A Bandwidth-Power Efficient Modulation Scheme Based on Quaternary Quasi-Orthogonal Sequences

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***Abstract*—A novel modulation scheme suitable for noncoherent demodulation based on quaternary quasi-orthogonal sequences is proposed. Compared to orthogonal modulation, the controlled quasi-orthogonality between the sequences allow significantly increased bandwidth efficiency with little or no degradation in power efficiency. A hardware efficient demodulator structure using fast Walsh transforms is also presented.**

***Index Terms*—Modulation, noncoherent demodulation, quasi-orthogonal sequences (QOSs).**

I. Introduction

ORTHOGONAL modulation (OM) schemes such as M-ary frequency shift keying (MFSK) [2] and M-ary Walsh modulation [3] combined with noncoherent demodulation are frequently used in communication systems where reliable phase recovery is not practically feasible or undesirable. It is well known that M-ary OM asymptotically achieves channel capacity under the AWGN channel as $M \rightarrow \infty$ with both coherent and noncoherent demodulation [2]. However, OM requires bandwidth which increases linearly with M for a given data rate, rendering them unsuitable for communication systems requiring bandwidth efficiency.

Modifying conventional OM schemes in order to increase the bandwidth efficiency by sacrificing the orthogonality among the signals have previously been addressed. Two examples include MFSK with nonorthogonal frequency spacing [2] and multitone MFSK (mMFSK) [4], both succeeding in increasing the bandwidth efficiency but only at the cost of significant loss in power efficiency.

In this letter, we propose a novel modulation scheme suitable for noncoherent demodulation based on quaternary quasi-orthogonal sequences (QOSs) [1] which will be referred to as quasi-orthogonal modulation (QOM). Compared to OM, we are able to achieve drastically increased bandwidth efficiency with little or no loss in power efficiency. A hardware efficient demodulator structure using the fast Walsh transforms (FWT) [5] is also presented.

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II. Quaternary QOSs

Let $\mathbf{x} = (x(0), x(1), \ldots, x(N-1))$ be a sequence of length N, where each $x(n)$ is referred to as a *chip*. The sequence $\mathbf{x}$ is called a *binary* sequence if $x(n) \in GF(2)$ and a *quaternary* sequence if $x(n) \in \mathbb{Z}_4$, the ring of integers modulo 4. Let $\mathcal{W}_m = \{\mathbf{w}_i | i = 0, 1, \ldots, 2^m - 1\}$ for a given positive integer m, be the set of binary Walsh sequences of length $N = 2^m$. Then, let $2\mathcal{W}_m = \{2\mathbf{w}_i | i = 0, 1, \ldots, 2^m - 1\}$ be the set of quaternary sequences derived from $\mathcal{W}_m$ by multiplying each sequence in $\mathcal{W}_m$ by 2 over $\mathbb{Z}_4$.

We define the correlation between two quaternary sequences $\mathbf{x}$ and $\mathbf{y}$ as $R_{\mathbf{x},\mathbf{y}} \triangleq \sum_{n=0}^{N-1} j^{x(n)-y(n)}$ with $j = \sqrt{-1}$. The set $\mathcal{F} = \{\mathbf{f}_i | i = 0, 1, \ldots, M-1\}$ of M quaternary QOSs of length $N = 2^m$ is then defined as a set of quaternary sequences satisfying the following two properties[1], [2] [1]:

1) $\mathcal{F}$ contains $2\mathcal{W}_m$;
2) for any two distinct sequences $\mathbf{f}_i, \mathbf{f}_l \in \mathcal{F}$, we have $|R_{\mathbf{f}_i,\mathbf{f}_l}| \leq \sqrt{N}$.

It was shown in [1] that a set of quaternary QOSs may be constructed based on an appropriate permutation of *Family* $\mathcal{A}$ sequences [6] and their cyclic shifts. The resulting set of quaternary QOSs of length N has a family size of $M = N^2$ and can be partitioned into N nonoverlapping equal size subsets $\mathcal{F}_L = \mathbf{c}_L \oplus_4 2\mathcal{W}_m = \{\mathbf{f}_{LN+i} | \mathbf{f}_{LN+i} = \mathbf{c}_L \oplus_4 2\mathbf{w}_i, 2\mathbf{w}_i \in 2\mathcal{W}_m, i = 0, 1, \ldots, N-1\}$, $L = 0, 1, \ldots, N-1$. Here, $\mathbf{c}_L$ is the defining *masking sequence*[3] for $\mathcal{F}_L$ and $\oplus_4$ denotes elementwise addition in $\mathbb{Z}_4$. The masking sequences for $N = 4, 8, 16,$ and 32 are given in Table I. Clearly, any two distinct QOSs contained in the same subset are orthogonal to each other. On the other hand, the correlation between any two QOSs contained in different subsets can be shown to take on only the four values $\pm\omega\sqrt{N}$, $\pm j\omega\sqrt{N}$ where $\omega = 1$ for even m and $\omega = e^{j\pi/4}$ for odd m [1].

III. System Model

For QOM, $k = \log_2 M$ data bits are used to choose a sequence from the set of M quaternary QOSs of length $N = \sqrt{M}$. The selected sequence is then transmitted using QPSK modulation[4]. The receiver consists of a chip pulse matched filter

[1]A third property called the *window property* is also included in [1] which places an upper bound on the absolute value of the *partial* correlation between any $\mathbf{f}_i$ in $\mathcal{F}$ but not in $2\mathcal{W}_m$ and any $2\mathbf{w}_i$ in $2\mathcal{W}_m$.

[2]If we change condition (b) to $|R_{\mathbf{f}_i,\mathbf{f}_l}| < \sqrt{N}$, then $\mathcal{F}$ reduces to $2\mathcal{W}_m$ since for any quaternary sequence $\mathbf{x}$ not contained in $2\mathcal{W}_m$, there exists at least one sequence $2\mathbf{w}_i \in 2\mathcal{W}_m$ such that $|R_{\mathbf{x},2\mathbf{w}_i}| \geq \sqrt{N}$ [1]. Thus, condition (b) implies that the correlation between any two distinct sequences in $\mathcal{F}$ should be as small as possible while not reducing $\mathcal{F}$ to $2\mathcal{W}_m$.

[3]For details on deriving the masking sequences, refer to [1].

[4]QPSK modulation of chip $f(n)$ is assumed to be given as $j^{f(n)}$.

TABLE I
MASKING SEQUENCES FOR QUATERNARY QOSS OF LENGTH N

N	Masking Sequences of Length N	
4	$c_0 = (0000)$, $c_1 = (0323)$,	$c_2 = (0233)$, $c_3 = (0332)$
8	$c_0 = (00000000)$,	$c_4 = (03211212)$
	$c_1 = (01212123)$,	$c_5 = (01321221)$
	$c_2 = (02112231)$,	$c_6 = (01122312)$
	$c_3 = (02223111)$,	$c_7 = (02131122)$
16	$c_0 = (0000000000000000)$,	$c_8 = (0130100321323001)$
	$c_1 = (0301210103230301)$,	$c_9 = (0211330231000013)$
	$c_2 = (0033021302310011)$,	$c_{10} = (0321101003031032)$
	$c_3 = (0002333100021113)$,	$c_{11} = (0033201311003120)$
	$c_4 = (0000002213131331)$,	$c_{12} = (0101303032102103)$
	$c_5 = (0103010310303212)$,	$c_{13} = (0312011023100330)$
	$c_6 = (0110033030211023)$,	$c_{14} = (0233112000313100)$
	$c_7 = (0310120110032330)$,	$c_{15} = (0020333131110200)$
32	$c_0 = (00000000000000000000000000000000)$	
	$c_1 = (01010123232301232301232301232323)$	
	$c_2 = (02130011203122332031223320312233)$	
	$c_3 = (02201331202031312200331122223333)$	
	$c_4 = (02222000313313112202220233313331)$	
	$c_5 = (02222022222000203313333131333111)$	
	$c_6 = (03232123323032122321230330101210)$	
	$c_7 = (02332231223102333320310013221102)$	
	$c_8 = (03233230321203012123321210120323)$	
	$c_9 = (02312233332213203120112200332031)$	
	$c_{10} = (03213210121223233030232321033210)$	
	$c_{11} = (03302310233221301221102332231203)$	
	$c_{12} = (01323201120321302332322312210330)$	
	$c_{13} = (02133322332220311122023102313300)$	
	$c_{14} = (01211232032332123032232132300301)$	
	$c_{15} = (03102312213223120332233003320112)$	
	$c_{16} = (00331302223331022013332202131122)$	
	$c_{17} = (02003133313320222022313313112022)$	
	$c_{18} = (02200202331133333113313122002222)$	
	$c_{19} = (03212103301230123232101023232323)$	
	$c_{20} = (03322312120110033203122323302132)$	
	$c_{21} = (03323023122321321223031021103023)$	
	$c_{22} = (01323223033012031221213032012332)$	
	$c_{23} = (01123221231232030332300121323023)$	
	$c_{24} = (00131322330202112231132233022033)$	
	$c_{25} = (02021331113322223333002220023131)$	
	$c_{26} = (03230323010323211210303232123212)$	
	$c_{27} = (01322130300332230312013232231221)$	
	$c_{28} = (00133122021133203302023313222213)$	
	$c_{29} = (03031032032132320123121223231230)$	
	$c_{30} = (00310233112013220233221313223302)$	
	$c_{31} = (00023313202213331113220231330222)$	

followed by M sequence matched filters. Without loss of generality, we assume that sequence $\mathbf{f}_0$ is transmitted. Assuming perfect chip synchronization at the receiver, the nth chip pulse matched filter output under the AWGN channel is given by $r(n) = j^{f_0(n)}e^{j\theta} + v(n)$, $n = 0, 1, \ldots, N-1$ where θ represents the random carrier phase assumed to be uniformly distributed over $[-\pi, \pi)$. Let E_b be the received energy per bit and N_0 be the single-sided power spectral density of the AWGN. Then, $v(n)$ is a zero mean complex Gaussian random variable with independent real and imaginary parts, each with variance $(2k/N \cdot E_b/N_0)^{-1}$ representing the contribution of the AWGN. The output of the sequence matched filter corresponding to sequence $\mathbf{f}_l$ is then given as

$$U_l = \frac{1}{N}\sum_{n=0}^{N-1} r(n) j^{-f_l(n)} \tag{1}$$

$$= \begin{cases} e^{j\theta} + z_0, & l = 0 \\ z_l, & l = 1, \ldots, \sqrt{M} - 1 \\ M^{-(1/4)} e^{j(\theta + \phi_{0,l})} + z_l, & l = \sqrt{M}, \ldots, M - 1. \end{cases} \tag{2}$$

Here, $\phi_{0,l}$ is the phase of $R_{\mathbf{f}_0, \mathbf{f}_l}$ and $z_l \triangleq \sum_{n=0}^{N-1} v(n) j^{-f_l(n)}$ are zero mean complex Gaussian random variables with independent real and imaginary parts, each with variance $(2kE_b/N_0)^{-1}$. If $\mathbf{f}_i$ and $\mathbf{f}_l$ are orthogonal to each other, then z_i and z_l are uncorrelated. However, if $\mathbf{f}_i$ and $\mathbf{f}_l$ are not orthogonal, then the correlation between z_i and z_l can be easily shown to be $e^{j\phi_{i,l}}/(M^{1/4}kE_b/N_0)$.

Note that $\lim_{M\to\infty} M^{-1/4} = 0$ and that the correlation between the noise terms $z_0, \ldots, z_{M-1}$ goes to zero as $M \to \infty$. This implies that the symbol error rate (SER) of QOM asymptotically approaches that of OM as $M \to \infty$. Therefore, as with OM, QOM asymptotically achieves channel capacity as $M \to \infty$ under AWGN.

IV. HARDWARE EFFICIENT DEMODULATOR STRUCTURE

Since $\mathbf{f}_{LN+i} = \mathbf{c}_L \oplus_4 2\mathbf{w}_i$, the sequence matched filter output (1) may be rewritten as $U_{LN+i} = 1/N \sum_{n=0}^{N-1} g_L(n)(-1)^{w_i(n)}$ where $g_L(n) \triangleq r(n) j^{-c_L(n)}$. Thus for a given $\mathbf{g}_L$, U_{LN+i} represents the Walsh transform of $\mathbf{g}_L$ and thus may be efficiently calculated using the FWT. Therefore, the M sequence matched filter outputs may be obtained by first calculating $\mathbf{g}_L$ for $L = 0, 1, \ldots, N-1$ and then applying an N-point FWT to each $\mathbf{g}_L$. Note that a complex N-point FWT requires $2N \log_2 N$ real addition operations [5]. Thus, for noncoherent demodulation, a QOM demodulator based on FWTs requires $N(2N \log_2 N) = M \log_2 M$ real addition operations, whereas a demodulator based on M separate sequence matched filters requires $2M(\sqrt{M} - 1)$ real addition operations.

V. PERFORMANCE

A. Symbol Error Rate

Since it is difficult to analytically evaluate the exact SER for general nonorthogonal modulation except for some special cases [2], we resort to the union bound. Without loss of generality, assume that $\mathbf{f}_0$ is transmitted. Then, since $\mathbf{f}_1, \ldots, \mathbf{f}_{\sqrt{M}-1}$ are orthogonal to $\mathbf{f}_0$, the pairwise error probability between $\mathbf{f}_0$ and $\mathbf{f}_1, \ldots, \mathbf{f}_{\sqrt{M}-1}$ is given by

$$P_{\text{orth}} \triangleq \Pr(|U_l| > |U_0|) = \frac{1}{2}\exp\left(-\frac{\gamma}{2}\right), \quad l = 1, \ldots, \sqrt{M} - 1 \tag{3}$$

where $\gamma \triangleq kE_b/N_0$ [2]. For the remaining sequences (not orthogonal to $\mathbf{f}_0$), the pairwise error probability is given by [2]

$$\begin{aligned} P_{\text{nonorth}} &\triangleq \Pr(|U_l| > |U_0|), \quad l = \sqrt{M}, \ldots, M - 1 \\ &= Q_1\left(\sqrt{(1-\beta)\frac{\gamma}{2}}, \sqrt{(1+\beta)\frac{\gamma}{2}}\right) \\ &\quad - \frac{1}{2}\exp\left(\frac{-\gamma}{2}\right) I_0\left(M^{-1/4}\frac{\gamma}{2}\right) \end{aligned} \tag{4}$$

where $\beta \triangleq \sqrt{1 - M^{-1/2}}$, $Q_1(a, b)$ is the first-order generalized Marcum Q-function and $I_0(x)$ is the ze-

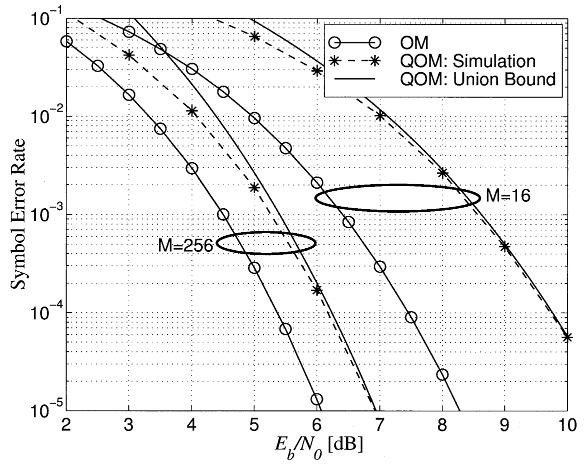

Fig. 1. SER performance of OM and QOM under AWGN with noncoherent demodulation.

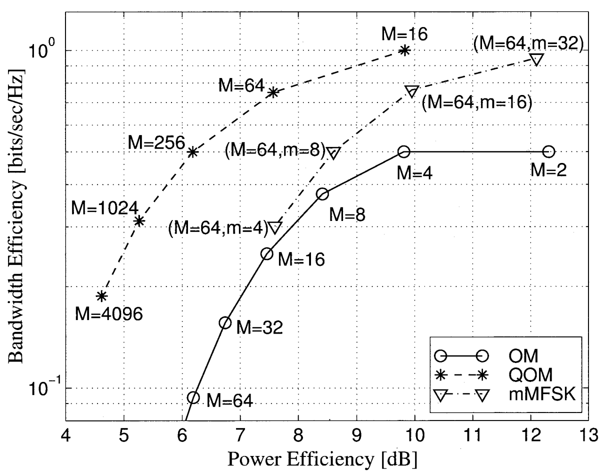

Fig. 2. Bandwidth efficiency vs. power efficiency under AWGN.

roth-order modified Bessel function of the first kind [7]. Hence, the union bound on the SER is given by $P_s \leq \left(\sqrt{M}-1\right) P_{\text{orth}} + \left(M-\sqrt{M}\right) P_{\text{nonorth}}$.

B. Bandwidth Efficiency

Let R/W be the bandwidth efficiency of a modulation scheme measured in bits/s/Hz [7] where R is the transmission bit rate in bits/s and W denotes the occupied bandwidth. It is well known that the bandwidth efficiency of OM is approximately given by $(\log_2 M)/M$ [7]. Note that for a given modulation order M, QOM requires $1/\sqrt{M}$ times as many chips per symbol as does orthogonal Walsh modulation. Hence, the bandwidth efficiency of QOM is $\sqrt{M}$ times larger than that of OM and is approximately given by $(\log_2 M)/\sqrt{M}$.

VI. NUMERICAL RESULTS

Fig. 1 shows the SER of OM and QOM with noncoherent demodulation under the AWGN channel. As expected, the SER of QOM approaches that of OM as M increases. For instance, the difference between QOM and OM in the E_b/N_0 required to achieve the SER value of 10^{-4} is 2.3 dB for $M = 16$, while the difference is only 0.8 dB for $M = 256$.

Since OM and QOM have different bandwidth efficiencies for a given modulation order, a more meaningful comparison of the two modulation schemes may be made on the bandwidth power efficiency plane [7]. Here, we measure the power efficiency by the E_b/N_0 required to achieve the SER value of 10^{-4}. Fig. 2 shows the bandwidth versus power efficiency curves for various modulation schemes including QOM. Note that by using QOM, significant improvements in bandwidth efficiency compared to OM may be achieved with minimal degradation in power efficiency. Specifically, we observe that by employing $4M$-ary QOM instead of M-ary OM ($M = 2^{2m}$, m a positive integer), the bandwidth efficiency is increased by a factor of $2^{m-1}(1 + 1/m)$ with negligible loss in power efficiency. The price paid for such improvement is the increase in demodulator complexity. Based on the results given in Section IV, it can easily be shown that the demodulator based on FWT for $4M$-ary QOM requires $2(1 + 1/m)$ times more real addition operations compared to a demodulator for M-ary OM. For example, by employing 256-ary QOM instead of 64-ary OM, the bandwidth efficiency is increased 16/3 fold with negligible loss in power efficiency but with a 8/3 fold increase in demodulator complexity. We believe that this is a very small price to pay for over a 5 fold increase in bandwidth efficiency.

A different approach aimed at improving the bandwidth efficiency of OM with noncoherent demodulation is the mMFSK [4]. Here, m tones out of the M FSK tones are simultaneously transmitted to send $\log_2 \binom{M}{m}$ bits per channel use. The bandwidth power efficiency curve for mMFSK for $M = 64$ is also shown in Fig. 2. The mMFSK also succeeds in obtaining increased bandwidth efficiency over OM but only at the cost of significant loss in power efficiency. For similar bandwidth efficiency, QOM is clearly more power efficient than mMFSK.

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