Sciencexpress Reports

Rapid growth of seed black holes in the early universe by supra-exponential accretion

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Mass accretion by black holes (BHs) is typically capped at the Eddington rate, when radiation's push balances gravity's pull. However, even exponential growth at the Eddington-limited *e*-folding time t_E ~ few×0.01 Gyr, is too slow to grow stellar-mass BH seeds into the supermassive luminous quasars that are observed when the universe is 1 Gyr old. We propose a dynamical mechanism that can trigger supra-exponential accretion in the early universe, when a BH seed is trapped in a star cluster fed by the ubiquitous dense cold gas flows. The high gas opacity traps the accretion radiation, while the low-mass BH's random motions suppress the formation of a slowly draining accretion disk. Supra-exponential growth can thus explain the puzzling emergence of supermassive BHs that power luminous quasars so soon after the Big Bang.

Optically bright quasars powered by accretion onto black holes (BHs) are now detected at redshifts as high as $z \sim 7$, when the universe was 6% of its current age (<1 Gyr) (*1*). Their luminosities imply supermassive BHs (SMBHs) with $M_\bullet \gtrsim 10^9\,M_\odot$ (*2*). The main obstacles to assembling such SMBHs so rapidly are the low masses of the hypothesized initial seed BHs, born of first-generation (Pop III) stars, coupled with the maximal growth rate for radiatively efficient accretion, the Eddington limit (*3–5*). Proposed ways to circumvent these limitations invoke super-Eddington accretion for brief periods of time (*6*); the ab initio formation of more massive BH seeds (*7–10*) from direct collapse of self-gravitating pre-galactic gas disks at high redshifts (*11–14*); and the formation of a very massive star from runaway stellar mergers in a dense cluster (*15, 16*). Discriminating between these scenarios is challenging, as seed formation redshifts ($z > 10$) are observationally inaccessible. Current data requires finely-tuned, continuous early BH growth and massive initial BH seeds (*17–20*). Recent results from high-resolution simulations of early star formation at $z \sim 15{-}18$ exacerbate the problem by indicating that efficient fragmentation and turbulence (*21–25*) lead to the efficient formation of stellar clusters embedded in the flow, which prevents the formation of massive seeds (>>10 $M_\odot$) by limiting the mass of their potential Pop III progenitor stars. On the other hand, theoretical and numerical results on larger scales suggest that ubiquitous dense cold gas flows (*26*) stream in along filaments and feed proto-galactic cores (*27, 28*). Adaptive mesh refinement simulations track the fate of these sites (collapsed $10^7\,M_\odot$ dark matter halos) from 1 Mpc scale at $z \sim 21$ with resolutions as low as $\sim 2\times10^{-10}$ pc in the central regions. These simulations find isothermal density cusps that reach extreme central densities, with an average density of $\rho_\infty \gtrsim 10^{-16}$ g cm^{-3} ($\gtrsim 10^9$ cm^{-3} for pure H) on 0.1 pc scales (*29*). They also reveal a marginally unstable central gas reservoir of few×$10^5\,M_\odot$ in the inner few pc (*30*), where the dynamical timescale is $\sim 10^6$ yr. Although these simulations are somewhat idealized, we adopt the properties of this high density environment as the initial conditions for the model presented here.

We consider a scenario in which a low-mass Pop III remnant BH remains embedded in a nuclear star cluster fed by dense cold gas flows (*26*) (Fig. 1, Table 1). The stars and gas are virialized in the cluster potential, and the BH is initially a test particle in equipartition with the stars. Gas within the accretion (capture) radius of the BH, $r_a = [2c^2/(c_\infty^2+v_\bullet^2)]r_g$, is dynamically bound to it, where $r_g = G\,M_\bullet/c^2$ is the gravitational radius of the BH, c_∞ is the gas sound speed in the cold flow far from the BH, which is a measure of the depth of the cluster's gravitational potential, and $v_\bullet$ is the BH velocity relative to the gas. Gas bound to the BH inside r_a is not necessarily accreted by it. Prompt accretion requires gas to flow from r_a into the BH on a plunging trajectory with low specific angular momentum $j < j_{\rm ISO} \simeq 4r_g c$, through the innermost stable periapse distance $r_{\rm ISO}$. It is this angular momentum barrier, rather than the Eddington limit, that is the main obstacle to supra-exponential growth.

The BH is more massive than a cluster star, so $v_\bullet^2 < c_\infty^2$, and the accretion flow is quasi-spherical. In the idealized case where the flow is radial and adiabatic, it is described by the Bondi solution (*31*), $\dot{M}_B = \left(\pi/\sqrt{2}\right) r_a^2 \rho_\infty c_\infty$ (adiabatic index $\Gamma = 4/3$ assumed), which can be written compactly in terms of $\mu = M_\bullet/M_i$, where M_i is the initial BH mass, as $\dot{\mu} = \mu^2/t_B$, with timescale $t_B = c_\infty^3/\,2^{3/2}\pi G^2 M_i \rho_\infty$. The stronger-than-linear dependence of the accretion rate on the BH mass leads to a solution that diverges supra-exponentially in a finite time t_B as $\mu(t) = 1/(1 - t/t_B)$. Physical flows, where gravitational energy is released as radiation, are not strictly adiabatic. As the mass accretion rate grows, the local luminosity can far exceed the Eddington luminosity $L_E = 4\pi c G\,M_\bullet/\kappa = \dot{M}_E c^2$ (κ is the gas opacity), for which radiation flux pressure balances gravity. However, radiation produced inside the photon-trapping radius $r_\gamma \sim (\dot{M}/\dot{M}_E) r_g$ is carried with the flow into the BH, because the local optical depth $\tau(r) \sim \kappa\rho(r) r$ makes photon diffusion outward slower than accretion inward (*32*) (which is a manifestation of the $\mathcal{O}(v/c)$ effect of relativistic beaming (*33*)). The luminosity L_∞ that escapes to infinity from $r \gtrsim r_\gamma$ translates to a lowered radiative efficiency $\eta_\gamma = L_\infty/\dot{M} c^2 \sim \min(r_g/r_\gamma, r_g/r_{\rm ISO})$, so it does not exceed $\sim L_E$, thereby allowing supra-exponential Bondi mass accretion rates (*34*). Detailed calculations show that $L_\infty \lesssim 0.6 L_E$ (*35*). The associated radiation pressure enters the dynamics of the flow as an effective reduction of gravity by a factor (1−0.6), and consequently, as a reduction of the accretion rate by $0.4^2 \simeq 1/6$. The supra-exponential divergence of spherical accretion is therefore

$$M_\bullet(t) = \frac{M_i}{1 - t/t_\infty}, t_\infty \simeq 6t_B = \frac{3}{2^{1/2}\pi}\frac{c_\infty^3}{G^2 M_i \rho_\infty}. \quad (1)$$

The typical lifetime of cold flow streams seen in simulations is $\gtrsim 10^7$ yr (*29*)-long enough for the $\sim 10^5\ M_\odot$ of gas in the marginally unstable reservoir on the few pc scale to accrete onto the growing BH, and yet short enough to be relevant for forming $z > 7$ quasars. As a demonstration of concept, we adopt the gas properties found in these simulations, and match a divergence time of $t_\infty \sim$ few$\times 10^7$ yr to the mean physical parameters of the $4 \times 10^4\ M_\odot$ of gas on the 0.25 pc scale. We further assume that half of that mass is in a star cluster with $1\ M_\odot$ stars in a non-singular distribution (a Plummer law), containing a $10\ M_\odot$ stellar BH, and that the escaped radiation from $r > r_\gamma$ does not substantially affect the cold flow on larger scales. This cluster, while dense, is dynamically stable on timescales >> t_∞. Table 1 lists the input physical parameters of the model, the derived gas and cluster properties, and the predicted accretion properties.

Simulations (*29*) find that the cold flow is nearly pressure-supported, and thus has little angular momentum. It is also plausible that there is little net rotation between the gas and the stars that were formed from it. However, unavoidable gravitational interactions of the low-mass BH with cluster stars accelerate it at orbital frequencies of $\Omega(r_\bullet) \sim \Omega_0$, which induce a velocity gradient across the capture radius. Gas captured by the BH then acquires specific angular momentum relative to it, j_a, due to the opposite velocity and density gradients and the velocity-skewed capture cross-section ($j_{a,v} \sim \Omega r_a^2$ and $j_{a,\rho} \sim (d\log\rho/d\log r)_{r_\bullet} \Omega r_a^2$) (*36*). Gas with $j_a > j_{\rm ISO}$ cannot plunge directly into the BH, but rather circularizes at a radius $r_c = j_a^2/G\,M_\bullet$, and accretion then proceeds on the slow viscous timescale rather than the fast, near-free fall timescale. Analytic and numeric results on the capture efficiency of angular momentum by inhomogeneous wind accretion are currently available only in the ballistic (supersonic) limit (*37*, *38*). We adapt these results to provide a rough estimate of the angular momentum in the accretion flow on the BH seed in the subsonic regime considered here (see details in the supplementary materials).

The formal divergence of $j_a/j_{\rm ISO} \propto M_\bullet$ in the test particle limit, where $\Omega \sim \Omega_0$, is reversed by the accretion itself. The non-rotating gas accreted by the BH from the cluster exerts a drag on it, $\ddot{r}_* = -2\gamma_a \dot{r}_\bullet$, where $\gamma_a = \dot{M}/2M_\bullet = M_\bullet(t)/2M_i t_\infty$. This provides additional damping beyond dissipation by dynamical friction against the gas and stars, $\gamma_{\rm df} \propto (M_\bullet(t)/M_c)\Omega_0$ (*39*), which is balanced by the 2-body fluctuations. Accretion damping drives the BH to sub-equipartition energy and angular momentum. 2-body interactions with the cluster stars can re-establish equipartition on the central relaxation timescale $t_{r0} \sim 2 \times 10^7$ yr only as long as the BH growth rate $2\gamma_a$ is slower than the relaxation rate, up to time $t_{\rm eq}/t_\infty = 1 - t_{r0}/t_\infty \sim 0.6$ (Eq. 1), when the BH has grown by a factor of only $t_\infty/t_{r0} \sim 2.5$ to $M_{\rm eq} \simeq 25\ M_\odot$. At later times, j_a is expected to fall below the extrapolated test particle limit, since both the BH wandering radius $\Delta_\bullet$ and the orbital frequency Ω are increasingly damped by dynamical friction and by the accretion drag, which are both $\propto M_\bullet(t)$.

Figure 2 shows the evolution of $j_a/j_{\rm ISO}$ with $M_\bullet$ for the cold flow cluster model of Table 1. A key property of acceleration-induced angular momentum accretion is the existence of a BH mass scale M_0 where $j_a(M_0) \rightarrow 0$ on typical orbits, because the density and velocity gradients cancel each other. For a cluster in dynamic equipartition and pressure-supported gas, M_0 depends weakly only on the shape of the density / potential cluster model near the origin: for the Plummer model, $M_0 \simeq 20\ M_\bullet$. This low mass scale is significant since $M_i < M_0 < M_{\rm eq}$, and therefore $j_a/j_{\rm ISO}$ remains low during the critical stage of early growth, before damping can become effective.

In addition, vector resonant relaxation (*40*, *41*), a rapid process of angular momentum relaxation that operates in nearly spherical potentials, further suppresses the growth of j_a by randomizing the BH's orbital orientation on a timescale (*42*) $t_{vRR} \sim 6 \times 10^5$ yr. This decreases j_a by a factor of $\sqrt{t_\infty / t_{vRR}} \sim 8$ over the divergence time. Randomization by resonant relaxation can be effective until time $t_{\rm rnd}/t_\infty = 1 - t_{vRR}/t_\infty \sim 0.98$. By that time, the BH has grown by a factor of $t_\infty/t_{vRR} \sim 60$ to $M_{\rm rnd} \simeq 600\ M_\odot$ and > 0.95 of its mass has been accreted from low-angular momentum gas in the absence of efficient equipartition. The effect of resonant relaxation can be estimated analytically in the early growth stages, up to $M_\bullet \sim M_{\rm eq}$, when both the growth rate and j_a can be approximated as near-constant: dynamical randomization quickly pushes $j_a/j_{\rm ISO}$ below 1 (independently of the decrease in $j_a/j_{\rm ISO}$ near $M_\bullet = M_0$) (Fig. 2). The damped and randomized motions of the BH suppress the accumulation of angular momentum in the accretion flow and allow Bondi accretion to proceed supra-exponentially.

The BH mass up to $t_{\rm rnd}$, $M_\bullet \le M_{\rm rnd} \sim 0.02 M_c$, is still small enough to justify both treating the BH as a test particle in the fixed potential of the gas and star cluster and representing cluster dynamics by a simple model. It is much more difficult to self-consistently predict the subsequent joint evolution of the BH and cluster. However, the physical arguments for the gradual deceleration of the BH and the decline of j_a beyond $M_{\rm eq} \sim 25\ M_\odot$, suggest that a substantial fraction of the available $10^5\ M_\odot$ gas reservoir can be accreted in $t_\infty \sim$ few$\times 10^7$ yr at $z > 15$. Even if the supra-exponential growth phase terminates with a modest BH seed of only $M_{\rm rnd} = 600\ M_\odot$ at $z = 16$ ($t \simeq 0.25$ Gyr for $H_0 = 0.7$, $\Omega = 1$, $\Omega_M = 0.28$), this would allow the subsequent Eddington-limited growth (with radiative efficiency $\eta_\gamma = 0.1$ and electron-scattering opacity $\kappa = 0.35$ cm^2 g^{-1}) of a $3.4 \times 10^8\ M_\odot$ SMBH by $z = 7$ ($t \simeq 0.78$ Gyr), and a $2.5 \times 10^{10}\ M_\odot$ one by $z = 6$ ($t \simeq 0.95$ Gyr). Even if the process operates efficiently only in 1-5% of the dark matter halos where the first stars form, it can adequately account for the SMBHs seen to be powering the detected luminous quasars at $z > 6$.

We conclude that low mass stellar BHs in very dense, low-angular momentum cold flows at redshifts $z > 15$ can be launched by stellar dynamical processes into a phase of supply-limited, supra-exponential accretion and can grow rapidly in $\sim$ few$\times 10^7$ yr into $\gtrsim 10^4\ M_\odot$ BH seeds. Subsequent slower Eddington-limited growth by disk accretion suffices to produce the supermassive BHs that power the brightest early quasars.

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ACKNOWLEDGMENTS

We thank P. Armitage, B. Bar-Or, M. Begelman, F. Bournaud, M. Colpi, A. Dekel. J.-P. Lasota, C. Reynolds, and N. Sapir for helpful discussions and

comments. T.A. acknowledges support by ERC Starting Grant No. 202996, DIP-BMBF Grant No. 71-0460-0101, and the I-CORE Program of the PBC and ISF (Center No. 1829/12). P.N. acknowledges support from a NASA-NSF TCAN award number 1332858. The authors thank the Kavli Institute for Theoretical Physics, UC Santa Barbara, where this work was initiated and supported in part by NSF Grant PHY11-2591. T.A. is grateful for the warm hospitality of A. Millán and L. Arkwright of La Posada San Marcos, Alájar, Spain, who hosted the Alájar Workshop where this work was continued.

Supplementary Materials
www.sciencemag.org/cgi/content/full/science.1251053/DC1
Supplementary Text
Fig. S1
References (*44*–*54*)

20 January 2014; accepted 21 July 2014
Published online 7 August 2014; 10.1126/science.1251053

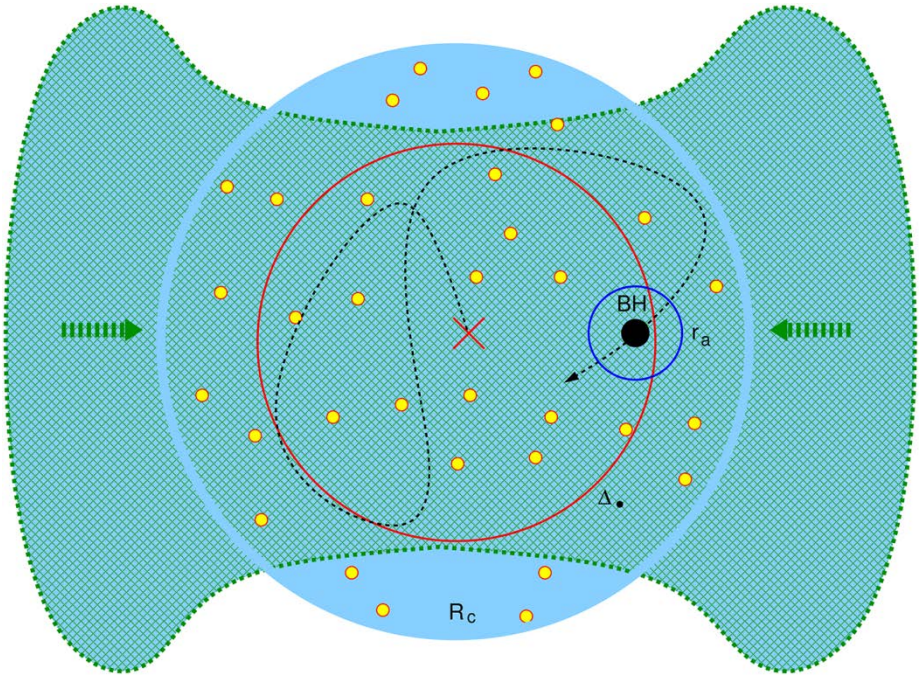

Fig. 1. A schematic depiction of accretion by a low-mass BH in a dense gas-rich cluster. Dense cold gas (green) flows to the center (red cross) of a stellar cluster (light blue region) of total mass $M_c = N_* M_* + M_g$ and radius R_c, which contains N_* stars (yellow circles) of mass M_* each with velocity dispersion σ_*, and gas of mass M_g. The gas is nearly-pressure supported and close to the virial temperature. A stellar BH (black circle) of mass $M_* < M_\bullet << M_c$, which is accreting from its capture radius r_a (blue circle), is initially in fluctuation-dissipation equilibrium with the stars, and is scattered by them (black dashed line) with velocity dispersion $\sigma_\bullet \sim \sqrt{M_* / M_\bullet}\,\sigma_*$ over a distance scale $\Delta_\bullet \sim \sqrt{M_* / M_\bullet}\,R_c$ (red circle).

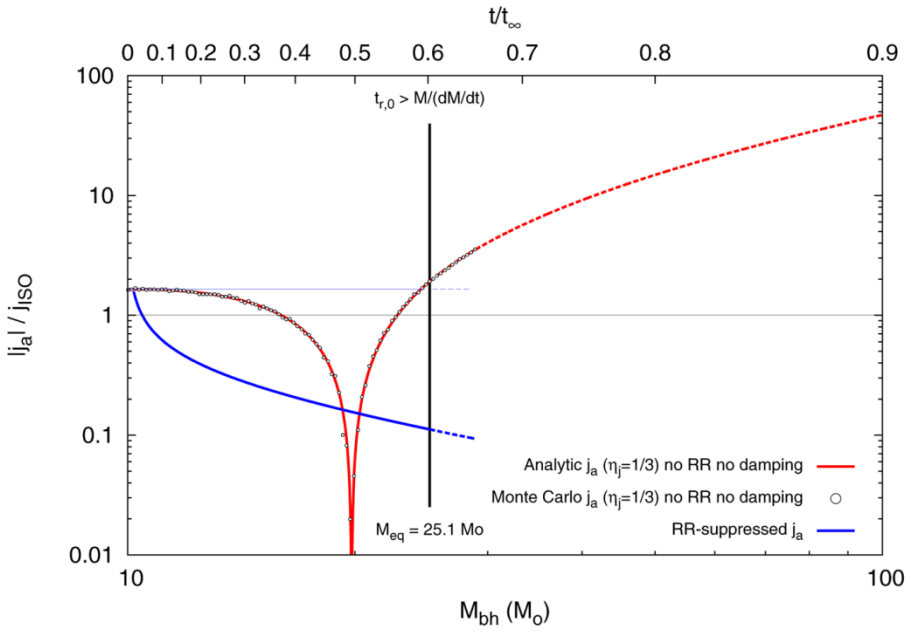

Fig. 2. The specific angular momentum ratio, j_a/j_{ISO}, in gas captured by the BH, as function of the BH mass $M_\bullet$ (and the corresponding t/t_∞ for Bondi accretion). The evolution of j_a/j_{ISO} during the initial stages of the BH growth is calculated in the ballistic wind accretion limit by a 1st-order analytic estimate with an angular momentum capture efficiency of $\eta_j = 1/3$ (red line), which is validated against results from a Monte Carlo integration over the exact capture cross-section (circles). j_a falls to zero at $M_0 \simeq 20\ M_\odot$, where the density and velocity gradients cancel each other. The vertical line at $M_{eq} \simeq 25\ M_\odot$ marks the transition to a dynamical regime where 2-body relaxation can no longer establish equipartition between the growing BH and the stars, and the acceleration frequency Ω is damped. The early dynamical suppression of the angular momentum down to $j_a < j_{ISO}$ by resonant relaxation of the BH orbit (thick blue line) is approximated by conservatively assuming a constant $j_a(t) = j_a(0)$ (thin blue line). At $M_\bullet > M_{eq}$, where the BH dynamics are sub-equipartition, the actual value of j_a/j_{ISO} is expected to progressively drop below the extrapolated one (dashed lines) (see detailed discussion in the supplementary materials).

Table 1. Supra-exponential BH growth model.

Property*	Notation	Value
Model parameters		
Initial BH mass	$M_\bullet$	$10\ M_\odot$
Star mass	M_*	$1\ M_\odot$
Core radius	R_c	0.25 pc
Stellar core mass	M_s	$2\times10^4\ M_\odot$
Gas mass in core	M_g	$2\times10^4\ M_\odot$
Total core mass	M_c	$4\times10^4\ M_\odot$
Cold flow adiabatic index	Γ	5/3
Derived cluster properties		
Mean gas density	ρ_∞	2.1×10^{-17} g cm^{-3}
Jeans sound speed	c_s	14.5 km s^{-1}
Velocity dispersion	σ_*	18.0 km s^{-1}
Initial BH rms velocity	$v_\bullet$	11.4 km s^{-1}
Initial BH rms scattering distance	$\Delta_\bullet$	6.5×10^{-2} pc
Cluster orbital frequency	Ω_0	1/5545 yr^{-1}
Central 2-body relaxation time	t_{r0}	1.6×10^7 yr
Vector resonant relaxation time	t_{vRR}	6.2×10^5 yr
Evaporation time	t_{evap}	8.0×10^9 yr
Collisional destruction time	t_{coll}	1.5×10^{11} yr
Gas reservoir dynamical time	t_{res}	$\sim10^6$ yr
Predicted accretion properties		
Initial accretion radius	$r_a^{\,0}$	2.5×10^{-4} pc
Mass divergence time	t_∞	3.5×10^7 yr
Initial specific accretion ang. mom.	j_a/j_{ISO}	1.7
Ang. mom. suppression by resonant relaxation	$\sqrt{t_\infty / t_{vRR}}$	7.6

*See also definitions and discussion in the supplementary materials.

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published online August 7, 2014

Editor's Summary

Science (print ISSN 0036-8075; online ISSN 1095-9203) is published weekly, except the last week in December, by the American Association for the Advancement of Science, 1200 New York Avenue NW, Washington, DC 20005.