

# Recognition of Arrows in Line Drawings based on the Aggregation of Geometric Criteria using the Choquet Integral

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## Abstract

*A new way to detect arrows in line drawings is proposed in this paper. Our approach is based on the definition of the structure of such a symbol. Signatures of angular areas are computed and axiomatic properties and geometric characteristics are checked using the Choquet integral. Finally an experimental application on line-drawing documents shows the interest of our approach.*

## 1. Introduction

Despite the large amount of technical documentation floating around, there have been relatively few accurate studies only focused on the detection of arrows in line drawings. It is well known that such a symbol brings precious information for the understanding of the document under consideration (objects pointed out, associated text boxes...) and in many recognition systems, it is important to have an accurate and powerful operator dedicated to the retrieval of this particular symbol. Dori and Velkovitch [6] have proposed a method for the location of dimensioning text from engineering drawings based on arrowhead recognition [5]. As in [14] the results provided are interesting but application dependent and required arrows having low distortions and belonging to a standard type as ISO or ANSI. In structural pattern recognition field the methods are usually based on graph matching to identify handwriting symbols in graphic documents [1, 11, 17]. In these approaches some rules are defined from structural primitives of features with a cost function associated to. Nevertheless the methods used are generally sensitive to the noise and the deformation of the objects. Valveny and Marti have recently proposed a deformable template Matching [18]. Such an approach is based on a probabilistic model composed of lines. Nonetheless this method is dedicated to symbols described by a set

of segments and it is not easy to extend it to manage with binary objects. Moreover the application of graph matching approaches on large documents should required several computing to take into account occluded objects (subgraph matching). In this paper a generic approach dedicated to the definition and the recognition of arrows is presented.

## 2. Arrow Properties

In this paper an arrow is assumed to be the union of two geometric parts: an isocel triangle  $T$  – defined by three points  $A(x_a, y_a)$ ,  $B(x_b, y_b)$  and  $C(x_c, y_c)$  – linked to a rectangle  $R = (EFGH)$  – defined by four points  $E(x_e, y_e)$ ,  $H(x_h, y_h)$ ,  $F(x_f, y_f)$  and  $G(x_g, y_g)$  –. We set  $c = d(A, B) = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$ ,  $b = d(B, C)$  and  $a = d(A, C)$  (figure 1).

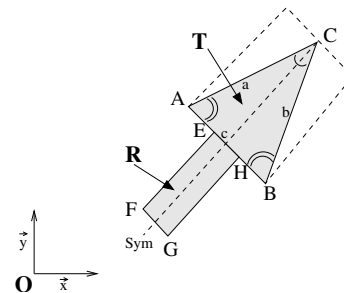


Figure 1. Arrow Description.

By definition a triangle is a 3-sided polygon having some basic geometric properties like [19]:

**[P1]** All triangles are convex.

**[P2]** A triangle  $T$  with two sides equal is called isocel.

**[P3]** The sum of the angles of a triangle is  $\pi$ .

**[P4]** 3 angles specify a triangle only modulo a scale size.

Beyer [3] and Baker [2] have given 110 formula to defined

the area of a triangle. In this paper the Heron's formula is used to compute this feature:

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

with  $s = \frac{1}{2}(a+b+c)$  the semi-perimeter.

The description of an arrow requires also the following property (symmetry):

**[P5]** An isocel triangle (not equilateral) has a unique angle bisector *Sym*, passing by *C*, which split it into two symmetrical parts. This bisector is also the median of the rectangle in a right built arrow.

### 3. Angular Signature Description

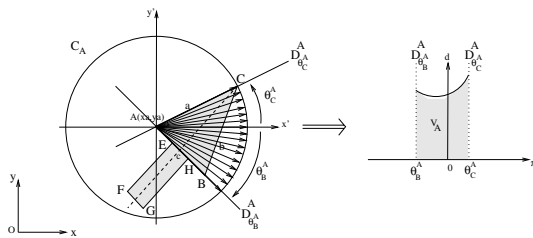
An arrow is assumed to be an isocel triangle *T* with a linked rectangle *R*, verifying [P5]. Let us take the point *A* of *T*. *Sec(a, c)* is assumed to be the sector defined from *[A, C]* and *[A, B]* which includes *T* from *A*. We set  $\theta_B^A = \arctan(y_B - y_A/x_B - x_A) + k\pi$  with  $x_B - x_A \neq 0$ , the angle between *(A, B)* and *(O, x)* and  $\theta_C^A = \arctan(y_C - y_A/x_C - x_A) + k\pi$  with  $y_C - y_A \neq 0$  the angle between *(A, C)* and *(O, x)*.

Remark: in the following expressions, the value of angle  $\theta$  is always supposed included in *Sec(a, c)* to simplify the expressions – that is extended in this paper to any sector –.

From [P1] we have for any segment joining two points in *T*, every point on the segment must also lie within *T*. That is extended to:

**[P6]** Let us take the pencil of lines, noted  $\mathcal{L}^A = \{D_\theta^A\}_{\theta \in [0, \pi]}$ , including *A*. There exists a set of segments  $I_A^\theta$  beared by pencil of lines contained in *Sec(a, c)* describing enterly *T*.

Let us note  $C_A$  the circle centered in *A* of radius  $r = \max(a, c)$  (figure 2). All the segments  $I_A^\theta$ , verifying [P6], are including in  $C_A$ :  $V_A = \{I_A^\theta\}_{\theta \in [\theta_B^A, \theta_C^A]}$  is set as the definition of *T* from *A* (figure 2).



**Figure 2. Signature  $V_A$ .**

The same scheme is performed to define  $V_B$ .  $V_C$  includes also the definition of *R* (that is using  $C_C$  centered in *C* of radius  $r = \max(a, d(C, F))$ ). In this case *T* and *R* are enterly described (*CEFGH* is convex).

In image processing  $V_A$ ,  $V_B$  and  $V_C$  will be computed in raster data and matched with their theoretical approximation function. The definition of such functions is now presented.

Let us consider any triangle  $T'$  defined by three points:  $X_1$ ,  $X_2$  and  $X_3$ .  $X_1$  is assumed to be the origin of the orthogonal frame,  $\theta'$  and  $\theta''$  the angles described by  $[X_1, X_2]$  and  $[X_1, X_3]$  from this frame. We also set  $x = d(X_1, X_2)$  and  $y = d(X_1, X_3)$  with  $d$  the Euclidean distance. The aim is to reach a continuous approximation, noted  $S_X$ , of the signature  $V_X$  previously described. That is to have a new representation of the segment  $[X_2, X_3]$  from  $X_1$ .

The continuous function  $S_{X_1}(\theta) : [\theta', \theta''] \rightarrow \mathbf{R}_*^+$ , associated to  $V_{X_1}$  is given by:

$$f(\theta, \langle x, y, \theta', \theta'' \rangle) = \frac{x \cdot y \cdot \sin(\theta' + \theta'')}{x \cdot \sin(\theta - \theta') - y \cdot \sin(\theta - \theta'')}$$

Let us now consider the points *A*, *B* and *C* of triangle *T*.

The continuous function  $S_A(\theta) : [\theta_B^A, \theta_C^A] \rightarrow \mathbf{R}_*^+$ , associated to  $V_A$ , is:

$$S_A(\theta) = f(\theta, \langle a, c, \theta_B^A, \theta_C^A \rangle)$$

The continuous function  $S_B(\theta) : [\theta_C^B, \theta_A^B] \rightarrow \mathbf{R}_*^+$ , associated to  $V_B$ , – in this case, the orthogonal frame is referred to  $B(x_B, y_B)$  –, corresponds to:

$$S_B(\theta) = f(\theta, \langle b, c, \theta_C^B, \theta_A^B \rangle)$$

The continuous function  $S_C(\theta) : [\theta_A^C, \theta_B^C] \rightarrow \mathbf{R}_*^+$  is defined as follows ( $a = b$ ):

If *R* is an empty rectangle:

$$S_C(\theta) = f(\theta, \langle a, a, \theta_A^C, \theta_B^C \rangle)$$

with  $\theta_A^C \leq \theta \leq \theta_B^C$  is defined from the frame  $C(x_C, y_C)$ .

else ( $R \neq \emptyset$ ), 5 triangles are processed (see figure 3):

$a' = d(C, E) = d(C, H)$ ,  $a'' = d(C, F) = d(C, G)$

The expression of  $S_C(\theta)$  is given by:

$$S_C(\theta) \rightarrow \begin{cases} \theta_A^C \leq \theta < \theta_E^C & f(\theta, \langle a, a', \theta_A^C, \theta_E^C \rangle) \\ \theta_E^C < \theta \leq \theta_F^C & f(\theta, \langle a', a'', \theta_E^C, \theta_F^C \rangle) \\ \theta_F^C < \theta \leq \theta_G^C & f(\theta, \langle a'', a'', \theta_F^C, \theta_G^C \rangle) \\ \theta_G^C < \theta \leq \theta_H^C & f(\theta, \langle a'', a', \theta_G^C, \theta_H^C \rangle) \\ \theta_H^C < \theta \leq \theta_B^C & f(\theta, \langle a', a, \theta_H^C, \theta_B^C \rangle) \end{cases}$$

**[P7]**  $V_A \Leftrightarrow S_A$ ,  $V_B \Leftrightarrow S_B$  and  $V_C \Leftrightarrow S_C$

□ Proofs of  $\Rightarrow$  and  $\Leftarrow$  are directly based on the differential definition of  $V_A$  ( $V_B$  and  $V_C$ ) and  $S_A$  ( $S_B$  and  $S_C$ ).

Now we can consider the global signature *S* associated to an arrow as the union of signatures: Let  $t_\pi$  be a translation of  $\pi$ ,  $S(T^{ABC}) = S_A \cup t_\pi(S_C) \cup S_B$  is a complete signature of *T*. *S* is unique and we can directly deduce from [P7] that: Let  $t_\pi$  be a translation of  $\pi$ ,  $V(T^{ABC}) = V_A \cup t_\pi(V_C) \cup V_B$  is a complete signature of *T* ( $V \Leftrightarrow T$ ).

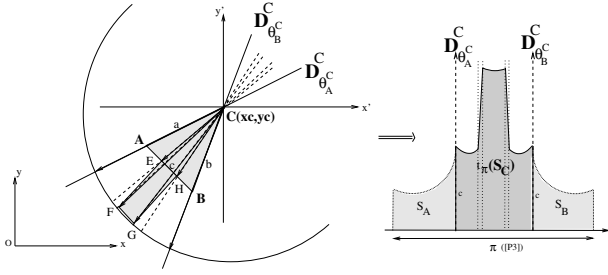


Figure 3. Signature  $S_C(\theta)$ .

The following properties are interesting to estimate criteria:

[P8]  $V_A \cap t_\pi(V_C) = V_B \cap t_\pi(V_C) = \{c\}$ ,  $V_A \cap V_B \cap V_C = \emptyset$ .

[P9]  $|V_A| = |V_B|$ .

□ From [P2,P6,P7],  $\int_{\theta_C^A}^{\theta_B^A} S_A(\theta)d\theta = \int_{\theta_C^B}^{\theta_B^B} S_B(\theta)d\theta$  is true.

[P10] There exists a symmetry, noted  $Sym$ , such that  $Sym(S_A) = S_B$ .

□ Let us consider  $V_A$  and  $V_B$  as being two sub-vector fields. From [P2,P5] we have  $\forall I_A^\theta \in V_A, \exists$  a unique  $I_B^{\theta'} \in V_B$  such that  $Sym(I_A^\theta) = I_B^{\theta'}$  and  $Sym(I_B^{\theta'}) = I_A^\theta$ .

[P11]  $S_C$  is a symmetric map. That is  $\forall I_C^\theta \in V_C, \exists$  a unique  $I_C^{\theta'} \in V_C$  such that  $Sym(I_C^\theta) = I_C^{\theta'}$ . There also exists a unique  $I_C^\theta$  such that  $Sym(I_C^{\theta'}) = I_C^\theta$ .

□ From [P2,P5,P7]; let us consider  $R = \emptyset$ , it is easy to show that  $S_C(\theta^C - \theta'_C) = S_C(\theta^C + \theta'_C)$  with  $\theta'_C = \frac{\theta_A^C + \theta_B^C}{2}$ . If  $R \neq \emptyset$  the reasoning is similar for each symmetric pair.

The signatures  $V$  and  $S$  keep fundamental geometric properties, useful in a pattern recognition process, as:

[P12]  $S$  is invariant by translation.

□ Let  $t_{\vec{u}}$  be a translation of vector  $\vec{u}$ :

$f(\theta, \langle d(X1, X2), d(X1, X3), \theta', \theta'' \rangle) =$

$f(\theta, \langle d(t_{\vec{u}}(X1), t_{\vec{u}}(X2)), d(t_{\vec{u}}(X1), t_{\vec{u}}(X3)), \theta', \theta'' \rangle)$

[P13] The angle of rotation can be deduced in  $S$ .

□ A rotation of  $\alpha$  is assumed to be a translation of  $\alpha$  in  $S$ :

$f(\theta, \langle x, y, \theta' + \alpha, \theta'' + \alpha \rangle) = f(\theta, \langle x, y, \theta', \theta'' \rangle)$

implies  $f$  is invariant by rotation.

[P14]  $S$ -normalized is invariant by scale factor.

□ Let  $k$  be the homothety factor, we have:

$f(\theta, \langle k \cdot x, k \cdot y, \theta', \theta'' \rangle) = k \cdot f(\theta, \langle x, y, \theta', \theta'' \rangle)$

## 4. Arrow Detection

### 4.1. Criteria

Let  $A, B$ , and  $C$  be a triplet of points to valuate. The previous part allows to define several measures to check an arrow. The set of criteria used here is the following:

$N = \{Sym, Card_{AB}, \bar{O}ver, Area, Proto\}$

$Sym$	$\max_{t \in [1, p/2]} \left\{ \frac{\sum_{i=1, p} \min(V^{t+i}, V^{t-i})}{\sum_{i=1, p} \max(V^{t+i}, V^{t-i})} \right\}$
$Card$	$\min( V_A ,  V_B ) / \max( V_A ,  V_B )$
$\bar{O}ver$	$1 - \frac{\sum_{i=1, p} \inf\{V_A^i, V_B^i, t_\pi(V_C^i)\} - 2 \cdot d(A, B)}{\sum_{i=1, p} \sup\{V_A^i, V_B^i, t_\pi(V_C^i)\}}$
$Area$	$1 - (K - H)/H$
$Proto$	$D(V, S)$

The degree of symmetry ( $Sym$ ) is computed to have an approximation of [P10] and [P11]. From [P9] the cardinality of  $V_A$  is close to the one of  $V_B$  ( $Card_{AB}$ ). An estimation of the degree of non-overlap ( $\bar{O}ver$ ) is required to check [P8]. Let  $K$  be the common area carried out from the scans during the definition of  $V_A, V_B$  and  $V_C$ , it should be close to the calculation of the Heron formula  $H$  from these points ( $Area$ ). At last, the numerical signature  $V$  given by the triplet is matched with its theoretical prototype  $S$  ([P7]).  $D$  is a distance and  $\bar{S}$  is a discrete approximation of  $S$ .

### 4.2. Choquet Integral

A first idea to merge these criteria may be to use a weighted sum. We can directly think that  $Sym, \bar{O}ver$  and  $Proto$  are the main criteria but what is the most important? Furthermore the criteria should be in correlation as they are defined from the basic structure of the arrow. Such a dependence can not be formalized using classical combinaisons. Our aggregation was based on the Choquet integral concept whose use was proposed by many authors (see [7]).

Let  $N = \{1, 2, \dots, n\}$  be a finite set of  $n$  criteria –  $n = 5$  in our study – and let  $A = \{a, b, \dots\}$  be a finite set of alternatives – that is the values reached from all the triplets of points found in a document –.

Let  $v$  be a monotonic set function – also called fuzzy measure – on  $N$  such that [12, 16]:  $v(\emptyset) = 0, v(N) = 1$  and  $S \subseteq T \Rightarrow v(S) \leq v(T)$ . Let us now consider an alternative  $x^i = (x_1^i, \dots, x_n^i)$ , where for any criterion  $j, x_j^i$  is the score of  $i$  related to  $j$  [10].

$\forall v \in \mathcal{P}_N$ , the Choquet integral of  $x \in \mathbb{R}^n$  is defined by:  $\mathcal{C}_v(x) := \sum_{i=1}^n x(i)[v(A_{(i)}) - v(A_{(i+1)})]$

where  $(\cdot)$  is a permutation of  $N$ , carried out from the values obtained by the application of each criterion on a triplet, such that  $x(1) \leq \dots \leq x(n)$  and  $A(i) = \{(i), \dots, (n)\}$  represents the  $[i..n]$  associated criteria in increasing order.

By definition  $v(\emptyset) = 0$  and  $v(\{1, \dots, n\}) = 1$ . The Lemme method is generally used to set the  $2^N - 2$  remaining values [7, 12] but can provide incoherent results [8]. The fuzzy measure was defined from samples (arrows and other symbols) from the optimal algorithm proposed by Grabisch [8]. Significant criteria, negative and positive interactions can be established from the  $2^N - 2$  values reached. The importance of each criteria [7] is based on the definition proposed by Shapley [15] extended to fuzzy measure in [13]. An importance index greater than 1

describes an attribute more important than the average:

Criterion	<i>Sym</i>	<i>Card</i>	<i>Over</i>	<i>Area</i>	<i>Proto</i>
Sh. value	0.72	0.51	<b>1.03</b>	0.76	<b>1.97</b>

This definition has been extended in [13] to measure the interaction index between criteria (values in  $[-1, 1]$ ):

$S, C$	-0.05	$S, \bar{O}$	-0.45	$S, A$	-0.14
$S, P$	<b>0.55</b>	$C, \bar{O}$	-0.11	$C, A$	0.19
$C, P$	<b>0.88</b>	$\bar{O}, A$	<b>0.54</b>	$\bar{O}, P$	0.02
$A, P$	0.41	—	—	—	—

Such tables show that the main criteria are  $\bar{O}_{ver}$  and *Proto* (not *Sym*!) and high correlations exist between criteria (between *Card* and *Proto* and *Area* and  $\bar{O}_{ver}$ ).

## 5. Arrow Recognition

### 5.1. Cardinal lists

For efficiency reasons, we use Bresenham's algorithm [4] in our implementation, as it is a fast method which minimizes error in drawing lines on integer grid points. First, let us suppose arrows are not occluded in the drawing and the angle of the sector defined from *C* is acute ( $Sec(a < b) < \pi/2$ ). Four scans of the plane from four cardinal directions are enough to locate any edge as a triangle is convex by definition ([P1]). All the segments  $V_{I_{ij}}$  ([P6]) are labeled with a level greater to 1 during the definition of the signature to limit the search to the location of new starting points. If the angle is obtuse the method may require the study of double points to rightly locate *C*.

### 5.2. Robustness to Noise

The starting area from *A*, *B* and *C* might be moved using noisy data. A disk search area ( $r = 1$ ) is set to compute the better starting points. Figure 4 shows a degraded arrow and the superimposed signatures reached. We can remark that the shape of the two signatures are close. Such a result is underlined by the high values reached from each criterion.

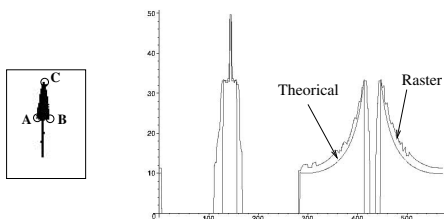


Figure 4. Signatures of Noisy Arrow.

Criterion	<i>Sym</i>	<i>Card</i>	<i>Over</i>	<i>Area</i>	<i>Proto</i>
Values	0.88	0.85	0.94	1.	0.88

## 5.3. Occlusions

If *A*, *B* and *C* are not occluded, our approach allows to find correctly an arrow even if parts of the symbol are occluded (figure 5.a) because the additive data do not interfere on the calculation of the general structure. However the edges of arrows can be occluded in line drawings (figure 5.b). Two approaches were studied to estimate the location of occlusion points. In the first hand, the new points were directly calculated from the intersection of the lines defined from the double points found in the cardinal lists (figure 5.c). In the other hand skeletonization and pruning steps [9] are required to manage with more important occlusions. In these case the junction edges found (figure 5.d) are assumed to be potential arrow points. The list of occlusion points found is appended to each cardinal list.

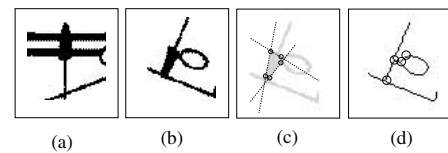


Figure 5. Occluded Arrows.

### 5.4. Rectangle Estimation

The rectangle (EFGH) is taken into account during the calculation of overlap and symmetry criteria. Nevertheless using noisy data an estimation of the rectangle is required to compute *Proto* properly. According to us an arrow without rectangle is assumed to be a degraded arrow and should have a lower recognition rate. Despite this consideration the lengthier the rectangle, the less the accuracy of *Proto* estimation is. Two boundaries are defined from  $V_C$  where the rectangle should be considered (see figure 6).

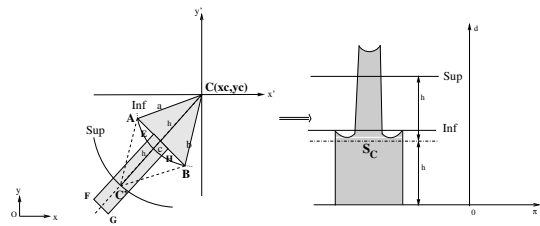


Figure 6. Rectangle signature boundaries.

$h$  is the altitude. The width [EH] of the rectangle is directly computed from  $V_c$  as the maximal symmetric pair from *Inf* up to *Sup* (calculated from the rhombus ( $CBC'A$ )). The new  $V_C$  signature is truncated of greater values. *Inf* may be greater than *Sup* when handling with obtuse angles. In this case, the two values are permuted.

## 5.5. Triplet Checking

The aim is to define a set of triplets to evaluate from four lists of points. So we have set some basic constraints to decrease drastically the number of choices:

- The points should be in the same connected component.
- Each point should be “visible” by the others.
- The selected points belong to distinct lists.
- If an arrow is found the points are removed from the cardinal lists.

When a triplet is found, three combinations of points are evaluated from the Choquet integral to determine which point is the more probable head of the arrow.

## 5.6. Line Drawing Application

Our approach has been applied on several types of line drawings with success and problems only occur with large occlusions. In the figure 7.a the circles represent the list of triplets found from the cardinal lists, the squares refer to the detected arrows (scores greater than 0.75 – fixed by the learning step –) and the letter *f* is a symbol badly recognized using classical weighted sum. Another example is provided in the figure 7.b.

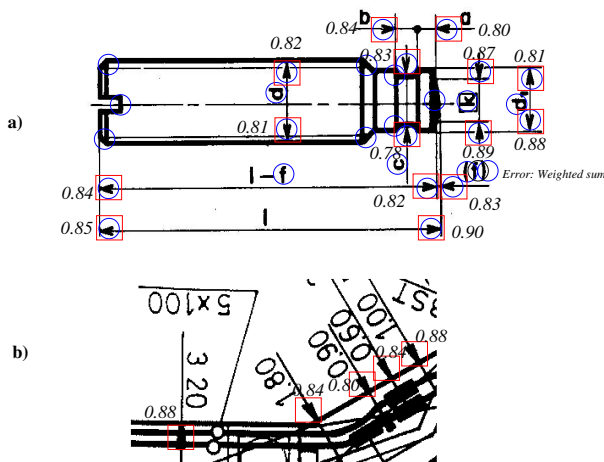


Figure 7. Detection of Arrows.

## 6. Conclusion

We have shown in this paper that the definition of both angular and theoretical signatures and the checking of axiomatic properties allowed to reach a robust detection of arrows. The experimental study shows the promising aspect of our approach. Currently, we focus on sub-pixel definition to decrease the aliasing given by bresenham’s algorithm and we want to extend our approach to the extraction of text boxes in line drawings.

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