Integrated production planning and route scheduling in pulp mill industry

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Abstract

In this paper we consider the complete supply chain for a large pulp producer in Sweden. The supply chain is divided up in two parts, the pulp production planning problem and the distribution to customers problem. We focus on the overall problem and the interaction between the partial problems and discuss how to solve these simultaneously in a supply chain logistic support system. We have modeled the partial problems as mixed integer optimization models and use specialized solution methods for each problem. The models become large scaled when we consider a realistic time horizon. We therefore use flexible ways to aggregate time periods to find good solutions within reasonable time limits.

1 Introduction

This paper considers planning the supply chain for Södra Cell AB, the world's leading manufacturer of market pulp intended for paper production. A large proportion of all pulp used in European non-integrated paper industries (writing paper, board, tissue etc.) originates from pulp mills owned by Södra Cell AB. With five pulp mills in use (three in Sweden and two in Norway), Södra Cell produces more than two million tonnes of pulp per year comprising around 15 different products that are distinguished by brightness and other properties. Södra Cell AB is a subsidiary of Södra, an economical association owned by more than 33,000 forest owners in southern Sweden.

Both short and and long-range planning of pulp production occurs throughout the year. This planning is currently performed manually, with a focus on meeting demand for products and ensuring sufficient supplies of raw material are maintained. On the other side of the supply chain is the customer demand, both domestic and outside Sweden. There is a total of about 175 delivery points spread across Europe. Also with manual planning and experience based schedules, they use cargo-ships to spread pulp to terminals accross Europe in order to satisfy demand.

The complete supply chain includes harvesting, transportation of logs, production at the mills, storage (at mills, in forests and at harbours), distribution to terminals by ship, storage at terminals and distribution by truck and train to customers. The planning of the supply chain is a complex task. It is difficult to satisfy all production restrictions, and at the same time keep the total supply chain cost low. Rapid growth in Södra Cell's business makes it increasingly important to accurately evaluate how decisions affect total costs and to estimate the value of specific production schedules. The manual scheduling strategies are based on solid experience incorporating knowledge of possible unplanned occurences such as stoppages in production and new unexpected orders. However, the components of the supply chain are continually changing as a result of new products, new demands and the development of new technologies. Södra Cell's production planners have identified the need for a new decision support system to improve the quality of their decision making and provide increased flexibility in their planning.

The purpose of this paper is to propose an overall process for supply chain planning at Södra Cell. This process includes optimization models and methods and an evaluation how optimization can be used to support the planners. We focus on the decision making for the short-range planning (three months). In the case of short-range planning high details about the solution is requried for a useful support system. With today's computer capacity it is not possible to solve an overall model with such high detail-level and we have therefore divided the supply chain in two major parts, the production planning and the distribution. This paper also describe the coordination between these models, and how to find a practical algorithm that coordinate the solutions from both problems.

We have developed mixed integer models for both the production planning prolem and the distribution problem. The models mostly reflect the real situation of the supply chain when each is taken separately. We use heuristic algorithms to find good solutions to the models and the solution time is the major limiting factor for the possibility to directly integrate them to one comprehensive model, for the com-

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plete supply chain. In this paper we suggest methods that finds solutions in accordance for both models.

The outline of this paper is as follows. We begin in Section 2 to give a problem formulation and a detailed survey of Södra Cell's supply chain. Then follows a summary of the production planning model in Section 3, followed by a summary of the ship scheduling model in Section 4. Approaches and considerations to integrate them is discussed in Section 5. We end this paper with a discussion about further work and concluding remarks in Section 6.

2 Problem formulation

Södra Cell's complete supply chain, from harvesting of trees to delivery of pulp-products, involves several important steps. A huge amount of pulp is produced, and so the savings generated by careful planning of both production and transportation can be significant. This section begins with a detailed description of the complete supply chain and continues with details about products, production plans and ship routes.

2.1 Components of the supply chain

The supply chain begins in Sweden's forests, typically in harvesting areas owned by the members of Södra. These areas are aggregated by Södra into domestic supply forest districts. The supply of raw material includes harvests of soft and hard wood from these forests, together with limited imports of raw materials from other countries, mainly Russia and the Baltic States. It is customary for Södra Cell to choose domestic logs over non-member deliveries. It is possible to store harvested timber within a forest district before it is transported to the production mills; there are costs associated with this storage. The byproduct from saw mills in the form of wood chips is another important raw-material used in pulp production.

The domestic logs are transported to the pulp mills by truck, while imported logs are delivered by cargo-ships. The production of pulp products from logs and chips at each mill takes place according to production recipes. There are limited local storage areas close to each mill for both manufactured pulp and the raw materials. Because of this, it is important to find good schedules that keep storage levels as even as possible during production.

Although the majority of export customers are located in Western Europe, deliveries are also made to North America. The wide spread of customer locations makes the planning of distribution a difficult but important task, not only because of the associated travel distances, but also because it is difficult to meet all the demands for different products with the limited range of production possibilities available. To distribute pulp, Södra uses three cargo-ships that they have hired on long term contracts. In addition, they occasionally hire additional boats for short trips, and make deliveries by train or truck. Close to each mill there is a harbour, either rented or owned by Södra. It is possible to store pulp (typically pulp destined for export) at these harbours. As well as exports, there are domestic customers whose demands need to be met using deliveries by truck.

The overall picture of the supply chain is shown in Figure 1, where the broken lines separates the production planning problem from the export distribution problem. It also shows the connection between these problems, the common storage at harbours located in connection with pulp mills.

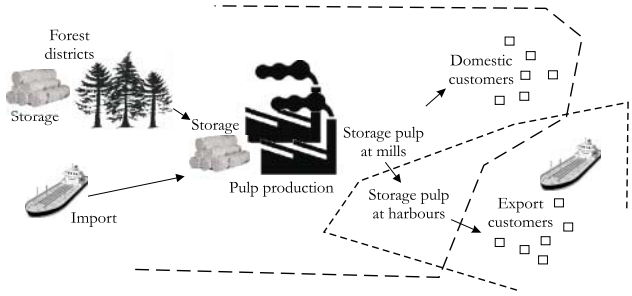

Figure 1: The modeled supply chain of Södra Cell.

2.2 Production plans

The production plans are created at different planning levels. First a planning period of one year is considered, whereby coarse schedules are produced. This is done mainly using forecasts of demand for the upcoming year. Second, a more detailed schedule for each pulp mill is developed, which shows what is produced in a shorter range with an accuracy usually of single days. These schedules are together supposed to meet determined deliveries and to have a balanced production according to supply of raw-materials.

At each pulp mill only one product (pulp type) can be produced at a time. Production is done in campaigns, which are sequences of days producing the same pulp type. There are many factors that affect the production, both obvious ones such as limited storage capacities and transportation flows, and also less obvious factors based on experience. When developing production plans, it is important to consider production changes between successive products and the lengths of the campaigns. Due to limitations at the pulp mills (running-in problems and other machine properties) and difficulties with providing a continuous supply of particular assortments, campaigns can have both minimum and maximum length restrictions. Figure 2 shows a possible production plan for the pulp mill at Mönsterås.

A production plan has two important cost factors: daily production costs and change-over costs between campaigns.

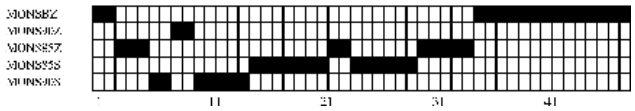

Figure 2: Example of a production plan. Here products from the pulp mill at Mönsterås are shown on the vertical axis and time periods (daily) on the horizontal axis. A marked square is interpreted as the production choice for each time period.

Both costs differ depending on what product is produced and at which pulp mill it runs. The pulp mills are different in the sense that there are differences in machinery and the consumption of raw materials differs between the pulp mills. The change-over cost is estimated from previous costs of equivalent changes and is dependent on the sequence of products involved. For example, a change from hardwood to softwood products costs more than a change between two softwood products. The change-over costs also vary with the change in brightness levels, with a change from high to low brightness costing less than a change in the opposite direction. Hence, large changes in brightness imply losses that can be interpreted as costs. A reduction in production efficiency, or the output of mixed products with lower market value, can also occur at the beginning and end of a campaign.

2.3 Recipes and products

There are four main assortments (classes) of wood used in the production; pulp wood of conifer, aspen, and birch and wood chips from sawmills. Aspen is treated specially in one class of its own, while the class birch includes all other hardwood. A recipe transforms, through several processes, different shares of assortments to form one unit of pulp. The outcome when using a given recipe is a specific pulp product. As an example of a recipe, we can take the combination 3/4 of conifer and 1/4 of wood chips with the transformation proportion of five tonnes of raw materials to one tonne of output pulp. There are eight other recipes frequently used. The number of recipes changes from time to time, but has lately been decreasing following a drive by Södra Cell to decrease their number of products.

Most recipes gives only one product as outcome. However, some recipes give an intermediate product that is 'strained' (separated) to produce several different products. For the sake of simplicity, straining is modelled as one special recipe with several outcome products, even though there are recipes that defines equal products, but only one at the time. The relationship between recipes and products is given in a transformation matrix. Because of varying equipment and restrictions on the products that can be produced at each mill, the transformation matrix is unique to each pulp mill. A possible recipe is shown in Figure 3.

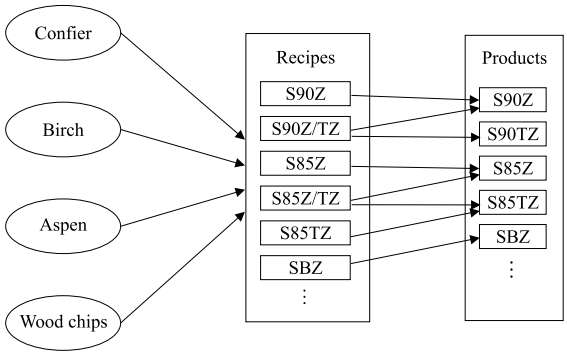

Figure 3: This figure illustrates the mechanism that defines a recipe. If the recipe is straining, different products are produced.

Occasionally, there are planned stops at the pulp mills due to, for example, cleaning and the upgrade of machinery. These stops are treated as recipes with no outcome product and no consumption of assortments. Usually Södra defines a span of days when a stop can occur, and it is then up to the industrial workers at the mill to choose suitable days within this span.

2.4 Distribution

There are very large quantities to deliver to customers located all around the world, especially within Europe. To accomplish this most pulp is distributed by cargo-ships. Therefore, Södra use three ships on long term contract, only for this purpose. If we look at the European market (where their ships sails), the chain from pulp mill to customer includes, loading at harbour in connection to pulp mill, unloading at terminal suitable for further delivery to customer location. In addition, they rent ships on the spot-market (with short term contract) and make some deliveries by train and some by trucks.

A route is determined by terminals in connection to pulp production locations (pulp mills) and by terminals around Europe with unload purpose only. There are around 20 unload terminals that Södra use on a regular basis and four terminals in connection to four of their pulp mills. A route consists normally by one up to three supply terminals and one or two destinations. We have chosed to consider a route with its beginning and end at a pulp mill. There is no back hauling incuded in the model. It is not profitable for Södra to consider back hauling more than in the case of their longest route (with destination in Italy) and therefore it is not nessecery for us to include in the model.

In addition to the ships there is a large use of trucks, railaway and prams. Prams are only used to move products from

terminals situated at ports to terminals located inland. For the domestic distribution only trucks and railway are used. It is also possible to use trucks and railway for direct transportation between mills and delivery points. This mode of transportation is also used for the distribution from terminals to delivery points.

3 Production plan model

In this section we sumarize the models developed to solve the production planning part of the supply chain. They can be found in [1]. We have developed two models for the production planning problem. The first model uses variables representing production plans for individual mills covering all three months. A natural solution approach which we have adopted for this model is column generation. A production plan essentially consists of a sequence of products to produce each day. The production plan can also be considered as a sequence of 'campaigns' (batches), where a campaign is the production in consecutive days of one single product. The subproblem used to generate production plans includes restrictions on the minimum and maximum lengths of each campaign, and takes into consideration the changeover costs of switching between products. This subproblem can be formulated as a shortest path problem. The column generation is integrated with a limited branch and bound search using constraint branching, see e.g. [3], to find integer solutions. We use a flexible approach where arbitrary time periods can be aggregated to allow the models to be solved within practical time limits. In the second model we use binary variables representing the explicit decisions of which product to produce on each day. The limitations of this simplified approach are that it does not consider the maximum lengths of campaigns and provides a lower level of detail in the production plan than is possible using the column generation approach. The model we present in the next sections covers only the column generation approach. We choose to exclude the direct approach model, since the models have most of their constaints and variables in common (except the variables and constaints defining the production plans, but these are easy in the direct approach).

3.1 Constraints and variables

Before developing the model, we present a summary of the symbols we will be using. We begin by listing the index sets, along with the typical choice of index variable associated with each set. Let

- M set of pulp mills, index j,
- T set of time periods, index t,
- A set of assortments (raw material types), index a,
- D set of domestic demand locations, index d,
- P set of products, indice p,
- F set of forest districts including imports, index i,
- Q_j set of production plans at pulp mill j, index q, and
- R_j set of recipes at pulp mill j, index r.

We introduce the flow and storage variables by means of two illustrations, Figure 4 and Figure 5. These variables are divided into two groups; the first contains assortment (raw material) variables, while the second focusses on products. The assortment variables are detailed in Figure 4, and flow variables are shown in Figure 5.

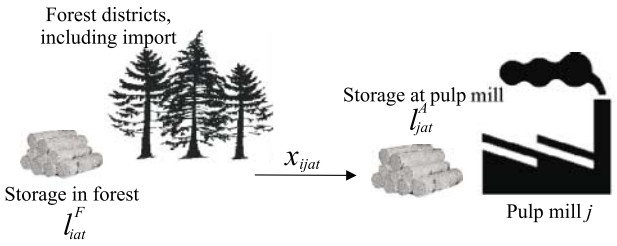

Figure 4: Assortment variables.

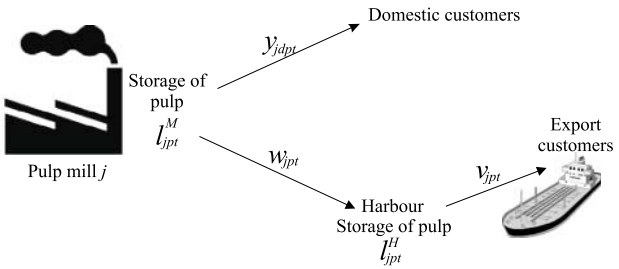

Figure 5: Variables concerning flows of pulp products.

- x_{ijat} flow from harvest district i to pulp mill j of assortment a during time period t
- f_{ja} target flow level of assortment a to pulp mill j
- l^F_{iat} quantity stored in harvest district (forest) i of assortment a during time period t
- l^A_{jat} quantity stored at pulp mill j of assortment a during time period t
- l^P_{jpt} quantity stored at pulp mill j of product p during time period t
- l^H_{jpt} quantity stored at Swedish harbour adjacent to mill j of product p during time period t
- y_{jdpt} amount of product p transported from pulp mill j to domestic demand location d during time period t

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w_{jpt} amount of product p transported from mill j to associated harbour during time period t

v_{jpt} total amount of product p transported from the harbour associated with pulp mill j to international customers during time period t

The column generation model has one binary variable for each production plan. Let these variables be defined as

z_{jq} 1, if production plan q at pulp mill j is used, 0 otherwise.

A production plan specifies which recipe is to be used in each time period. The contents of possible production plan q at mill j are specified by constants δ_{jqrt}, where

δ_{jqrt} 1 if recipe r is used in production plan q during time period t at pulp mill j, 0 otherwise.

Using knowledge of the exchange between assortments and products at each pulp mill, flow conservation constraints can be constructed. The separation of δ_{jqrt} from the amount of assortments and pulp products (i.e. R^{in}_{jra} and R^{out}_{jrp}) is made to facilitate the solution methods discussed shortly.

The known supply of logs is based on a forecast for the 90 day planning period and is modeled as daily harvest quantities in each area. There are transportation limits in each district and at each pulp mill that are considered to be fixed. Each recipe at a specific mill specifies the amount of raw materials required per day and and the quantity of pulp that is produced per day. The full set of these and other parameters are as follows:

H_{iat} amount of assortment a harvested in district i during time period t

D^D_{dpt} amount of product p demanded at domestic demand location d during time period t

D^E_{pt} amount of product p demanded for export during time period t

T^M_j maximum amount of assortment transported to pulp mill j during one time period

T^D_i max amount over all assortments that may be transported from district i during one period

R^{in}_{jra} amount of assortment a used in one time period when running recipe r at pulp mill j

R^{out}_{jrp} amount of product p produced in one time period when running recipe r at pulp mill j

The constraints follow and are commented below.

$$l^F_{iat} = l^F_{ia,t-1} + H_{iat} - \sum_{j\in M} x_{ijat} \quad (1)$$

$$l^H_{jpt} = l^H_{jp,t-1} + w_{jpt} - v_{jpt} \quad (2)$$

$$\sum_{a\in A} f_{ja} \le \frac{10}{11} T^M_j \quad (3)$$

$$0.9 f_{ja} \ge \sum_{i\in F} x_{ijat} \le 1.1 f_{ja} \quad (4)$$

$$\sum_{j\in M}\sum_{a\in A} x_{ijat} \le T^D_i \quad (5)$$

$$\sum_{j\in M} y_{jdpt} = D^D_{dpt} \quad (6)$$

$$\sum_{j\in M} v_{jpt} = D^E_{pt} \quad (7)$$

$$\sum_{q\in Q_j} z_{jq} = 1 \quad (8)$$

$$l^A_{ja,t-1} - \sum_{q\in Q_j}\sum_{r\in R_j} R^{in}_{jra}\delta_{jqrt} z_{jq} + \sum_{i\in F} x_{ijat} - l^A_{jat} = 0 \quad (9)$$

$$l^P_{jp,t-1} + \sum_{q\in Q_j}\sum_{r\in R_j} R^{out}_{jrp}\delta_{jqrt} z_{jq} - w_{jpt} - \sum_{d\in D} y_{jdpt} - l^P_{jpt} = 0 \quad (10)$$

First there are flow conservation constraints in storage locations in forest district (1) and at domestic harbours (2). It is not realistic for inflow to vary more than approximately 10% from a fixed level. This is mostly dependent on factors in the transport planning. The constraints (3) set a maximum inflow level for each mill and the constraints (4) give the feasible interval of daily inflow changes. Capacity constraints for outflow from each forest district are given in (5) and there are constraints to meet demand from domestic (6) as well as export (7) customers.

In practice it is possible to, in certain cases, delay distribution and thereby go below the minimum storage limits. We allow this, but at the cost of a daily fee. This is handled with punished slack variables added to the constraints (6) and (7). This gives a more robust model, since the production and demand volumes almost always are of about equal size. Note that any deficit in one time period increases the demand in the following time period. It is also possible to relax the storage limits with an associated cost, estimated from the risk of being under the minimum limits. The production plan specific constraints are: flow conservation of assortments at pulp mills (9), flow conservation of products at pulp mills (10), convexity constraints that require each pulp mill to use exactly one production plan (8), and the binary restrictions.

The objective function (below) includes all costs of transportation, storage, and, through the last sum ($\sum_{j,q} C^Z_{jq} z_{jq}$), the production plan costs (being changes in production between successive recipes and usage of recipe costs). The summations are defined over the ranges for the indices specifed earlier.

$$z = \sum_{i,j,a,t} C^X_{ij} x_{ijat} + \sum_{j,d,p,t} C^Y_{jd} y_{jdpt} + \sum_{j,p,t} C^W_j w_{jpt} +$$

$$\sum_{i,a,t} C^F_{ia} l^F_{iat} + \sum_{j,a,t} C^A_{ja} l^A_{jat} + \sum_{j,p,t} C^H_{jp} l^H_{jpt} + \sum_{j,p,t} C^P_{jp} l^P_{jpt} + \sum_{j,p,t} C^V_{jpt} v_{jpt} + \sum_{j,q} C^Z_{jq} z_{jq}$$

The number of constraints is highly dependent on the length of the planning horizon and the length of each time period. For example, to schedule production defining every time period to be one day for a period of 90 days gives a model with more than 28,000 constraints and 90,000 continuous variables. In addition we have the binary variables for each production plan.

3.2 Solution methods

In this section we give the idea behind the solution methods for the column generation approached model. The explicit model is naturally solved with a standard branch and bound algorithm.

Given an initial set of production plans at each pulp mill, we solve a restricted master problem including all variables and constraints, but where the production variables are allowed to be fractional.

The solution to this linear program gives dual variables which are then used to obtain new columns (new production plans) to be included in the restricted master problem. The column generation subproblem is a shortest path problem for each pulp mill, where the network is defined with one node for each recipe and each time period, and the links represent feasible changes of recipes from one period to another. The link costs in the network are defined by the production costs for the corresponding recipe, by the changeover costs between recipes, and by the dual variables from the restricted master problem. A possiblity is also to add restrictions in the production, for example regarding the length of a campains with the same recipe, by modifying the origins and destinations of the links in the network.

The column generation procedure stops when no more production plans can be generated that improves the restricted master problem. The optimal solution is certainly not feasible with respect to the binary restrictions on the production variables. The interpretation of this fractional solution is that more than one production plan (recipe) is used in the same time period. Since we have constraints restricting the number of production plans to at most one in a period, a traditional branching over the production plan variables will not be fruitful. We therefore suggest to use a heuristic including, so called, constrained branching to obtain a feasible solution. The heuristic can be interpreted as a depth-first search strategy without recourse.

The size of the subproblem is strongly connected to the number of possible campaigns. Taken three mills with 5 recipes at each and all with possible lengths between 2 and 30 days gives a network with more than 300,000 arcs when there are 90 days to schedule. The possible cost concerning shifts in form of lower quality and other losses in production can directly be added to the change-over arcs.

Initial trials to solve the master problem with 90 time periods, three pulp mills and ten domestic locations gave excessively long solution times. The major time consuming part was the solution of the linear relaxation. What most affects the number of constraints and thereby also the solution time is the number of time periods. Therefore, a possible way to decrease the solution time is to aggregate time periods. While aggregation can speed up the solution procedure, important information is lost in the process. Hence, it is important to consider how the aggregation should be made and where the accuracy is to be emphasised, that is to consider if some periods are more important than others. We suggest a flexible way to make the aggregation, where arbitrary sets of the time periods can be aggregated. In our implementation we can e.g. do no aggregation in month one, aggregate two time periods into one in the second month etc.

4 Ship scheduling model

In this model we consider the distribution of pulp to export customers (as well as domestic customers) with route scheduling for ships a central part of modeling. The actual supply is assumed to be known, and the basic model idea is to satisfy the demand in an optimal way. Södra deliver very large quantities of pulp every year and if measured with costs, the distribution is one of the major cost factors of the supply chain.

We model the route-scheduling problem with an explicit mixed integer optimization model with integer decisions for each ship type, each route and the first time-period that is covered by the route. That is, what is left if the integer variables were not in the model would be an assignment problem with additional comprehensive constraints supplying upper bounds on the routes. More details can be found in [2]. Another approach can be found in [4].

4.1 Mathematical model

The model we present is specially constructed to find good feasible routes for the ships in the distribution problem, throughout all the planning horizon. To accomplish this, we use a time discretization that gives as detailed route description as possible (dependent of the solution time requirements and the length of the horizon). Therefore, we use three detail levels in time: weeks, days and periods. We let one day consist of a number of periods and, of course, one week spans a set of days. With this system we aggregate some parts of the supply chain to weekly basis, keep the supply and demand on daily basis, while all the route information is handled with the period system. One can thus change the

detail level in three steps, and thereby decide where the accuracy is to be emphasized. Begining with sets, we have

- S set of ship types
- P set of products
- C set of customers
- P_r^C set of products demanded by customer r
- T set of time periods
- D set of days
- W set of weeks
- T_d^D set of time periods during day $d \in D$
- T_w^W set of time periods during week $w \in W$
- M set of terminals close to pulp mill
- P_j^M set of products supplied by pulp mill j
- J_H set of harbour terminals
- J_L set of land terminals
- Ω set of routes, with start and end at a mill terminal
- T_q^Ω ordered set of time periods, relative to start period for route $q \in \Omega$,when the ship is at the respective terminal
- J_{qt}^Ω $\in M \cup J_H,$ the terminal that route $q \in \Omega$ harbours during its $t \in T_q^\Omega$'s period

We use continuous variables for all flow and storage levels. Also the actual flow on routes that are used are modeled continuously. What is complicating is to make the selection of routes, which by nature is a integer (or binary) decision, if a particular route is used or not. Before the constraints are stated, we give a complete list of variables and parameters (capital letters) used in the model.

- x_{jrpw}^{train} flow by train from harbour $j \in M$ to customer r with product p during week t
- x_{jrpw}^{truck} flow by truck from harbour $j \in M$ to customer r with product p during week t
- x_{spqvt} in/out flow at harbour J_{qv}^Ω with product p on route q with start at time period t during time period $t+v$, $v \in T_q^\Omega$
- z_{sqt} number of ships of type s using route $q \in \Omega_s$ and begins during time period t
- y_{sjt} number of ship of type s that are harboured at terminal $j \in M$ during time period t
- l_{jpw}^M quantity stored at harbour $j \in M$ of product p during week w
- l_{kpw}^H quantity stored at harbour terminal $k \in J_H$ of product p during week w
- l_{kpw}^L quantity stored at land terminal $k \in J_L$ of product p during week w
- l_{rpw}^C quantity stored at customer r of product p during week w
- u_{krpw} flow from terminal $k \in J_H \cup J_L$ to customer r of product p during week w
- v_{hkpw} flow from harbour terminal $h \in J_H$ to land terminal $k \in J_L$ of product p during week w
- S_{jpd} amount of product p delivered to harbour $j \in M$ during day d
- B_k transport capacity to harbour terminal $k \in J_H \cup M$
- B_k^L transport capacity to land terminal $k \in J_L$
- D_{rpd} demand of product p by customer r during day d
- K_s ship type capacity
- I_{sjt} number of ship of type s that are initialized at terminal $j \in M$ during time period t

This model is basically about connecting supply locations to demand locations throughout a horizon. This supply chain model has therefore a set of flow consevations constraints and limitations for the flow on links between different locations. Furthermore, the main modeling items are the route-defining constraints and the modeling for limitations thereof. We simplify the description by introducing some general matricies F_x where x is any number. A detailed description can be found in [2].

$$l_{jpw}^M = l_{jp,w-1}^M - F_1(x, x^{train}, S) \quad (11)$$

$$l_{jpw}^H = l_{jp,w-1}^H - F_2(x, u) \quad (12)$$

$$F_3(x) \leq B_k \quad (13)$$

$$F_4(v) \leq B_k^L \quad (14)$$

$$l_{kp,w-1}^L = l_{kpw}^L - F_5(v, u) \quad (15)$$

$$l_{rpw}^C = l_{rp,w-1}^C - F_6(u, x^{train}, x^{truck}) \quad (16)$$

$$F_7(x_{spqvt}) \leq K_s z_{sqt} \quad (17)$$

$$y_{sjt} = y_{sj,t-1} + F_8(I, z) \quad (18)$$

$$\sum_{j \in M} y_{sj,|T|} = -F_9(z_{sqe}) \sum_{j \in M} \sum_{t \in T} I_{sjt} \quad (19)$$

The constraints in (11) gives flow conservation at harbours in connection with pulp mills. These constraints are aggregated down to weekly basis. That is, there is one constraint for each week, mill and product even though supply is measured on daily and routes on periodical basis. The same conditions holds for the flow conservation at harbour terminals, (12). Limitations in form of upper bounds at all terminals can occure. These are modeled with (13) which gives limited (simultaneous) capacity at harbour terminals and pulp mills, while (14) gives restrictions for the transport capacity to land terminals. With (15) flow conservation at land terminals is kept. The demand locations are considered as physical locations with different possible connection to terminals. The connections between terminals and customers and the direct transport with train and truck are both modeled in one flow conservation constraint, (16), this for each export customer demand. If one ship of type s is running on a specific route q during a time period t, then

$z_{sqt} = 1$ and the constraint (17) determines the limited ship capacity. The constraints (18) gives restrictions such that each ship only uses one route at the time. The parameter I_{sjt} determines the startup location and period. The last constraints, (19) The constraints are formulated as easy as possible. The linear objective function cover all the costs for flows, storage and boat usage. It is based on the lowest discretization of time periods, so it can include additional costs associated with for instance evening hours.

4.2 Solution methods

The model size is dependent on the number of terminals, ships, products and time periods. To make it solvable within reasonable time for different time discretizations we suggest a solution method based on dividing it up in smaller (with less time periods) subproblems and partially solve these smaller problem repeatedly.

First an initial set of periods is selected. Then the model over this time period is solved with a specified accuracy using branch and bound on the integer variables. An in advance defined part of the received solution is then saved and considered as determined. The next set of time periods extends the horizon with new days but still have a common interval with the previous period. We resolve the common interval to give possibility for ships to use routes beginning and ending at any time. This repeated solving continues until the whole planning horizon is covered and it is then possible to solve the linear problem to find optimal flows for a set of determined routes.

If we solve the problem with a case including 20 terminals and three ships distributing around 20 products over a time-horizon of 90 days, then each 30 days sub-problem size is around 100,000 variables of which 10,000 are binary variables. The number of constaints is dependent on the length of a week. In this case, with a week length of six days, it gives 10,000 linear constraints.

5 Integration of production and ship scheduling

The two problems have one model component in common, the storage level in connection to pulp mills. This is what connects the models and give rise to possible interaction. We can choose which problem is the most important and decide in what order they should be solved for different purpose. The most desirable situation is to let both models run simultaneously (or iterative one at the time, adjusting eachother towards an optimal solution). This is highly difficult in practise because the number of binary decisions increases and the models are both close to the limit of what the computer capacities of today handles. Instead only one or two iterations between the models are possible.

We can choose to solve the production planning problem first and then given the supply throughout the planning horizon solve the distribution problem. Instead of this we can do the opposite, to give priority to the distribution, optimize the route schedule and with fixed routes calculate when the supply needs to be available. Both ideas have advantages and disadvantages but what is important is to decide what is a more easy adjustment to make. If we look at the planning today, production plans are determined early and followed in production very careful. The distribution planning is adjusted continuously. That is, one suggestion in line with their business today would be to first solve the distribution problem and follow by production planning. Given this plan, the distribution problem can be resolved with new backgroud.

A second approach is to formulate an itegrated model where detailed ship scheduling is removed but where the distribution is included. Such a model would increase the production planning model with many more constraints and continuous variables but to a much less extent than with a full integration of the ship scheduling (which also include a large number of binary variables). In this model, the ship scheduling is replaced by a direct flow between mills and terminals. By adding constraints on shipping capacity it could be possible to approximate the shipping. Once the production and overall distribution is decided it is possible to find detailed routes for the ships. This approach is possible as it is possible to adjust the solution using less or more distribution by trucks and railway.

6 Further work and concluding remarks

The overall supply chain problem faced by Södra is a very difficult problem for which there are large potential savings. Today the planning is done manually but with two proposed optimization models it is possible to use these as decision support. An important aspect is that the models are solvable within practical time limits.

To solve the optimization problem it has been required to use sophisticated solution methods based on column generation and special branch and bound techniques. Innovative modelling to include aggregated time periods and strenghening of the LP-relaxation has also been needed.

A full integration of the planning can provide even larger savings. Today it is not possible to integrate the two models into one integrated. However, as they are very closely related and use the same type of constraints as the common factor it is relatively straightforward to use a solution approach where they are solved iteratively.

Further research includes a model where the production

planning model is expanded with a full distribution model but where detailed ship scheduling with binary variables are exluded. This scheduling can later be solved in a second more detailed step. This approach is also supported by the fact that production planning and overall is more static and the ship scheduling more operational.

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