Sequencing Groups of Jobs under Precedence Constraints

Yakov M. Shafransky
Institute of Engineering Cybernetics, National Academy of Sciences of Belarus
shafr@newman.bas-net.by

Abstract

The paper considers scheduling models with precedence constraints under the group availability condition. The main result is an extension of the techniques developed for the problems under job availability to those under group availability.

A number of scheduling problems may be formulated as problems of optimal sequencing under precedence constraints [1, 2, 3]. Priority-generating functions introduced at the end of 70th and the technique of their minimization under precedence constraints (see [3] for review) allowed us to understand the nature of algorithms developed earlier for various known sequencing problems (see [4, 5, 6, 7, 8, 9]) and to solve several new problems. The concept of a priority-generating function was introduced by Shafransky [10, 11]. Similar concepts were introduced and published some later by Burdyuk and Reva [12, 13] and Monma and Sidney [14]. An algorithm for minimizing an arbitrary priority-generating function over a set of permutations under series-parallel precedence constraints was proposed by Gordon and Shafransky [11, 15]; similar approach was proposed by Monma and Sidney [14]. A number of papers were devoted to a technique for minimizing priority-generating functions under arbitrary precedence constraints [16, 17, 18, 19, 20]. Note that for arbitrary precedence constraints the problem is NP-hard in the strong sense. The developed technique was later extended for solving multi-criteria sequencing problems under both precedence and group technology constraints [21].

In the latter paper, a scheduling model with job availability is considered, i.e., the model in which each job is available for further processing immediately after its processing on some machine is finished. On the other hand, models with group availability are widespread, in which jobs of the same group (or batch) may leave the processing machine only together after the processing of the last job of the group is finished.

The paper considers group availability models with precedence constraints. To extend the techniques developed for job availability models for solving problems under group availability, we need to show that under these new conditions the objective function of a problem is still priority-generating.

Now we proceed with the main problem formulation.

Let P_n be the set of all permutations of elements of finite set $N = \{1, 2, \ldots, n\}$ and N is partitioned into q subsets (groups): $N = N_1 \cup N_2 \cup \ldots \cup N_q$, $N_l \cap N_k = \emptyset$ for

$l \neq k$. There is a precedence relation $\stackrel{k}{\rightarrow}$ defined for each subset N_k and represented by an acyclic graph $G_k = (N_k, U_k)$. There is also a precedence relation $\stackrel{0}{\rightarrow}$ defined over set $N_0 = \{N_1, N_2, \ldots, N_q\}$ and represented by an acyclic graph $G_0 = (N_0, U_0)$.

Permutation $\pi = (i_1, i_2, \ldots, i_n)$, $i_\nu \in N$, $\nu = 1, 2, \ldots, n$, is called feasible if the following conditions hold:

(a) for any $k \in \{1, 2, \ldots, q\}$ conditions $i_\nu, i_\mu \in N_k$ and $\nu < \mu$ imply $i_j \in N_k$ for every $\nu < j < \mu$ (i.e., elements from the same group should be situated contiguously in every feasible permutation);

(b) for any $k \in \{1, 2, \ldots, q\}$ conditions $i_\nu, i_\mu \in N_k$ and $i_\nu \stackrel{k}{\rightarrow} i_\mu$ imply $\nu < \mu$ (i.e., a mutual disposition of elements of any group k should not contradict to the relation $\stackrel{k}{\rightarrow}$);

(c) for any $k, l \in \{1, 2, \ldots, q\}$, $k \neq l$, conditions $i_\nu \in N_k$, $i_\mu \in N_l$ and $N_k \stackrel{0}{\rightarrow} N_l$ imply $\nu < \mu$ (i.e., a mutual disposition of elements from different groups should not contradict to the relation $\stackrel{0}{\rightarrow}$).

Denote a set of all feasible permutations by $P_n(G_0, G_1, \ldots, G_q)$. Let function $F(\pi)$ be defined over such a set P' that $P_n(G_0, G_1, \ldots, G_q) \subseteq P'$. Consider the following sequencing problem.

For a function $F(\pi)$ defined over a set P' it is necessary to find such a permutation π^* that

$$\pi^* \in P_n(G_0, G_1, \ldots, G_q)$$

and

$$F(\pi^*) = \min\{F(\pi) | \pi \in P_n(G_0, G_1, \ldots, G_q)\}.$$

Permutation π^* is called optimal.

The elements of set N are associated with jobs, and a feasible permutation π represents a feasible schedule. Thus, the problem is to find a feasible schedule that minimizes a given objective function under the condition that the completion time of a job is the completion time of a group the job belongs to.

Now we give the notion of a priority-generating function.

Let $\hat{P}$ be the set of all permutations $\pi_r = (i_1, i_2, \ldots, i_r)$, $r = 0, 1, \ldots, n$, of the elements of a set $N = \{1, 2, \ldots, n\}$. Here r is the length of a permutation π_r, $\{\pi_0\} = \emptyset$ and $\{\pi\}$ denotes the set of all elements, which the permutation π consists of. For a permutation set $P \subseteq \hat{P}$ we denote by $S[P]$ the set of all subpermutations or strings from P, i.e., $\pi' \in S[P]$ if and only if there exist $\sigma, \chi \in \hat{P}$ such that $(\sigma, \pi', \chi) \in P$.

Given a function $F(\pi)$ defined over a set $P' \subseteq \hat{P}$, suppose that there exists a function $\omega(\pi)$ defined over set $S[P], P \subseteq P'$, and possessing the following property: for any $\alpha, \beta \in S[P]$ and any permutations $\pi' = (\sigma, \alpha, \beta, \chi)$ and $\pi'' = (\sigma, \beta, \alpha, \chi)$ such that $\pi', \pi'' \in P$, the inequality $\omega(\alpha) > \omega(\beta)$ implies $F(\pi') \leq F(\pi'')$ and the equality $\omega(\alpha) = \omega(\beta)$ implies $F(\pi') = F(\pi'')$.

In this case we say that $F(\pi)$ is a priority-generating function and $\omega(\pi)$ is its priority function.

The main result of the paper is establishing the fact that most of the functions that are priority-generating under job availability preserve this property under group availability. The paper gives corresponding proofs and presents a priority function for each specific function to be minimized.

References

[1] Brucker, P.: Scheduling algorithms. Springer (1998) Berlin.

[2] Lawler, E.L., Lenstra, J.K., Rinnooy Kan, A.H.G., Shmoys, D.B.: Sequencing and scheduling: algorithms and complexity. In: Handbook in Operations Research and Management Science, Vol.4, Logistics of Production and Inventory, Graves S.C, Rinnooy Kan A.H.G., Zipkin P.H. (eds.) Amsterdam, North-Holland (1993) 445–522.

[3] Tanaev, V.S., Gordon, V.S., Shafransky, Y.M.: Scheduling Theory. Single-Stage Systems. Kluwer academic publishers (1994) Dordrecht-Boston-London.

[4] Adolphson, D.L.: Single machine job sequencing with precedence constraints. SIAM J. Comput. (1977) 6, N 1, 40–54.

[5] Adolphson, D.L., Hu, T.C.: Optimal linear ordering. SIAM J. Appl. Math. (1973) 25, N 3, 403–423.

[6] Kurisu, T.: Two-machine scheduling under required precedence among jobs. J. Oper. Res. Soc. Jap. (1976) 19, N 1, 1–13.

[7] Lawler, E.L.: Optimal sequencing of jobs subject to series-parallel precedence constraints. Preprint BW 54, Mathematische Centrum, Amsterdam (1975).

[8] Lawler, E.L.: Sequencing jobs to minimize total weighted completion time subject to precedence constraints. Ann. Discr. Math. (1978) 2, 75–90.

[9] Sidney, J.B.: Decomposition algorithms for single-machine sequencing with precedence relations and deferral costs. Oper. Res. (1975) 23, N 2, 283–298.

[10] Shafransky, Y.M.: Optimization for deterministic scheduling systems with tree-like partial order. Vestsi Akademii Navuk BSSR. Ser. fiz.-mat. navuk (1978) N 2, 119 (in Russian).

[11] Gordon, V.S., Shafransky, Y.M.: On a class of scheduling theory problems with partially ordered jobs. In: Proceedings of the 4th All-Union Conference on Theoretical Cybernetics Problems, Novosibirsk (1977) 101-103 (in Russian).

[12] Burdyuk, V.Y., Reva, V.N.: A method for optimizing functions over permutations under constraints. Kibernetika (1980) N 1, 99–103 (in Russian).

[13] Reva, V.N.: On an algorithm for optimizing a function over the permutations of a partially ordered set. In: Actual Problems of Computers and Programming, Dnepropetrovsk (1979) 92–95 (in Russian).

[14] Monma, C.L., Sidney, J.B.: Sequencing with series-parallel precedence constraints. Math. Oper. Res. (1979) 4, N 3, 215–224.

[15] Gordon, V.S., Shafransky, Y.M.: Optimal ordering with series-parallel precedence constraints. Doklady Akademii Nauk BSSR (1978) 22, N 3, 244-247 (in Russian).

[16] Shafransky, Y.M.: On a problem of minimizing functions over a set of permutations of partially ordered elements. I, II. Vestsi Akademii Navuk BSSR. Ser. fiz.-mat.navuk (1980) N 5, 132; (1982) N 1, 113 (in Russian).

[17] Shafransky, Y.M.: On an algorithm for finding the minimum of priority-generating functions over special sets of permutations. I, II. Vestsi Akademii Navuk BSSR. Ser. fiz.-mat.navuk (1982) N 3, 38–42; (1983) N 1, 15–20 (in Russian).

[18] Monma, C.L.: Sequencing with general precedence constraints. Discr. Appl. Math. (1981) 3, N 2, 137–150.

[19] Sidney, J.B., Steiner, G.: Optimal sequencing by modular decomposition: polynomial algorithms. Oper. Res.(1986) 34, 606–612.

[20] Monma, C.L., Sidney, J.B.: Optimal sequencing via modular decomposition: Characterization of sequencing functions. Mathematics Oper. Res. (1987) 12, 22–31.

[21] Janiak, A., Shafransky, Y.M., Tuzikov, A.V.: Sequencing with ordered criteria, precedence and group technology constraints. Technical Report. Wroclaw Institute of Technology (1993).