mission parameters used in the simulations are summarized in Table 1.

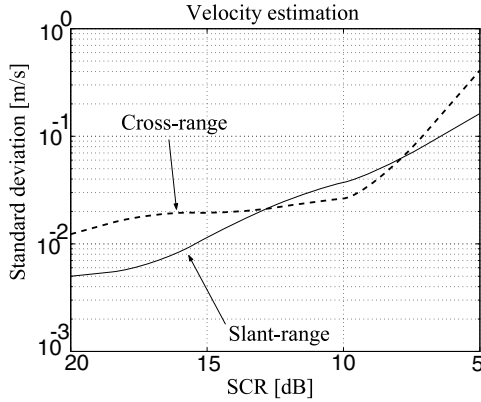
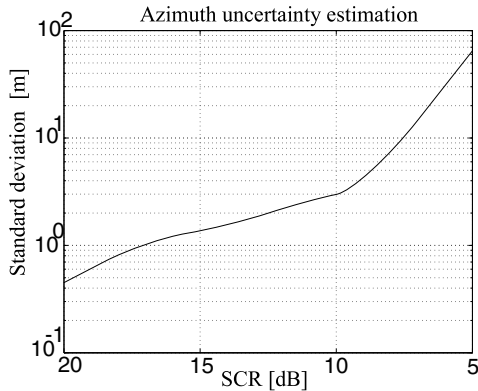
Table 2 summarizes the estimation results for the three considered targets. The signal-to-clutter ratio (SCR) was set to 20dB. As can be seen, if the target is point-like, or, if it has some predominant scatterers, the proposed scheme performs quite well and enables the velocity estimation, focusing and repositioning with accuracy high enough for most applications.

**Table 3: Mission parameters (MSTAR data).**

Parameter	Value
Carrier frequency	9.6 GHz
Chirp bandwidth	250 MHz
Altitude	12 km
Velocity	637 km/h
Look angle	$15^\circ$
Oversampling factor	2

**Table 1: Mission parameters used in the simulation.**

Parameter	Value
Carrier frequency	5 GHz
Chirp bandwidth	100 MHz
Altitude	12 km
Velocity	637 km/h
Look angle	$20^\circ$

**Figure 2: Velocity estimation as function of the SCR.****Figure 3: Azimuth uncertainty estimation.****Table 2: Simulation results (SCR=20 dB).**

Target type	$v_x$ error	$v_y$ error	$x_0$ error	$y_0$ error
point-like	0,005m/s	0,012m/s	0,02	0,45m
$7 \times 12$ m extended	0,3m/s	4,2m/s	0,18	28,3m
$7 \times 12$ m extended (one predominant)	0,041m/s	0,29m/s	0,07	1,6m

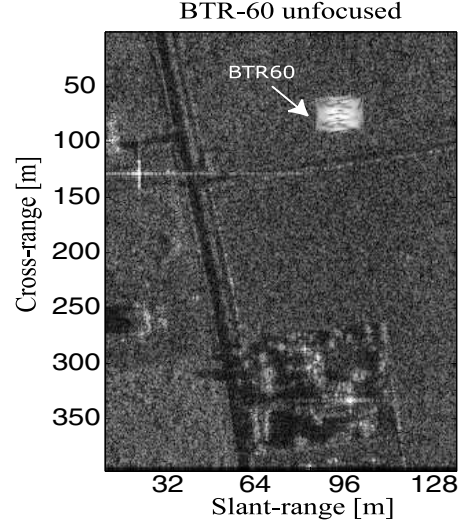
**Figure 4: Target area focused using the wavefront reconstruction algorithm with static ground parameters.**

Fig. 2 and Fig. 3 plot the velocity estimation and the azimuth uncertainty estimation, respectively, as function of the SCR for 64 Monte-Carlo runs. These results show that if the SCR is higher than 10dB, and assuming that the targets are detectable, the algorithm allows the velocity and initial coordinates estimation with accuracy enough for most applications.

### A. Real Data

We now present results using a BTR-60 vehicle with simulated movement and a background scene from Hunstville-Alabama, both taken from the MSTAR data.

The mission parameters are presented in Table 3. The SCR is approximately 20 dB. The velocity of the BTR-60 is  $(v_x, v_y) = (16.58, 2)$  m/s ( $v_x$  induces a Doppler-shift 12 times greater than the Nyquist limit).

Fig. 4 presents the target area focused with static ground parameters. As expected, the vehicle appears misplaced, blurred and defocused. If correctly processed, the BTR-60 image should appear positioned at coordinates  $(x_0, y_0) = (90, 124)$  m. After detection, we digitally spotlighted the BTR-60 signature and resynthesized it.

Fig. 5 displays the resynthesized signature and the estimated  $(\hat{x}', u_0) = (87, 123)$ m. Using the estimated coordinates, we then proceeded with the algorithm to estimate the remaining parameters. The estimated velocity and initial coordinates of the BTR-60 are, respectively,  $(\hat{v}_x, \hat{v}_y) =$

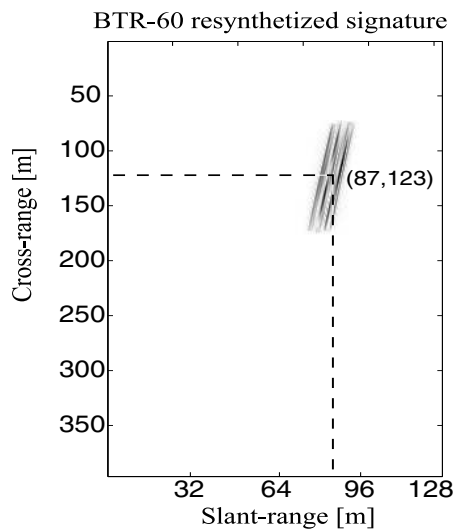


Figure 5: Resynthesized signature.

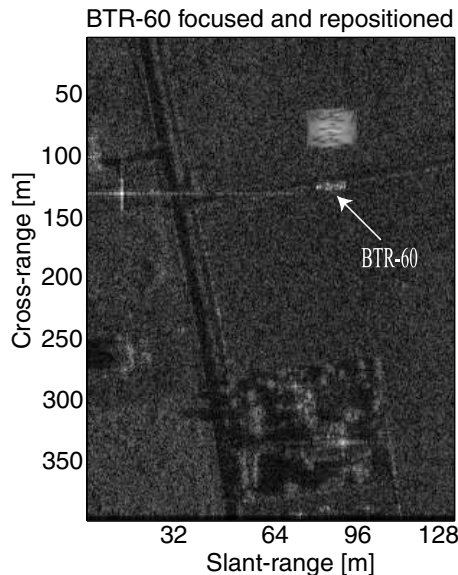


Figure 6: Focused and repositioned BTR-60.

(16.53, 6.2) m/s and  $(\hat{x}_0, \hat{y}_0) = (87.6, 129)$  m.

Using the estimated velocity and initial coordinates we focused and repositioned the BTR-60 as shown in Fig. 6.

Notice that since we are using a single SAR sensor and because the moving target spectra (although 6 times folded) is completely superpositioned on that from the clutter, we cannot remove the defocused BTR-60 from the original image. To achieve this goal the moving target spectrum should not overlap completely the clutter spectrum. Another possibility would be to have data from more than one SAR sensor [7].

The obtained results for the BTR-60 under study are summarized on Table 4. As can be seen, the proposed method-

Table 4: Estimation results for BTR-60 (SCR= 20 dB).

Target type	$v_x$ error	$v_y$ error	$x_0$ error	$y_0$ error
BTR-60	0,005m/s	4,2m/s	2,4m	5m

ology yields velocity and position estimates accurate enough for focusing and repositioning the vehicle.

From the results presented in this section, we conclude that the suggested strategy works well for point-like targets or extended targets exhibiting some predominant targets. When this is not the case, the algorithm still gives good results for the slant-range velocity. It produces, however, large errors on the cross-range velocity estimation and on the estimated cross-range initial position. Nevertheless, the presented methodology can be used to estimate the velocity and initial coordinates of most man-made targets, since they typically exhibit predominant scatterers.

## 4 CONCLUSIONS

In this paper we presented a spatial based methodology for the estimation of all the moving target parameters using a single sensor. The main algorithm samples the spatial domain, compressed in the slant-range direction and uncompressed in the cross-range direction, to acquire data along the signature curve that depends of the moving target kinematics. To achieve efficiency and simplicity we derived the ML estimator of the velocity parameters by assuming white noise. This assumption led us to a matched filter type solution. The method was successfully tested using a combination of simulated and real data, with all targets inducing Doppler-shifts beyond the Nyquist bound imposed by the mission PRF.

## 5 ACKNOWLEDGMENTS

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