Original Article

Institution of MECHANICAL ENGINEERS

Journal of MECHANICAL ENGINEERING SCIENCE

Proc IMechE Part C:
J Mechanical Engineering Science
227(8) 1870–1884

DOI: 10.1177/0954406212466354
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Design and development of strain wave generating cam for a new concept 'harmonic drive'

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Abstract

A split cam design is proposed to solve the problem of assembly of the single piece cam in the flexible raced bearing of an earlier proposed novel harmonic drive system, which shows better torque characteristics and capacities in comparison to the conventional one of same size with oval-shaped strain wave generating cam. The cam profile has circular arcs at two working zones at 180° phases. The proposed profile shape is identified as the cause of trouble in assembly if the cam is made single piece. The split cam is made of two identical pieces having circular arc edges. These pieces constitute the cam in assembly after putting it inside the inner race of the flex bearing and adjusted by an adjuster. The design, kinematics, and the assembly method of the proposed split cam are presented in this article. The split cam arrangement not only solves the assembly problem but also gives a scope of fine adjustment of center distance (eccentricity). Such an adjustment is not possible in conventional oval wave generating cam. Stresses in flex gear cups assembled with both type cams at load and no-load conditions are estimated using finite element method. Some results are verified experimentally. Although the flex gear cup with the proposed split cam experiences lower stresses at load transmitting active gear contact zones, it shows higher stresses at some non-active zones (where teeth are free of load). It is apparent from results that stresses at those non-active zones do not increase substantially with the increase in torque, as they are away from active zones.

Keywords

Strain wave gearing, harmonic drive, involute profile, flex gear, ring gear, wave generator cam, split cam, longitudinal and lateral stresses in flex gear cup

Date received: 21 May 2012; accepted: 9 October 2012

Introduction

Figure 1(a) shows the feature of a conventional harmonic drive (HD). The reason of choosing an oval to shape the cam is lying in the fact that with natural deflection along the line through the center, a circular ring resumes an elliptical-like shape. Thereby, the assembly of the cam in the flex gear (FG) (The term 'flex gear' (FG) instead of 'flex spline' (FS) is used in this article. This is because HDs with the standard and corrected *involute gears* are dealt in this article.) cup is very easy. Various attempts are made to improve the performance of the strain wave gearing (SWG) system, commonly known as HD, since its introduction by Musser[1,2] in late 1950s. The synthesis of suitable tooth profiles to work with the conventional oval-shaped cam[3–5] is one of them. Consequently, attempts are made to optimize deflections of teeth, rim, bearing, length of the flex spline (FS) cup, etc.[6–10] Some researchers have proposed electromagnetic actuation techniques[11,12] to replace the solid oval cam. They have concentrated on the cam performance rather than improving teeth profiles. HDs with most common involute teeth[8,9] can perform only at a low torque capacity. It is obvious due to the fact that the gearing becomes poorer with pure involute teeth of deformed FG and circular ring gear (RG). In recent time, Dong et al.[13] have investigated further kinematics of HDs in a more rigorous way targeting the better tooth design to improve performances. Vladis[14] has considered tooth deformations to show the phase change of the tooth contact zone with wave generator

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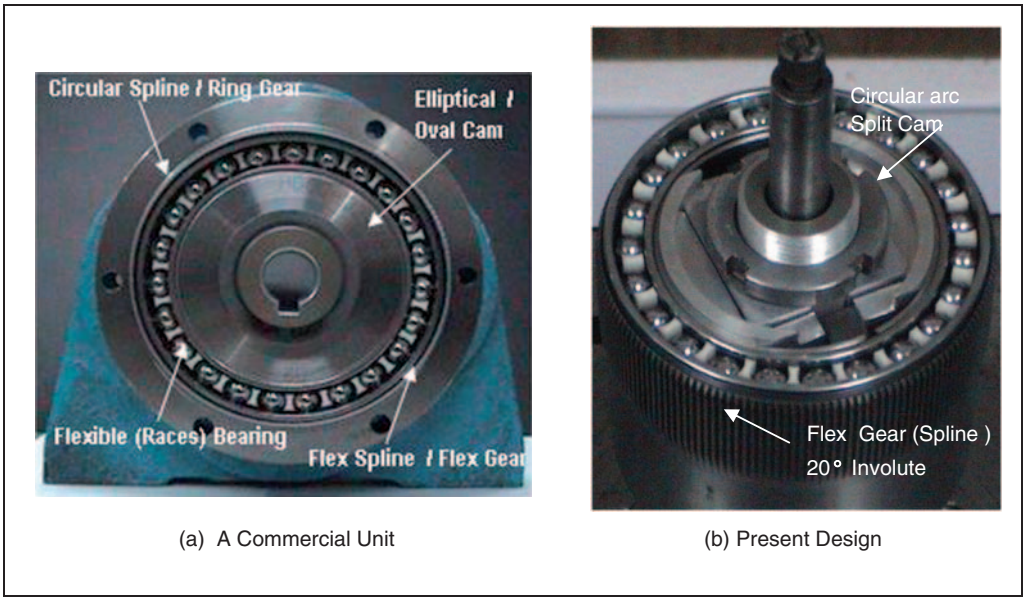


Figure 1. Photographic views of HD (SWG) units.
HD: harmonic drive; SWG: strain wave gearing.

cam (WGC) with the change in torque. To improve the performance and life, tribological aspects are also rigorously investigated by Ueura et al.[15] On the other hand, FS of composite material is considered to kinematic borne vibrations in HDs.[16]

Routh and Maiti[17] have shown that the contact pattern for the internal–external involute gear set in a conventional SWG or HD with the oval-shaped WGC does not maintain the gearing laws properly. Therefore, HDs with involute gears and the conventional oval cam are used for a high ratio but moderate torque output in less precision and relatively low-cost applications. Patterns of tooth contacts and interferences,[18,19] error in kinematics,[20] and dynamic behaviors[21,22] have also been investigated earlier. In synthesizing base curves for elliptical gears, Figliolini and Angeles[23] have mentioned about HDs indicating FS as an elliptical gear. However, it is to be noted that the FS, i.e. FG of a HD is not an elliptical or an oval-shaped gear meshing with a circular internal gear. Rather, the FG is a circular gear, the rim (as well as the base circle) of which is deformed by the oval cam to perform in HDs. The approach of elliptical gear design cannot directly be adopted in designing FGs for HDs. At the most the approach and the theory may help in finding the optimum contact patterns.

A new WGC for the HD to work with pure involute gear pair is proposed by Maiti[24,25] in an earlier investigation. The proposed cam profile ideally is a combination of circular and elliptical arcs. The circular portion maintains the involute teeth contact pattern satisfying gearing laws. The basic idea is that the pitch circle is stretched keeping a certain amount of its circular portion intact, such that the geometrical center of the new cam coincides with the center of the whole unit which is also the center of the RG. The circular portion of the pitch curve is having radius equal to the original pitch circle radius of the pinion, but shifted along the cam's major axis by the center distance. The circular portion of the cam is made accordingly at a constant inward distance from the new pitch curve. Remaining portions of the WGC can be connected by any suitable arcs other than an elliptical curve which is considered in this investigation. Another condition which is also to be satisfied, for smooth gearing, is that the slope of the circular arc and the joining arc at their junction points (four such zones in a cam) must be equal. In design, described in earlier investigations,[24,25] joining arcs are taken as a part of an ellipse equally spread in two sides of the minor axis. The center of the considered ellipse is found to be situated at a certain distance (along the *Y*-axis as in Figure 12 in 'Geometry of the pitch curve of the proposed WGC') away from the center of the unit. The cam profile is inwardly shifted curve which is parallel to the pitch curve. This means that the curve is shifted by a constant difference along radii vectors of the curvature of the pitch curve. The amount of shift is equal to the summation of the dedendum height of FG tooth and, total thickness of the rim and bearing thickness along the radial direction.

In the original proposal of the novel HD by Maiti[24,25] initially, a single piece cam was considered. It was very difficult to assemble the new proposed cam in contrary to the assembly of conventional elliptical-like cam in the flexible bearing and the whole subassembly in the cup of FG. Later, a split cam design[26] is found to be a good solution to eliminate the assembly problem (Figure 1(b)). Two identical pieces, with circular arcs at outer periphery, are used to construct the proposed cam. These two pieces are put inside the

Table 1. Specification of considered commercial 'HD' with oval cam.

Nomenclature	Description	Parametric value	Remarks
i_1	Transmission/reduction ratio	78	
T_{o1}	Output torque	118/167/294 Nm Rated/intermittent/ maximum starting	$i_1 = -\frac{Z_p}{Z_g - Z_p}$
Z_g	Number of teeth in RG	158	Involute tooth profile
Z_p	Number of teeth in FG	156	
α_o	Pressure angle	30°	
m	Normal module	0.529 mm (48 DP)	Addendum = module × factor
a_{fg}	Addendum factor of RG	0.6	
a_{fp}	Addendum factor of FG	0.456	
$2a$	Major axis of the cam (ellipse)	80.85 mm	Including flex bearing
$2b$	Minor axis of the cam (ellipse)	78.66 mm	

HD: harmonic drive; RG: ring gear; and FG: flex gear.

flexible bearing and they are pushed apart by a taper adjuster to form the desired cam. Methods of profile design and steps of construction are established in this investigation considering all aspects of design and performance requirements. The selection of included angle of the arc is an important task. In next step, the stresses in FG cup at load and no-load conditions are estimated analytically using finite element method (FEM) in ANSYS® environment. Some results are also verified experimentally. Cases of both the conventional cam and the proposed split cam working with same gear set, i.e. FG and RG, are examined.

Proposed wave generator split cam

The proposed SWG or HD system is made identical in overall size, specification (except gears), and shape of a standard commercially available unit. The commercial unit has involute gears. In the proposed HD, FG and RG have also equal numbers of involute teeth. However, involute teeth specifications (with respect to teeth heights, profile shift, etc.) are different from the conventional one, to match with the proposed cam. The specified data of the commercial unit (as measured and from manufacturer's catalogue) are as in Table 1. Figure 2(a) shows the gear teeth shapes of the RG and FG of the commercial unit.

Geometry

The geometric design of the outer profile of the proposed WGC, established earlier by Maiti[24–26] and Nema,[27] can be realized from the pitch curve design presented briefly in 'Geometry of the pitch curve of the proposed WGC'. In designing split cam first of all, a profile following that design method is to be selected. Keeping the perimeter of the proposed cam equal to that of the commercial oval cam and satisfying all necessary gearing and kinematic conditions, parametric values for the suitable cam are calculated. The 20° involute teeth system, as it would be more critical with respect to the avoidance of tip interference with low tooth difference (two in this case)[19,25] is also considered for the present design. It is to be noted that with 30° involute teeth system, contact ratio is less for same proportions of teeth with 20° involute, although other gearing features are better. The gear set of both 30° and 20° involute teeth systems for the proposed HD are manufactured indigenously using wire electrical discharge machining (EDM) and spark erosion EDM. Teeth profiles are shown in Figure 2(b).

In estimating the profile of elliptical-shaped portions of the proposed WGC, the pitch curve of FG is designed. A suitable value of semi major axis of the ellipse – 'a', is selected (Figure 12 in 'Geometry of the pitch curve of the proposed WGC').[24–29] It is studied and found that an initial value of 'a' is to be taken smaller than r_p by a factor of 3.5 to 4 times the module. Then, with varying 'b' – the semi minor axis, the point of tangency D with the pitch circle of the pinion and corresponding values of 'C' and ψ are determined. The value of 'A' is set equal to the center distance in internal gearing with the unstretched FG, following equation (1). In the next phase, the circumference of the pitch curve is compared with the circumference of the pitch circle of the pinion solving equation (11). Finally, tip interference ('Check for tooth interference in gearing with proposed cam') and contact ratio ('Contact ratio') are verified. In case that any or both of them are not satisfied, 'a' is varied, the whole process is repeated and a suitable solution is found.

A typical set of result is presented in Table 2. The actual profile of the WGC will be a curve inwardly shifted from the generated pitch curve, with constant difference, as mentioned earlier. Referring to 'Geometry of the pitch curve of the proposed WGC', the amount of shift is a_s times the module

(as shown in Figure 13). It depends on the rim thickness that will satisfy the flexion as well as foundation rigidity and the bearing thickness along the radial direction. It is to be verified that the inward shift, which is along the radius of curvature vector of the pitch curve for each and every point, must not cross the center of curvature. It is to be suitably less than the radius of curvature.

For larger circular arcs, i.e. for larger ψ, values of some parameters including the possibility of avoidance of tip interferences, improve. However, as elliptical portions become smaller, very short spaces are available for teeth disengagement to engagement (and vice versa) of the teeth pair. In reality, this may cause assembly and kinematic problems. Again keeping the contact ratio satisfactory, if the addendums of both gears are shortened, then the above situation improves. Figure 3 shows the variation of minor axis and the shift of ellipse center from the cam center satisfying all design and kinematic conditions, for a selected semi major axis – '*a*'. With the increase in '*a*', '*C*' increases and '*b*' decreases. Usually, in internal gearing, full depth pinion teeth and gear teeth means addendum heights are 1 module times for the pinion and is 0.8 times module for gear teeth. In normal cases, i.e. for an ordinary internal gearing with 20° involute teeth, the difference in teeth numbers of the RG and the pinion is to be minimum 8 to avoid tip interference.[19] Fortunately, two teeth difference is possible in case of HD with the new proposed cam as the FG is deformed at engagement and disengagement zones.[19,25] However, this is not possible with the conventional cam (say of elliptical shape). In that case, for two teeth difference, pressure angle is taken 30°. Full depth gears are considered in the present analysis. To be in safe side, keeping the contact ratio equal to 1.4 or above, addendums of both the pinion teeth and the gear teeth were taken as 0.75 resulting in better allowances for avoiding tip interferences. It is to be noted that a design beyond the range of limiting values of the semi major axis and the semi minor axis, as in the illustrations Figure 3, are not possible. Figures 4 and 5 show the variation in ranges of other important design parameters, such as the tip interference, the angular spread of circular arc, the ellipse–circular arc intersection, etc. Both θ_p–γ (°) and ψ (°) have their highest values at the limiting lowest value of '*a*' (Figure 4). This is better for the

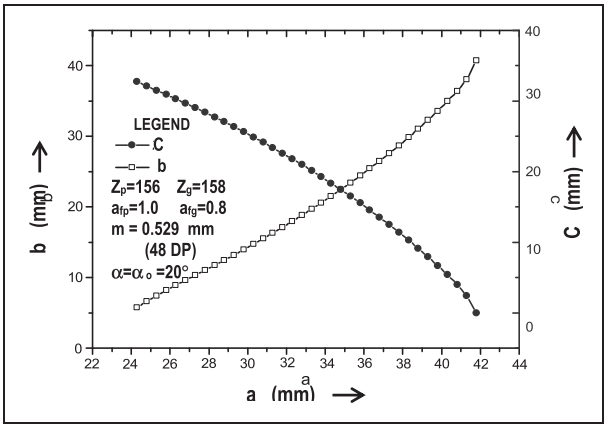


Figure 3. Variation of *C* and *b* with *a* in the proposed cam profile (Table 2).

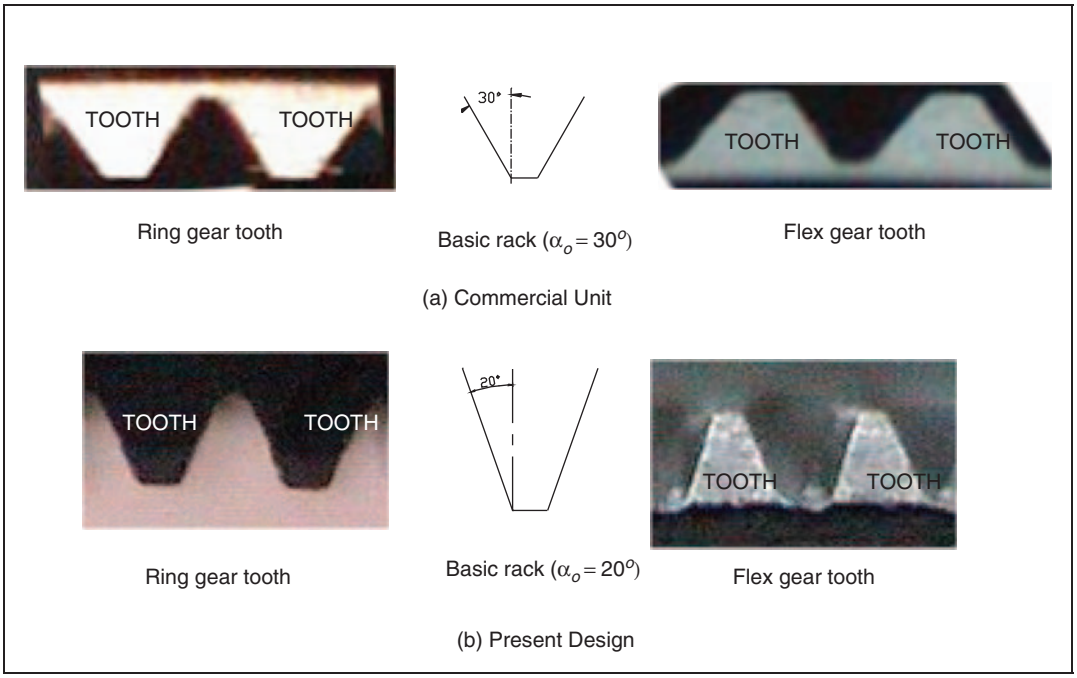


Figure 2. Teeth of HD gears.
HD: harmonic drive.

avoidance of tooth tip interference. However, spaces between the FG tooth tips and the RG tooth tips at engagement and disengagement zones become smaller. Compromising all aspects values of '*a*', '*b*', and '*C*' at the middle of their ranges are to be chosen for the cam design.

Any set of data presented in Table 2 can be taken for the new cam design. However, the data shown in bold face letters (Table 2) are considered, following the above arguments, for the proposed cam for constructing the HD prototype to be tested. The cam profile, including the bearing, i.e. the resumed shape of the inside wall of the FG on assembly, is shown in Figure 6. It is to be noted that the cam length along the pseudo major axis of the cam profile curve is larger than that of conventional elliptical-like cam. This is due to the difference in positions of the working pitch points of the two units as they have teeth of different pressure angles and corrections.

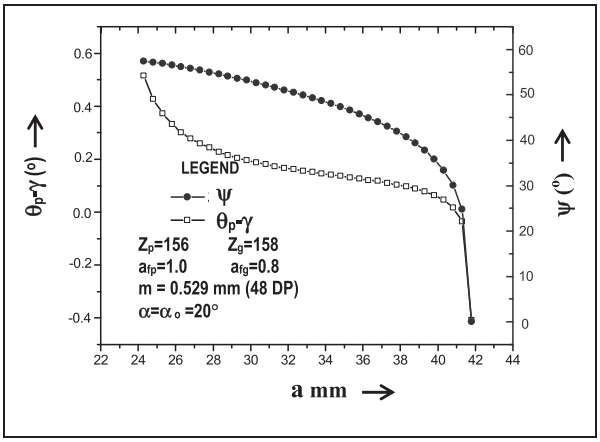


Figure 4. Variation of $\theta p - \gamma$ (°) and ψ (°) with a in the proposed cam profile (Table 2).

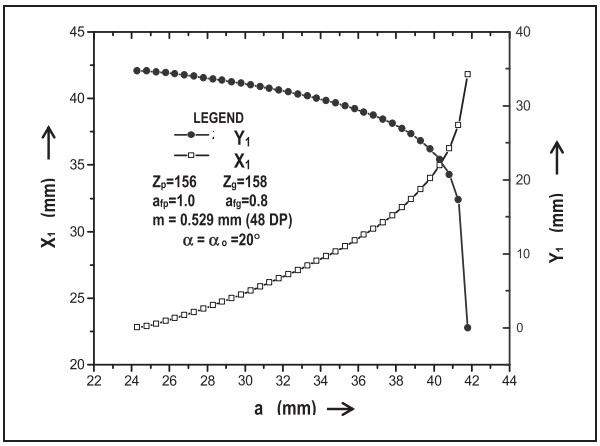


Figure 5. Variation of x_1 and y_1 with a in the proposed cam profile (Table 2 and Figure 12).

Assembly of the new cam

The assembly of the conventional cam of oval shape into the FG cup is not a problem. Exploded views of the conventional oval-shaped cam assembly as well as the new proposed cam assembly are shown in Figure 7. An oval cam (1) is made of a single solid piece. It is put on a hub (2) with a coupling (3) (like an Oldham coupling) in between. Then, a circlip (4) is put to fix the subassembly. This assembled cam can easily be inserted inside the ball bearing (5) with thin and flexible inner and outer races specifically made for such HD units. Finally, the cam unit can easily be put inside the cup-shaped FG cup (6), which is simply pressed diametrically by hand for easy insertion. It may be attributed to the fact that the natural

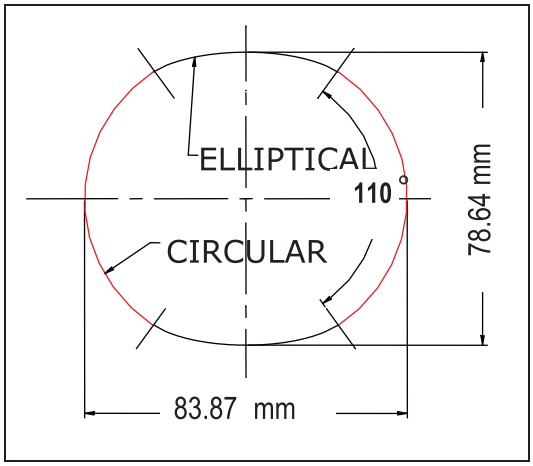


Figure 6. Geometric shape of the pitch curve of FG with the proposed new cam.
FG: flex gear.

Table 2. Data of new split cam with gear pair of present design (Appendix 2 and Figures 12 and 13).

a (mm)	b (mm)	C (mm)	ψ (°)	θ_p (°)	γ (°)	$\theta_p - \gamma$ (°)	x_1 (mm)	y_1 (mm)	C_r
30.80	15.49	24.16	52.15	65.017	64.823	0.193	25.848	8.417	1.490
28.30	11.71	27.73	54.56	65.348	65.114	0.234	24.453	5.891	1.490
27.80	**11.0**	**28.39**	**55**	**65.363**	**65.115**	**0.247**	**24.196**	**5.409**	**1.490**
27.30	10.28	29.04	55.42	65.364	65.101	0.262	23.948	4.931	1.490
26.80	9.576	29.68	55.82	65.336	65.055	0.281	23.707	4.461	1.490

$Z_g = 158$; $Z_p = 156$; $\alpha_o = 20°$ involute; $m = 0.529$ mm (48 DP); $a_{fg} = 0.75$; and $a_{fp} = 0.75$.

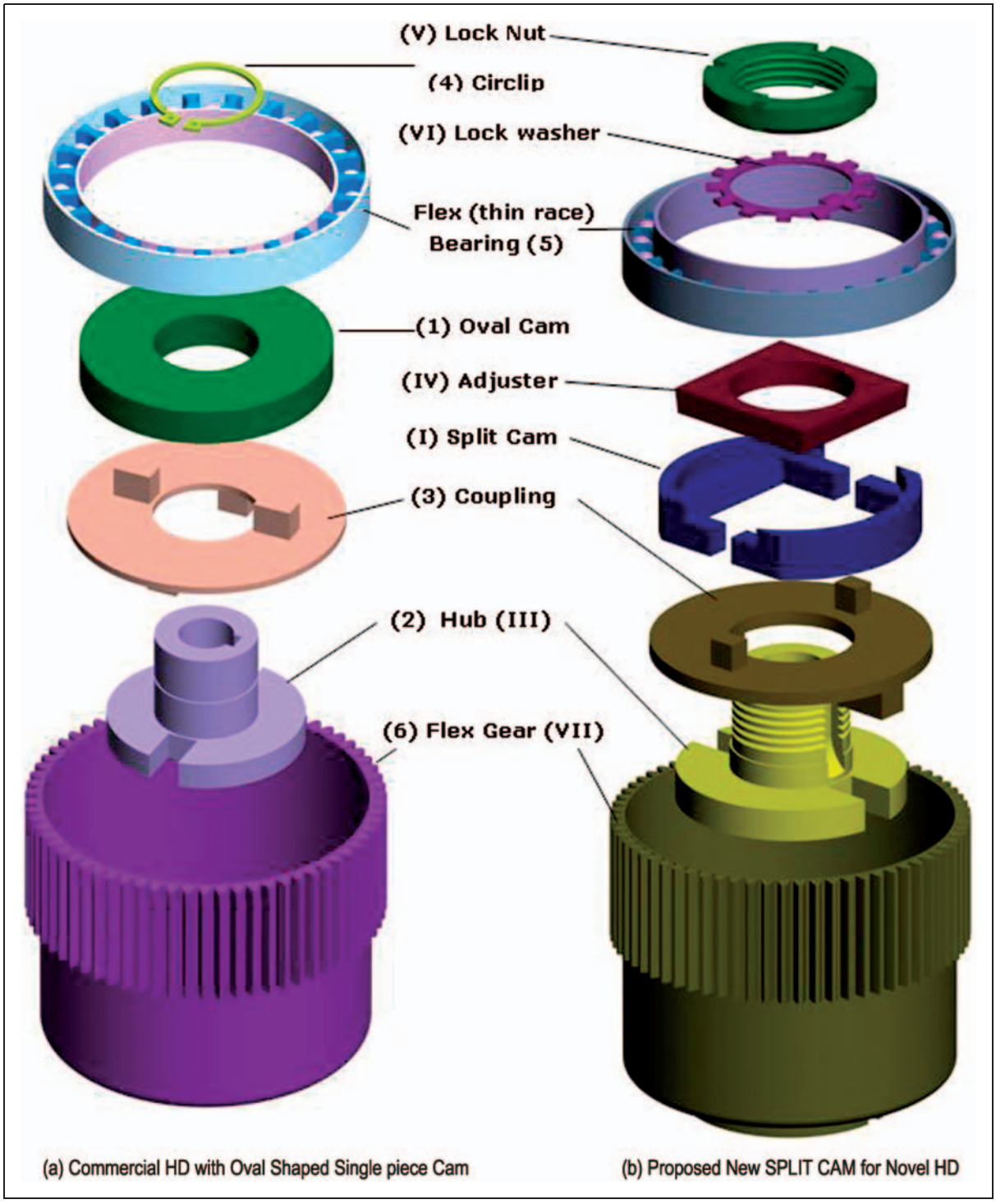
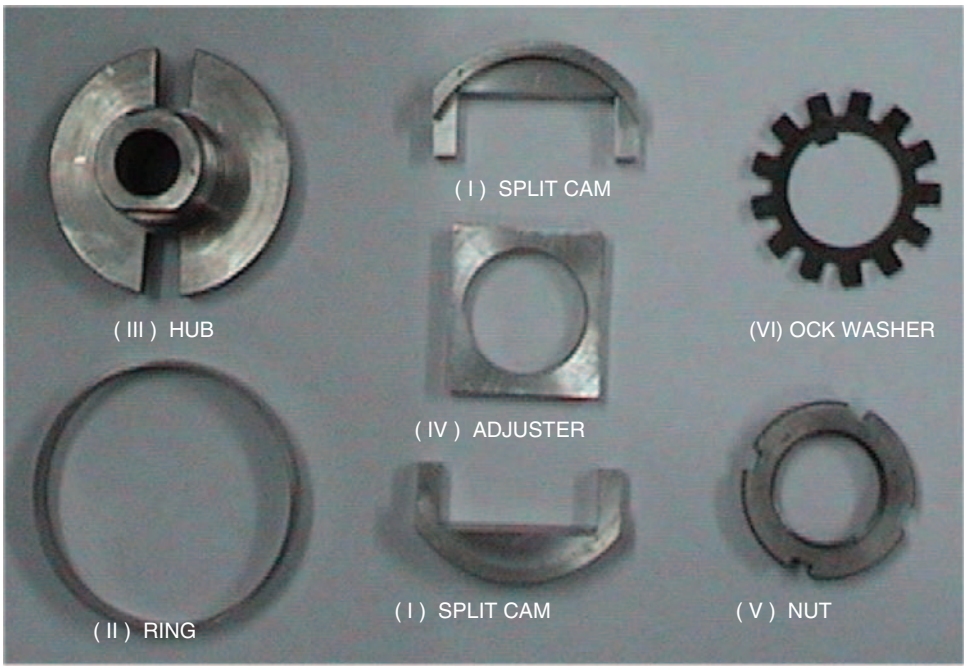


Figure 7. Exploded views of the conventional and the proposed split cam.

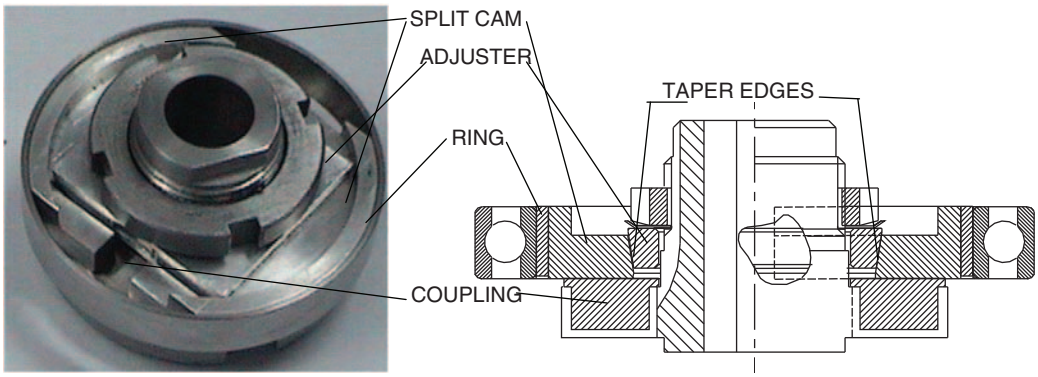
deflection of a circular ring takes the shape of an ellipse or very close to that shape.

The proposed new cam if made a single piece is, however, difficult to insert in the flexible bearing and in the FG. This is because the shape of the periphery is a combination of circular and elliptical arcs, and the similar shape does not take place in the natural deflection of the FG cup. To overcome this assembly problem, the cam is split into two identical pieces and assembled inside the flex bearing to make the cam subassembly inside the FG cup. The exploded view in Figure 7(b) will help in understanding the assembly sequence of the new proposed cam with the bearing and FG. Figure 8(a) and (b) shows the photographic views of components and the assembly of the cam, respectively. Figure 8(c) shows the sectional view of the cam assembly. Referring to these illustrations, the assembly method is as follows.

Two U-shaped identical pieces, named as 'split cam' (I), have outer surfaces of circular arcs of a



(a) Components of the proposed cam (Coupling is not shown)



(b) Photographic view (Without bearing) (c) Sectional view (With Flexible bearing)

Subassembly of the proposed cam

Figure 8. Components and subassembly of the proposed split cam.

spread slightly less than 2ψ (as in Table 2 and in Figure 12). They are placed inside a thin ring (II). The ring has inner radius equal to the outer radius of the split cam. Split cams and the ring are placed on the hub (III) (which is different from that (2) in case of the conventional oval cam) with the coupling (3) in between (Figure 7(b)). The inside edges of these split cams are made taper in such a way that when pressed by the adjuster (IV) in between two pieces, they move apart to give the desired shape of the new cam. The distance along the major axis of the cam can be adjusted by tightening or loosening a thin nut (V) for the perfect meshing of the pitch curve of FG with the pitch circle of RG, and to control backlash. The adjuster nut is finally locked by a lock washer (VI). It is to be noted that as the taper angle is very small, the adjuster gets locked inside the split cams. However, it comes out easily, when the nut is loose, with light hammering on it and on split cams as at taper contact surfaces decoupling forces act due to the spring actions of the flexed bearing races. The same spring force also balances the split cams to be at their places on assembly. Obviously, there will be changes in stresses in bearing races and FG cup. However, variations in stresses and pressure will be small as such an adjustment is of very small amount.

The outside of the thin ring has the equal periphery of that of the inside of the inner race of the bearing, i.e. equal to the conventional oval cam. The whole assembly process is to be conducted inside the inner ring of the flexible bearing (5) which is put inside the open end of the FG cup (VII). The thin ring – II (shown in Figure 8 but not shown in Figure 7(b)) is optional and it may be eliminated by making the outer radius of the split cam equal to the inner radius of the inner race of the bearing or inside radius of a bush bearing as the case may be. Over the assembly facility such a split cam is having the advantage of center distance adjustment which can control the backlash. This design has been patented.[26]

Performance

Indigenously built prototypes of the proposed HD with novel cam and gear set as well as the conventional unit were tested to examine their performances. Experiments were conducted[28–30] to examine the workability of the proposed gear system at first. Next, strains developed in various parts of FGs of both the commercial unit and the proposed unit are also examined experimentally. Finally, stresses in the FG cup are estimated using FEM analysis in ANSYS® environment taking in to account the design for torsional behavior, similar to the theoretical investigation by Rhéaume et al.,[31] of the FG cup and compared with experimental results.

Experimental verification

Experiments were conducted at both no load and with load (torque). Distinct differences in patterns of strain developed at FGs of both conventional unit and the proposed comparable units were recorded. Experimental results are shown in Figure 9. Strain gages were fixed inside the FG cup, i.e. on the thin wall, in a circumferential direction and it was very close to teeth (zone 3, Figure 9-inset). Apparently, the cup wall is more strained when operated with the new cam. But the trend is that with the increase in torque, the strain near teeth contact zones reduces. It is approaching zero in case of the new cam where as it is increasing in negative direction in the case of the conventional oval cam. Obviously, the strain will reverse its sign at a higher torque but the magnitude will be less than that in commercial unit. This means that for the higher torque range, the proposed unit will experience low stress at teeth contact zones. Further investigations are in progress.

Figures 10 and 11 show FEM results[30] on von Mises stresses developed in the FG cup with insertions of the conventional oval cam and the proposed cam, respectively, into the FG cup. The maximum stress in case of the oval cam is about 117 MPa and that occurs

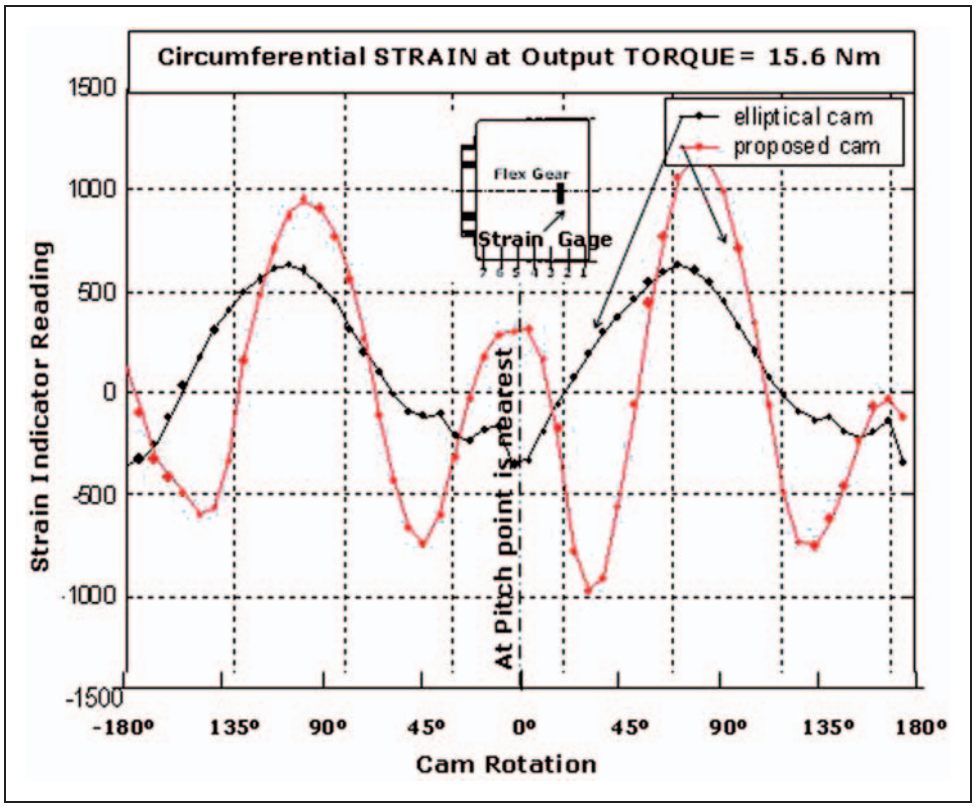


Figure 9. Comparison of strains at FGs – commercial unit versus proposed unit.
FG: flex gear.

along the major axis, i.e. where the tooth contact takes place.

When the oval cam of the conventional unit (i.e. with original gears of the commercial unit) is replaced by the proposed new cam, the unit works but with low contact ratio as presented in Table 3. The performance is not as good as it is with the original oval cam. This is due to the shift of pitch curve. Using methods of estimation as in an earlier investigation by Maiti and Roy[19] and in subsequent further research

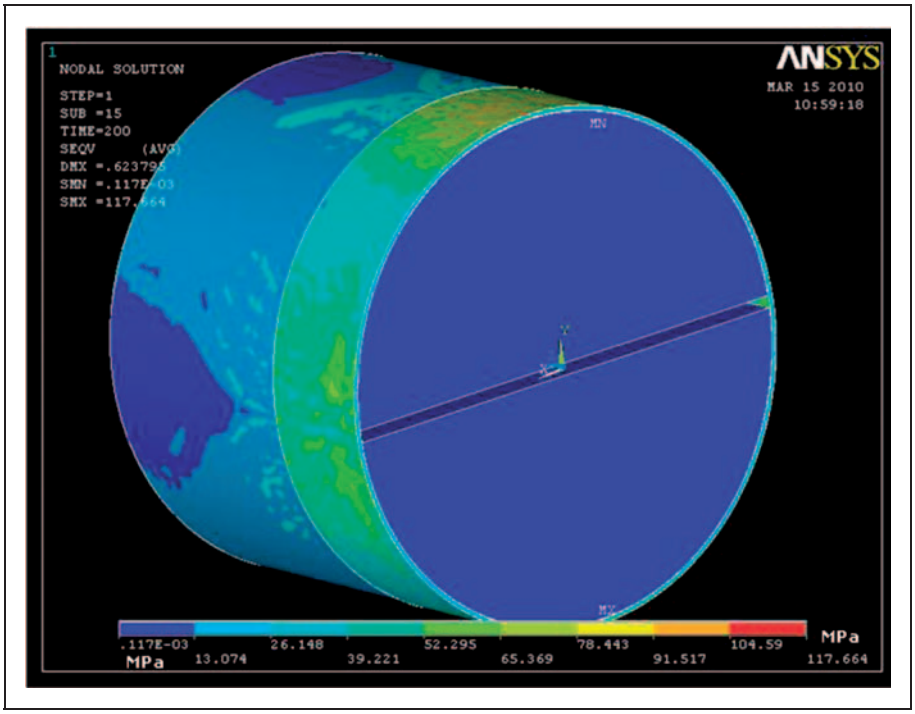


Figure 10. The von Mises stresses developed in FG cup with the insertion of the conventional oval cam. FG: flex gear.

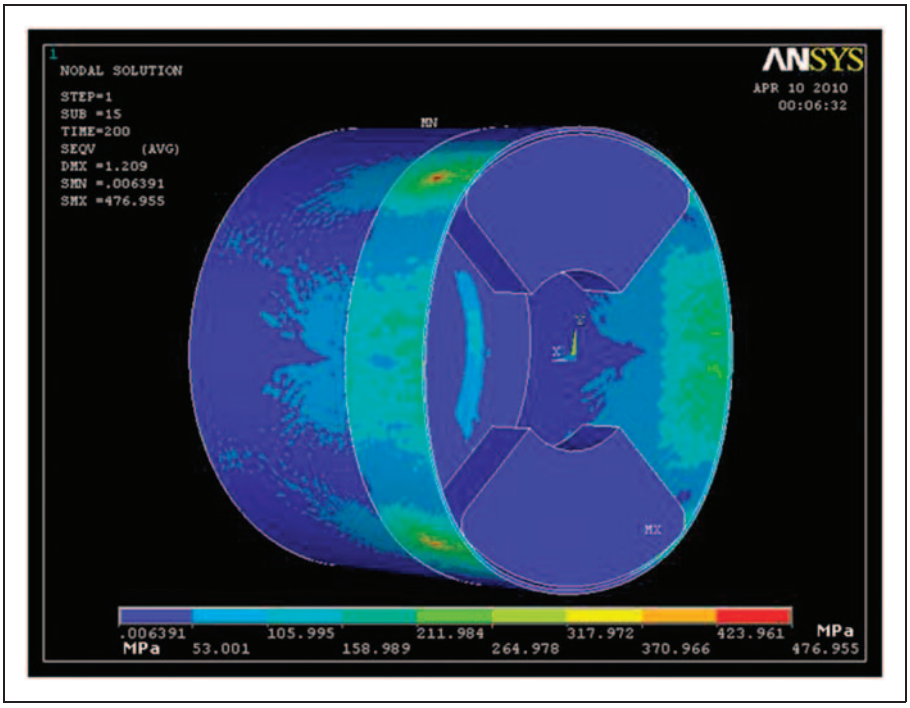


Figure 11. The von Mises stresses developed in FG cup with the insertion of the proposed split cam. FG: flex gear.

Table 3. Data of new split cam (ref. Table 2 & Figure 6) with commercial gear pair.

$Z_g = 158$, $Z_p = 156$, $\alpha_0 = 30^\circ$ Involute, $m = 0.529$ mm (*48DP*), $a_{fg} = 0.6$, $a_{fp} = 0.456$									
a (mm)	b (mm)	C (mm)	ψ (°)	θ_p (°)	γ (°)	$\theta_p - \gamma$ (°)	x_1 (mm)	y_1 (mm)	C_r
27.80	11.0	28.39	55	61.487	61.11	0.377	24.20	5.40	**0.782**

report,[28] it can be shown that with the new cam addendum factors for both the RG and the FG of standard pressure angles such as 20° and 30° involute, can be increased resulting in the increase in contact ratios.

Summary

The problem of assembly, due to the geometric shape, of the single piece cam in an earlier proposed novel strain wave generator subassembly with FG cup for a HD is solved by splitting the cam into two parts. The geometric design method is established and presented in this article. The constructional features and the assembly method of the cam are also presented. In the earlier proposal, the geometric shape of the periphery of new wave generator (cam) for the HD consisted of elliptical and circular arcs. In the proposed split cam solution, both identical split parts of the cam assembly have circular arcs which maintain the original circular shapes of the FG at the two diametrically opposite contact zones. An introduction of taper adjuster completes the cam assembly as well as facilitates center distance or eccentricity adjustment. However, the rest portions of the assembled cam take oval shapes. This has no effect on the performance of HDs and on the avoidance of tip interferences during engagement and disengagement. The improvement in performances and stresses in contact zones are also realized experimentally.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Acknowledgments

Rathindranath Maiti acknowledges the Department of Science and Technology, Government of India, for the R&D project grant no. SR/S3/RM-04/2002, dated: 11 July 2002; and contributions made by associated project staffs and students.

References

1. Musser CW. *Strain wave gearing*. US Patent 2.906.143, USA, 1955.
2. Musser CW. Breakthrough in mechanical drive design: the harmonic drive. *Mach Des* 1960; 14: 160–172.
3. Kiyosawa Y and Ishikawa S. Performance of strain wave gearing using a new tooth profile. *ASME International Power Transmission and Gearing Conference*, Vol. 11, Book No. 10288B1989.
4. Kondo K and Takada J. Study on tooth profiles of the harmonic drive. *ASME J Mech Des* 1990; 112: 131–137.
5. Peter J. Investigation of engagement of harmonic drives – part II. *Acta Tech (Budapest)* 1982; 94(3–4): 223–233.
6. Kikuchi M, Nitta R, Kiyosawa Y, et al. Stress analysis of cup type strain wave gearing. *Key Eng Mater* 2003; 243–244: 129–134.
7. Degen R and Slatter R. Zero-backlash micro-gears and actuators for micro-assembly applications. A technical paper of 'harmonic drive AG, Germany', www.harmonicdrive.de, 2007.
8. Harmonic Drive AG, Germany. Product catalogues. www.harmonicdrive.de, 2007.
9. Harmonic Drive AG. New drive, special issue. www.harmonicdrive.de, 2007.
10. Slatter R and Koenen H. Light weight harmonic drive gears for service robots. A technical paper of 'Harmonic Drive AG, Germany', www.harmonicdrive.de, 2007.
11. King T and Xu W. The design and characteristics of piezomotors using flexure-hinged displacement amplifiers. *Rob Auton Syst* 1996; 19: 189–197.
12. Xu L, Zhu C and Qin L. Parametric vibration for electro mechanical integrated electrostatic harmonic drive. *Mechatronics* 2007; 17: 31–43.
13. Dong H, Ting K-L and Wang D. Kinematic fundamentals of planar harmonic drives. *ASME J Mech Des* 2011; 133: 1–8.
14. Vladis K. Analytical investigation of the change in phase angle between the wave generator and the teeth meshing zone in high-torque mechanical harmonic drives. *Mech Mach Theory* 1997; 32(5): 533–538.
15. Ueura K, Kiyosawa Y, Kurogi J, et al. Tribological aspects of a strain wave gearing system with specific reference to its space application. *Proc IMechE Part J: J Eng Tribol* 2008; 222(8): 1051–1061.
16. Jean HS and Oh SH. A study on stress and vibration analysis of a steel and hybrid flexspline for harmonic drive. *Compos Struct* 1999; 47: 827–833.
17. Routh B and Maiti R. On a gearing problem in conventional harmonic drives with involute toothed gear set. In: *Proceedings of the ASME IDETC/CIE conference*, Washington, DC, USA, 29–31 August 2011.
18. Shu X-L. Determination of load sharing factor for planetary gearing with small tooth number difference. *Mech Mach Theory* 1995; 30(2): 313–321.

19. Maiti R and Roy AK. Minimum tooth difference in internal external involute gear pair. *Mech Mach Theory* 1996; 31: 475–485.
20. Ghorbel FH, Gandhi PS and Alpeter F. On the kinematic errors in harmonic drive gears. *ASME J Mech Des* 2001; 123: 90–97.
21. Gandhi PS and Ghorbel FH. High-speed precision tracking with harmonic drive systems. *Int J Control Eng* 2005; 78(2): 112–121.
22. Tuttle TD and Seering W. A nonlinear model of a harmonic drive gear transmission. *IEEE Trans Rob Autom* 1996; 12(3): 368–373.
23. Figliolini G and Angeles J. Synthesis of the base curves for N-lobed elliptical gears. *ASME J Mech Des* 2005; 127(5): 997–1005.
24. Maiti R. *An improved drive system such as a two gear epicyclic drive similar to harmonic drive system*. Indian Patent 186322, 1995.
25. Maiti R. A novel harmonic drive with pure involute tooth pair. *ASME J Mech Des* 2004; 126(1): 178–182.
26. Maiti R. An improved split cam assembly and an improved harmonic drive (strain wave gearing) system comprising the cam assembly. Indian Patent application 1457/KOL/2009, 2009.
27. Nema V. *Analysis of flex gear of a new concept harmonic drive*. Masters (MTech) Thesis, IIT, Kharagpur, India, 2003.
28. Maiti R and Ray MC. Design, development and performance study of a new concept harmonic drive. Project completion report, Ref. SR/S3/RM-04/2002, dated: 11 July 2002, Department of Science and Technology, Government of India, India, 2005.
29. Biswas I. *Some studies on stresses in thin rimmed gears with specific reference to its application in harmoinc drive*. Masters (MS) Thesis, IIT, Kharagpur, India, 2005.
30. Mahanto BS. *Some studies on stresses in cup shaped flex gears of harmonic Drive*. Masters (MTech) Thesis, IIT, Kharagpur, India, 2010.
31. Rhéaume FE, Champliaud H and Liu Z. Understanding and modelling the torsional stiffness of harmonic drives through finite-element method. *Proc IMechE Part C: J Mech Eng Sci* 2009; 223(2): 515–524.

Appendix 1

Notation

a	semi major axis of an ellipse
a_{fg}	addendum factor of RG
a_{fp}	addendum factor of FG
A	center distance/eccentricity
b	semi minor axis of an ellipse
C	shift of the center of ellipse (Figure 12)
i_1	transmission/reduction ratio
m	normal module
T_{o1}	output torque
Z_g	number of teeth in RG
Z_p	number of teeth in FG

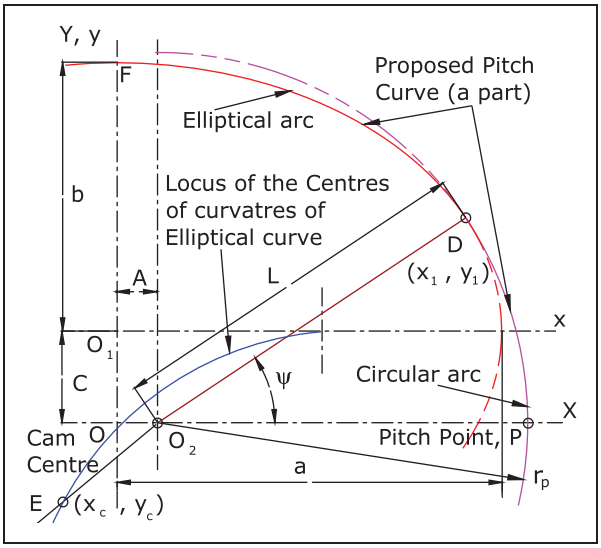


Figure 12. Geometry of the proposed pitch curve of FG. FG: flex gear.

α	working pressure angle
α_o	standard (basic) pressure angle
ψ	angle of spread of circular arc portion in an identical quarter of cam

(Note: Many parameters related in geometrical constructions are presented in figures).

Appendix 2

Geometry of the pitch curve of the proposed WGC

Figure 12 shows the constructional geometry of the pitch curve of the FG on assembly of the proposed WGC. The cam profile is a curve which is constant difference inwardly shifted pitch curve. 'A' is the center distance between the RG and the undeflected FG, i.e. the pinion in mesh. It is expressed as

$$A = \frac{(Z_g - Z_p)m}{2} \tag{1}$$

where Z_g, Z_p, and m are the number of teeth in RG, number of teeth in FG, and normal module, respectively.

The required ellipse, with respect to the xO_1y axes, can be described as

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \tag{2}$$

where a and b are the semi major axis and semi minor axis, respectively, of the elliptical curve in the proposed cam profile.

Let (x_1, y_1) be the coordinates, with respect to the xO_1y axes, of the junction point D of the ellipse and

the circular arc. Or in other words, D is the point of tangency of the ellipse and circular arc. The coordinates of E, the center of curvature, (x_c, y_c) of the ellipse at that point is given by

$$x_c = x_1^3\left(\frac{a^2-b^2}{a^4}\right)$$
$$y_c = y_1^3\left(\frac{b^2-a^2}{b^4}\right) \tag{3}$$

Intersection of the radius of curvature with the line $x = A$ is expressed as (with respect to the xO_1y)

$$y = y_1 + \left(\frac{y_c - y_1}{x_c - x_1}\right)(A - x_1) = C \tag{4}$$

where C is the distance of O from O_1, i.e. $|C|$ is the shift of the major axis of ellipse from the center of the RG (Figure 12).

The distance $O_2D = L$ is expressed as

$$L = \sqrt{(x_1 - A)^2 + (y_1 - C)^2} \tag{5}$$

where C is calculated using equation (4)

To satisfy the conditions of the profile of the pitch curve and kinematics, O_2 must be the center of the circular arc portion of the pitch curve, i.e. $L = r_p$, the pitch circle radius of undeformed FG.

The angular span ψ of the circular portion in a quarter of pitch curve is derived as

$$\psi = \tan^{-1}\left(\frac{y_1 - C}{x_1 - A}\right) \tag{6}$$

From the length equality criteria

$$2\Pi r_p = 4r_p\Psi + 4S_{\mathrm{DA}} \tag{7}$$

where S_{DA} is the peripheral length of the elliptical curve in the symmetrical quarter of the cam profile, i.e. arc length FD (Figure 12).

On rearranging

$$S_{\mathrm{DA}} = r_p\left(\frac{\pi}{2} - \psi\right) \tag{8}$$

Considering the equation of ellipse S_{DAE}, the peripheral length of the pitch curve in the symmetrical quarter of the cam profile is expressed as

$$S_{\mathrm{DAE}} = \int_0^{x_1} \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \tag{9}$$

Ideally

$$S_{\mathrm{DA}} = S_{\mathrm{DAE}} \tag{10}$$

Evaluating the integral in equation (9)

$$a\left(\lambda - \frac{1}{2}e^2\mu_1 - \frac{1}{8}e^4\mu_2 - \frac{1}{16}e^6\mu_3 - \ldots\right) = r_p\left(\frac{\pi}{2} - \psi\right) \tag{11}$$

where $\lambda = \sin^{-1}\left(\frac{x_1}{a}\right)$, e is the elliptical constant given by

$$\frac{b}{a} = \sqrt{1 - e^2} \tag{12}$$

and

$$\mu_1 = 1/4(2\lambda - \sin 2\lambda)$$
$$\mu_2 = 3/16(2\lambda - \sin 2\lambda) - 1/4\sin^3\lambda\cos\lambda$$
$$\mu_3 = 5/32(2\lambda - \sin 2\lambda) - 5/24\sin^3\lambda\cos\lambda - 1/6\sin^5\lambda\cos\lambda$$

Check for tooth interference in gearing with proposed cam

In the present analysis, it is assumed that the deviation in geometric properties of the FG teeth under all applied loads is negligibly small with respect to the phenomenon 'tooth tip interference'. The gearing action in HDs is such that the tip interference during engagement is more critical, as reported by Maiti and Roy.[19] In the earlier work, the theories for avoidance of tip interferences with elliptical cam of HD with truncated pure involute gears are established. The same analysis is suitably adapted for this proposed cam[25] also.

Figure 13 shows a condition when the tip interference occurs during the engagement in a HD with the proposed cam and with the fixed RG and the FG as output. Let T be the point of intersection of the addendum circle of the RG and the addendum curve of the FG of deflected rim. T_g' and T_p' are current positions of T_g and T_p, respectively, which they will resume after reaching the contact at pitch point P due to cam rotation. To avoid the engagement tip interference T_p' should cross T first before T_g' reaches there. In case of HDs, the tooth of pinion which could interfere, comes out of the tip zone of RG in exaggerated manner, when it travels on the elliptical curve of the proposed cam or on the oval path in case of conventional cam. Thus, two teeth difference is possible in case of HDs.

Mathematically, the condition to avoid the tip interference can expressed as

$$\theta_p > \gamma \tag{13}$$

when T_g' has reached at T.

These angles are derived as follows. The line MT, which is along the radius of curvature, intersects the elliptical pitch curve at K of coordinate (X_k, Y_k) with respect to the central axis (XOY).

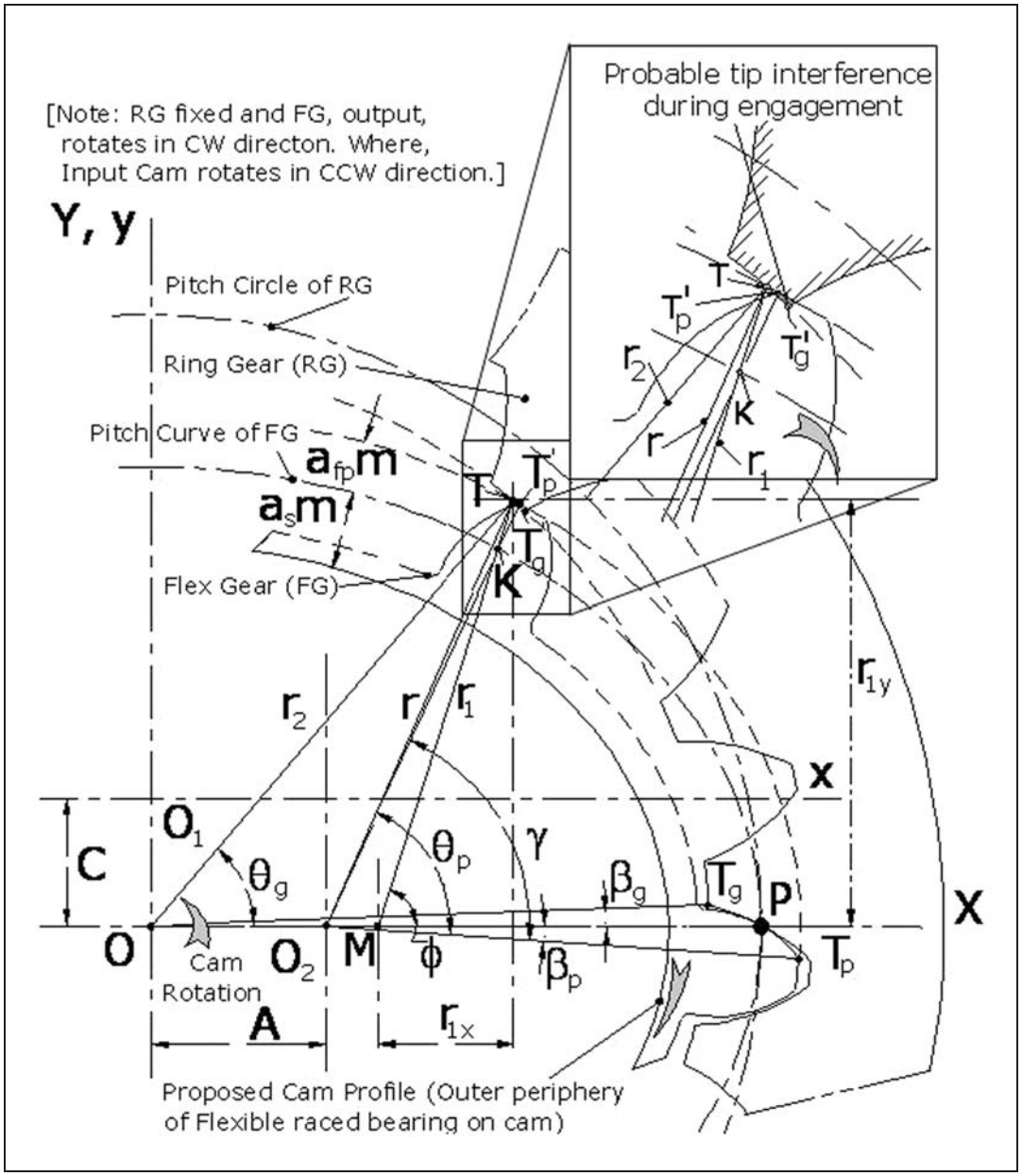


Figure 13. Geometry to check the tip interference with the proposed cam.

For any value of x_k $(=X_k)$ with respect to the ellipse axes (i.e. xO_1y axes), the value of Y coordinate, Y_{ke} is calculated using equation (2) and C is calculated using equation (4). Y_k, with respect to the central axis (XOY), the parameters used for pitch curve design, is expressed as

$$Y_k = Y_{ke} - C \tag{14}$$

Similarly, the center of curvature (X_c, Y_c) with respect to the central axis, on the MT line (or its extension) is calculated using equations (3) and (4). The coordinate (X_m, Y_m) of point M is expressed as

$$Y_m = 0$$

and

$$X_m = X_k + (Y_m - Y_k)\left(\frac{X_c - X_k}{Y_c - Y_k}\right) \tag{15}$$

Angle Φ is given by

$$\Phi = \tan^{-1}\left(\frac{Y_m - Y_k}{X_m - X_k}\right) \tag{16}$$

The distance $\mathrm{MT} = r_1$ is calculated as

$$r_1 = \sqrt{(X_m - X_k)^2 + (Y_m - Y_k)^2} + a_{fp}m \tag{17}$$

where a_{fp} is the addendum factor of the FG (i.e. pinion) tooth (a_{fp} = pinion addendum/module).

The X and Y components of r_1 are expressed as

$$\begin{aligned} r_{1x} &= r_1 \cos\phi \\ r_{1y} &= r_1 \sin\phi \end{aligned} \tag{18}$$

The distance OT ($=r_2$) is derived as

$$r_2 = \sqrt{(r_{1x} + X_m)^2 + r_{1y}^2} \tag{19}$$

Now to calculate the angle θ_p (the critical condition of tip interference), r_2 must be equal to r_{ag}, the tip circle radius of RG. Therefore, O_2T ($=r$) must be expressed as

$$r = \sqrt{(x_m + r_{1x} - A)^2 + r_{1y}^2} \tag{20}$$

To calculate r for which $r_2 = r_{ag}$, equations (14) to (19) are to be solved iteratively.

The angles θ_p and θ_g are calculated as

$$\begin{aligned} \theta_p &= \cos^{-1}\left(\frac{r_{ag}^2 - A^2 - r^2}{2Ar}\right) + \beta_p \\ \theta_g &= \cos^{-1}\left(\frac{r_{ag}^2 + A^2 - r^2}{2Ar}\right) + \beta_g \end{aligned} \tag{21}$$

where

$$\begin{aligned} \beta_p &= (inv\alpha_{tp} - inv\alpha) \\ \beta_g &= (inv\alpha - inv\alpha_{tg}) \end{aligned}$$

In which α is the working pressure angle. The pressure angles α_{tp} and α_{tg} at the tip of the pinion and gear (internal) teeth, respectively, are given by

$$\begin{aligned} \cos\alpha_{tp} &= \left[\frac{Z_p}{Z_p + 2a_{fp}}\right]\cos\alpha_o \\ \cos\alpha_{tg} &= \left[\frac{Z_G}{Z_G - 2a_{fG}}\right]\cos\alpha_o \end{aligned} \tag{22}$$

Now, if the RG were given a rotation of angle θ_g, the corresponding rotation of the circular pinion would be $(Z_g/Z_p)\ \theta_g$. The corresponding rotation γ of the deflected pinion can be obtained from the condition that the arc length of the circle with radius equal to r_p and the center at O_2 (Figure 13) over the angle $(Z_g/Z_p)\ \theta_g$ is equal to the length of the arc on the profile of the proposed cam, from the common starting point on the cam. If the final coordinate on the contour of the proposed curve after the rotation is $(X,\ Y)$, then

$$\gamma = \tan^{-1}\left(\frac{Y}{X - A}\right) + \beta_p \tag{23}$$

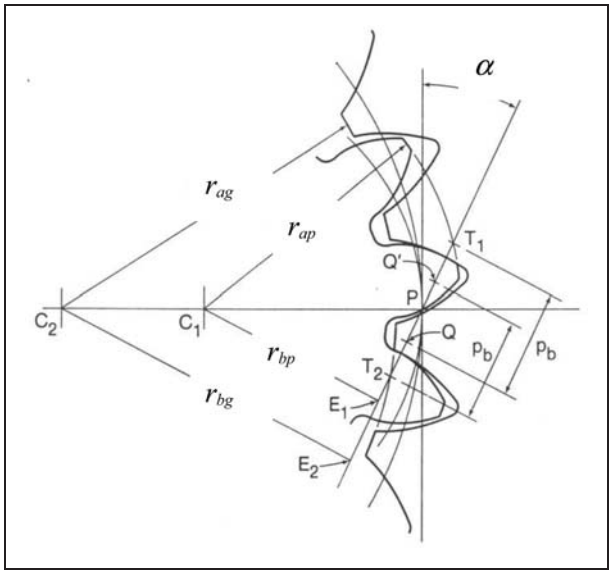


Figure 14. Geometry to verify the contact ratio.

Hence to avoid the tip interference, the necessary condition is

$$\theta_p > \left[\tan^{-1}\left(\frac{y}{x - A}\right) + \beta_p\right] \tag{24}$$

Appendix 3

Mystery in contact ratio of commercial HD with elliptical cam and involute gears

Contact ratio. As the contact portion is same as that of circular gears the contact ratio can be verified using the standard formulae for internal involute gearing.

Referring to Figure 14 (and for the RG as driver and rotating clockwise)

$$\text{The contact ratio} = \frac{\begin{pmatrix}\text{Approach contact} \\ + \text{recesses contact}\end{pmatrix}}{\text{Base pitch}} = \frac{T_1P + PT_2}{P_b}$$

The relation can be finally derived as

$$C_r = \frac{\left(\sqrt{r_{ap}^2 - r_{bp}^2} - r_p\sin\alpha\right) + \left(r_g\sin\alpha - \sqrt{r_{ag}^2 - r_{bg}^2}\right)}{\pi m\cos\alpha_o} \tag{26}$$

As the pitch curve of the FG becomes elliptical at the contact region in commercial conventional HD, the laws of contact ratio of involute gearing is violated. However, as radii of curvatures of the pitch curve of the FG at the contact region are close to the pitch circle radius of the undeformed FG, the contact ratio can be estimated using the standard formula as in equation (26) in 'Contact ratio'. The avoidance of tip interference and other geometric and

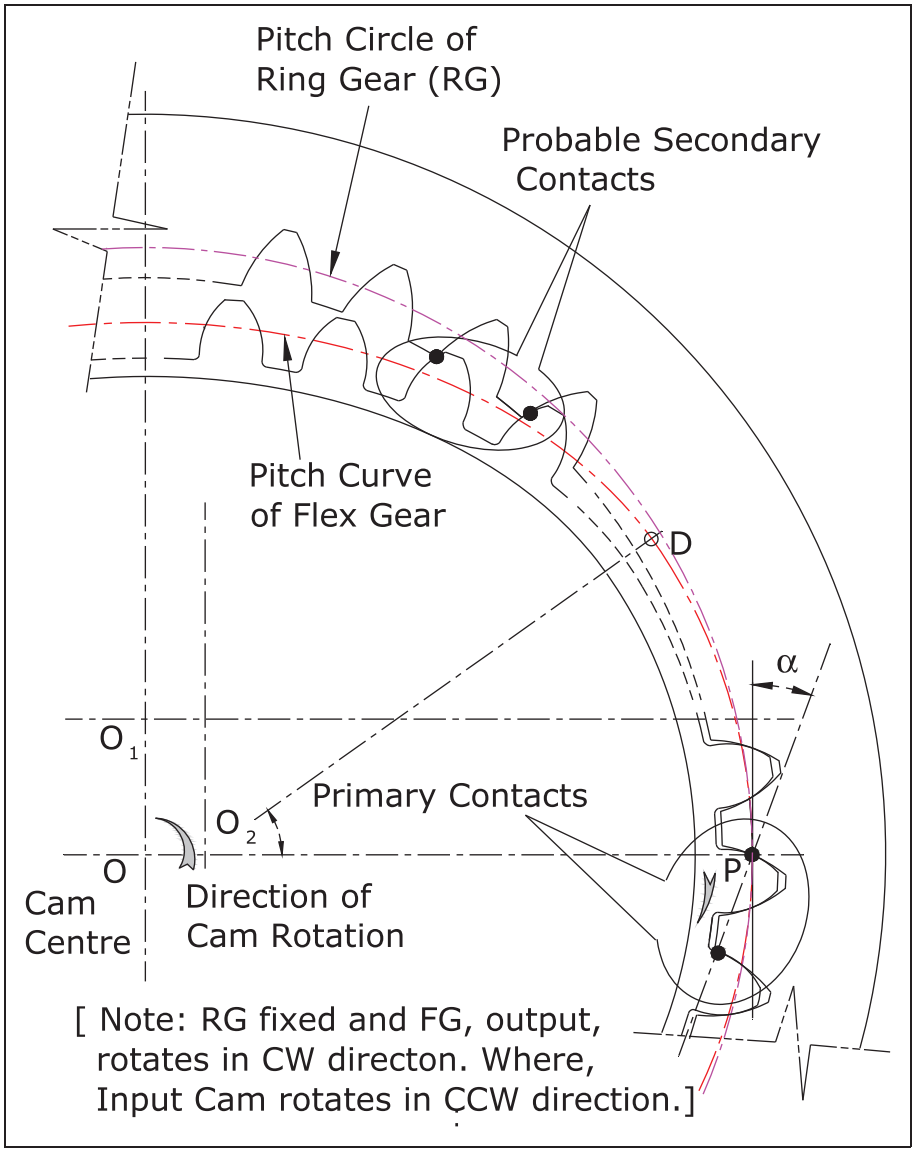


Figure 15. Possible secondary contact in HDs.
HD: harmonic drive.

kinematic relations with elliptical cam are discussed in an earlier work by Maiti and Roy.[19]

For the commercial unit, the tooth profile is involute as mentioned in the product catalogue and also examined by the present investigators. The contact ratio, calculated for a single contact region, is 0.782 (Table 3), which is less than 1. As such, the gearing is not possible. But the unit works well. It is apparently a mystery. One may be tempted to consider that as there are two contacts probably the contact ratio criteria is satisfied therein. But it is not true. The number of teeth of RG is exactly divisible by the number of contacts. This means that phases of contact at both the contacts are same. Or in other words, contacts at both the zones occur at same time and also disengagements occur at the same time. Therefore, the contact ratio cannot be enhanced by phase addition.

It raises a question how the gearing laws are maintained and the slip in motion is avoided! In such gearing, multi-teeth contacts occur as shown by Shu[18] and as also indicated by Vladis.[14] However, Long analyzed general cases with circular gear and with higher working pressure angle. In the case of the particular commercial HD under analyses, the working pressure angle is not changed, as measured, at the contacts along the major axis of the elliptical cam. As revealed by the present authors, probably the secondary contacts of teeth pairs occur away from the pitch point, at either in engagement zone and or in disengagement zone (Figure 15) or at both depending on the driving arrangement and backlash. This is possible as the FG tooth bends at the root and tooth leans toward the surface of the RG tooth profile and force contacts occur even at no load, as shown in a recent investigation by Routh and Maiti.[17] However, the gearing laws may not be maintained at the secondary contact although the contact ratio will improve.