Computing True Alternatives and not only a Single Best Solution

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Abstract

In a "Decision Support System with Multiple Choice Structure" (one or several) computer programs propose a clear handful of interesting candidate solutions, and a human has the final choice amongst these alternatives. Traditional "k-best algorithms" are not good tools for such systems, because the k best solutions are often merely micro mutations of each other and not true alternatives.

We present structural results on an approach where the alternatives are generated one after the other by introducing penalties: The best solution is computed, then certain building blocks are punished in the objective function, the best solution under this modified objective function is computed, again certain building blocks are punished, and so on. We concentrate on a monotonicity result for k=2 for sum-type objective functions and discuss several applications.

1 Motivation

Suppose, someone has a real world problem: Typically the situation is too complex to find an exact solution immediately. So he or she makes or uses a model of the problem situation. An optimal solution for this model is generated (often by a computer), and this solution for the model is translated back to a solution for the original problem. Often this "translation back" does not work in practice. If there is enough time left, the model may be adapted more precisely and a new solution generated, hopefully a bettter one. This process may be iterated several rounds, until the original problem is solved satisfactorily.

In realtime situations there is normally no time for readjustment or repeated rounds. Here a "MULTIPLE CHOICE SYSTEM" may help: For the model a clear handful of true alternatives and not only a single best solution is generated (by the computer). These candidate solutions are translated back to solutions for the original problem, and the user has the final choice among them. In the game of chess the author was very successful with such multiple choice approaches.

Four decades ago already, mathematicians started to investigate "K-BEST ALGORITHMS". In a problem with discrete (finite) set of admissible solutions not only the best solution is computed but the k best ones. Here k is some natural number. Faced

with reality, these k-best algorithms had and have severe problems of acceptance, because the k solutions are typically very similar to each other - merely micro mutations and not true alternatives.

The task of finding true alternatives is rather different from the one to generate k best solutions. In our talk we present structural results on an approach where the alternatives are generated one after the other by introducing penalties: The best solution is computed, then certain building blocks (of this best solution, but maybe also others) are punished in the objective function, then the best solution under this modified objective function is computed, again certain building blocks are punished, and so on. In the talk we concentrate on the basic case k = 2; the best solution and one alternative are computed.

2 A Monotonicity Result for Sum-Type Problems

Let E be a finite set, and let S be a set of feasible subsets of E. Let $w : E \to IR$ and $u : E \to IR$ be two positive weighting functions on E. For every element $B \in S$ we define $w(B)$ to be the sum of the $w(e)$ for all $e \in B$. Analogously $u(B)$ is defined. For positive ε we define $f_\varepsilon(B) := w(B) + \varepsilon * u(B)$ for all $B \in S$.

Theorem 1 *Let $B(\varepsilon)$ be some set $B \in S$ with minimum value for f_ε. Three statements of monotonicity hold.*

(i) $u(B(\varepsilon))$ is weakly monotonically decreasing in ε.

(ii) $w(B(\varepsilon))$ is weakly monotonically increasing in ε.

(iii) $w(B(\varepsilon)) - u(B(\varepsilon))$ is weakly monotonically increasing in ε.

The original task was to minimize $w(.)$ over S. $u(.)$ is a penalty function. ε is the penalty parameter. Let $B(0)$ be an optimal set in S with respect to the minimization of $w(.)$. Of special interest are the settings

(i) $u(e) = w(e)$ for all $e \in B(0)$,

(ii) $u(e) = 1$ for all $e \in B(0)$, (results in this case have been achieved for generalized matroids by Althöfer & Wenzel [1])

and, for appropriate subsets or supersets N of $B(0)$,

(i') $u(e) = w(e)$ for all $e \in N$,

(ii') $u(e) = 1$ for all $e \in N$.

All edges e not mentioned in (i) to (ii') get $u(e) = 0$.

3 Applications of the Theorem

(i) Complete Bipartite Matching

There is a complete bipartite graph with n vertices on each side and positive real-valued cost $c(i,j)$ for each pair (i,j). Complete matchings with small cost sum have to be found. First of all a minimum cost matching $M(0)$ is computed. Then all pairs in this matching are punished by setting $c'(i,j) = (1+\varepsilon)c(i,j)$ for them. Here $\varepsilon > 0$ is a penalty parameter, chosen by the user. All other pairs keep their original cost, hence $c'(i,j) = c(i,j)$ for them. After this a minimum cost matching $M(\varepsilon)$ with respect to c' is computed. $M(0)$ and $M(\varepsilon)$ are the two alternatives, presented to the user. Let $C(\varepsilon)$ be the sum of the original c-weights of all pairs in the intersection of $M(0)$ and $M(\varepsilon)$. Then $C(\varepsilon)$ is weakly monotonically decreasing in ε.

(ii) Traveling Salesman
The problem with n cities and edge lengths $w(e)$ is wellknown. Let $T(0)$ be a shortest tour through all cities, and put $u(e) = 1$ for all edges e in $T(0)$. Let $T(\varepsilon)$ be any shortest tour for penalty parameter $\varepsilon > 0$. Then the number of joint edges of $T(0)$ and $T(\varepsilon)$ is weakly monotonically decreasing in ε. Observe that the statement of our theorem does not depend on the degree of difficulty to compute $T(0)$ and the $T(\varepsilon)$.

(iii) Time Table
Consider the design of time tables where the objective is to minimize cost. The penalty function may punish building blocks which occur in the optimal solution and also elements which are similar to such building blocks.

4 Open Questions

(i) Mathematically it is a challenge to prove analogous monotonicity results for situations where the objective function is no longer sum-type.

(ii) Practitioners may examine which are good choices for the penalty functions u and penalty degrees ε. Concerning shortest paths, there is an interesting example from commercial practice. In a 1997 version of a vehicle routing program of the software company AND all segments of the shortest (or "best") route are punished by substituting $w(e)$ by $1.2 * w(e)$.

Reference

[1] I. Althöfer, W. Wenzel. Two-best solutions under distance constraints: the model and exemplary results for matroids. Advances in Applied Mathematics **22** (1999), 155-185.