A Method for more specific Simulation of Operative Temperature in Thermal Analysis Programmes

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SUMMARY:
Simulation of energy consumption of buildings on hourly basis is closely connected to the thermal indoor climate. The operative temperature can be used as a simple measure for thermal environment. The operative temperature is a function of the air temperature, the mean radiant temperature and the relative air velocity. However, in many programs the model for calculating the mean radiant temperature has traditionally been based on the calculation of an area weighted mean value independently of the location in the room. In practice the location of the person in the room has a significant influence and inhomogeneous radiation plays an important role for the usability and functionality of the room. In order to calculate the mean radiant temperature as function of the person´s location in the room one needs to calculate the angular factor between the person and the surfaces. The angular factor between surfaces can easily be calculated; however the angular factor between a person and a surface cannot be found directly from a formula, but needs to be approximated in order to be used in computer simulations. The paper describes different kinds of validations that have been done to compare different models for calculating the angular factor and shows how a matrix solution for the angular factor can easily be implemented in programs for dynamic building thermal analysis.

1. Introduction

Nowadays many new and old buildings have large glass surfaces, which create a lot of consequences for the thermal indoor climate. Designing rooms with big glass areas has a great effect on the thermal comfort in the rooms. The outside air temperature is closely related to the temperatures on the surface of the walls and the windows. Since the insulation of walls in Scandinavia is 4 to 15 times better than the insulation of the glass, the inside surface temperature is much more affected on the inside of the glass than on the inside of the wall. On a cold day this will result in the surface temperature of the glass being quite lower than the rest of the surfaces and occupants being exposed to inhomogeneous radiation. Many other effects can result in inhomogeneous radiation, by way of example may be cited solar radiation transmitted through glass, hot radiators, hot floor/roof, etc, and the consequences of this can be that the functionality and productivity of the room can be reduced.

When calculating the consequences for the thermal indoor climate, the operative temperature can be used as a simple measure for the heat loss from a person. The operative temperature is a function of the air temperature, the mean radiant temperature and the relative air velocity.

In most of the programs for dynamic building thermal analysis, the model for the operative temperature has traditionally been based on a relatively simple model independently of the location of the person in the room. In reality this model will not be able to give a correct picture of the thermal condition for a person in a room since the person’s location plays an important role. Another factor is the real consequences of shading devices. A person behind a window directly exposed to the direct sun light will have a completely different operative temperature than a person behind a shading device. The author has made many simulations with different programs and the indication is that this effect is not being taken into account in the right way, since the effect of a shading device only shows minor effect on the results of the air- and operative temperature.

The air temperature and the temperature on the surfaces can easily be calculated. When one wants to find the angular factor between two surfaces, one can find a specific equation for the angular factor to be used in the calculations. However, the real problem is calculating the mean value of the angular factor between a sedentary/standing person and a vertical/horizontal rectangle since there are no direct formula for calculation of the angular factor between the surface and a person.

This paper describes the background for the calculation of the angular factor between a sedentary/ standing person and a vertical/horizontal rectangle. From this study a model for calculation of the operative temperature dependent on the location of the person in the room will be set up, and from this make a proposal for an improvement of the computer program package BSim used for analyzing indoor climate and energy consumption. Since the improvement has not yet been implemented in BSim it has not been possible to make any simulation showing the effect on operative temperatures and energy consumption.

2. Description of the computer program package BSim

As part of the author's Ph.D. study from 1982 to 1987 at the Thermal Insulation Laboratory, Danish Technical University, with the subject utilization of passive solar energy under Danish climate conditions, the author used the software BLAST (Hittle, 1979) and SUNCODE (Palmiter et al., 1985). One of the important differences between the two programs is the solution of the in stationary heat conduction through walls.

In BLAST the response factor method has been used to make the approximation to Fourier's equation. The method is fast, once the response factors have been calculated. The calculations of the factors are, however, complicated. In SUNCODE an explicit difference method was used for solving Fourier's equation which resulted in limits on the time step, especially for thin light materials.

One of the advantages of using a difference method rather than the response factor method is that the solution model is much simpler and gives the user much better possibility to follow the process inside the constructions, this being of importance for example in floor heating and combined hygrothermal simulation in buildings/walls. From an overall aspect it was decided to use the principles from SUNCODE for the heat balance model and combine it into a Danish program *tsbi2*. For the in stationary heat conduction through the wall a full implicit solution model was chosen and the program was developed into the *tsbi3* program released in 1990 (Johnsen et al., 1993; Johnsen et al., 1994). Later the program has been incorporated as *tsbi5* into the computer program software package (BSim, 2005).

BSim is being used for research and commercial calculations by consulting engineers in the field of heating and air-conditioning. The program provides means for detailed, combined hygrothermal simulations of buildings and constructions. BSim can be used for analyzing indoor climate, energy consumption, passive solar energy, automatic control functions, etc. in connection with the planning and design of buildings, energy-conservation measures, renovation of buildings, and heating and air-conditioning systems. The program is the most used program in this field in Denmark. The BSim package (BSim, 2005) consists of several modules:

- SimView – Graphic model editor and input generator
- *tsbi5* – Hygro-Thermal building simulation core
- Simligth – Tool for analysis of daylight conditions in simple rooms
- XSun – Graphical tool for analysis of direct sunlight and shadowing
- SimPV – a simple tool for calculation of the electrical yield from PV systems
- NatVent – Analysis of single zone natural ventilation
- SimDxf – a simple tool which makes it possible to import CAD drawings in DXF format

In the module *tsbi5* it is possible to make a combined transient thermal and transient indoor humidity and surface humidity simulation. This paper deals with a suggestion on how to improve the model in *tsbi5* for the operative temperature for a specific location in the room, since *tsbi5* only calculates the mean radiant temperature as an area mean parameter.

3. The operative- and mean radiant temperature

To find a simple measure for the heat loss from a person and a simple measure for the thermal indoor climate, the operative temperature can be calculated, (ISO 7730, 1994):

$$t_o = A \cdot t_a + (1 - A) \cdot t_r \qquad (1)$$

Where t_o is the operative temperature [ºC], t_a is the air temperature [ºC], t_r is the mean radiant temperature [ºC], A is a factor accordance to the relative air velocity (A=0.5 for v_{ar}=0.2 m/s, A=0.6 for v_{ar}= 0.2-0.6 m/s, A=0.7 for v_{ar}= 0.6-1.0 m/s).

The mean radiant temperature between a person and the surrounding surfaces:

$$\bar{t}_{r,P} = F_{P-1} \cdot t_1 + F_{P-2} \cdot t_2 + ... + F_{P-n} \cdot t_n \qquad (2)$$

Where $t_{r,P}$ is the mean radiant temperature for a person [ºC], t_i is the temperature of surface i [ºC], $F_{P\text{-}i}$ is the angular factor between a person and surface i [–]

The surface temperature can easily be measured or calculated. However, the angular factor between a sedentary/standing person and a rectangle on the wall, floor or ceiling is far more complicated calculating. In most cases the angular factor is read from a figure since there is no exact formula for calculation of the angular factor between a person and a surface. In the following a proposal for solving this problem shall be discussed.

4. The projected area factor and the angle factor

In order to calculate the angle factor between a person and a surface $F_{P\text{-}A}$ it is necessary to use *the projected area factors* f_p (Fanger, 1970):

$$f_p = \frac{A_p}{A_{eff}} \quad \wedge \quad F_{P-A2} = function(f_p) \tag{3}$$

Where A_{eff} is the effective radiation area of the subject, $F_{P\text{-}A2}$ is the angle factor between the person and the sphere *(A_2)*, A_p is projected area of a person on a plane perpendicular to the direction to d_{A2}

At the time when this theory (Fanger, 1970) was developed it was only possible to find a person´s projected area A_p from experiments. Later in this paper newer methods will be described in which the human body has been modelled by using a detailed 3D geometry and radiation model.

f_p as a function of *(α, β)* can be determined from experiment or from 3D geometry and radiation models. The angle factor $F_{P\text{-}A2}$ is a function of the dimensionless relationship *a/c* and *b/c*, figure 1. In this way a simple diagram can be used to find the angle factor for a rectangle of any size, placed as shown in figure 1 with the normal at the corner point passing through the centre of the person.

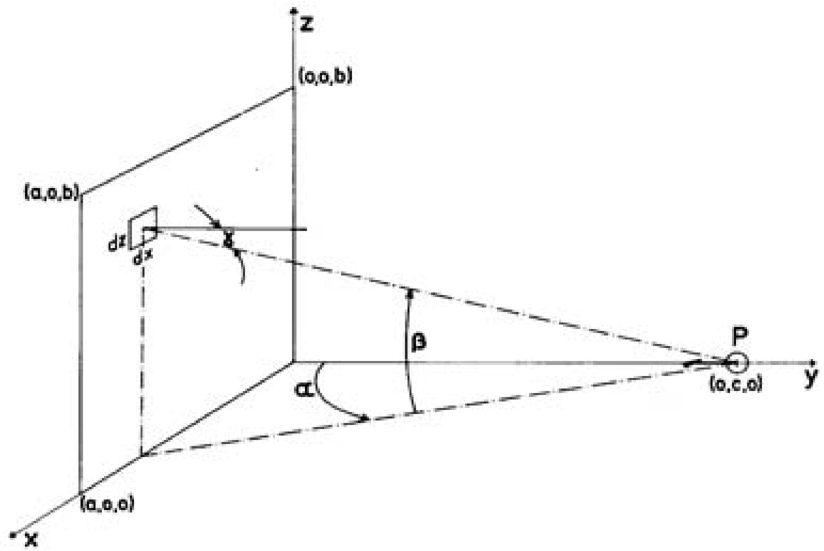

FIG. 1: The geometric principle for development of the evaluation of the angle factor between a person and a rectangle (Fanger, 1970).

To find an angle factor in a room in which a person is facing one of the walls, there will, according to the book (Fanger, 1970), be 6 cases, which means that 6 diagrams have to be drawn: 4 vertical surfaces and 2 horizontal surfaces. In most cases the location of the person in the room will be known, however the person's orientation will change from time to time. This means that it will be natural to find the mean value of the angle factor for a person rotating around himself $\bar{F}_{P-A}$ ($0 < \alpha < 2\pi$), meaning that for a vertical rectangle the angle factor can be found by using formulary (4) (Fanger, 1970):

$$\bar{F}_{P-A} = \frac{1}{2\cdot\pi^2}\int_{x/y=0}^{x/y=a/c}\int_{z/y=0}^{z/y=b/c}\int_{\alpha=0}^{\alpha=2\pi}\frac{f_p}{\left[1+\left(\frac{x}{y}\right)^2+\left(\frac{z}{y}\right)^2\right]^{\frac{3}{2}}}\,d\left(\frac{x}{y}\right)d\left(\frac{z}{y}\right)d\alpha \tag{4}$$

Since we now calculate the mean value of the angle factor between the person and the rectangle, where the person now is rotating around his own axis, we need to mark the angle factor as an average: $\bar{F}_{P-A}$. This simplification reduces the number of diagrams which need to be drawn from 6 to 2 – one for horizontal rectangles and one for vertical.

5. Experiments to predict projected area factor

Several researchers have used different kinds of photographic methods to find the projected area factors f_p. Fanger (Fanger, 1970) is the researcher who did the most extensive experiments to determine f_p. Fanger performed his experiments for measuring A_p using a photographic technique of the persons from many different directions. From a given viewing angle each photo provided the projected area of the body. By taking photos it was possible to find the projected area of the body for all these angles – and then the projected area factors f_p can be calculated. In the study ten males and ten females participated in standing and sedentary positions for azimuth (α) and altitude (β) angels between 0º to 180º and 0º to 90º, respectively. The persons were a fair sample of the adult population.

In the experiment Fanger wanted for solar radiation to determine the projected area factor f_p, which is why the radiation rays should be as close as possible to be parallel, meaning that the distance between the person and the camera has to be infinite. In Fanger´s case he positioned the camera in a distance of about seven meters and was then able to make his experiment with quite a good approximation to the simulation of parallel rays from the subject to the radiant source. Fanger presented his results as a form of graphs.

Fanger showed that the projected area factor f_p can be used in practice independently of the clothing, size and sex of the person, however one has to distinguish between a standing and a sedentary human body. The consequence of this is that all the mean values of the f_p-values can be plotted into one diagram, e.g. for a sedentary human body, see figure 2.

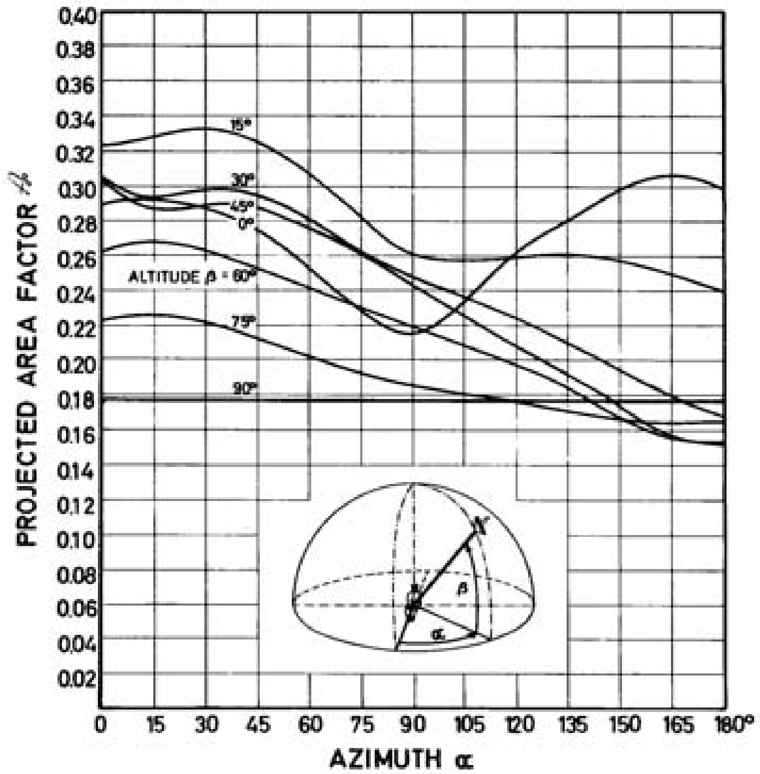

FIG. 2: Projected area factor for seated persons, nude and clothed to an infinitely small surface (Fanger, 1970).

The diagram in figure 2 for the projected area factor shows the factor as a function of the azimuth α and the altitude β to an infinitely small surface. Since equation (4) uses the *projected area factor –* f_p, it is impossible to solve the equation exact since f_p is a function of the azimuth α and the altitude β in an irregular way because it is based on experimental measurement from the human body. However it is possible from the actual measured data to make integration using equation (4) over all the angles for the azimuth α and the altitude β and draw a diagram for the angle factors between a human body and vertical or horizontal rectangles based on the mean values of f_p shown for a sedentary person in figure 2. If we look at the case with a fixed position for a person in a room but the facing of the person is unknown, the mean value of the angle factor can be found when the person rotates around a vertical axis. An example of the mean value of the angle factor between a sedentary human body and a vertical rectangle is shown in figure 3. Since the figure shows the mean value for the person rotating around his own axis, only one diagram is necessary. If the orientation for the person has been fixed it will be necessary to use four diagrams.

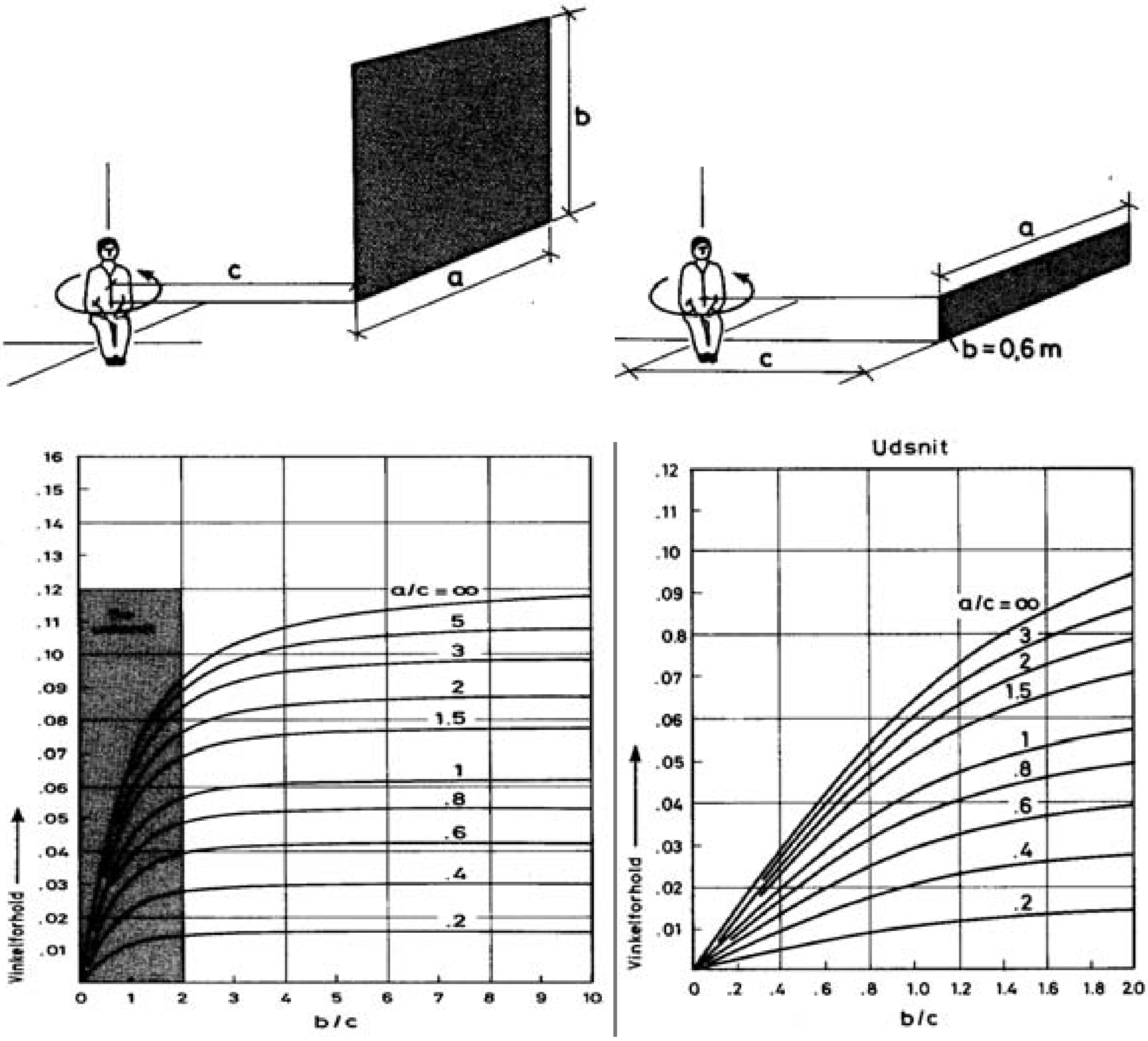

FIG. 3: Mean value of angular factor between a sedentary person and a vertical surface when the person is rotated around a vertical axis. (Fanger, 1970).

6. Geometry model and numerical simulation model used to predict projected area factor

In the last ten years computer animation programs have been developed for cinemas and computer games for which detailed 3D models have been developed for the human body geometry. This means that body posture can be generated in the general case from these software tools in order to make detailed modelling of the human radiation heat exchange with the surrounding surfaces. As a consequence of this experimental investigation of the thermal radiation from the human body will be used less since the computerized models are superior. This will create a new world of investigations in which the possibilities are nearly unlimited.

In the paper (Kubaha et al., 2004) they made a detailed study for human projected area factors for detailed direct and diffuse solar radiation analysis using commercial software (Curious Labs, 2000) in which they modelled persons in standing or sedentary posture. The software was used to generate a detailed 3D model of the human body. The model of the human body was extremely detailed and consisted of 10,995 small surface elements, which made it possible to make a radiation simulation with sufficient details.

In order to compare the 3D geometry model with a simulation model – the human body was transformed to 3D humanoid geometries that were imported into a thermal analysis software package (ThermoAnalytics, 2001). The software uses ray tracing technique to predict the absorbed short-wave radiation energy for all the surface elements.

This method (Kubaha et al., 2004) gives the possibility for a detailed analysis of the human projected area factors. The authors made an impressive verification and validation process in which they first verified the new 3D geometry model against simulation results from the thermal analysis software package (ThermoAnalytics, 2001). The comparison showed that the predictions with the 3D geometry model agreed well with the results of the ray tracing simulations in the whole range of angles for the azimuth *(α)* and altitude *(β)*. This comparison will not be described further in this paper since the importance in this matter is a calculation of the angle factor between the whole human body in a standing/sedentary position and the surrounding surfaces.

Since experimental data are only available for the whole body it is necessary, in order to make a comparison with these measurements, to make integration over the whole body surface for the 3D geometric model. Since Fanger (1970) produced the most experimental data with the best details these have been the best materials for the comparison. The validation shows that for the most altitude and azimuth angels the relative error has only been approximately five percent. The agreement has been good for both standing and sedentary postures, however for the sedentary posture the projected area factors were higher than ten percent relative error for an altitude angle of $\beta = 15°$ and $\alpha < 60° / \alpha > 300°$.

One of the reasons for differences in results could be that the experiments were performed in a finite distance between the camera and the persons (7 m) – as opposed to the predicted results that was using an infinite distance between the radiant source and the human being, which resulted in parallel rays.

7. Selecting of model for calculation of the angle factor

Since the results have shown such fine agreement comparing the two models developed by (Kubaha et al., 2004) with Fanger´s data (Fanger, 1970) it has been decided to use Fanger´s data in this study.

However, since the projected area factor for a seated person in figure 2 cannot be used directly to solve equation (4), for the case illustrated in figure 3, it is necessary to go through an integration of the projected area factor to find the angular factor. Seeing that the data were only in an irregular form in figure 2 and the data behind were not available, it was not possible to find the value for f_p directly. To solve this problem it was decided to find f_p in a reverse process, in which the data from figure 3 were being used to find the solution for the angular factor $F_{P\text{-}A}$ from the diagram for a specific case, depending on the position of the seated person, the size of the window, and the distance from the window.

These results were then entered into equation (4), and iterations have been used to find a solution for the projected area factor f_p. This value of f_p could only be used for this one special case. For other values, the error becomes increasingly larger. In order to solve this problem, a 20 by 26 matrix was created for the specific case in figure 3, and iterated solutions for all the projected area factors f_p. In this way, the error was reduced to less than one per cent.

Equation systems can then be created for the operative temperature for a general case. Real data for the specific case can be entered to calculate the thermal indoor climate, without having to spend time reading tables and making calculations. If the results show that the operative temperature under special winter conditions will be too low, the user can quickly make alternative calculations with other assumptions. For example, the glass in the window can be changed to a better quality with a lesser loss of heat. However the purpose for this paper is not to make a small program for analysing a stationary situation but to implement the final model including more features in (BSim, 2005). By looking at different stationary results under different climate and building conditions it will be possible to get a good idea about what kind of features will be wanted.

As a platform for setting up the equations the mathematical program software (Mathcad 14, 2007) and (Matlab version 7.5, 2007) have been used. It is the plan that – when the model has been created to the wanted form – the model can be rewritten to C++, which is being used in the BSim package. In this model other kinds of features have to be included in a more specific way in order to find the operative temperature depending on the location of the person in the room: direct solar radiation to a sedentary/standing human body, effect from shading devices, hot radiators, hot floor/roof, air velocity, etc.

8. Practical example – The operative temperature in an office

As a little demonstration of the principles an office has been simulated during winter conditions. The office is facing south and has the following dimensions (inside measurements): area 12 m^2, width 2.8 m, height 3.0 m and depth 4.4 m. The U-value of the exterior wall is 0.6 W/m^2 K and of the glass in the window changed to, respectively, single glass (6.0 W/m^2 K), double glass (3.0 W/m^2 K) and triple glass (1.6 W/m^2 K). The outside temperature is fixed to -12 ºC and the inside air temperature as well as the temperature on the inside surfaces set to 21 ºC. The simulations have been done for a person situated in the middle line of the room 0.4 m from the window and to the far side of the room. A picture and a plan of the room are shown in figure 4.

FIG.4: To the left a picture of the room and to the right a plan of the room.

The results of the calculations of the mean radiant temperature (left) and the operative temperature (right) have been shown in figure 5. The figure to the left shows 3 pairs of curves for the three different kinds of glass types: The mean radiant temperature respective as 1) a function of the distance from the window and 2) an area weighed mean value independently of the distance – fixed value. As expected the mean radiant temperature is lower near the window and increasing as the distance from the window is being increased. Better U-value of the glass reduces the inconvenience from the cold radiation of the window. These results will give a more correct picture of the thermal condition in the office in which the location of the person has been taken into account. This is as opposed to what has been done in many programs for dynamic building thermal analysis, in which the mean radiant temperature has been calculated as an area weighted mean value independently of the location in the room – see the three curves with fixed values (horizontal curves). If a person is seated 0.4 m from a double glassed window the difference between the mean radiant temperature as function of distance (18.6ºC) and independently of the distance (20.1ºC) will be 1.5ºC. For the operative temperature the similar numbers will be (19.8ºC), (20.6ºC) and 0.8ºC, respectively. Increasing glass area and lower U-value will increase the differences.

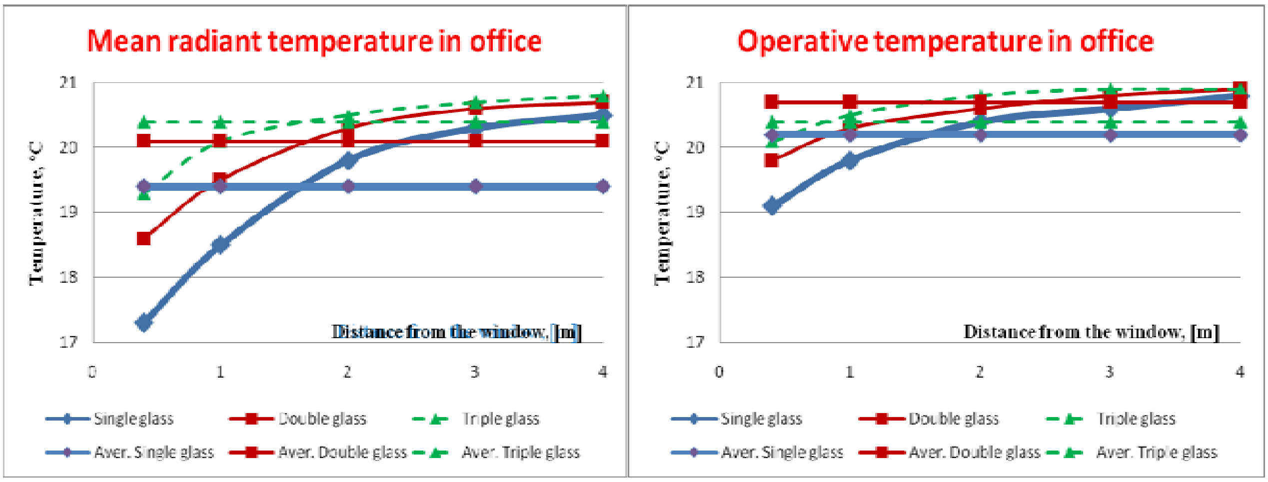

FIG.5: The mean radiant temperature and the operative temperature as a function of the distance from the window

In practice it will be natural for people to have a higher air temperature in the room if they are situated nearby a cold window in order to compensate for being exposed to the inhomogeneous radiation and the lower operative temperature. As a result of this the energy consumption for heating will be increased in order to keep satisfactory indoor thermal environment.

This problem could have been shown already in the design phase, if the simulation had taken into account the specific location of the person in the room. One can argue that this will complicate the calculations of the energy / thermal indoor climate too much, since the location will often be changed in the life time of the building. However it is natural for human beings to wish to sit nearby the window and have the view and daylight, and if it is too cold the person can easily solve part of the problem by turning up the heat and by this way increase the air temperature. The individuals in the building will normally only have very little or no idea about the total heating demand in the building and will more focus on their own personal comfort.

9. Conclusion

The possibility to simulate the operative temperature depending on the location of the person in the room has been a neglected topic in many simulation programs for thermal simulation. The result can be a deviation of the operative temperature with up to several degrees Celsius near for example a large window compared to a mean operative temperature based on the calculation of the mean radiant temperature as an area weighted mean value. The energy consumption in the winter situation can also be increased significantly if the persons in the room turn up the thermostat in order to increase the air temperature as compensation to cold radiation from a glass surface. The consequences energy wise of the inhomogeneous radiation could have been analysed in the design phase, if the necessary design tool had been available where the exact location of the person could have been used as input to the program.

The paper shows a possible way to calculate the angular formula using a matrix model. The matrix model is based on Fanger´s data (Fanger, 1970) since the validation done by (Kubaha et al., 2004) has shown such fine agreement comparing the two models developed by (Kubaha et al., 2004) with Fanger´s data (Fanger, 1970). The model has been set up in the software (Mathcad 14, 2007) and (Matlab version 7.5, 2007), which have shown to be a good platform for performing the necessary equations in the right order and built up knowledge on how the model is working. The model can later on be rewritten to for example to C++ and implemented to the BSim package. The model needs more development in order to take into account direct solar radiation through windows, airflow, PMV-values, etc. In addition the model will need more testing of other possible models for calculation of the angular factor.

10. References

Hittle D.C. (1979). BLAST – The Building Loads Analysis and System Thermodynamics Program, CERL – the US Army Construction Engineering Research Laboratory, Champaign, Illinois.

BSim. (2005). Danish Building Research Institute, Hørsholm, Denmark.

Curious Labs. (2000). POSER4: the premier 3D character animation and figure design tool. Curious Labs, Santa Cruz

Fanger P.O. (1970). Thermal comfort, McGraw-Hill, New York.

ISO 7730. (1994), Moderate thermal environment – Determination of the PMV and PPD indices and specification of conditions for thermal comfort, Reference number ISO 7730:1994(E), CH-1211 Genève 20, Switzerland

Johnsen K., Grau K. and Christensen J.E. (1993). *tsbi3*, Edb-program til beregning af bygninger og installationer, Brugervejledning. Danish Building Research Institute, Hørsholm, Denmark.

Johnsen K. and Grau K. (1994). *tsbi3*, comuter program for thermal simulation of buildings, user´s guide. Danish Building Research Institute, Hørsholm, Denmark.

Kubaha K., Fiala D., Toftum J. and Taki A. H. (2004). Human projected area factors for detailed direct and diffuse solar radiation analysis. *Int J Biometeorol,* No. 49, 113-129.

Mathcad 14. (2007). PTC Corporate Headquarters, Needham, USA

Matlab version 7.5 (2007), MathWorks Inc., Natick, MA, USA

Palmiter L., Wheeling T. and Ecotope Inc. (1985), SUNCODE-PC – A Program User´s Manual, Ecotope Group, Seattle, Washington, USA

Thermo Analytics. (2001). RadTherm technical documentation. Thermo Analytics, Calumet