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Rotated Spreading Sequences for Broadband Multicarrier-CDMA

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Abstract— **In this paper we present an extension of the pure Walsh-Hadamard spreading sequences for MC-CDMA. By rotating the phase of the modulated signal before spreading, the signal space is exploited more efficiently. After the spreading the Euclidean distance of each superposed constellation is distributed more favorable in the signal space than without rotating. The novel approach of this paper combines the more favorable distributed Euclidean distance, caused by rotation, and a multi-user detector (MUD). A MUD does exploit the more favorable distributed Euclidean distance, which a single-user detector, like e.g. the MMSE, is not able to do. The novel combination of these two techniques offers up to 3 dB gain for 4-QAM in comparison to the non rotated Walsh-Hadamard sequences in an MC-CDMA system. The performance gain gets lost in a coded system applying a convolutional code with a Hamming distance of ten. A gain of 1 dB can be retrieved by applying convolutional codes with higher code rates, that offer a lower Hamming distance of three for a full loaded system.**

I. Introduction

The advantages of multicarrier (MC) modulation systems and code division multiple access (CDMA) schemes are combined in MC-CDMA. Therefore MC-CDMA is a promising candidate in the downlink for the fourth generation of wireless mobile radio systems [1].

The data of different users is transmitted at the same time, at the same frequency and in the same space in a MC-CDMA downlink system. However there is a significant performance difference between a full loaded system and a single user system. The multiple access interference caused by the other active users degrades the performance significantly. Therefore it is of much interest to improve the performance for the most critical case, the full loaded system. There are various multiuser detectors that could improve the performance in comparison to single user detection techniques [2].

In this paper an enhanced MC-CDM access system is presented, which exploits additional diversity more efficiently at each subcarrier. A phase shift of each spreading sequence at the transmitter offers the additional diversity gain. The phase shift can be reversed easily at the receiver. The diversity gain on each subcarrier reduces the risk of loosing parts of the energy of different signals in a deep fade on distinct subcarriers. The performance is enhanced significantly by applying a multi-user detector (MUD).

A simple exemplary system that does not need a MUD is an MC-CDMA system applying BPSK with two users and a spreading length of two. The signal of the first user is not rotated, the signal of the second user is rotated by $\pi/2$ in the signal space. The two user signals can not interfere with each other, as one is in the real and the other is in the imaginary domain of the signal space. A spreader with real Walsh-Hadamard spreading sequences add the real and the imaginary part of the different user signals and spreads them over the two subcarriers. In comparison to a system that does not apply the phase shift this exemplary system is free of multiple access interference. In this paper we apply a MUD in a system with a 4-QAM modulator and longer spreading sequences with variable load of users.

The paper is organized as follows. Section II introduces the concept and the signal model of the system. Section III describes the rotation of the Walsh-Hadamard matrix. Section IV derives the used maximum likelihood sequence estimator for the uncoded system and the maximum likelihood sequence-by-sequence estimator for the coded system. In Section V simulation results for an uncoded and two differently coded systems are presented. Section VI summarizes and discusses the presented results.

II. Concept and Signal Model

This paper introduces an additional method to enhance the multiuser detection process. It is demonstrated by means of a maximum likelihood detector. Fig. 1 depicts the concept for the downlink. Each data stream is modulated and then multiplied by a rotation matrix $\mathbf{D}(\mathbf{u})$, which is introduced in more detail in Section III. Then the K_q user signals, limited by the size of the spreading matrix L, are spread by a Walsh-Hadamard sequence $\mathbf{W}_l$ and summed up. This is done in

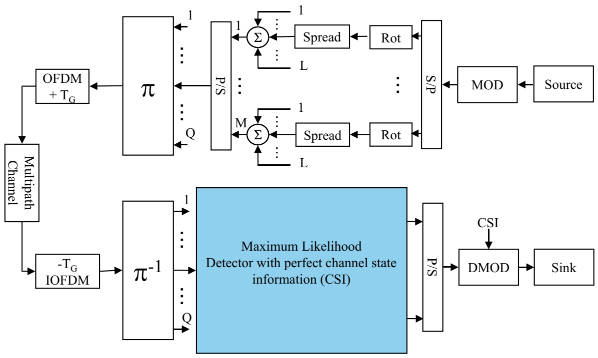

Fig. 1. The MC-CDMA system with rotated transformation of the signal

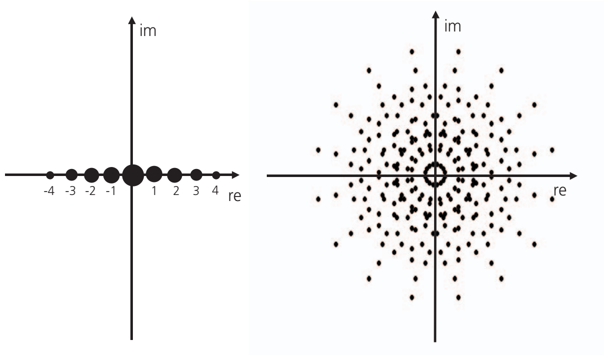

Fig. 2. On the left the non rotated and spread BPSK symbols of eight users. On the right the rotated and spread BPSK symbols of eight users [3].

parallel for M different user groups. After the parallel-to-serial conversion an interleaver scrambles all different subcarriers. Finally, the OFDM modulation is done and the guard interval is added. The signal is transmitted over a frequency selective fading channel. At the receiver the signal is OFDM demodulated, the guard interval is removed and the signal is deinterleaved.

A multiuser detector, which is presented by the exemplary maximum likelihood sequence estimator in more detail in Section IV, detects the signal sequence in groups of L subcarriers. After the parallel-to-serial conversion the different user signals are finally demodulated.

III. Rotated Spreading

Rotated spreading sequences offer diversity gain by using the two-dimensional space more efficiently. For a modulation cardinality C and a spreading block size L, there are C^L different possible points on each subcarrier and none of them should be zero. All signals of a user group distribute their energy over the subcarriers used by this group, and therefore offer diversity. Fig. 2 depicts the BPSK modulated spread signals with and without rotation transform. From the signal constellation in the two-dimensional space it can be seen that the energy of the rotated signals are spread over the whole plane. The non rotated signals are just concentrated on nine different points. Moreover most of the energy is focused on the center of the coordinate system (more than 20% of the total).

Rotated spreading is defined as a rotated transform [3] by column rotation of the original spreading matrix by specific angles:

$$\begin{aligned} \mathbf{W}_{rot} &= \mathbf{W}_{org} \cdot \mathbf{D}(\mathbf{u}), \qquad (1) \\ \text{with } \mathbf{u} &= [u_1, u_2, \ldots, u_L]^T, u_i = e^{j\frac{2\pi}{B}\cdot\frac{w(i)}{L}}, i = 1, \ldots, L \end{aligned}$$

where $\mathbf{W}_{org}$ is the Walsh-Hadamard matrix, that is used for spreading the different user signals of a user group and $\mathbf{W}_{rot}$ is the rotated transform of $\mathbf{W}_{org}$. $\mathbf{D}(\mathbf{u})$ defines a square matrix with the elements $\mathbf{u}$ on the diagonal. $w(i)$ defines a sequence of L values, $0, \ldots, L-1$. B is a constant, that is defined by the modulation cardinality. For a PSK modulated system $B = 2^{\alpha}$, where α is given as the number of bits per data symbol. This derives the step size of the rotation angle, that is $2\pi/B$.

IV. Detector

In the first part of this section the maximum likelihood sequence estimator (MLSE) is derived for an uncoded MC-CDMA system. In the second part the maximum likelihood symbol-by-symbol estimator (MLSSE) for a coded MC-CDMA system is derived.

A. Uncoded MLSE

The optimum detection technique exploits the maximum a posteriori (MAP) criterion or the maximum likelihood (ML) criterion [4]. We assume that all possible transmitted sequences are equally propable a priori. Therefore the MAP and the ML estimator are equivalent. The maximum likelihood sequence estimation (MLSE) optimally estimates the transmitted data sequence $\mathbf{d}$. The possible transmitted data symbol vectors are $\mathbf{d}_\mu$, $\mu = 1, \ldots, C^{K_q}$, where C^{K_q} is the number of possible transmitted data symbol vectors. The MLSE minimizes the sequence error probability, i.e., the data symbol vector error probability, which is equivalent to maximizing the conditional probability $P\{\mathbf{d}_\mu|\mathbf{r}\}$ that $\mathbf{d}_\mu$ was transmitted by a given received signal $\mathbf{r}$. The estimate of $\mathbf{d}$ obtained with MLSE is

$$\hat{\mathbf{d}} = \arg \max_{\mathbf{d}_\mu}, P\{\mathbf{d}_\mu|\mathbf{r}\}, \qquad (2)$$

with arg denoting the argument of the function. By using Bayes' rule [5], the conditional probability $P\{\mathbf{d}_\mu|\mathbf{r}\}$ can be written as

$$P\{\mathbf{d}_\mu|\mathbf{r}\} = \frac{p(\mathbf{r}|\mathbf{d}_\mu)P\{\mathbf{d}_\mu\}}{p(\mathbf{r})}, \;\; \mu = 1, \ldots, M_d^{K_q}. \qquad (3)$$

When assuming that all vectors $\mathbf{d}_\mu$, $\mu = 1, \ldots, M_d^{K_q}$, are equally probable and by noting that the denominator in (3) is independent of the transmitted data symbol vector, the decision rule based on finding the sequence that maximizes $P\{\mathbf{d}_\mu|\mathbf{r}\}$ is equivalent to finding the sequence that maximizes $p(\mathbf{r}|\mathbf{d}_\mu)$. The conditional probability density function $p(\mathbf{r}|\mathbf{d}_\mu)$ of the received vector $\mathbf{r}$ given $\mathbf{d}_\mu$ is referred to as a likelihood function. With independent complex-valued white Gaussian noise on the subcarriers, the elements of the received vector r are statistically independent and $p(\mathbf{r}|\mathbf{d}_\mu)$ may be expressed as [5]

$$p(\mathbf{r}|\mathbf{d}_\mu) = \left(\frac{1}{\pi\sigma^2}\right)^L \exp\left(-\frac{1}{\sigma^2}\|\mathbf{r} - \mathbf{HCd}_\mu\|^2\right), \quad \mu = 1, \ldots, {M_d}^{K_q}. \tag{4}$$

The conditional probability $P(\mathbf{d}_\mu|\mathbf{r})$ is maximized by minimizing the squared Euclidean distance:

$$\Delta^2(\mathbf{d}_\mu, \mathbf{r}) = \|\mathbf{r} - \mathbf{H}\ \mathbf{W}_{rot}\ \mathbf{d}_\mu\|^2, \quad \mu = 1, \ldots, M_d^{K_q}, \tag{5}$$

between the received and all possible transmitted sequences. The most likely transmitted data vector can be expressed as [4]

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}_\mu} \Delta^2(\mathbf{d}_\mu, \mathbf{r}). \tag{6}$$

B. Coded MLSSE

The MLSSE minimizes the symbol error probability, which is equivalent to maximizing the conditional probability $P\{d_\mu^{(k)}|\mathbf{r}\}$ that $d_\mu^{(k)}$ was transmitted given $\mathbf{r}$. The estimate of $d^{(k)}$ obtained by MLSSE is

$$\hat{d}^{(k)} = \arg\max_{d_\mu^{(k)}} P\{d_\mu^{(k)}|\mathbf{r}\}. \tag{7}$$

The conditional probability $P\{d_\mu^{(k)}|\mathbf{r}\}$ is given by

$$P\{d_\mu^{(k)}|\mathbf{r}\} = \sum_{\substack{\forall \mathbf{d}_\mu \text{ with same} \\ \text{realization of } d_\mu^{(k)}}} P\{\mathbf{d}_\mu|\mathbf{r}\}, \quad \mu, \ldots, M_d^{K_q}, \tag{8}$$

where the probability $P\{d_\mu^{(k)}|\mathbf{r}\}$ is the union of all mutually exclusive events $P\{\mathbf{d}_\mu|\mathbf{r}\}$ with the same realization of $d_\mu^{(k)}$ [5]. By using Bayes' rule and assuming that all data symbols $d_\mu^{(k)}$ are equally probable and by noting that $p(\mathbf{r})$ is independent of the transmitted data symbol, the decision rule based on finding the symbol that maximizes $P\{d_\mu^{(k)}|\mathbf{r}\}$ is equivalent to finding the symbol that maximizes $p(\mathbf{r}|d_\mu^{(k)})$. Thus, with (7) and (8), by ignoring the constant factor $1/(\pi\sigma^2)^L$ which is independent of the transmitted data symbol, the most likely transmitted data symbol is

$$\hat{d}^{(k)} = \arg\max_{d_\mu^{(k)}} \sum_{\substack{\forall \mathbf{d}_\mu \text{ with same} \\ \text{realization of } d_\mu^{(k)}}} \exp\left(-\frac{1}{\sigma^2}\Delta^2(\mathbf{d}_\mu, \mathbf{r})\right). \tag{9}$$

The MAP decoder uses soft log-likelihood ratios (LLRs) of all possible symbols that are inherently produced by the MLSSE unit. The LLRs for coded MC-CDMA mobile radio systems applying MLSSE are given by

$$\mathcal{L}^{(k)} = \ln\left(\frac{\sum_{\forall \mathbf{d}_\mu \in \mathcal{D}_+^{(k)}} \exp\left(-\frac{1}{\sigma^2}\Delta^2(\mathbf{d}_\mu, \mathbf{r})\right)}{\sum_{\forall \mathbf{d}_\mu \in \mathcal{D}_-^{(k)}} \exp\left(-\frac{1}{\sigma^2}\Delta^2(\mathbf{d}_\mu, \mathbf{r})\right)}\right). \tag{10}$$

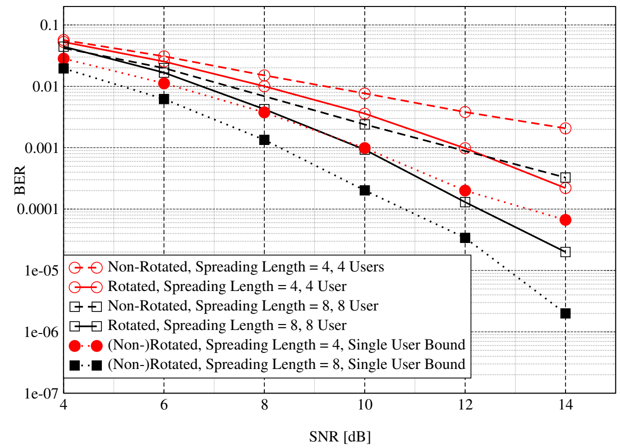

Fig. 3. Performance comparison of rotated and non rotated spreading codes with a spreading length of four and eight.

V. Simulation Results

In this section simulation results for an uncoded and a coded system for the independent Rayleigh fading channel are presented.

A. Uncoded MLSE

The MLSE detector in an uncoded MC-CDMA system is applied for different spreading sequences for a 4-QAM modulated system. Fig. 3 shows the system with and without rotated transforms for a spreading length L of four and eight. The performance of the system is given for a full loaded system, where the spreading length equals the number of users. As the reference curve the single user bound for rotated and non-rotated transforms is given. The single user bound does not differ between the rotated and the non-rotated case. For a full loaded system with $L = 8$ the performance improves by 2 dB at a BER of $2 \cdot 10^{-3}$. For $L = 4$ and a full loaded system the improvement versus the non-rotated scheme is about 3 dB at a BER of $2 \cdot 10^{-3}$. The remaining loss for the rotated transform against the single user bound is about 2 dB for $L = 4$ or 1.5 dB for $L = 8$ and does not increase notably for higher SNRs.

B. Coded MLSSE

The parameters of the encouraging results of the uncoded MLSE rotated transforms were applied for a coded MLSSE system with a convolutional code of rate $\frac{1}{2}$ and memory size

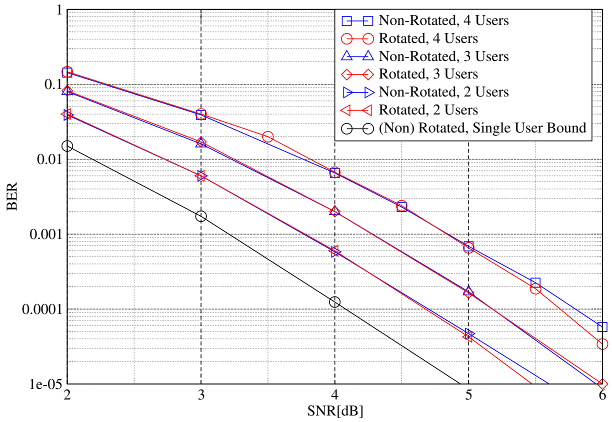

Fig. 4. Performance comparison of rotated and non rotated spreading codes with a spreading length of four in a coded system with a code rate of $\frac{1}{2}$ and different user load.

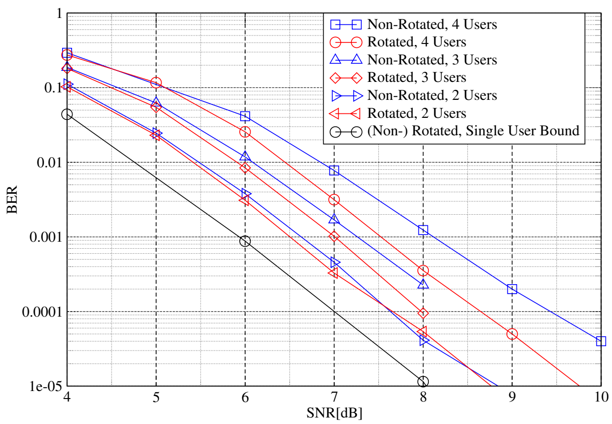

Fig. 5. Performance comparison of rotated and non rotated spreading codes with a spreading length of four in a coded system with a code rate of $\frac{5}{6}$ and different user load.

six. The minimum Hamming distance of this code is ten. The simulated system used a spreading code length of four, 512 subcarriers, 16 data symbols per user and OFDM symbol, and 64 OFDM symbols per frame. All sorts of user load from the single user bound to the full loaded system are compared. The results in Fig. 4 are sobering. For all cases no gain can be found. The increase of diversity caused by the rotated transforms cannot be exploited. The codebit interleaver distributes the signal in the signal space very efficiently. The increase of the Hamming distance by the rotated transforms does not lead to any further gains. In addition a different code with a higher code rate of $\frac{5}{6}$ was used. This encoder has also a memory size of six, but the Hamming distance is only three. The gain caused by applying rotated transforms increases with the load of the system. Fig. 5 shows the improvement of the system performance for the rotated transforms for the full loaded case at a BER of 10^{-4} is nearly 1 dB.

VI. Summary

By applying rotated transforms the Euclidean distance for each superposed constellation is distributed more favorable in the signal space. The advancement is given by combining the multi-user detector with the rotated transform which exploits the more favorable distribution of the Euclidean distances of each superposed sequence.

This approach is applicable for every system that applies code division multiplex in the time, frequency or space domain. In this paper the procedure has been exemplarily demonstrated for an uncoded and coded MC-CDMA system. A multiuser detector, like the used MLSE-detector, enhances the performance for an uncoded full-loaded MC-CDMA system with 4-QAM modulation and a spreading length of eight by at least 2 dB for eight users and for a spreading length of four for four users by about 3 dB at a BER $2 \cdot 10^{-3}$.

For a coded system, the performance depends on the Hamming distance of the chosen code. A convolutional code with a Hamming distance of ten does not offer any notable extra gains that the multiuser detector could exploit. However higher code rates of convolutional codes do not offer higher Hamming distances. Rotated transforms improve these coded systems by up to 1 dB at a BER of 10^{-4} for a full loaded system with four users.

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