## A Computer Program for the Specification of Axial Compressor Airfoils

Aerospace Research Laboratories

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# A COMPUTER PROGRAM FOR THE SPECIFICATION OF AXIAL COMPRESSOR AIRFOILS 

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DECEMBER 1972

PROJECT 7065

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AEROSPACE RESEARCH LABORATORIES
AIR FORCE SYSTEMS COMMAND UNITED STATES AIR FORCE WRIGHT-PATTERSON AIR FORCE BASE, OHIO

## FOREWORD

This report was prepared by lst Lt. George R. Frost and Dr. Arthur J. Vennerstrom of the Fluid Dynamics Facilities Research Laboratory, Aerospace Research Laboratories, UrichtPatterson Air Force Base, Ohio, and by Mr. Richard M. Hearsey of the University of Dayton Research Institute.

The report presents resuits from a portion of the effort of the Fluid Machinery Research Group, supervised by Dr. Arthur J. Wennerstrom and was conducted under Work Unit 09 of Project 7065, "Aercspace Simulation Techniques Research" under the overall direction of Mr. Elmer G. Johnson.

## ABSTRACT

This report is a revision and extension of ARL 70-0046. It describes the analysis in, and the use of, a computer program which has been developed for use in the design of axial compressor airfoils suitable for operation at high subsonic and supersonic fach numbers. Four rather versatile camber line shapes and two thickness distributions are mathematically derived. These camber lines provide the capability of defining a wide variety of blades, from those of continuously positive camber to the so-called "S-blades", including many of the intermediate possibilities. A method is presented whereby the airfoils are specified on arbitrary axisymmetric streamsurfaces and then accurately redetermined in Cartesian coordinates on planes normal to the stacking axis. The program determines the coordinates, calculates section properties, and provides precision plots of the specified blade sections and the resulting blade projections on the above-mentioned planes.

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## SECTION I

## INTRODUCTION

The Fluid Dynamics Facilities Research Laboratory of the Aerospace Research Laboratories (ARL) is engaged in a program to find means of designing axial compressors capable of efficient performance at Mach numbers and pressure ratios beyond the current state-of-the-art. In conjunction with this effort, it was determined that general improvements could be made over some of the blade-design techniques now in popular use for high relative Mach numbers.

It is desirable when designing a compressor blade to create blade sections upon arbitrary surfaces of revoluition that correspond to streamsurfaces indicated by the compressor design calculations. However, for manufacturing purposes, it is desirable to have blade surface coordinates for sections through the blade that are plane and perpendicular to the stacking axis. A method is presented whereby the biade is designed by creating sections on the streamsurfaces, and then determining coordinates for plane sections through the resultant blade.

Several objectives guide the aerodynamic design of any airfoil, whether for a compressor or any other purpose. First, and usually foremost, is the desire to keep the rate of diffusion as low as possible on the airfoil surfaces. While this objective does not change with Mach number, the shape of airfoil required to accomplish this car change significantly with Mach rumber. At supersonic Mach numbers there is generally little camber, and sometimes negative camber, in the leading edge region of a compressor blade. At these Mach numbers, minimizing diffusion rates means minimjzing the suction surface supersonic expansion upstream of the inevitable passage shock or even producing a limited amount of supersonic diffusion upstream of the shock location. There are two principal methods of obtaining supersonic diffusion. When using blades having zero or positive camber near the leading edge, the diffusion may be obtained by a convergence of the annulus walls. If this is impractical, for example, in a high-aspect-ratio supersonic fan rotor, the same result can be accomplished by negative camber in the leading edge region of the airfoil. Blades which have negative camber followed by a region of positive camber have been called "S-blades".

A second objective is to define the airfoil in such a manner that its shape can be continuously varied to accommodate varying conditions along the blade span, thus maintaining resonable diffusion rates along each blade element. For example, it would be possible to define a type of profile
having a very attraciive diffucion distribution for a hub section, but which could not be adapted to tip conditions, or vice versa. This would obvicusly be undesirable. It would generally be impractical to attempt to blend dirfoil sections of more than one type alonf the span of a blade. A third objective for a desirable compressor blade is that the parameters determining its aerodynamic characteristics should be variable over a wide range within which there is little danger of producing shapes that are mechanically undesirable.

This report is a revision and extension of an earlier report (Reference 1) which presented two blade profiles which satisfy all the above requirements. They are composed of a common thickness distribution applied to two different camber lines: one a fourth-order polynomial, the other defined by two exponential functions. The thickness distribution consists of two third-order polynomials: one between the leading edge and the point of maximum thickness, and one from there to the trailing edge.

In addition to these camber lines, two other camber lines have been incorporated into the associated computer program. One of these is the familiar circular-arc camber line, upon whish is placed the double-circular-arc thickness distribution. The second camber line may be composed of any pair of circular arcs, either of which may degenerate to a straight line, allowing for a wide range of design flexibility. This latter camber line, referred to as the multiple-circular-arc camber line in this report, uses the common thickness distribution of the original blade profiles and, as a special case, allows this thickness distribution to be applied to a simple circulararc camber line.

The various camber lines and thickness distributions are described in Sections II and III.

In order to specify a blade for manufacture it is desirable to have coordinates for plane sections throurh the blade. For maximum accuracy, blade profiles should be desipned on the arbitrary surfaces of revolution that typical current design procedures indicate. The method of satisfyinf, these two requirements consists first of designing blade profiles on surfaces of revolution. Then a thin, uniformly-flexible beam is passed (mathematically) through like points on each profile and the intersection of the beam with the "manufacturing. planes" yields the coordinates of the "manufacturing sections." The original report and computer program presented Equation (4?) Reference 1, incorrectly which resulted in an improper determination of the Cartesian coordinates of the specified streamsurface blade sections for any non-cylindrical streamsurface. This led to associated errors in the determination of the manufacturing sections. (The mafnitude of thas error
increases with streamsurface slope ancle and generally would not result in a blade leading or trailing edge ancle error of more than about one degree.) The version of the propram presented in this report incorporates the correct analysis as described in Section IV.

The fifth section of this report describes calculations to determine various properties of both the streamsurface blade profiles and the manufacturinf, sections. Several blade characteristics required for aerodynamic analysis of a resulting blade row are presented in Section VI.

In addition to the new features and the analysis correction already mentioned, the computer program (presented in Sections VII through I\%) has been modified in several other respects since the original report was prepared. The procram now accommodates curvilinear computine stations, as compared to the strictly linear stations previously required. An additional section property, the torsional constant, is calculated for the manufacturing sections to facilitate structural analysis of the blade. Finally, a blade may now be stacked also at either the leading or trailing edge, in addition to the previous capability of stackine at, or offset from, the section centroid.

The seventh section of this report presents a peneral user's guide for the computer program, and Section VIII presents several examples for the use of the propram. The actual codins, and a synopsis of the essential steps in the prorram loric, as well as information which may facilitate its implementation on a computing, system, are presented in Section IY.
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## SECTION II

## the section camber line

To obtain a camber line varying smoothly in curvature and satisfying the requirements of a compressor blade, it would be convenient to describe it mathematically in equation form. The type and order of such an equation will in turn be influenced by the boundary conditions which are imposed. The equation must be of high enough order to accommodate all of the necessary boundary conditions. Its order should be no higher than absolutely necessary, however, to minimize the number of singularities which can occur within its useful working range. In the following discussjon, we shall consider an equation of the form $y=f(x)$ where the variable $x$ is nondimensional and varies from 0 at the blade leading edge to 1.0 at the blade trailing edge. Furthermore, instead of defining the x-axis as the chord line, which is usual, $x$ has been defined as the axial direction in the blade-element plane. This definition of the x-axis was chosen because it was fround to lead to the simplest treatment of boundary conditions.

Three boundary corditions, or their equivalent, are absolutely necessary. One condition fixes a point on the line with respect to the coordinate system, another defines the slope at the leading edge, and the third defines the slope at the trailing edge. These three alone are sufficient to define a unique circular-arc camber line between the edge points. However, these boundary conditions are insufficient to define a camber line for a blade which will meet the special requirements of efficient operation at relatively high Mach numbers.

A characteristic usually desired for supersonic compressor blades is very little camber in the leading portion of the blade. Dependinf upon the overall aerodynamic design, the desired leadinr-edpe camber may be negative, zero, or slightly positive. A convenient way of controlling this is to impose a boundary condition on the second derivative of the camber line at the leadink edre. To put it in dimensionless form, we specify the ratio between second derivatives at the leading edge and the point where the absolute value of the second derivative is a maximum. A fifth condition was felt advisable for an efficient compressor blade. Namely, the first four conditions used alone tend to produce a blade whose curvature is highest at the trailinr edre. This could result in large deviation ancles and correcpondincly high losses. Therefore, a condition was imposed on the second derivative at the trailing edre whereby it.; value could be specified as something less than the maximum. Thic boundary condition was phrased in exactly the sane manner as at the leadine edpe. The resulting five conditions may be uritten

$$
\begin{align*}
\text { At } x=0: \quad y & =0  \tag{1}\\
y^{\prime} & =\tan \alpha_{1} \\
y^{\prime \prime} & =P\left(y^{\prime \prime}\right)_{\max } \\
\text { At } x=1: \quad y^{\prime} & =\tan \alpha_{2}  \tag{2}\\
y^{\prime \prime} & =Q\left(y^{\prime \prime}\right)_{\max }
\end{align*}
$$

where prime (') and double prime (") refer to first and second derivatives respectively.

## 1. POLYNOMIAL CAMBER LINE

The five stated boundary conditions could be applied to a great number of equations to produce a camber line. A simple fourth order polynomial was chosen for the blade having primarily positive camber because it was convenient, and proved quite satisfactory. Since the second derivative of a fourth order polynomial is obviously second order and two of the boundary conditions apply to the second derivative, it was convenient to start by phrasing this in the form of a parabola. In standard form this can be written

$$
\begin{equation*}
(x-h)^{2}=4 a\left(y^{\prime \prime}-k\right) \tag{3}
\end{equation*}
$$

or

$$
y^{\prime \prime}=\frac{1}{4 a}(x-h)^{2}+k
$$

where $h$ is the position on the $x$-axis where the second derivative is a maximum (or minimum) and $k$ is the value which the second derivative has at this point. Integrating twice, one obtains

$$
\begin{gather*}
y^{\prime}=\frac{1}{12}(x-h)^{3}+k x+b  \tag{4}\\
y=\frac{1}{48 a}(x-h)^{4}+\frac{k}{2} x^{2}+b x+c \tag{5}
\end{gather*}
$$

Application of the five boundary conditions to these equations produces the following result

$$
\text { At } \begin{align*}
x=0, & y \tag{6}
\end{align*} \quad=0
$$

$$
\begin{gathered}
\text { At } x=0, y^{\prime}=\tan \alpha_{1} \\
b=\frac{h^{3}}{12 a}+\tan \alpha_{1} \\
\text { At } x=0, y^{\prime \prime}=P\left(y^{\prime \prime}\right)_{\max } \\
k=-\frac{h^{2}}{4 a(1-P)} \\
\text { At } x=\frac{1, y^{\prime}}{}=\tan \alpha_{2} \\
4\left(\tan \alpha_{1}-\tan \alpha_{2}\right) \\
{\left[\frac{P}{1-P} h^{2}+h-\frac{1}{3}\right]} \\
\text { At } x=1, y^{\prime \prime}=Q\left(y^{\prime \prime}\right)_{\max } \\
h=\frac{1}{1+\sqrt{\frac{1-Q}{1-P}}}
\end{gathered}
$$

The values of the input parameters at which the equations degenerate are easily ascertained by examining the derived constants. The values of the two blade angles may be anything between, and less than, $\pm 90$ degrees, measured fron the axial direction. Note that when the two blade angles are the same, this fourth-order camber line becomes a straight line, regardless of what values $P$ and $Q$ may have. The parameter $Q$ may be given any value detween zero and 1.0. In practice, a value of 0.5 has proven very satisfactory for most applications. Singularities associated with the parameter $P$ are partially dependent upon $Q$. At $P=1.0$, a singularity exists independently of $Q$. However, if $Q=1.0$, the equations also degenerate at $P=-2.0$ and if $Q=0$, the equations degenerate at $P=-3.0$. In general, P should be greater than -2.0 and less than 1.0. For most applications, a value of $P=0$ is quite satisfactory. Toward the hub of a compressor, it may be desirable to give $P$ small positive values. Toward the tip, one might wish to use slightly negative values of $P$. However, this particular camber line does not produce an attractive "S-blade," especially
if the overall section camber is nearly zero. Nerative values of $P$ are only useful for delayine the onset of poaitive curvature by producing very slight negative cambers at the leadinf edre.

Firures 1 through 5 illustrate the general characteristics of this camber line. In Fifure $l_{\text {, }}$ the distribution of the second derivative of the camber line is shown for various values of $P$ and blade angles characteristic of mid spar. Firure 2 shows the actual camber lines which correspond to the curves shown in Figure 1. Figure 3 is similar to Figure 2 but shows the influence of $P$ on a camber line more typical of a compressor hub section. Firure 4 illustrates the influence of $Q$ for a camber line typical of mid span with $P=z e r o$. Finally, Figure 5 shows the different camber lines which result from varying the blade outlet angle with all other parameters fixed at typical values.

## 2. EXPONENTIAL CAMBER LINE

The exponential camber line is the result of efforts to find a mathematical expression capable of defininf, a satisfactory "S-blade." An "S-blade" should satisfy all of the basic criteria which led to the preceding polynomial camber line and, in addition, several conditions unique to the " $S$ " confipuration. Namely, it should be possible to locate the inflection point anywhere on the camber line, the blade angle at that point should be independently specifiable, and the method must work for overall blade cambers which are positive, zero, or negative. Furthermore, the transition across the inflection point should occur smoothly but rapidly so that a long straight region is avoided in the middle of the blade. It is also highly desirable that the equations allow a smooth transition from the "S" configuration to a more conventional one such as characterized by the polynomial camber line. To the list of five boundary conditions presented earlier, one is added:

$$
\begin{equation*}
\text { At } x=s, y^{\prime}=\tan \alpha_{s} \tag{11}
\end{equation*}
$$

Numerous equations, including polynomials, were examined with respect to the six boundary conditions. It soon became evident that more satisfactory results could be obtained by defining the camber line with two equations, one either side of the inflection point. As with the previous blade, it was easiest to choose an equation by examining equations for the second derivative of the camber line and then integrating twice to obtain the coordinate equation. The equation selected for both portions of the camber line is

$$
\begin{equation*}
y^{\prime \prime}=b(x-s) e^{a(x-5)} \tag{12}
\end{equation*}
$$

One set of constants applies from 0 to $s$ and another set applies from s to 1.0. The maximum value of $\mathrm{y}^{\prime \prime}$ cecurs where $\mathrm{y}^{\prime \prime}=0$, which is at

$$
\begin{equation*}
x=s-\frac{1}{a} \tag{13}
\end{equation*}
$$

Note that the constant a is positive ahead of the inflection point 5 , and is nepative after it. The maximum (or minimum) value of the camber line second derivative for each camber line segment is

$$
\begin{equation*}
y_{\text {max }}^{\prime \prime}=-\frac{b}{a e} \tag{14}
\end{equation*}
$$

Integrating the second-derivative equation twice, one obtains

$$
\begin{gather*}
y^{\prime}=\frac{b}{a^{2}} e^{a(x-s)}[a(x-s)-1]+c  \tag{15}\\
y=\frac{b}{a^{3}} e^{a(x-s)}[a(x-s)-2]+c(x-s)+d \tag{16}
\end{gather*}
$$

This equation contains four arbitrary constants, and there are two sets to be determined as described above. Because there are different equations describing the two portions of the camber line, a seventh condition is added to the six already stated. At the inflection point, the coordinates of the camber line are the same on both lines.

Applying the four conditions appropriate for the leading portion of the camber line produces the following result:

$$
\text { At } \begin{align*}
x=0, & y
\end{align*}=0 .
$$

$$
\begin{align*}
& \text { At } x=0, y^{\prime}=\tan \alpha_{1} \\
& c_{1}=\tan \alpha_{1}+\left(a_{1} s+1\right) \frac{b_{1}}{a_{1}^{2}} e^{-a_{1} s} \\
& \text { At } x=0, y^{\prime \prime}=P\left(y^{\prime \prime} \text { max }\right) \\
& P=a_{1} s e^{\left(1-a_{1} s\right)}  \tag{IB}\\
& \text { At } x=s, y^{\prime}=\tan \alpha_{s}  \tag{19}\\
& c_{1}=\tan \alpha_{s}+\frac{b_{1}}{a_{1}^{2}}
\end{align*}
$$

Equating the two expressions for $c_{1}$ yields

$$
\begin{equation*}
b_{1}=\frac{a_{1}^{2}\left(\tan \alpha_{1}-\tan \alpha_{s}\right)}{1-\left(a_{1} s+1\right) e^{-a_{1} s}} \tag{20}
\end{equation*}
$$

Applying the four conditions appropriate for the rearward portion of the camber line produces the following result:

$$
\text { At } \begin{align*}
x=s, \quad y^{\prime} & =\tan \alpha_{s} \\
c_{2} & =\tan \alpha_{s}+\frac{b_{2}}{a_{2}^{2}} \tag{21}
\end{align*}
$$

At $\mathrm{x}=\mathrm{s}, \mathrm{y}=\mathrm{y}$ obtained from Equation (16) for the leading portion of the camber line.

$$
\begin{align*}
& d_{2}=2\left(\frac{b_{2}}{a_{2}^{3}}-\frac{b_{1}}{a_{1}^{3}}\right)+s\left(c_{1}-c_{2}\right)+d_{1} \\
& \text { At } x=1, y^{\prime}=\tan \alpha_{2} \\
& c_{2}=\tan \alpha_{2}-\frac{b_{2}}{a_{2}^{2}} e^{a_{2}(1-s)}\left[a_{2}(1-s)-1\right] \tag{22}
\end{align*}
$$

Equating the two expressions for $c_{2}$ yields

$$
\begin{gather*}
b_{2}=\frac{a_{2}^{2}\left(\tan \alpha_{2}-\tan \alpha_{s}\right)}{1+\left[a_{2}(1-s)-1\right] e^{a_{2}(1-s)}}  \tag{23}\\
\text { At } x=1, y^{\prime \prime}=Q\left(y_{\text {max }}^{\prime}\right) \\
Q=(s-1) a_{2} e^{1+a_{2}(1-s)} \tag{24}
\end{gather*}
$$

The equations obtained to determine $a_{1}$ and $a_{2}$ (Equations (18) and (24))are implicit, and require iterative solution. In each case, there are two solutions for the constant (assuming $P$ and $Q$ to be less than unity). Because it is desired that the magnitude of the second derivative reach its maximum value between the inflection point and leading or trailing edge, as appropriate, it follows that

$$
\begin{equation*}
a_{1}>\frac{1}{s} \tag{25}
\end{equation*}
$$

and

$$
a_{2}<\frac{1}{s-1}
$$

Figures 6 through 10 illustrate the general characteristic of this camber line. Figure 6 shows the distribution of the second derivative of the camber line for various values of $P$ and Q. The other parameters that are held constant describe a blade perhaps typical of mid-span conditions for a high-speed machine. Figure 7 shows the corresponding camber line. Figure 8 shows camber lines for a section having zero overall camber and hence equal negative and positive cambers. The eifect of varying the parameters $P$ and $Q$ is shown. Figure 9 shows camber lines for a similar section, except that the effect of varying the location of the inflection point $S$ is illustrated. Camber lines for three sections having no negative camber are shown in Figure 10. As the "inflection point" is shifted rearwards from the leading edge, the camber line changes from one of continuous positive camber to that of a "J-blade."

## 3. CIRCULAR-ARC CAMBER LINE

The circular-arc camber line has been considered because of the wide application of the double-circular-arc airfoil in turbo-machinery of the more conventional variety and because of its potential use, particularly with respect to stators, in
high performance compressor stages. Of the original five boundary conditions of Equations (1) and (2), those involving y" are superfluous to the definition oi a unique circular-arc camber line.

The equation of this camber line is of the form

$$
\begin{equation*}
\left(x-x_{0}\right)^{2}+\left(y-y_{0}\right)^{2}=R^{2} \tag{26}
\end{equation*}
$$

where $\left(x_{0}, y_{0}\right)$ is the center of, and $R$ the radius of, the circle of which the desired arc is a portion.

From the boundary condition at $x=1$ and the knowledge that the slope of a radial ine at any point must equal the negative reciprocal of the slope of the tangent to the arc at that point, one obtains

$$
\begin{equation*}
\tan \alpha_{2}=-\frac{\left(1-x_{0}\right)}{y_{2}-y_{0}} \tag{27}
\end{equation*}
$$

Where $y_{2}$ is the $y$ coordinate of the arc at $x=1$. Solving for $x_{0}$,

$$
\begin{equation*}
x_{0}=\left(y_{2}-y_{0}\right) \tan \alpha_{2}+1 \tag{28}
\end{equation*}
$$

At the point ( $1, y_{2}$ ), Equation (26) may be arranged to yield

$$
\begin{equation*}
\left(y_{2}-y_{0}\right)^{2}=R^{2}-x_{0}^{2}+2 x_{0}-1 \tag{29}
\end{equation*}
$$

From the boundary condition which fixes the leading edge of the arc at the origin of the coordinate system, and Equation (26), one obtains

$$
\begin{equation*}
y_{0}^{2}=R^{2}-x_{0}^{2} \tag{30}
\end{equation*}
$$

Substituting Equations (28) and (30) into Equation (29),

$$
\begin{equation*}
\left(y_{2}-y_{0}\right)^{2}=y_{0}^{2}+2\left(y_{2}-y_{0}\right) \tan \alpha_{2}+1 \tag{31}
\end{equation*}
$$

Expanding the left hand side of Equation (31) and solving for $y_{0}$,

$$
\begin{equation*}
y_{0}=\frac{y_{2}^{2}-2 y_{2} \tan \alpha_{2}-1}{2\left(y_{2}-\tan \alpha_{2}\right)} \tag{32}
\end{equation*}
$$

Using this expression for $\mathrm{y}_{0}$ in Equation (28),

$$
\begin{equation*}
x_{0}=\frac{\left(y_{2}^{2}-1\right) \tan \alpha_{2}+2 y_{2}}{2\left(y_{2}-\tan \alpha_{2}\right)} \tag{33}
\end{equation*}
$$

and substituting Equations (32) and (33) into (30) and solving for $R^{2}$,

$$
\begin{equation*}
R^{2}=\frac{\left(y_{2}^{2}+l\right)^{2} \sec ^{2} \alpha_{2}}{4\left(y_{2}-\tan \alpha_{2}\right)^{2}} \tag{34}
\end{equation*}
$$

The above equations (Equations (32), (33) and (34)) give the necessary camber line constants in "erms of the as yet undetermined $y_{2}$. The third boundary condition, fixing the slope at the origin, is employed to express $y_{2}$ in terms of known quantities. From this boundary condition,

$$
\begin{equation*}
x_{0}=-y_{0} \tan \alpha_{1} \tag{35}
\end{equation*}
$$

Combining Equations (28) and (33) and solving for $y_{2}$,

$$
\begin{equation*}
y_{2}=\frac{y_{0}\left(\tan \alpha_{2}-\tan \alpha_{1}\right)-1}{\tan \alpha_{2}} \tag{36}
\end{equation*}
$$

Substituting this expression for $y_{2}$ into Equation (32) yields, after considerable simplification,

$$
\begin{equation*}
y_{0}=\frac{-\tan \alpha_{1} \sec ^{2} \alpha_{2} \pm \tan \alpha_{2} \sec \alpha_{1} \sec \alpha_{2}}{\tan ^{2} \alpha_{1}-\tan ^{2} \alpha_{2}} \tag{37}
\end{equation*}
$$

The proper sign of the second terp ( $(T)$ of Equation (37) is determined by the realization that $\left|y_{0}\right|$ must approach infinity as $\alpha_{2}$ approaches $\alpha_{1}$. Therefore,

$$
\begin{equation*}
y_{0}=\frac{-\left(\tan \alpha_{1} \sec ^{2} \alpha_{2}+\tan \alpha_{1} \sec \alpha_{1} \sec \alpha_{2}\right)}{\tan ^{2} \alpha_{1}-\tan ^{2} \alpha_{2}} \tag{38}
\end{equation*}
$$

Substitution of Equation (38) into Equation (35) determines $x_{0}$ in terms of known quantities. Substitution of Equation (38) into (36) determines $\mathrm{y}_{2}$ in terms of the known quantities $\alpha_{1}$ and $\alpha_{2}$, which, when used in Equation (34), yields an expression for $R^{2}$ in terms of $\alpha_{1}$ and $\alpha_{2}$. This completes the determination of the three constants ( $\mathrm{x}_{0}, \mathrm{y}_{0}$, and $\mathrm{R}^{2}$ ) of the circular arc camber line.

## 4. MULTTPLE-CIRCULAR-ARC CAMBER LINE

The multiple-circular-arc camber line incorporates and extends the circular-arc definition scheme as previously described to include the potential for defining a camber line composed of two segments, either or both of which may be either circular arcs or straight lines, in principle. The most interesting applications of this capability probably involve the creation of "S-blades," composed of two circular arcs, and "J-blades" composed of a straight-line and circular-arc combination.

The specification of this two-segment camber line requires the boundary condition of Equation (11) as well as the three boundary conditions necessary for the circular-arc camber line. The circular-arc portions are computed in the manner of the subsection "Circular-Arc Camber Line" using the inflection angle and the inlet (or outlet) angle. Each segment is scaled, and positioned with respect to the origin (if necessary). A straight-line segment is created when the angle at the inflection point is identical to the inlet (or outlet) angle.

One other feature of the programming associated with the multiple-circular-arc camber line should be noted: an ordinary circular-arc camber line will be produced if the inflection point is specified at the blade leading edge. Since the standard thickness distribution (described in Section III) is applied to the multiple-circular-arc camber line, this feature enables the program user to define a blade using the circular-arc camber line and either the standard or double circular-arc thickness distribution.

Figure 11 shows various types of camber lines which may be specified using the multiple-circular-arc camber line option.

Figure 12 shows camber lines for similar "S-blade" sections, illustrating the effect of varying the location of the inflection point. Camber lines for three "J-blade" sections with different inflection locations are shown in Figure 13.

## THE SECTION THICKNESS DISTRIBUTION

## 1. STANDARD THICKNESS DISTRIBUTION

For the same reasons that it was desirable to define the camber line in equation form, it is also practical to define the thickness distribution this way. Five features were particularly desired for a supersonic compressor airfoil. First, the maximum thickness should be arbitrary. Second, it snould be possible to locate the maximum thickness at any point on the downstream half of the airfoil. Third, there shouid be no discontinuity in curvature, meaning that derivatives through the second should be continuous. Fourth, for mechanical reasons, the airfoil should have a thickness distribution which is continuously convex; i.e., the second derivative should not change sign. Fifth, the airfoil should have the slenderest leading edge that is compatible with the first four conditions.

Numerous equations were examined with these objectives in mind. However, no single equation was found which seemed to satisfy all of the foregoing requirements to a satisfactory extent. The method finally selected consisted of defining the thickness distribution with two equations: one from the leading edge to the point of maximum thickness and one from there to the trailing edge. At the roint of juncture, the thickness and the first and second derivatives of thickness were equated. In order to prevent a reflex curvature from occurring in the thickness distribution near the leading edge, the second derivative of thickness was set equal to zero at the leading edge. Thickness was considered to be distributed normal to a camber line of unit length. Maximum thickness was located at a point $Z$ on this axis which, it should be noted, is not the chord line. The thickness of the leading and trailing edges is independently specified so that it need not be the same at both edges. At the leading edge, the blade surface is completed with a circular arc. At the trajing edge, the blade was truncated by connecting the two end points with a straight line.

The boundary conditions chosen for the two segments of the thickness distribution may be summarized as follows.

For the Leading Portion:

$$
\text { At } \begin{align*}
x=0, y & =y_{0}  \tag{39}\\
y^{\prime \prime} & =0 \\
\text { At } x=z, y & =T / 2 \\
y^{\prime} & =0
\end{align*}
$$

For the Trailing Portion:

$$
\begin{align*}
\text { At } x=z, \quad y & =T / 2  \tag{40}\\
y^{\prime} & =0 \\
y^{\prime \prime} & =y^{\prime \prime} \text { (leading portion) } \\
\text { At } x=1.0, y & =y_{1}
\end{align*}
$$

Note that y represents the half-thickness of the blade element and $T$ is the total value of the maximum thickiess. Both are scaled in proportion to a camber line length of unity. The parameter $Z$ denotes the position where maximum thickness occurs on the camber line. For the leading section, a simple thirdorder polynomial was used in the form

$$
\begin{align*}
& y=a x^{3}+b x^{2}+c x+d  \tag{41}\\
& y^{\prime}=3 a x^{2}+2 b x+c \\
& y^{\prime \prime}=6 a x+2 b
\end{align*}
$$

Applying the boundary conditions for this portion produces the following result valid from 0 to $Z$ :

$$
\begin{align*}
& a=-\frac{T / 2-y_{0}}{2 Z^{3}}  \tag{42}\\
& b=0 \\
& c=\frac{3\left(T / 2-y_{0}\right)}{2 z}  \tag{43}\\
& d=y_{1} \tag{44}
\end{align*}
$$

Also, at $x=Z$

$$
\begin{equation*}
y^{\prime \prime}=-\frac{3\left(T / 2-y_{0}\right)}{Z^{2}} \tag{45}
\end{equation*}
$$

Which is used as a boundary condition for the trailing portion of the blade. A similar third-order equation was used to define the trailinf portion of the thickness distribution. This was arranged

$$
\begin{align*}
& y=e(x-Z)^{3}+f(x-Z)^{2}+g(x-Z)+h  \tag{46}\\
& y^{\prime}=3 e(x-Z)^{2}+2 f(x-Z)+g \\
& y^{\prime \prime}=6 e(x-Z)+2 f
\end{align*}
$$

Applying the boundary conditions relating to the trailing portion leads to the following resuit valid from $x=z$ to 1.0 .

$$
\begin{align*}
& e=\frac{3\left(T / 2-y_{0}\right)}{2 Z^{2}(1-Z)}-\frac{\left(T / 2-y_{1}\right)}{(1-Z)^{3}}  \tag{47}\\
& f=-\frac{3\left(T / 2-y_{0}\right)}{2 Z^{2}}  \tag{48}\\
& g=0  \tag{49}\\
& h=T / 2
\end{align*}
$$

As the equations for the two portions of the thickness distribution are cubics, potentially there is the possibility of introducing inflection points into the thickness distribution. One boundary condition applied to the equation describing the forward portion of the thickness was that the second derivative be zero at the leading edge. Hence, because a cublc has only one point of inflection, the thickness distribution forward of the point of maximum thickness cannot contain an inflection. Examination of the equation for the second portion of the thickness distribution yields the result that there is a minimum value of $Z$ (the point of maximum thickness) such that the inflection point of this equation is not on the blade surface. This is given by

$$
\begin{equation*}
z_{\min }=\frac{1}{1+\sqrt{\frac{1-2 y_{1} / T}{1-2 y_{0} / T}}} \tag{50}
\end{equation*}
$$

Thus if the leading and trailing edge thicknesses are equal, $Z_{\text {min }}$ is 0.5 , and will increase as the ratio of the trailing egine to leading edge thicknesses increases.

Figure 14 shows thickness distributions for three values of $Z$, with the leading and trailing edge thicknesses equal in each case.
2. DOUBLE-CIRCULAR-ARC THICKNESS DISTRIBUTION

The determination of the double-circular-arc thickness distribution is essentially a geometrical exercise involving the translation and rotation of coordinate systems and the use of polar coordinates for the sake of simplicity and convenience.

The most desirable coordinate system would define the circular arc camber line as part of a circle centered at the origin, such that the camber line represents the portion of the circle in the range

$$
\begin{equation*}
\frac{\pi}{2}-\frac{\phi}{2} \leq \psi \leq \frac{\pi}{2}+\frac{\phi}{2} \tag{51}
\end{equation*}
$$

when $\psi$ is measured counterclockwise from the positive x axis and, from geometry,

$$
\begin{equation*}
\phi=\alpha_{1}-\alpha_{2} \tag{52}
\end{equation*}
$$

This combined translation-rotation is accomplished by the coordinate change

$$
\begin{align*}
& x^{\prime}=\left(x-x_{1}\right) \cos \rho+\left(y-y_{1}\right) \sin \rho  \tag{53}\\
& y^{\prime}=-\left(x-x_{1}\right) \sin \rho+\left(y-y_{1}\right) \cos \rho
\end{align*}
$$

where

$$
\begin{equation*}
\rho=N+\frac{\phi}{2} \tag{54}
\end{equation*}
$$

From the geometry of the camber line,

$$
\begin{equation*}
W=\sin ^{-1}\left[\frac{\left(x_{0}-1\right)}{R}\right] \tag{55}
\end{equation*}
$$

In the new coordinate system, then,

$$
\begin{equation*}
x^{\prime^{2}}+y^{t^{2}}=R^{2} \tag{56}
\end{equation*}
$$

In this system, either surface of the airfoil may be expressed as a portion of a circle centered on the $y^{\prime}$ axis. That is,

$$
\begin{equation*}
x^{\prime 2}+\left(y^{\prime}-y_{s}\right)^{2}=R_{s}^{2} \tag{57}
\end{equation*}
$$

or, expanding,

$$
\begin{equation*}
x^{\prime^{2}}+y^{\prime 2}-2 y^{\prime} y_{s}+y_{s}^{2}=R_{s}^{2} \tag{58}
\end{equation*}
$$

In cylindrical polar coordinates, Equations (56) and (58) may be expressed as follows

$$
\begin{gather*}
r_{2}=R  \tag{59}\\
r_{2}^{2}+y_{s}^{2}-2 r_{2} y_{s} \sin \psi=R_{s}^{2} \tag{60}
\end{gather*}
$$

where the relevant range of $\psi$ is as given in Equation (51).
The desired thickness is the difference ( $\Delta r$ ) between the expressions for the two arcs. First, however, Equation (60) must be solved for $r_{2}$ rather than $r_{2}{ }^{2}$. Using the binomial theorem,

$$
\begin{equation*}
r_{2}=y_{s} \sin \psi \pm \sqrt{R_{s}^{2}-y_{s}^{2} \cos ^{2} \psi} \tag{61}
\end{equation*}
$$

If it is stipulated that Equation (61) represents the surface of the blade which lies further from the origin than does the camber line, then the thickness $t$ in a positive sense is given by the expression $r_{2}-r_{1}$.

$$
\begin{equation*}
t=y_{s} \sin \psi \pm \sqrt{R_{s}^{2}-y_{s}^{2} \cos ^{2} \psi}-R \tag{62}
\end{equation*}
$$

The proper sign of the second term ( + ) is chosen by requiring that at $\psi=\pi / 2$,

$$
\begin{equation*}
t=y_{s}+R_{s}-R \tag{63}
\end{equation*}
$$

Therefore, Equation (62) becomes

$$
\begin{equation*}
t=y_{s} \sin \psi+\sqrt{R_{s}^{2}-y_{s}^{2} \cos ^{2} \psi}-R \tag{64}
\end{equation*}
$$

A further rotation of the coordinate system such that

$$
\begin{equation*}
\mu=\psi-\left(\frac{\pi}{2}-\frac{\phi}{2}\right) \tag{65}
\end{equation*}
$$

yields

$$
\begin{equation*}
t=y_{s} \cos \left[\frac{\phi}{2}-\mu\right]+\sqrt{R_{s}^{2}-y_{s}^{2} \sin ^{2}\left[\frac{\phi}{2}-\mu\right]}-R \tag{66}
\end{equation*}
$$

in the range $0 \leq \mu \leq \phi$.
Expressing Equation (66) in terms of the normalized arc length, s,

$$
\begin{equation*}
t=y_{s} \cos \left[\left(\frac{1}{2}-s\right) \phi\right]+\sqrt{R_{s}^{2}-y_{s}^{2} \sin ^{2}\left[\left(\frac{1}{2}-s\right) \phi\right]}-R \tag{67}
\end{equation*}
$$

in the range $0 \leq \mathrm{s} \leq 1$.
The expressions for $y_{S}$ and $R_{S}$ in Equation (67) can be determined geometrically in terms of $T / 2$ and $r_{0}$, the edge radius.

$$
\begin{equation*}
\left.R_{s}=r_{0}+\frac{R \sin \phi}{\sin [A r c c o s}\left(\frac{1-A^{2}}{1+A^{2}}\right)\right] \tag{68}
\end{equation*}
$$

$$
\begin{equation*}
y_{s}=R-R_{s}+\frac{T}{2} \tag{69}
\end{equation*}
$$

where

$$
\begin{equation*}
A=\frac{\left(T / 2-r_{0}\right)+R(1-\cos \phi / 2)}{R \sin \phi / 2} \tag{70}
\end{equation*}
$$

Equations (68), (69) and (70) yield all the unknown constants of the thickness distribution in terms of known quantities.

## SECTION IV

## CARTESIAN COORDINATES FOR THE BLADE

The two previous sections have described the methods derived to specify individual blade sections. When located as desired relative to the blade stacking axis, the section coordinates are the coordinates of the streamsurface blade section. A series of such sections on all given stream surfaces specifies the envelope of the blade, but the surface coordinates are not in a convenient form for manufacturing purposes. A method of determining the coordinates of plane sections through the blade at a series of locations along the stacking axis is now described.

The method relies upon the use of the "spline-curve" for interpolation (or extrapolation) of the coordinates of the blade suriaces and therefore the theory of the curve is given. The same interpolation scheme is also used elsewhere in the computer program.

Given the Cartesian coordinates $x$ and $y$ for a series of points, the problem is to find a curve $y=f(x)$ that passes through each given point. A series of algebraic cubics is used, one equation applying between each adjacent pair of points. The four coefficients for each cubic permit the matching of the first and second derivatives of the two equations applying at each point (except the first and last), in addition to passing the curves through the specified points. Two additional ciditions are required to establish the coefficients. Those chosen here are that the second derivatives of the equations at the first and last points are zero. Other conditions are possible, but less appropriate for the current purpose. A curve of the form described is that theoretically assumed by a thin, uniformly-flexible beam pinned at each specified point, hence the name "spline-curve."

By applying the conditions described above, the following system of equations may be produced. In the interval between any two adjacent points, such that $x_{n-1} \leq x \leq x_{n}$, the equation of the spline-curve is

$$
\begin{gather*}
y=\frac{M_{n-1}}{6} \frac{\left(x_{n}-x\right)^{3}}{\left(x_{n}-x_{n-1}\right)}+\frac{M_{n}}{6} \frac{\left(x-x_{n-1}\right)^{3}}{\left(x_{n}-x_{n-1}\right)}+\left(x_{n}-x\right)\left[\frac{y_{n-1}}{x_{n}-x_{n-1}}-\frac{M_{n-1}}{6}\left(x_{n}-x_{n-1}\right)\right]+ \\
\left(x-x_{n-1}\right)  \tag{71}\\
{\left[\frac{y_{n}}{x_{n}-x_{n-1}}-\frac{M_{n}}{6}\left(x_{n}-x_{n-1}\right)\right]}
\end{gather*}
$$

and the slope of the spline-curve ls
$\frac{d y}{d x}=-\frac{M_{n-1}}{2} \frac{\left(x_{n}-x\right)^{2}}{\left(x_{n}-x_{n-1}\right)}+\frac{M_{n}}{2} \frac{\left(x-x_{n-1}\right)^{2}}{\left(x_{n}-x_{n-1}\right)}+\left(\frac{y_{n}-y_{n-1}}{x_{n}-x_{n-1}}\right)-\left(\frac{M_{n}-M_{n-1}}{6}\right)\left(x_{n}-x_{n-1}\right)$
The coefficients $M$ are riven by $M_{1}=M_{N}=0.0$ where $N$ is the
total number of specified points, and

$$
\begin{align*}
& M_{n}=\frac{D_{n}-B_{n} M_{n+1}}{A_{n}}  \tag{73}\\
& A_{1}=1.0 \\
& B_{1}=0.0 \\
& D_{1}=0.0 \\
& A_{n}=\frac{\left(x_{n+1}-x_{n-1}\right)}{3}-\frac{\left(x_{n}-x_{n-1}\right)}{6} \frac{B_{n-1}}{A_{n-1}} \\
& B_{n}=\frac{x_{n+1}-x_{n}}{6} \\
& D_{n}=\frac{y_{n+1}-y_{n}}{x_{n+1}-x_{n}} \cdot \frac{y_{n}-y_{n-1}}{x_{n}-x_{n-1}}-\frac{\left(x_{n}-x_{n-1}\right)}{6} \frac{D_{n-1}}{A_{n-1}}
\end{align*}
$$

The determination of the Cartesian coordinates of the manufacturing sections involves two steps. First, Cartesian coordinates for the points defining the streamsurfaces are obtained. Second, coordinates for the manufacturing sections are interpolated (or extrapolated) at the desired plane sections through the blade.

Figure 15 illustrates the geometry involved in the determination of the Cartesian coordinates of the streamsurface sections. The radius of the streamsurface is defined at a number of axial locations. A spline-curve is passed through these points, and the slope of the streamsurface is calculated (as the slope of the spline-curve) at 100 points distributed uniformly on the $x$-axis between the first and last given points. Then an $m-x$ table is constructed by numerically integrating for the streamsurface using

$$
\begin{equation*}
m_{n}=m_{n-1}+\left(x_{n}-x_{n-1}\right) \sqrt{1+\left[\left(\frac{d r}{d x_{n}}+\frac{d r}{d x_{n-1}}\right) / 2\right]^{2}} \tag{74}
\end{equation*}
$$

The $m$ coordinate of any point on the streamsurface blade section is the "axial" coordinate of the point derived from the camber line and thickness distribution. Thus linear interpolation of the $m-x$ table yields the corresponding $x$ coordinate. Use of a 100 -point lable assures that linear interpolation is adequate. The $r-x$ table given enables the radius of the streamsurface at any value of $x$ to be found. Spline-curve interpolation is used for this purpose.

The "y" coordinate of any point derived from the camber line and thickness distribution must now be related to the circumferential direction on a streamsurface. This can be accomplished by determining the angle $\varepsilon$ (defined in Figure 15) in the following manner. First, the origin of the m-system is shifted so that the origin ( $m=0$ ) corresponds to the location of the centroid of the section in the cascade plane. The camber line slopes in the cascade plane must then be reproduced on a streamsurface. This requires that at each point

$$
\begin{equation*}
r \frac{d \varepsilon}{d m}=\frac{d y}{d x}=y^{\prime} \tag{75}
\end{equation*}
$$

It is possible to determine the variation of $\varepsilon$ with $m$ by integrating Equation (75) with respect to m,

$$
\begin{equation*}
\varepsilon(m)-\varepsilon_{0}=\int_{0}^{m} \frac{y^{\prime}}{r} d m \tag{76}
\end{equation*}
$$

where both the streamsurface radius and the camber line slope are known functions of $x$, hence $m$.

$$
\text { At } m=0
$$

$$
\begin{equation*}
\varepsilon_{0}=\frac{y_{d}}{r_{d}} \tag{77}
\end{equation*}
$$

where $y_{d}$ is the distance between the section centroid and the camber line measured in the $y$ direction in the cascade plane, and $r_{d}$ is the streamsurface radius.

Therefore, from Equations (76) and (77),

$$
\begin{equation*}
\varepsilon(m)=\int_{0}^{m} \frac{y^{\prime}}{r} d m+\frac{y_{d}}{r_{d}} \tag{78}
\end{equation*}
$$

For each point on the blade surface in the cascade plane, the appropriate $\varepsilon$ is determined through a six step procedure:
(1) $m$ is determined from the $m-x$ table
(2) $\varepsilon_{\text {camber }}$ line is determined from Equation (78)
(3) $\Delta y$ is determined from $y=y_{\text {surface }}-y_{\text {camber }}$ line in the cascade plane
(4) $r$ is determined from the value of $m$ in conjunction with the $m-x$ and $r-x$ tables
(5) $\Delta \varepsilon$ is determined from $\Delta \varepsilon=\Delta y / r$
(6) $\varepsilon_{\text {surface }}=\varepsilon_{\text {camber line }}+\Delta \varepsilon$

The two remaining Cartesian coordinates of the point then follow from

$$
\begin{equation*}
z=r \cos \varepsilon \tag{79}
\end{equation*}
$$

and

$$
y=r \sin \varepsilon
$$

The interpolation of $x$ and $y$ coordinates for the manufacturing sections (on constant-z planes) is achieved by fitting the spline-curve through corresponding points on each streamsurface section. The same number of points is used to define each blade section. Hence, the spline-curve is used first to fit $x$ as a function of $z$ for the first point on the suction
surface of each section. The value of $x$ is then interpolated at each desired value of 2 . Then this is repeated for the coordinate $y$, and then for all other corresponding points describing the streamsurface sections. Thus an equal number of similarly distributed points describing the manufacturing sections is obtained.

## SECTION V

## SECTION PROPERTIES

The stacking axis of the blade is passed through each streamsurface section either at its leading or trailing edge, or at a point specified relative to the centroid of the section. Because the streamsurface sections are in general not planar, the centroids of the manufacturing sections will not generally lie precisely on the stacking axis when the streamsurface sections are stacked about their centroids. By determining the locations of the centroids of the manufacturing sections so obtained, it is possjible to estimate the offsets that must be applied when restacking the streamsurface sections in order that the centroids of the manufacturing sections will be located as desired relative to the stacking axis. Thus there is a requirement to determine the locations of the centroids of both the streamsurface and manufacturing sections if the blade is to be stacked on the centroids.

To further assist in any mechanical analysis of the blade, the areas, second moments of area, principal axes, and principal second moments of area for both the streamsurface and manufacturing sections are also determined. The methods used to determine these quantities are given below.

## 1. STREAMSURFACE SECTIONS

The location of the centroid of an area relative to an arbitrary origin is given by

$$
\begin{equation*}
\bar{x}=\frac{\int x d A}{\int d A} \text { and } \bar{y}=\frac{\int y d A}{\int d A} \tag{80}
\end{equation*}
$$

In the computer program these integrals are performed numerically by considering the section to be composed of a semicircle formed by the leading edge and a series of quadrilaterals. The quadrilaterals are defined by the series of points defining the camber line and the values of the section thickness at these points, as shown in Figure 16. The origin for the calculation is the leading edge of the blade and each quadrilateral is assumed to be a rectangle. The length of one side is the camber line length enclosed by the quadrilateral, and the length of the other is the mean section thickness at the ends of the quadrilateral. Hence the above relationships are evaluated as

$$
\begin{gather*}
\int x d A=\frac{\pi y_{0}^{2}}{2}\left(y_{0}-\frac{4}{3} \frac{\cos \alpha_{1}}{\pi} y_{0}\right)+\sum_{1, N-1}\left(\frac{x m_{n}+x m_{n+1}}{2}\left(s_{n+1}-s_{n}\right) \frac{\left(t_{n+1}-t_{n}\right)}{2}\right)  \tag{81}\\
\int y d A=-\frac{\pi y_{0}^{2}}{2} \frac{4 y_{0}}{3 \pi} \sin \alpha_{1}+\sum_{1, N-1}\left(\frac{y m_{n}+y m_{n+1}}{2}\left(s_{n+1}-s_{n}\right) \frac{\left(t_{n}+t_{n+1}\right)}{2}\right) \\
\int d A=\frac{\pi y_{0}^{2}}{2}+\sum_{1, N-1}\left(s_{n+1}-s_{n}\right) \frac{\left(t_{n}+t_{n+1}\right)}{2}
\end{gather*}
$$

For the case of the double-circular-arc blade, a term representing the contribution of the trailing edge semicircle, similar to the term for the leading edge contribution, is added to each of Equations (81), (82) and (83).

The second moments of area $I_{x}$ and $I_{y}$, and products of inertia, $I_{x y}$, are found by again ${ }^{x}$ considering the section to be composed of Xy series of rectangles, as shown in Figure 16. (The contribution of the edge(s) is neglected.) The principal second moments of one rectange about its centroid are

$$
\begin{align*}
& I_{x^{\prime}}=\left(s_{n+1}-s_{n}\right)\left(\frac{t_{n+1}+t_{n}}{2}\right)^{3} / 12  \tag{84}\\
& I_{y^{\prime}}=\left(\frac{t_{n+1}+t_{n}}{2}\right)\left(s_{n+1}-s_{n}\right)^{3} / 12 \tag{85}
\end{align*}
$$

The mean camber line angle, $C$, at the two ends of the rectangle is used to determine the second moments of area of the rectangle about its centroid from

$$
\begin{align*}
& I_{x}=\frac{I_{x^{\prime}}+I_{y^{\prime}}}{2}+\frac{I_{x^{\prime}}-I y^{\prime}}{2} \cos (2 C)  \tag{86}\\
& I_{y}=\frac{I^{\prime}+I^{\prime}}{2}-\frac{{ }^{\prime} x^{\prime}-I}{} y^{\prime}  \tag{87}\\
& 2 \\
& \cos (2 C)
\end{align*}
$$

The product of inertia of the rectangle is also determined from

$$
\begin{equation*}
I_{x y}=\frac{\left(I_{x}-I_{y}\right) \cos (2 C)-I_{x^{\prime}}+I_{y^{\prime}}}{2 \sin (2 C)} \tag{88}
\end{equation*}
$$

The contributions of the rectangle to the second moments of area of the section about its centroid are given by

$$
\begin{equation*}
\Delta I_{y}(\text { section centroid })=I_{y}+A(x-\bar{x})^{2} \tag{90}
\end{equation*}
$$

where $A$ is the area of the rectancle, and $\bar{x}$ and $\bar{y}$ are the coordinates of the center (that is, centroid) of the rectangle. Also, the contribution of the rectangle to the product of inertia of the section about its centroid is given by

$$
\begin{equation*}
\Delta I_{x y}(\text { section centroid })=I_{x y}+A(x-\bar{x})(y-\bar{y}) \tag{91}
\end{equation*}
$$

The summation of all such contributions yields the second moments of area and product of inertia of the section about its centroid.

The inclination of the principal axes of the section, $B$, is given by

$$
\begin{equation*}
B=\frac{1}{2} \arctan \left(\frac{2 I x y}{I_{y^{-I}} x}\right) \tag{92}
\end{equation*}
$$

The principal second moments of area of the section about its centroid are given by

$$
\begin{align*}
& \tilde{I}_{x p}=\frac{I_{x}+I_{y}}{2}+\frac{I_{x}-I_{y}}{2} \cos (2 B)-I_{x y} \sin (2 B)  \tag{93}\\
& I_{y p}=\frac{I_{x}+I_{y}}{2}-\frac{I_{x}-I_{y}}{2} \cos (2 B)+I_{x y} \sin (2 B) \tag{94}
\end{align*}
$$

The various calculations described above are made for a "normalized" streamsurface blade section (that is one having an axial chord of unity), and are subsequently scaled as appropriate, according to the specified chord length.

## 2. MANUFACTURING SECTIONS

The methods used to determine the properties of the manufacturing sections are essentially the same as those described above for the streamsurface sections. However, in this case the section geometry is described by the coordinates of a series of points on the two blade surfaces as shown in Figure 16.

The section is again considered to be composed of a semicircle at the leading edge and a number of rectangles. The radius of the leading edge, $y_{0}$, is calculated to be a half of the distance between the first points on the blade pressure and suction surfaces. The angle of the blade section at the leading edge is obtained from the first two points on each blade surface, thus

$$
\begin{equation*}
\alpha=\arctan \left(\frac{y p_{2}+y s_{2}-y p_{1}-y s_{1}}{x p_{2}+x s_{2}-x p_{1}-x s_{1}}\right) \tag{95}
\end{equation*}
$$

The lengths of the sides of each assumed rectange are determined from the coordinates of the points at the four corners, thus

$$
\begin{align*}
& \ell_{n}=\frac{1}{2} \quad\left\{\begin{array}{l}
{\left[\left(x s_{n+1}-x p_{n+1}\right)^{2}+\left(y s_{n+1}-y p_{n+1}\right)^{2}\right]}
\end{array}\right.  \tag{96}\\
&\left.\sqrt{\left[\left(x s_{n}-x p_{n}\right)^{2}+\left(y s_{n}-y p_{n}\right)^{2}\right]}\right\} \\
& m_{n}=\frac{1}{2} \quad\left\{\sqrt{\left[\left(x s_{n+1}-x s_{n}\right)^{2}+\left(y s_{n+1}-y s_{n}\right)^{2}\right]}+\right.  \tag{97}\\
&\left.\sqrt{\left[\left(x p_{n+1}-x p_{n}\right)^{2}+\left(y p_{n+1}-y p_{n}\right)^{2}\right]}\right\}
\end{align*}
$$

Hence, the section area and the location of the centroid are determined from

$$
\begin{equation*}
\int \mathrm{dA}=\frac{\pi y_{0}^{2}}{2}+\sum_{1, N-1}\left(\ell_{n} m_{n}\right) \tag{98}
\end{equation*}
$$

$$
\begin{align*}
& \int x d A=\frac{\pi y_{0}^{2}}{2}\left[\left(x p_{1}+x s_{1}\right) / 2 \cdot 0-\frac{4 y_{0}}{3 n} \cos \alpha\right]+  \tag{99}\\
& \sum_{1, N-1}\left[\ell_{n} m_{n}\left(x s_{n}+x p_{n}+x p_{n+1}+x s_{n+1}\right) / 4.0\right] \\
& \int y d A=\frac{\pi y_{0}^{2}}{2}\left[\left(y p_{1}+y s_{1}\right) / 2.0-\frac{4 y_{0}}{3 \pi} \sin \alpha\right]+  \tag{100}\\
& \sum_{1, N-1}\left[\ell_{n} m_{n}\left(y s_{n}+y s_{n+1}+y p_{n}+y p_{n+1}\right) / 4.0\right]
\end{align*}
$$

As for the streamurface sections, an additional term is inserted in each of Equations (98), (99) and (100) for the double-circular-arc blade, to account for the contribution of the trailing edre semi-circle.

The second moments of area and product of inertia of the section are again found by first determinine the principal second moments of each rectanple about its centroid, and then determining the contribution of each rectanfle to the second moments and product of inertia of the section. Referring again to Figure 16, we have for one rectancie

$$
\begin{align*}
& I_{x^{\prime}}=m \ell^{3} / 12  \tag{101}\\
& I_{y^{\prime}}=\ell m^{3} / 12 \tag{102}
\end{align*}
$$

In order to determine the second moment: of area of the rectanfle about is centroid in directions parallel to the axis of the section coordinates, the anfle of inclination of the rectangle is found from

$$
\begin{equation*}
A=\arctan \left(\frac{y p_{n+1}+y s_{n+1}-y p_{n}-y s_{n}}{x p_{n+1}+x s_{n+1}-x p_{n}-x s_{n}}\right) \tag{103}
\end{equation*}
$$

Then the second momente of area and product of inertia for the rectanmle are found usine Equations (86), (87) and (88) imain, and the contributions of the rectancle to the properties
of the sections are given by Equations (89), (90) and (91). Equations (92), (93) and (94) are applied to give the inclination of the princinal axes, and the principal second moments of the section about its centroid.

An additional section property, the torsional constant, is computed for the manufacturing sections. Following Reference 2 the constant $K$ is determined from

$$
\begin{equation*}
K=\frac{\frac{1}{3} F}{\left(1+\frac{4}{3} \frac{F}{S u^{2}}\right)} \tag{104}
\end{equation*}
$$

where

$$
\begin{equation*}
F=\int_{0}^{u} t^{3} d u \tag{105}
\end{equation*}
$$

and $d u, t$, and $S$ are the elemental length along the section median line, the thickness normal to the median line, and the section area, respectively.

## SECTION VI

## BLADE CHARACTERISTICS

## 1. BLADE VOLUME CALCULATION

Except when IPRINT = 1, a calculation of the volume enclosed by the blade between the irmermost and outermost streamsurfaces is made. This is done from the Cartesian coordinates of the points describing the streamsurface blade sections. The total volume is obtained by summing the volumes enclosed between each adjacent pair of streamsurfaces. These volumes in turn are obtained by summing two types of subvolume. First, there is the volume enclosed by the leading edge radius. This is determined by finding the mean area of the two leading edge semicircles on the two streamsurfaces, and multiplying by the mean height at the two ends of the semicircle diameters. The diameter of the leading edge is calculated to be the distance between the first points on the section pressure and suction surfaces using the streamsurface Cartesian coordinates. Thus the areas computed are those projected onto a constant-z plane, and the volume is given directly by multiplying by the mean $z$ dimension. Sccond, there is a series of volumes enclosed by corresponding quadrilaterals on the two streamsurfaces. Again, the sub-volume is determined from the mean of the areas of the two quadrilaterals projected onto a constant-z plane, and the mean distance between them. The quadrilaterals are each defined by two adjacent points on the section suction surface and the two corresponding points on the pressure surface. The area of the quadrilaterals is obtained by assuming them to be rectangles of side $\ell$ and $m$, as defined in Equations (96) and (97), except that in this case Cartesian coordinates of the streamsurface sections are used. The distance between the two rectangles is taken to be the mean of the distance between the four corresponding corners of the rectangle. Thus the total volume, $V$, is given by

where $J$ is the total number of streamsurfaces, $z$ is the mean coordinate at each end of the leading edge semicfrcle diameter, $A_{r}$ is the mean of the two leading edge semicircle areas, $N$ is the number of points defining each blade surface, and $z_{n}$ is the mean z-coordinate at the four corners of the rectangle. ${ }^{n}$ Equation (106) is modified to contain a term similar to the first to represent the trailing edge contribution for the double-circulararc blade.

## 2. CALCULATIONS FOR AEKODYNAMIC ANALYSIS

Certain aerodynamic analyses of the blade require a description of the blade on cylindrical surfaces. As an option, the computer program will produce quantities necessary for this description at a specified streamsurface-computing station intersection.

First, the angular position of the camber line with respect to the stack axis at a streamsurface-computing station intersection is specified in terms of $\Phi$, which is the value of $\varepsilon_{\text {camber }}$ line on the streamsurface at that particular intersection.

The physical passage blockage (b) due to the presence of the blades is determined as a percentage of the passage circumference in terms of the number of blades in the blade row, $N$, and $\sigma$, the angle subtended on the cylindrical surface by each blade.

$$
\begin{equation*}
b_{b}=\frac{N_{0}}{2 \pi} \tag{107}
\end{equation*}
$$

The blade lean angle, $\in$, with respect to the radial direction at a given point is obtained from the slope of a spline-surve fit through the y-z Cartesian coordinates of the streamsurface section camber lines at the particular axial location. Thus

$$
\begin{equation*}
\epsilon=\Phi-\operatorname{Arctan}\left(\frac{d y}{d z}\right) \tag{108}
\end{equation*}
$$

Two other quantities are needed to produce the proper mean-camber line angle on the cylindrical surface: the local computing station inclination, $\mu$, obtained from the specified station description; and the local streamsurface inclination, $\gamma$, obtained from the specified $r-x$ streamsurface description. Together with the camber line angle on the streamsurface, $\alpha_{n}^{0}$ :

$$
\begin{equation*}
\tan \alpha_{*}^{\circ}=\frac{\tan \gamma \tan \epsilon+\tan \alpha_{\star} / \cos \gamma}{1-\tan \mu \tan \gamma} \tag{109}
\end{equation*}
$$

Equation (109) is taken from Reference 3.

## SECTION VII

## USE OF THE COMPUTER PROGRAM

Basic information required by the user to run the computer program is given in this section. The various input data items are defined first, and the input data format is then specified. A description of the output data that may be expected is given. (Implementation of the program on a computing system is not discussed here, but in the section entitled "Computer Program Details", )

1. DEFINITION OF INPUT DATA ITEMS

TITLE

NLINES

NSTNS

NZ

NSPEC

NPOINT

NBLADE
ISTAK

An alphanumeric title of 72 characters that may be used to identify a run.

The number of streamsurfaces which are defined on and on which blade sections will be designed. Must satisfy $2 \leq$ NLINES $\leq 15$.

The number of computing stations at which the streamsurface radii are specified. Must satisfy $3 \leq$ NSTNS $\leq 10$ 。

The number of constant-z planes on which manufacturing (Cartesian) coordinates for the blade are required. Must satisfy $3 \leq N Z \leq 15$.

The number of radially-disposed points at which the parameters of the blade sections are specified. Must satisfy $1 \leq$ NSPEC $\leq 15$.

The number of points that will be generated to specify the pressure and suction surfaces of each blade section. Must satisfy $2 \leq N P O I N T \leq 80$. Generally, no less than 30 should be used. It will be advantageous to specify 80 points when precision plots of the sections are to be produced directly by the program.

The number of blades in the blade row.
If ISTAK $=0$, the blade will be stacked at the leading edge.

If ISTAK $=1$, the blade will be stacked at the trailing edge.

If ISTAK $=2$, the blade will be stacked at, or offset from, the section centroid.

IFUNCH

ISECN

IFCORD

IFPLO'T

An interer which indicates whether the quartitie. necessary for aerodynamic anajysis of the blode are to be produced on punched cards.

If ISECN $=0$, the blade will be conctracted using the polynomial camber line and the standard thickness distribution.

If ISECN $=1$, the exponential camber line and the standard thickness distribution will ve used.

If ISECI $=2$, a blade using the circular are camber line and the double-circular-are thickness distribution will be used.

If $\operatorname{ISECN}=3$, the atandard thickness distribution and any of the camber lines described in the sub-section, "Multiple Circular-Arc Camber Lines," may be used.

If $\operatorname{IFCORD}=0$, the meridional projection of the streamsurface blade section chords are specified.

If $I F C O R D=1$, the streamsurface blade section chords are specified.

Where CALCOMP software is incorporated into the computing system, IFPLOi specifies the creation of precision plots. (Further information regarding the requirements for this are riven in the section entitled "Computer Program Details".)

If $\operatorname{IFPLOT}=0$, no plots will be produced.
If IFPLOT $=1$, a plot of the streamsurface sections will be produced. All NLINES section are shown superimposed. The origin for each section plot is ofrset from the centroid of the section by distances specified by DELX and DELY.

If TFPLOT $=2$, a plot of the manufacturing. sections will be produced. The orlgin is the blade stacking axis, and all $N Z$ sections are shown superimposed.

If IFPLOT $=3$, both of the plots described for IFPLOT $=1$ and 2 will be produced.
if lpllof $=4$, indivilual plote of each of the manufacturly section: wlll bo produced. The axee are rotated clockwise by the section :tapper anple for each plot.

The input data $1:$ alway: listed by the program. Detalls of the streamsurface and manufacturing sections are printed as prescribed by IPRINT.

If IPRIHT $=0$, detail: of streamsurface and momfacturiref sections are printed.

If I!RINT $=1$, detalls of streamsurface sections are printed.

If IPRINT $=2$, details of the manufacturing sections are printed.

ZMWER, ZOURER
$\therefore$ CALE

OTACKX

PLTSZE

KPTS

IFANGS

The NZ manufacturing sections are equispaced between $z$ equals ZINNER and ZOUTER.

When precision plots are produced, SCALE is the scale factor employed.

This is the axial coordinate of the stacking axis for the blade, relative to the same origin as used for the station locations, XSTA.

The size (inches) of the plotter to be used in the creation of precision plots.

The number of points provided to specify the shape of a computing, station.

If KPTS $=1$, the computing station is upright and linear.

If KPTS $=2$, the computing station is linear and either upright or inclined.

If KPTS > 2, a spline curve is fit through the points provided to specify the shape of the station.

If IFANGS $=0$, the calculations of the quantities required for aerodynamic analysis will be omitted at a particular computing station.

If $\operatorname{IFANGS}=1$, these calculations will be performed at that station.

An array of KPTS axial coordinates (relative to an arbitrary origin) which, together with RSTA, specify the shape of a particular computing station.

An array of KPTS radii which, together with XSTA, specify the shape of a parcicular computing station.

The streamsurface radii at NLINES locations at each of the NSTNS stations.

The variation of properties of the streamsurface blade sections is specified as a function of streamsurface number. The various quantities are then interpolated (or extrapolated) at each streamsurface. The streamsurfaces are numbered consecutively from the innermost outward, starting with 1.0. ZR must increase monotonically, there being NSPEC values in all.

The blade inlet angle. (Figure 17 illustrates the geometry of a typical blade section.)

The blade outlet angle.
If $\operatorname{ISECN}=0, \mathrm{PP}$ is the ratio of the second derivative of the camber line at the leading edge to its maximum value. Must satisfy $-2.0<\mathrm{PP}<1.0$.

If $\operatorname{ISECN}=1, \mathrm{PP}$ is the ratio of the second derivative of the camber line at the leading edge to its maximum value forward of the inflection point. Must satisfy $0.0<\mathrm{PP} \leq 1.0$.

If ISECN $=2$ or $3, \mathrm{PP}$ is superfluous.
If $\operatorname{ISECN}=0, Q Q$ is the ratio of the second derivative of the camber line at the trailing edre to its maximum value. Must satisfy $0.0 \leq Q Q \leq 1.0$.

If $\operatorname{ISECN}=\mathrm{J}, \mathrm{Q} Q$ is the ratio of the second derivative of the camber line at the trailing edpe to its maximum value rearward of the inflection point. Must satisfy $0.0<Q Q \leq 1.0$.

NOTE: Some comments regarding useful values for $P P, Q Q, S$ and $B S$ are made in the section entitled "Examples of Use of Computer Program."

If $\operatorname{ISECN}=2$ or $3, Q Q$ is superflucus.

| RLE | The ratio of blade leading edge radius to chord. |
| :---: | :---: |
| TC | The ratio of blade maximum thickness to chord. |
| TE | The ratio of blade trailing edge half-thickness to chord. |
|  | If $\mathrm{ISECN}=2, \mathrm{TE}$ is superfluous. |
| 2 | The location of the blade maximum thickness, as a fraction of camber line length. |
|  | If $\mathrm{ISECN}=2, \mathrm{Z}$ is superfluous. |
| CORD | If $\operatorname{IFCORD}=0$, CORD is the meridional projection of the blade chord. |
|  | If $\mathrm{IFCORD}=1, \mathrm{CORD}$ is the blade chord. |
| $\begin{aligned} & \text { DELX, } \\ & \text {, } \end{aligned}$ | The stacking axis passes through the streamsurface blade sections, offset from the centroids, leading, or trailing edge by DELX and DELY in the $x$ and $y$ directions respectively. |
| S, BS | If ISECN $=1$ or $3, S$ and $B S$ are used to specify the location of the inflection point (as a fraction of the meridionally-projected chord length) and the change in camber angle from the leading edge to the inflection point. If the absolute value of the angle at the inflection point is larger than the absolute value of $B 1, B S$ should have the same sign as Bl; otherwise, Bl and BS should be of opposite sign. |

## 2. INPUT DATA FORMAT

Data input is by punched card, and three formats are used. The first card only is alphanumeric, using the first 72 columns on the card. This is followed by one card of integers. Integers are each placed in a field of three locations, starting in Column 1 of the card. No decimal point may be used, and the number must occupy the right-most locations of the allocated field. Real numbers are each placed in fields of 12 locations, starting in Column $I$ of the card. Decimal. points should be included to ensure correct interpretation of the data, and the numbers may be placed anywhere in the allocated fields.

In the following chart, one line corresponds to one card.
TITLE
NLINES NSTNS NZ NSPEC NPOINT NBLADE ISTAK IPUNCH ISECN IFCORD IFPLOT IPRINT ZINNER ZOUTER SCALE STACKX PLTSZE

## KPTS IFANGS

XSmA RSTA
R
ZR BI B2 PP QQ RLE
TC TE Z CORD DELX DELY
repeated NSPEC times
$S B S-$ if $I S E C N=1$ or 3 only
Listings of sample input data decks are included under "Examples of Use of Computer Program."

## 3. OUTPUT DATA

Printed output from the program may be considered to consist of four sections: a printout of the input data, details of the blade sections on each streamsurface, a listing of quantities required for aerodynamic analysis, and details of the manufacturing sections determined on the constant-z planes. These are briefly described below. In the explanation which follows, parenthetical statements are understood to refer to the particular case of the double-circular-arc blade (ISECN = 2).

The input data printout includes all quantities read in, and is self-explanatory.

Details of the streamsurface blade sections are printed if IPRINT $=0$ or 1 . Listed first are the parameters defining the blade section. These are interpolated at the streamsurface from the tables read in. Then follow details of the blade section in "normalized" form. The blade section geometry is given for the section specified, except that the meridional project of the chord is unity. For this section of the output, the coordinate origin is the blade leading edge. The following quantities are given: blade chord; stagger angle; camber angle; section area; location of centroid of the section; second moments of area of the section about the centroid; orientation of the principal axes; and the principal second moments of area of the section about the centroid. Then are listed the coordinates of the camber line, the camber line angle, the section thickness, and the coordinates of the blade surfaces. NPOINT values are given.

A lineprinter plot of the normalized section follows. The scales for the plot are arranged so that the section just fills the page, so that the scales will generally differ from one plot to another. "Dimensional" details of the blade section are given next. The normalized data given previously is scaled to give a blade section as defined by IFCORD and CORD. For this section of the output, the coordinates are with respect to the blade stacking axis. The following quantities are given: blade chord; radius and location of center of leading (and trailing) edge(s); section area; the second moments of area of the section about the centroid; and the principal second moments of area of the section about the centroid. The coordinates of NPOINT points on the blade surfaces are then listed, followed by the coordinates of 31 points distributed at (roughly) six degree intervals around the leading (and trailing) edges. Finally, the coordinates of the blade surfaces and points around the leading (and trailing) edge(s) is (are) shown in Cartesian form.

The quantities required for aerodynamic analysis are printed at all computing stations specified by the IFANGS parameter. The radius, section angle, blade lean angle, blade blockage, and relacive angular location of the camber line are printed at each streamsurface intersection with the particular computing station.

Details of the manufacturing sections are printed if IPRINT $=0$ or 2. At each value of $z$ specified by ZINNER, ZOUTER and NZ, section properties and coordinates are given. The origin for the coordinates is the blade stacking axis. The following quantities are given: section area; the location of the centroid of the section; the second moments of area of the section about the centroid; the principal second moments of area of the section about the centroid; the orientation of the principal axes; and the section torsional constant. Then the coordinates of NPOINT points on the blade section surfaces are listed, followed by 31 points around the leading (and trailing) edge(s).

Precision plots are produced if IFPLOT $=1,2,3$ or 4 as described under the definition of IFPLOT given previously.

If IPUNCH $=1$, the program punches the quantities required for aerodynamic analysis, together with identifying indices denoting station number and streamsurface number, on cards in the following format: 5 fields each of 12 locations for the quantities themselves, followed by 2 fields each of 3 locations for the indices.

SECTION VIII

## EXAMPLES OF USE OF THE PROGRAM

## 1. INTRODUCTION

This section shows the use of the program to generate four blades: one based upon the polynomial camber line; another upon the exponential camber line; a double-circular-arc biade; and one based upon the multiple-circular-arc camber line. The same basic aerodynamic design is assumed for the first two and last cases, although all blades could not satisfy the same aerodynamic requirements. Figure 18 is a meridional section through the compressor annulus, showing eight streamsurfaces (including the hub and casing) defining the flow, and the locations of six planes on which manufacturing sections are to be determined. Usually, more streamlines would be used to define the flow and thus the blade, by means of the streamsurface blade sections. Also, it would normally be advantageous to determine more than six manufacturing sections. However, these numbers were selected for the sample runs of the program as they exemplify the program adequately and lend clarity to the resulting figures. Jncluded on Figure 18 are the approximate locations of the blade leading and trailing edges for the polynomial camber line case. The exponential and multiple-circular-arc camber line cases would appear slightly different for reasons discissed below.

## 2. POLYNOMIAL CAMBER LINE

a. Input Data

The input data deck used for this example is listed below. Some points of interest are noted in the order in which they occur in the input.

As mentioned previously, eight streamsurface blade sections are used to define the blade. The streamsurface radii are specified at eight locations, the first and last of which are outside, and well clear of, the rotor. This ensures that the boundary conditions imposed on the spline-curve (zero curvature at the end points) have little influence on the shape of the curve representing the streansurface in the region of interest, that is, within the blade. The parameters defining the streamsurface blade sections are given at eight locations; that is, directly at each streamsurface. Thus the interpolation of the parameters at each streamsurface is reduced in this cace to merely reading the appropriate row of data from the table. Fifty points are specified to define each blade surface. This number serves the purposes of this example well, but it would be advantageous to use more points where the precisionplot output is to be directly incorporated in the manufacturing
procedure. Specification of the meridionally-profected chord (rather than actual chord) is iefected. All optional cections of the output are to be printed. Superimposed plote of the otreamsurface and manufacturine sections are to be produced at a scale of five times full size.

Blade inlet and outlet angles would normally be determined from an aerodynamic analysis of the flow throuph the blade row. Desirable incidence ancles for this profile type are probably near to one half the leading edpe vedpe anfle. Considerations of static pressure (or velocity) distributione within the blade channel, and chokine flow limits, will influence the selection of incidence angles, depending upon the selected acrodynamic analysis method. Deviation angles for the rofile have not been empirically determined as yet, and therefore the best means of estimating deviation angles is probably to use Carter's Rule, or a similar formulation. Some consideration of the camber line shape (point of maximum camber, for examile) should be made. The parameters PP and QQ are set to 0.0 and 0.5 on all streamsurfaces, respectively. It is anticipated that the zero curvature leading edge configuration ( $P P=0.0$ ) will be frequently specified for this profile type. The specification of a second derivative at the tralling edge that is less than its maximum value $(Q Q=0.5)$ is made to prevent the curvature of the camber line from rising to a maximum at the trailing edre. It is believed that this could cause excessive deviation ancies. The blade thickness distribution of a rotor blade is determined by aerodynamic and mechanical factors. The leading edge radius and trailing edge half-thickness are set to approximately .005 inch, which is probably a practical minimum. Maximum thickness/chord ratios specified vary from $1.8 \%$ at the hub to $3.1 \%$ at the casing. These figures are typical of those aerodynamically acceptable for a high Mach number rotor blade, and yield a blade with a reasonable distribution of crosssectional area. The maximum thickness is placed at 0.6 of the camber line from the leading, edge, which helps to maintain a small leadine edpe wedre angle.

FIGJRE EXAYRLE HLADE OESITN－POLYNDMIAL SAYGER LINE

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9.12

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$\begin{array}{ll}1.3307 & 5.5 \\ 1.0204 & 5.75\end{array}$
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7.0669
7.3844
7.0994
8.6459
8.3065
8.5023
9.4

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## b. Output Data

The input data specified all optional output data, and a selection for each section of the output is shown below. Some points of interest are as follows.

Shown first is the printout of the input data; all input items are listed. This is followed by details of the first streamsurface blade section. The parameters defining the blade section are followed by details of, and a line-printer plot of, the normalized blade section. Then appears a specification of the section scaled to the desired dimensions. First there are the fifty points specified for each blade surface. These are printed two per line, and if an odd number of points is specified, coordinates of the last point are not printed. Next follow the coordinates of the 31 points describing the leading edge radius. The final data for the streamsurface section are the equivalent Cartesian coordinates for the points just mentioned. It will be seen that the first and last points on the leading edge radius coincide with the first points on the pressure and suction surfaces.

The same format is repeated for each of the remaining sireamsurface blade sections, and a printout of the volume enclosed by the blade, but these results have not been reproduced here. The calculations for aerodynamic analysis are printed next for those computing stations for which they were requested by the IFANGS parameter, presented in order of increasing radial streamsurface location.

Details of the six manufacturing sections defining the blade follow. Reproduced below is the output relating to the first (innermost) section. Properties of the section are followed by the coordinates of 50 points on each blade surface, and 31 points around the leading edge. It will be noted that the section centroid is not calculated to be exactly on the stacking axis (the origin for the coordinates) although the offsets DELX and DELY were all zero. This is a reflection of the fact that the streamsurface sections is not planar. If it were desired to have the centroids of the manufacturing sections lie precisely on the stacking axis, the program would be rerun with DELX and DELY offsets specified so that they counteract the mislocation of the centroids previously determined. This iterative procedure might require more than one loop, depending upon the accuracy desired. Use of the centroid offsets alo permits the final blade to be leaned for stress redistribution or other purposes.

The input data specified superimposed precision plots of both the streamsurface sections and manufacturing sections. These plots are reproduced (at reduced size) as Figures 19 and

20, respectively. It is of interest to refer also to Figure 18. The innermost manufacturing plane is well below the lowest point on the hub streamsurface section in the plane of the stacking axis, that is, the plane of Figure 18. The Cartesian coordinates of the first streamsurface blade section show that the lowest point on the section (the 20 th point on the leading edge) is at $z=6.66478$. The radius of the streamsurface at this point is 6.75 (approximately), and the innermost manufacturing plane is at $z=6.5$. Thus, at the leading edge the extrapolation required to define the manufacturing section is somewhat smaller than might at first appear. At the trailing edge, extrapolation is required to define the first two manufacturing sections. The streamsurface radius at the casing and z-coordinate for the outermost manufacturing plane both equal 9.0 at the blade inlet. However, the z-coordinate of the leading edge of the blade is 8.8357 , so that the outermost manufacturing section too is defined completely by extrapolation. Of course, portions of the blade that are defined by extrapolation would not appear on a final blade, but would facilitate manufacture.


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| 7.4420 | 60.1380 |  | 8.6890 |  | －0¢09 |  | －． 1040 |
| 7．725c | 61.8545 |  | 9.88 ¢6 |  | ． 0858 |  | －． 1101 |
| E．01ES | E3．5＜41 |  | 9．9633 |  | －0805 |  | －． 11167 |
| 8.3160 | 64.9430 |  | 8.3483 |  | ． 0737 |  | －． 1226 |
| $\text { £. © } 23 \mathrm{z}$ | $66.0450$ |  | $7.9518$ |  | －0666 |  | －． .1276 |
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| $6.9872$ | 51.4198 |  | 3.3368 |  | ． 136 |  | －． 0208 |
| 7．233E | 53.4525 |  | 3．6562 |  | ．1332 |  | － 0239 |
| 7.484 C | 55.5857 |  | 3．9569 |  | ． 1303 |  | －0352 |
| 7.7421 | 27.7541 |  | 4.3673 |  | －E2E2 |  | －． 3776 |
| 8．00E3 | 59．e658 |  | 4.5293 |  | －1 cee |  | －．C3J2 |
| 8.2777 | E1．6C34 |  | 4.3872 |  | －1．331 |  | －． 9328 |
| $8.555 t$ | $63.1635$ |  | $4.8712$ |  | $.11 c ̧$ |  | －． 0354 |
| $\text { g. } 8417$ | 54.4945 |  | $6.2324$ |  | $\text { - } 1155$ |  | －．C？ 26 |
|  | STATICh | 5 | hLAgER | CF | 6ACIT $=$ | ${ }^{*}$ |  |
| RADIU | section angle |  | $\begin{array}{r} \text { LEAN } \\ \text { ANGLE } \end{array}$ |  | ロと先き 9L JCKRUE |  | theta |
| 7．6786 | 40.5336 |  | －2．35ct |  | －1：18 |  | － 44175 |
| 7.6951 | 43．4E14 |  | －6．703\％ |  | －11＊ |  | －8418 |
| 7．5164 | 46.3585 |  | －3．1874 |  | ＋1154 |  | － 1433 |
| 7.7436 | 49.0142 |  | －3．4\％35 |  | －1731 |  |  |
| 7．9776 | 51.9301 |  | －3．1．17 |  | －1111 |  | － 14.6 A |
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a. Input Data

As mentioned previously, the overall design incorporated in the polynomial camber line example was also assumed for the exponential camber line example. The discussion of the input data therefore highlights the changes made to the polynomial camber line example data in order to create the exponential camber line example.

The exponential camber line is specified by setting ISECN equal to 1 . For this example, chords rather than meridionally-projected chords are specified. The chord lengths used are taken from the results of the polynomial camber line example, and because of the different camber line configurations, the resulting meridionally-projected chords are somewhat smaller for the exponential camber line case. The parameters PP and $Q Q$ are set equal to 0.5 for all sections. These values will probably be satisfactory for most applications of the blade profile. The only remaining change to the polynomial camber line input data is to specify the distributions of the inflection point and change in camber angle from blade inlet to the inflection point. The distributions used for the example are quite arbitrary, but representative of what might be indicated by a typical aerodynamic analysis. The inflection point is located at the leading edge on the innermost streamsurface section, and moved rearward .05 per streamsurface, to . 35 (of the meridionally-projected chord) at the casing. (As a value of $s$ of zero is not acceptable, the value used for the innermost section is .01.) For the three innermost sections, the inflection angle is set equal to the inlet angle. Thus, these sections have straight leading edge portions (of length from zero to. I of the meridionally-projected chord), followed by portions of continuously-positive camber. The remaining sections have inflection angles that are greater than the section inlet angles, the difference in angles rising from zero at the third section (as described above) to seven degrees at the casing. Thus, these sections have reverse and positive camber, the reverse camber increasing with radius, as the inflection point also moves rearward.

The input data deck is listed below.
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| $\begin{aligned} & 6.0 \\ & .03543 \\ & 0.25 \end{aligned}$ | $\begin{aligned} & 65.623 \\ & .0 .155 \\ & 2.6 i \end{aligned}$ | $\begin{aligned} & 28.327 \\ & ن .6 \end{aligned}$ | $\begin{aligned} & 0.5 \\ & 3.22147 \end{aligned}$ | 0.5 | -00155 |
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| $7 \cdot u$ | 66.471 | 34.37 |  | 0.5 |  |
| $\begin{aligned} & 17 \\ & \text { us us } 3 \end{aligned}$ | .9 .15153 +.72 | J. 6 | $3.29428$ | $0 \cdot 5$ | -US153 |
| 8.0 | 67.1 | 42.52 | 0.5 | 0.5 |  |
| $\cdot 03129$ J.35 | $\begin{aligned} & 0.3015 u \\ & 7,3 \end{aligned}$ | U. 6 | 3.380331 | 0.5 | . 00150 |

b. Output Data

The input calls for all optional printed output. As this is identical in format to that shown for the polynomial camber line example, none has been reproduced here.

Figures 21 and 22 show (at reduced size) the superimposed streamsurface section and manufacturing section plots. The comments regarding extrapolation made in connection with the polynomial camber line example agair apply. A comparison of some intermediate sections from both plots shows well the value of an accurate method of determining manufacturing coordinate data from streamsurface design data. For blades where the streamsurface section form is critical throughout the section, any approximate method based upon blade edge geometry only is unlikely to be satisfactory.
4. DOUBLE-CIRCULARR-ARC BLADE
a. Input Da.ta

This example illustrates several program features not illustrated by the previous examples. First, this example illustrates curvilinear computing stations. Secondly, the blade is described in terms of negative inlet angle, illustrating that the program accommodates blades specified in either sense. This allows stators to be specified in the opposite sense to rotors, if desired. Finally, the blade is stacked at its trailing edge, rather than at, or offset slightly from, the centroid.

The double-circular-arc blade is specified by setting ISECN equal to 2. The particular example illustrates a novel design concept in that the leading edge is swept from hub and tip toward mid-passage, a concept which is described in further detail in Reference 4.

The input data is listed below.

| FIGURE |  | EXAMPLE | CASE－double | CIRCULAR ARC blade |
| :---: | :---: | :---: | :---: | :---: |
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| 2．7879」 | 7.62720 u |  |  |  |
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| 3．1152us | 7.8728 ua |  |  |  |
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| 3.1746 uc | $8.28620{ }^{0}$ |  |  |  |
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| 7.627230 |  |  |  |  |
| 7.7472 us |  |  |  |  |
| 7．87く8い |  |  |  |  |
| 8．6．45ui |  |  |  |  |
| 8.142000 |  |  |  |  |
| 8.286200 |  |  |  |  |
| 8.4392 J |  |  |  |  |
| 8.6114 Ji |  |  |  |  |
| 81 |  |  |  |  |
| 3.340925 | 7.732600 |  |  |  |
| 3.48116 | 7.83348 l |  |  |  |
| 3.5864 Ju | 7．940981 |  |  |  |
| 3.65090 | 8.054948 |  |  |  |
| 3.668 .75 | 8.174438 |  |  |  |
| 3．63i95． | 8.298995 |  |  |  |
| 3.530450 | 8.428176 |  |  |  |
| 3．3428．j． | 8.5638 si |  |  |  |
| 7.732 buu |  |  |  |  |
| 7.83348 c |  |  |  |  |
| 7.944981 |  |  |  |  |
| 8.65431 .8 |  |  |  |  |
| 8.174438 |  |  |  |  |
| 8.298995 |  |  |  |  |
| 8.428176 |  |  |  |  |
| 8.56380 |  |  |  |  |
| 81 |  |  |  |  |
| 3.89395 | 7.7467 .30 |  |  |  |
| 3.9874 J | 7.847329 |  |  |  |
| 4.05750 | 7.955332 |  |  |  |
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| 4．112．5L | 8.189528 |  |  |  |
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| 4．6203」 | 8.439458 |  |  |  |
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\hline
\end{tabular}
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b. Output Data

The printed output for this example is similar in most respects to that shown for the polynomial camber line. The section entitled "Dimensional Results" is shown for streamsurface section 1 to illustrate that the section coordinates are presented relative to the stack axis which, in this case, has been specified at the blade trailing edge. Two other minor differences in comparison to the polynomial camber line output are related to the trailing edge: The printout contains the coordinate location of the center of the trailing edge as well as the 31 points distributed around the trailing edge. The latter feature also appears with the printout of the manufacturing sections (not reproduced here).

Figures 23 and 24 show (at reduced size) streamsurface section and manufacturing section plots, respectively. The comments regarding extrapolation made in connection with the polynomial camber line example again apply.
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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23 | 7．5c17EEtoc | －2．2074cE＋00 | 7．45331E－01 | 7．6E193E4C0 | －8．6e707E－04 | －3．51222E－03 |
| 24 | 7.55187 Etcc | －ごこCEEEE＋00 | 7.453 こ1E－01 | $7.62194 \mathrm{t}+00$ | －1．19159E－03 | －2．93E10E－03 |
| 25 | $7.5 ¢ 1 \leq C E t C C$ | －z．zctueeto | $7.45254 \mathrm{E}-01$ | 7．EE196E＋CO | －1．5E7E4E－03 | －4．31522E－03 |
| 2E | 7． 5 SE11E＋C | －2． $20 \leq 5 ¢ \mathrm{E}+0 \mathrm{C}$ | 7．451？1E－01 | 7． 6 E1G7E＋00 | －1．5E233E－03 | －4．E5EG1E－03 |
| 27 | 7．5ci24Etct |  | $7.44 \mathrm{CEEE01}$ | 7．6E19cE＋00 | －2．40131E－03 | －4．95333E－03 |
| 28 | 7．5cz37Etcc | －c．a 0 CEEAC | 7.447 CEE－01 | 7．EECOCEFCG | －2．8EG42E－03 | －5． $20106 \mathrm{E}-03$ |
| 29 | 7．ESE4SEtCC | －z．coutietoc | 7．4444E－01 | 7． 6 E20とE＋00 | －3．3E1こ7E－03 | －E．39725E－03 |
| 30 | 7．5EE6EEACC | －こ．こC4てくE＋OC | 7．441ここE－01 | 7．EE204E＋00 | －3．87120E－93 | －5．53963E－03 |
| 31 | 7．5¢E6¢Efoc | －2．20392E＋0C | 7．439CEE－01 | 7．EECOSE＋CC | －4．20152E－03 | －5．61283E－03 |

## a. Input Data

The same overall design incorporated in the first two examples was also assumed for the multiple-circular-arc camber line example. The discussion of the input data will be limited to changes made to the exponential camber line example to create the multiple-circular-arc camber line example.

The multiple-circular-arc camber line is specified by setting ISECN equal to 3 . The only other changes made to the exponential camber line example are the distributions of inflection point and the change in camber angle from blade inlet to the inflection point. The inflection point has been arbitrarily set at 0.3 (of the meridionally-projected chord) on all streamsurfaces. The change in camber angle has been specified in such a way that the blade has positive camber in the leading segment at the hub, gradually decreasing to zero camber (straight line leading segment) on streamsurface 4, and producing sections with reverse camber on the four outermost streamsurfaces.

The input data deck is listed below.

|  | figure | mult | IPLE | CIECULAR | arc camber | INE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 88 | 8 E 8 | 85030 | 2 | 031 | 30 |  |
| E. 5 |  | 9.0 |  | 5.0 | 0.0 | 12.0 |
| 1 C |  |  |  |  |  |  |
| -1.4357 |  | 0.0 |  |  |  |  |
| E. 623 |  |  |  |  |  |  |
| 6.974 E |  |  |  |  |  |  |
| 7.3197 |  |  |  |  |  |  |
| 7.6645 |  |  |  |  |  |  |
| 8.0126 |  |  |  |  |  |  |
| 8.3681 |  |  |  |  |  |  |
| 8.7351 |  |  |  |  |  |  |
| 9.12 |  |  |  |  |  |  |
| 2 ¢ |  |  |  |  |  |  |
| -1.6357 |  | 0.0 |  |  |  |  |
| -1.02E4 |  | 6.75 |  |  |  |  |
| 6.75 |  |  |  |  |  |  |
| 7.068 C |  |  |  |  |  |  |
| 7.3844 |  |  |  |  |  |  |
| 7.6994 |  |  |  |  |  |  |
| 8.0159 |  |  |  |  |  |  |
| 8.3365 |  |  |  |  |  |  |
| 8.6825 |  |  |  |  |  |  |
| 9.0 |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |
| -. 5767 |  | 0.0 |  |  |  |  |
| E.8864 |  |  |  |  |  |  |
| 7.1634 |  |  |  |  |  |  |
| 7.4426 |  |  |  |  |  |  |
| 7.7255 |  |  |  |  |  |  |
| 8.016 S |  |  |  |  |  |  |
| 8.316 |  |  |  |  |  |  |
| 8.6232 |  |  |  |  |  |  |
| 8.938 c |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |
| -. 2087 |  | 0.0 |  |  |  |  |
| E.9872 |  |  |  |  |  |  |
| 7.2338 |  |  |  |  |  |  |
| 7.484 C |  |  |  |  |  |  |
| 7.7421 |  |  |  |  |  |  |

```
    8.0063
8.2777
8.555%
8.8417
    1
.20003
3.0786
7.295C
7.5164
7.7436
7.977 C
8.2201
    8.4732
    8.7345
    1 1
.6093
7.149E
7.3387
7.5325
7.7325
7.9403
8.158?
8.3885
8.6315
    C
2.0623
1.0265
7.207€
7.375c
7.54ce
7.7301
7.918!
8.1197
8.3377
8.584E
    1 C
1.427I 0.0
7.2715
7.4214
7.576&
```

7.7386
7.9054
8.0905
8.2651
8.4 E24
1.0
.04857
0.3
2.0
.0453
0.3
3.0
.04243
0.3
4.0
. 0397 C
0.3
5.0
.03731
0.3
E. 0
. 0350 3
0.3
7.0
.03317
0.3
8.0
. 03125
0.3

| -57. | -8.809 | 0.0 | 0.5 | . 00175 |
| :---: | :---: | :---: | :---: | :---: |
| . 00175 | $0 . E$ | 2.1109 |  |  |
| 5.471 |  |  |  |  |
| -58.954 | -12.614 | 0.0 | 0.5 | . 00171 |
| . 00171 | 0.6 | 2.0525 |  |  |
| 3.E19 |  |  |  |  |
| - E0.858 | -16.336 | 0.0 | 0.5 | . 00167 |
| . 00167 | 0.6 | 1.9940 |  |  |
| 1.80 |  |  |  |  |
| -ez.ere | -19.965 | 0.0 | 0.5 | . 00163 |
| -C0163 | 0.6 | 1.9356 |  |  |
| 0.0 |  |  |  |  |
| -64.348 | -23.758 | 0.0 | 0.5 | . 00159 |
| . 00159 | 0.6 | 1.8771 |  |  |
| -2.131 |  |  |  |  |
| -E5.623 | -28.327 | 0.0 | 0.5 | . 00155 |
| . 00155 | 0.6 | 1.8187 |  |  |
| -5.425 |  |  |  |  |
| -EE.471 | -34.37 | 0.0 | 0.5 | . 00153 |
| -1015 | 0.6 | 1.7603 |  |  |
| -10.62 |  |  |  |  |
| -67.1 | -42.52 | 0.0 | 0.5 | . 00150 |
| -CC15C | 0.6 | 1.7018 |  |  |
| -18.141 |  |  |  |  |

b. Output Data

The input data calls for all optional printed output. As this is identical in format to that shown for the polynomial camber line, none has been reproduced here.

Figures 25 and 26 show (at reduced size) the superimposed streamsurface section and manufacturing section plots, respectively. The comments regarding extrapolation made in connection with the polynomial camber line example are again applicable.

## SECTION IX

## COMPUTER PROGRAM DETAILS

## 1. IMPLEMENTATION OF THE COMPUTER PROGRAM

The program is written in FORTRAN IV and was develcped on an IBM 7094/7044 Direct Couple System (incorporating the version 13 IBSYS Operating System) and a CDC 6000 Series System (incorporating the version 3.3 SCOPE Operating System). When loaded into core, the program (and resident system) occupies about 32 K of storage, so that the program will probably not be usable without modification on a relatively small computer. Apart from this limitation, the program should be compatible with the majority of modern computing systems. The program consists of a main program, and Subroutines $B Q, C Q, D 1$, $E Q, F Q$ and $G Q$. The seven decks have been given the identifiers A, B, C, D, E, F and G, respectively. Listings of the decks are shown below, and the deck set-up required for the CDC 6000 Series System is also presented.

The program uses thiee numerical system units for its input and output routines. Input is drawn from the card unit via READ statements referring to Unit LOG 1 ; output is sent to the line printer by WRITE statements referring to Unit LOG 2; and punched output is produced via WRITE statements referenced to Unit LOG 3. Units LOG 1, LOG 2, and LOG 3 are set equal to 5, 6, and 7, respectively on cards A1140-60 in the FORTRAN programming. On the CDC 6000 Series System, the "PROGRAM" card must also establish the input and output linkages. On other computing systems, the "PROGRAM" card may not be required, and the input-output files may be established via control cards.

The prograri as presented herein utilizes an on-line precision plotting capability available at the program development site. Calls to four subroutines not included in the deck are included in the program. These calls are executed only if precision plots are specified in the input data, and the entry pcints expressed are part of standard CALCOMP software normally supplied to users of CALCOMP precision plotting equipment. The various call statements used in the program are explained below, to facilitate modification should the need arise.

CALL PLOT (XPLOT, YPLOT, N)
The majority of plotting is done using this form of call. The parameters XPLOT and YPLOT are the "x" and "y" coordinates (in inches) on the paper to which the pen is being directed. The parameter $N$ indicates pen up or down, $|N|=3$ or $|N|=2$ respectively, and will cause XPLOT or YPLOT to be assigned
as the origin for further coordinates if $N$ is negative.
CALL SYMBOL (X, Y, H, TEXT, THETA, N)
This call is used to title the plots. The parameters $X$ and $Y$ are the coordinates (in inches) of the lower left hand corner of the first character, $H$ is the character height (in inches), TEXT is the character to be printed, THETA is the angle of the lettering with respect to the " $x$ " axis and $N$ is the total number of characters to be printed.

CALL NUMBER (X, Y, H, F, THETA, N)
This call causes the printing of the number $F$. The parameters $X, Y, H$, and THETA are used as for CALL SYMBOL. The parameter $N$ indicates the number of digits following the decimal point if positive, or truncation to an integer if equal to -l.

CALL PLOTE
This call terminates the tape.
In the event the program is used on a computing system which does not include CALCOMP software, and the operating system will not execute a program with unsatisfied external references, dummy entry points may be supplied by adding to the deck a subroutine such as the following:

SUBROUTINE PLOT
$A=A$
ENTRY SYMBOL
ENTRY NUMBER
ENTRY PLOTE
RETURN
END
2. DECK SETUP FOR CDC 6000 SERIES SYSTEM

The deck setup required to run the program on a CDC 6000 Series System incorporating the SCOPE 3.3 Operating System is shcom below. Production runs of the program would usually employ relocatable binary forms of the routines produced from the source decks to avoid having to compile the FORTRAN for each run and the waste of associated computer resources.

FTN.
LGO.
7/8/9 End of Record
SOURCE DECK A
SOURCE DECK B
SOURCE DECK C
SOURCE DECK D
SOURCE DECK E
SOURCE DECK F
SOURCE DECK G
7/8/9 End of Record
DATA DECK
$6 / 7 / 8 / 9$ End of Job
3. FORTRAN PROGRAM LISTING

A listing of the FORTRAN program appears on the following pages with each subroutine started on a new page.



PROGRAM BLADE (INPUT, OUTPUT, PUNCH, TAPES=INPUT, TAPEG=OUTPUT, TAPET=

+14
4
3
3

```
n \(\begin{array}{ll}0 & n \\ 11 & 11 \\ N & 10 \\ 0 & 0 \\ 0 & 0 \\ -1 & 1\end{array}\)
```


## $P I=3.1415926536$

 $A$$A$

| 0 |
| :--- |
| - |
| - |


 SION)

FORMAT (2F12.
FORMAT (2F12.0)
FORMAT (F12.U)


IF (NLINES.GT. IOUM) IOUM=NLINES
IF (iNCT.LE.54-NLINES)GO TO 50
WRITE(LOG2,4J)
FORMAT(IHi)
LNCT $=1$
LNCT $=$ LNCT + IUUM+6

-
$+$
FORMAT $(2 X, 1,1$ jX, 17 HCOMPUTING STATION, I3, $5 X, 28$ HNUMBER OF DESCRIBING



IF (K.LE.KPTS(I).AND.K.LE.NLINES) WRITE(LOG2,56) XSTA(K,I), RSTA(K,I
 IF (K.GT.K.TS(I).AND.K.LE.NLINES) WRITE(LOG2,58) K,R(I,K) CONTINUE M
ज




$$
\text { FORMAT ( } 3 \text { X, F8. } 4,2 X, \text { F8. } 4 \text { ) }
$$







IF(IFPLOT.EQ. D.OR.IFPLOT.EQ.2.OR.IFPLOT.EQ.4)GO TO 260






$33 \mathrm{LNCT}=1$


IF(I.GT•31) GO TO 43







TC(J)=A BS(ATAN(W1/SQRT(1.-W1**2))-ATAN(W2/SQRT(1。-W2**2)))/(2.*PI)
*FLOAT (NSLADE)
(F (IFANGS (I), EQ.

CONTINUE
$C A L L \cdot C Q(Z C A M B(1, I), Y C A M B(1, I), N L I N E S, Z C A M S(1, I)$, XDUM,RLE,NLINES,1)
IF(LNET + IJUM +NLINES.LEE.59) GO TO 463 WRITE(LOG2,4i)
LNCT=2

$$
\begin{aligned}
& \text { I GHRAOIUS, } 5 X, 7 H S E C T I O N, 6 X, ~ 4 H L E A N, 9 X, 5 H S L A O E, 7 X, 5 H T H E T A, /, 48 X, \\
& 2 \text { 5HANGLE, } 5 X, 5 \text { HANGLE, } 7 X, 8 \text { HBLOCKAGE, } /, 2 X)
\end{aligned}
$$

$$
\begin{aligned}
& \text { DO } 4 \dot{O} 5=1, N L I N E S \\
& E P S=(T H E T A(J, I)-A \\
& A L P H B=A L P H A(J, I)
\end{aligned}
$$

$$
E P S=(T H E T A(J, I)-A T A N(R L E(J))) * C 1
$$

ALP = (ATAN $((\operatorname{TANPHI}(I, J) * T A N(E P S / C 1)+A L P H B * S Q R T(1 .+T A N P H I(I, J) * * 2)) /$ 1 (1.-TANPHI (I, J)*ZR(J))))*C1
IF (IPUNG CONTINUE
466 FORMAT(3」X,5F12.4)
467 FORMAT(5F12.7,2I3)











 AXIS) ONSTANT, $20 \mathrm{X}, 1 \mathrm{H}=, 1 \mathrm{PE} 12.4, \mathrm{f}, 2 \mathrm{X}$ )



547 IF (ISEGN.EQ.2) WRITE(LOG2, 62U) I,XSEMI(J,I),YSEMI(J,I),XSEMJ(J,I), 1 YSE゙MJ(J,I)
IF (IS末CN,EQ.2) GO TO OSG IF (IS:CN,EQ.Z) GO TO OS(I
WRITE(LOG2, GOJ) I,XSEMI $J, I), Y S E M I(J, I)$ COITINUE

$$
\begin{aligned}
& \text { IF ISECN.NE. } 2 \\
& \text { UO } \triangle 85 \quad I=2,3 U
\end{aligned}
$$

$$
\text { YPLOT }=Y \text { ScMJ }(J, I) * S C A L E
$$

$$
\begin{aligned}
& X P_{L O T}=X S E M J(J, I) * S C A L E \\
& Y P L O T=Y S E M J(J, I) * S C A L E
\end{aligned}
$$

$$
\begin{aligned}
& \text { CALL PLOT (XPLOT,YPLOT,2) } \\
& \text { DO OGU II=1,NPOINT } \\
& I=N P O I N T+1-I I \\
& \text { XPLOT } X X P(J, I) * S C A L E \\
& Y P L O T=Y P(J, I) * S C A L E \\
& \text { BAI PI OT XPIOT.YPLOT.2) }
\end{aligned}
$$

ALL PLOT (XPLOT, YPLOT, 2)

$$
\begin{aligned}
& \text { DO } 7 \cup I=2, J J \\
& X P L O T=X S E M I(J, I) * S C A L E \\
& Y P L O T=Y S E M I(J, I) * S C A L E \\
& C A L L Y L O T(X P L O T, Y P L O T, 2) \\
& X P L O T=X S(J, 1) * S C A L E \\
& Y P L O T=Y S(J, 1) * S C A L E
\end{aligned}
$$

[^3]\[

$$
\begin{aligned}
& Y P L O T=Y S(J, 1) * S C A L E \\
& \text { CALL PLOY }(X P L O T, Y P L O T, 2)
\end{aligned}
$$
\]


 ब
CALL NUMBER(28.U,L.U,U,35,STAGER,N.C,3) SNSTG=SIN(STAGÉR/CI)
COSSTG=COS(STAGER/C1)
PRLOT $=4 \cdot 75 *$ SINSTG/COSSTG F(ABS (XPLOT).LE.22.J)GO TO 72u $X P L O T=22.0$
CALL PLOT(22.U,S゙・2S,-3)
SINSTG=SIN(STAGERR1CI)
COSSTG=COS(STAGER/C1)
YPLOT $=4.75$
$X P L O T=22.0$
$Y P L O T=-22 . U / S I N S T G * C O S S T G$
CALL PLOT $X P L O T, Y P L O T, 3)$
XPLOT $=-X P L O T$
YPLOT =-YPLOT
CALL PLOT(XPLOT, YPLOT, 2 )
CALL PLOT $X$ XPLOT, YPLOT, 3 )
XPLOT $=-X P L O T$
YPLOT $=-Y P L O T$
CALL PLOT
YALL PLOT(XPLOT,YPLOT,2)
XPLOT $=22 . \ddot{0}$
$Y D L O T=-22 . j * S I N S T G / C O S S T G$
YALL PLOT(XPLOT,YPLOT,2)
XPLOT $=22 . \ddot{0}$
$Y D L O T=-22 . j * S I N S T G / C O S S T G$
IF (ABS (YPLOT).LE.4.75)GO TO 730
YPLOT $=-4.75$
XPLOT $=4.75 /$ SINSTG*COSSTG
IF (ABS(YPLOT) .LE.4.75)GO TO 730
YPLOT $=-4.75$
XPLOT $=4.75 /$ SINSTG*COSSTG
ALL PLOT (XPLOT, YPLOT, 3)
$Y P L O T=-Y P L O T$
$X P L O T=S C A L E *(X S(J, 1) * \operatorname{COSSTG}+Y S(J, 1) * S I N S T G)$
$Y P L O T=S C A L E *(Y S(J, 1) * C O S S T G-X S(J, 1) * S I N S T G)$
$X P L O T=S C A L E *(X S(J, I) * C O S S T G+Y S(J, I) * S I N S T G)$
YPLOT =SCALE* (YS $(J, I) * \operatorname{COSSTG-XS}(J, I) * S I N S T G)$
ALL PLOT (XPLOT, YPLOT,2)
XPLOT $=$ SCALE* (XSEMJ (J,I)*COSSTG+YSEMJ(J,I)*SINSTG) YPLOT=SCALE* (YSEMJ (J,I)*COSSTG-XSEMJ(J,I)*SINSTG) ALL PLOT (XPLOT, YPLOT,2)
XPLOT $=$ SCALE* $(X P(J, I) * C O S S T G+Y P(J, I) * S I N S T G)$
$Y P$ LOT SCALE* (YP $(J, I) * C O S S T G-X P(J, I) * S I N S T G)$
$75 \cup$
CALL PLOT $X P L O T, Y P L O T, 2)$
DO $76 . J I=2, J U$

##   <br> 











${ }^{\circ}$
110 A $=G 3 * * 2 *(\operatorname{TAN}(B E T A 1 / C 1)-T A N(S B / C 1)) /(1.0-(A 1 * S Q+1.0) * E X P(-A 1 * S Q))$
$\begin{array}{rr}3 & \omega \\ \rightarrow & -1 \\ H & -1\end{array}$












00 17」 JJ=1, 11
GO TO ISEC1, (14J, 15u)
PHI (JJ) =SQRT(1. $1+(0 A / 12 \cdot u *(X X-H) * * 3+X K 2 * 2 \cdot 1 * X X+R) * * 2)$
Nan

$$
\square \square
$$

## -G) $=\operatorname{SQRT}(1 \cdot \mathrm{U}+($ F5 (A1, 81,CC1) )**2)

## \section*{$$
017
$$ <br> <br> GO TO 17 j ( 10 (F5 (A2, $2, C(21) * * 2)$

 <br> PHI (JJ) $=\operatorname{SQRT}(1 .+(F 5(A 2, B 2, C C 21) * * 2)$}
$X X=X X-0 E L X$
$S(J)=S(J-1)+(P H I(1)+P H I(11)+4 \cdot 0 *(P H I(2)+P H I(4)+P H I(6)+P H I(8)+P H I(1$ iu) + + , (PHI - ${ }_{-1}$




IF

$$
r 2=1 / c 0
$$

$$
\begin{aligned}
& T 2=T / 2 \cdot \\
& T P R I \dot{I} 2=T \\
& C 2=2 \cdot{ }^{*} C 1 \\
& \triangle F O P M=(T
\end{aligned}
$$

$$
\begin{aligned}
& 2= \\
& { }^{2}=T 2
\end{aligned}
$$

## 2-YZË:RO

$$
\begin{aligned}
& C 2=2 \cdot * C 1 \\
& A F O R M=(T P R I M 2+R O I U S *(1,-\operatorname{COS}((B E T A 1-B E T A 2) / C 2))) / X M L C * 2 . \\
& P H I S=A C O S((1,-A F U R M * * 2) /(1,+A F O R M * * 2))
\end{aligned}
$$ $K S=Y Z E R O+X M L C / 2 \cdot / S I N(P H I S)$

> BFORM = (KDIUS* (1.-COS ( (BETA1-BETA2)/C2))-TPRIM2)/XMLC*2. PHIP $=A \operatorname{COS}((1,-B F O R M * * 2) /(1 \oplus+B F O R M * * 2))$

> PHI2=A3S((8ETA2-BETA2)/C1)
> $X M(1)=U . \hat{U}$
$I F(I S E C N . N E, 3)$ GO TO 186
> YMMTJ. U
> $X M M=0 . J$
> $2=S Q * F L O A T(N)$
> N
H
H
H
> IF (IL.LE.1) I2=N+1 OELX=SQ/FLOAT (I 2-1)

> IF (I3.NE.I2) I $3=1$ DELXX=(1.-SQ)/FLOAT $(N-I 3)$ F(I2.EU. $(N+1))$ DELX=DELXX $J=1, N$
> /S (N)

237 IF (ISECN.EQ.2) GO TO
IF (SN.GT,Z)GO TO $19{ }^{\circ}$
以及



## THICK2（J）$=(A T * S N * * 2+こ T) * S N+D T$

 3$N$
0
0
$\vdots$
0
0
0
0
$S N=S N-2$（ $\quad$（ $* S N+F T) * S N * * 2+H T$

| THICK2（J）$=(E T * S N+F T) * S N * * 2+H T$ |  |
| :---: | :---: |
| IF（ISECN．EQ．3）GO TO 233 |  |
| GO TO ISEC2，（21ú，220） |  |
| $Y M(J)=0 A 48 *(X M(J)-H) * * 4+X K 2 * X M(J) * * 2+8 * X M(J)+C$ |  |
|  |  |
| GO TO 24 J |  |
| $X D=X M(J)-S Q$ |  |
| IF（XD．GT．U．u）GO TO 230 |  |
| $Y M(J)=F 3(A 1,81, C C 1, E 1)$ |  |
| $Y P R I M E=F 5(A 1, B 1, C C 1)$ |  |
| GO TO 24 U |  |
| $Y M(J)=F 3(A 2, B 2, C O 2,02)$ |  |
| YPRIME＝F5（A2，82，CC2） |  |
| GO TO 240 |  |
| IF（（XM（J）－SQ）．GT•U．U̇•OR．XM（J）．EQ．O．O．AND．SQ．EQ．U．J）GO TO 235 |  |
| IF（EETA1．EQ．BETAS）GO TO 239 |  |
| 8TA1＝BETA1 |  |
| BTA2＝BETA3 |  |
| RUIUS＝RDIUS1 |  |
| $\mathrm{Y}_{1}=\mathrm{Y}_{1} 1$ |  |
| $\mathrm{X} 1=\mathrm{X} 11$ |  |
| GO TO 238 |  |
| IF（BETA2．EQ．BETA3）GO TO 239 |  |


| $\bigcirc$ | $\bigcirc$ | $\square$ | $\bigcirc$ | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\sigma$ | 3 | $\stackrel{-}{\square}$ | N | n |
| －1 | N | $N$ | $N$ | 4 |



5）$* \mathrm{PHI} 2$
－ROIUS
（2）





$262 D X(I K)=D X(I K-1)+A X$ $Y M M=J \cdot u$ 3
$\vdots$
2
2
2

- 5 ciJ I
$X A B=0 X(I K)$
F ISËCN.EQ
F(ISECN.EQ.E) GO TO 263

$F(I S E C N \cdot E Q \cdot 1)$ GO TO 264
$F(I S E C N \cdot E Q \cdot 2)$ GO TO 268
$F(I S E C N \cdot E Q \cdot 3)$ GO TO 266
(IK) $=\left(0 A 48 *(X A B-H) * * 4+X A B * * 2^{*} X K 2+3 * X A B+C\right) * F C S L M N$
$S I(I K, 1)=0 A / 12 * *(X A B-H) * * 3+X K 2^{*} 2 . * X A B+B$
O TO 270


 RUIUS=RDIUSI



## 

BTAI=BETA3
BTAC=BETAZ

 HNM MNMNN




##  









 in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in in






$Y \operatorname{SEMI}(I B L, J)=Y C+R L E * \operatorname{COS}(E P S / C 1)$
ETS=EPS-6.U
IF IPRINT.EO.2) GO TO 486
WRITE(LOG2,47U)J,XSEMI (IBL, J), YSEMI (IBL, J)
FORMAT $43 X, I 5,1$ PE17.5,1PE14.5)
PHISS=PHIS-ABS ( $(3 E T A 1-E E T A Z) / C 2)$
PHIPP=A SS ( (BETA1-BETA2) )/C2-PHIP

=1,31
$I F(J . N E \cdot 1) \quad G 0 \quad$ TO 485
$X S E M I(I B L, J)=X P(I B L, 1)$
$Y S E M I(I B L, J)=Y P(I B L, 1)$
$X S E M J(I B L, J)=X S(I B L, N)$
$Y S E M J(I B L, J)=Y S(I B L, N)$
$E P S=E P S-P H I P P * C 1$
$E P S 2=E P S 2-P H I S S * C 1$
$G O$ TO 489
$I F(J, N E .31)$ GO TO 486
$X S E M I(I B L, J)=X S(I B L, 1)$
$Y S E M I(I B L, J)=Y S(I B L, 1)$
$Y S E M J(I B L, J)=Y P(I B L, N)$
$X S E M J(I B L, J)=X P(I B L, N)$
$G O T O 489$
YSEMI $(I B L, J)=Y P(I B L, 1)$
$X S E M J(I S L, J)=X S(I B L, N)$
$Y S E M J(I B L, J)=Y S(I B L, N)$
$E P S=E P S-P H I P P * C 1$
$E P S 2=E P S 2-P H I S S * C 1$
$G O$ TO 489
$I F(J, N E .31)$ GO TO 486
$X S E M I(I B L, J)=X S(I B L, 1)$
$Y S E M I(I B L, J)=Y S(I B L, 1)$
$Y S E M J(I B L, J)=Y P(I B L, N)$
$X S E M J(I B L, J)=X P(I B L, N)$
$G O$ TO 489
DO $489 \mathrm{J=1,31}$
IF $(J, N E, 1)$ GO TO 485
$X S E M I(I B L, J)=X P(I B L, 1)$
GO TO 489 XSEMI (IBL,J) $=X C-R L E * S I N(E P S / C 1)$
$Y S E M I(I B L, J)=Y C+R L E * C O S(E P S / C 1)$
XSEMJ (IBL,J) $=X T C+R L E * C O S(E P S 2 / C 1)$
$Y S E M J(I B L, J)=Y T C+R L E * S I N(E P S 2 / C 1)$
EPS =EPS-DELEP
EPSZEPS2-DELEP
CONTINUE
IF (IPRINT,EQ.2) GO TO 499
WRITE(LOG2,491)
FORMAT $2 X, /, 39 X, 44 H P O I N T S$ DESCRIBING LEAOING AND TRAILING EDGES, $1,25 \mathrm{X}, 12 \mathrm{HLEADING}$ EOGE, $22 \mathrm{X}, 13 \mathrm{HTRAILING}$ EOGE, $/, 2 \mathrm{X}, 9 \mathrm{HPOINT}$ NO., 4 X , $28 \mathrm{X}, 1 \mathrm{HX}, 14 \mathrm{X}, 1 \mathrm{HY}, 12 \mathrm{X}, 8 \mathrm{X}, 1 \mathrm{HX}, 14 \mathrm{X}, 1 \mathrm{HY}, /, 2 \mathrm{X})$
$\begin{array}{ll}20 & 1 \\ & 0 \\ 5 & 0\end{array}$
in
$\infty$
.7

| 0 |
| :--- |
| 0 |
| $\mathbf{n}$ |


| 0 |
| :--- |
| $\infty$ |
| + |




WRITE(LOG2, 492) (J,XSEMI(IBL,J), $\operatorname{YSEMI}(I B L, J), X S E M J(I B L, J)$, 1 YSEMJ (IBL,J), J=1,31)
$\begin{array}{ll}492 & \text { FORMAT } 6 X, I 2,7 X, 1 P E 17.5,1 P E 14 \cdot 5,2 X, 1 P E 17.5,1 P E 14,5) \\ 499 & \text { SSURF=AXIALC } \\ \text { SSZ }\end{array}$















| 3 | 3 \% | 3 | 2 | '3 | In | 0 | $N$ | $\infty$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pm$ ¢ | 0 0 | N | $\infty$ | $\sigma$ | $\sigma$ | 0 | $\sigma$ | ¢ | $\sigma$ |




SUBROUTINE D1(XDATA, YDATA, NDATA,XIN,YOUT,NXY,NTYPE)
REAL M
OIMENSION M(15), A(15), B(15), D(15), XDATA(1), YDATA $(1), X I N(1), Y O U T(1)$ RLAL
OIMCNSION M(15), A(15), B(15), D(15), XDATA(1), YDATA $(1), X I N(1), Y O U T(1)$
$I F(N D A T A-1) I U, 1,3,30$ IF (NDATA-1) iu, 1:3,30
00 Zu $I=1, N X Y$

## YOUT (I) = YOATA(1)

IF (NDATA-2)5i,5u,40
IF (NTYPE) $18 \mathrm{G}, 18 \mathrm{O}, 5 \mathrm{u}$
$\xrightarrow{-111}$

$I F(X I N(I)-X D A T A(J)) 10 J, 12 u, 90$
$I F(X I N(I)-X D A T A(J+1)) 12 u, 120,100$
$J=J+1$
$I F(J-N D A T A) 80,11 \cup, 110$
$J=1$
$G O T O$ TO
YOJT $(I)=Y D A T A(J)+(Y O A T A(J+1)-Y D A Y A$
$I F(X I N(I)-X D A T A(J)) 10 J, 12 u, 90$
$I F(X I N(I)-X D A T A(J+1)) 12 u, 120,100$
$J=J+1$
$I F(J-N D A T A) 80,11 \cup, 110$
$J=1$
$G O T O$ TO
YOJT $(I)=Y D A T A(J)+(Y O A T A(J+1)-Y D A Y A$
1I)-XDATA(J))
3
$n$
0
0
0
0
130 YOUT $(I)=$ YOATA $(1)+($ YDATA $(2)-Y U A T A(1)) /(X O A T A(2)-X D A T A(1)) *(X I N(I)-X$ 10ATA(1))
YOUT(I) = YDATA (NDATA-1) + (YDATA (NDATA)-YDATA(NDATA-1))/(XDATA(NDATA)
1-XOATA(NDATA-1) * (XIN(I) -XDATA(NDATA-1)) $I F(I-N X Y) 16 u, 176,170$
$I=I+1$
GO TO Bu
RETURN
a (1) =ueu
$N=$ NOA TA -1
00 i9」 $I=2, N$
$A(I)=\{\operatorname{XDATA}(I+1)-X D A T A(I-1)) / 3 . E-(\operatorname{XDATA}(I)-X D A T A(I-1)) * B(I-1) /(6 . U$
1*A $(I-1))$
$B(I)=(X D A T A(I+1)-X D A T A(I)) / 6 \cdot \dot{U}$
$B(I)=(X O A T A(I+1)-X D A T A(I)) / 6 \cdot \cup$
$190 \mathrm{D}(\mathrm{I})=(\mathrm{YOATA}(I+i)-Y D A T A(I)) /(X D A T A(I+1)-X D A T A(I))-(Y D A T A(I)-Y D A T A(I$


 لیا لוֹ
RËAL LINE
 $X H A X=(X M I N+X M A X-X H+X L) / 2 \cdot U$
$X H=X H-X L+X M A X$


\footnotetext{
$X M A X=(Y M I N+Y M A X-Y H+Y L) / 2.0$




##  

隹


##  








SUBROUTINE FQ(ISTAK,PLTSZE,ITRIG,TITLE,IKDUM) SUBROUTINE FQRISTAK,PLTSZE,ITRIG,TITLE,IKOUN


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## 4. PROGRAM LOGIC

The analysis which has been described in this report is performed primarily in the main program and Subroutine $B Q$, while the other subroutine performs specific tasks supportive of the desired objectives. Subroutine $B Q$ concerns the details of defining and describing the blade on the streamsurface; the main program utilizes this information to stack the blade and obtain the manufacturing section description. Subroutine CQ is used to find slopes of various "spline-curves" at particular points. Dl is the curve-fitting routine, EQ produces a plot in the printed portion of the output, and $F Q$ is used to draw streamsurface and/or manufacturing section plot axes if IFPLOT = 1, 2 or 3. Subroutine $G Q$ is used to compute the center coordinate, the radius of curvature, and the arc length for the circulararc and multiple-circular-arc camber lines.

A description of the calculation procedure employed in the main program and in Subroutine $B Q$ is described below. The steps of this procedure are keyed to their location in the program through the associated deck serialization, which appears parenthetically.

1. The input data is read and printed. (A1190-A2160)
2. If precision plotting is specified, the plot is initialized and if IFPLOT $=1$ or 3 , the axes for the superimposed streamsurface plots are produced. (A2170-A2200)
3. A loop performed for each streamsurface section is commenced. This loop creates the blade on the streamsurface. (A22\%)
4. The axial locations of the intersections of a particular streamsurface with all computing stations are determined, the interval between the first and last points is subdivided into 99 intervals, and the axial coordinates of the 100 resulting locations obtained. (A2220-A2290)
5. The slope of the streamsurface at each of the 100 locations obtained in 4. above is derived. (A2300)
6. The slope of the computing stations at each of the streamsurface computing station intersection points is derived. (A2310)
7. Using Equation (74), the streamsurface length (in the meridional projection) is obtained over the 99 intervals of 4., providing an $x-m$ table for the streamsurface. (A2320-A2340)
8. The parameters defining the streamsurface blade section are Interpolated (or extrapolated) from the input tables. If NSPEC $=1$, they are taken to be radially uniform. If NSPEC $=2$,
linear interpolation is employed. If NSPEC >2, spline-curve interpolation is used. (A2350-A.2500)
9. The design of the streamsurface section is initiated. (A2510-A2540)
10. If IPRINT $=0$ or 1 , the parameters defining the streamsurface blade section are printed. (B1230-B1410)
11. If ISECN $=0$, the soefficients of the quartic camber line are computed using Equations (6), (7), (8), (9) and (10). The chord length corresponding to a meridional camber line chord of unity is determined. (B1450-B1510)
12. If ISECN $=1$, the coefficients for the two segments of the exponential camber line are computed using Equations (17), (18), (19), (20), (21), (22), (23) and (24). Equations (18) and (24) require iterative solution, and up to 50 attempts to satisfy the equations are permitted. The equations are taken to be satisfied when the parameter P or 0 , as appropriate, is given to within 0.0001 of its specified value. (Should the iterative procedure fail, a diagnostic is printed and the job is terminated. This will not normally occur.) The chord length corresponding to a meridional camber line chord of unity is determined. (B1530-B2050)
13. If ISECN $=3$, the constants of the equations for the two segments of the multiple-circular-arc camber line are determined from Equations (38), (36), (35), and (34) respectively, in Subroutine GQ. These constants are scaled and the arc center offset, if required, and the arc length of each segment determined cunulatively. The chord length corresponding to a meridional camber line chord of unity is determined. (B2070-B2300)

NOTE: Each segment is described by a proportion of the NPOINT camber line points based on its proportion of the meridionally projected chord length. If this number of points for the leading segment is 0 or $l$, this segment is abandoned, and one circular arc is produced for the entire camber line.
14. If ISECN $=2$, the constants of the equation for the circular-arc camber line and the camber line arc length are computed as for ISECN $=3$. The chord length corresponding to a meridional chord of unity, that corresponding to an overall blade chord of unity, and an associated scale factor (described in 15.) are determined for the double-circular-arc blade. (B2320-B2350)
15. For $\operatorname{ISECN}=0,1$ or 3 , the chord length corresponding to an overall blade chord of unity as well as a scaling factor which relates the ratio of dimensions for an overall meridional chord of unity to dimensions for a camber line meridional
chord of unity are determined. (B2370-B2380)
16. If $\operatorname{IFCORD}=1$, the specified chord is divided by the chord corresponding to a unity meridional chord, giving the desired meridional chord. The three blade thickness descriptors are scaled so that they apply to a blade having a camber line meridional chord of unity. ( $\mathrm{B} 2390-\mathrm{B} 2420$ )
17. The coefficients of the two thickness equations of the standard thickness distribution are computed using Equations (42), (43), (44), (47), (48) and (49). (B2470-B2520)
18. The length of the section camber line for ISECN $=0$ or 1 is determjened by numerical integration. The meridional chord (of unity) is divided into (NPOINT-1) uniform intervals, where NPOINT is the number of points specified in the input to define each blade surface. The integration for the curve length between each adjacent pair of points is obtained using Simpson's Rule, the interval being subdivided into 10 equal intervals. If the polynomial camber line is specified, the gradient of the camber line is given by Equation (4). If the exponential camber line is specified, the gradient of the camber line is given by Equation (15), different coefficients applying forward and rearward of the inflection point. These data enable a location on the camber line to be expressed as a fraction of camber line length. (B2540-B2730)
19. At each of the NPOINT points, equispaced on the meridional chord, the section half-thickness is computed from Equations (41), (46) or (67), and the coordinate and slope of the camber line are computed, from either Equations (5) and (4) (polynomial), (16) and (15) (exponential), or (25) and the derivative of (26) solved for ( $x-x_{0}$ ), choosing the appropriate branch of the square root involved, for the circular-arc and multiple-circulararc camber lines. (B2970-B3470)
20. The coordinates of NPOINT points on each blade surface are determined by combining the data derived in 19. The scaling factor derived in 14. or 15. is applied so that the blade coordinates correspond to an overall blade meridional chord of unity. (B3480-B3520)
21. The slope of the camber line is expressed in degrees. The camber line coordinates, lengths, and the section half-thickness are scaled by the factor derived in 14. or 15. (B3530-B3570)
22. The section area and the location of the centroid are determined using Equations (81), (82) and (83). (B3580-B3730)
23. The camber line is redefined in terms of 100 points to accure cufficient points for accurate linear interpolations in the determination of $\varepsilon$, needed for the eventual trans-
24. The scaled camber line is interpolated to yield its description over 99 equal axial intervals. (B4250-B4260)
25. The camber line $y$-coordinate and slope at the axial location of the centroid or centroid-offset are determined by linear interpolation. (B4270-B4280)
26. The product of inertia and second moments of area of the section about the centroid are determined using Equations (84), (85), (86), (87), (88), (89), (90) and (91). (B4310-B4470)
27. The orientation of the principal axes and the principal second moments of area of the section about the centroid are determined using Equations (92), (93) and (94). (B4480-B4520)
28. If IPRINT $=0$ or 1 , details of the normalized blade section (meridional chord unity) are printed, (B4570-B4850)
29. If IPRINT $=0$ or 1 , a line-printer plot of the section is made. (B4870-B4940)
30. Section properties determined in 22., 26., and 27. are scaled according to the specified section chord to produce "dimensional" results. (B4950~B5120)
31. If IPRINT $=0$ or 1 , section properiies are printed. (B5150-B5420)
32. The coordinates of the NPOINT points on the blade surfaces are scaled according to the specified chord, and the origin is shifted to the stacking axis. The stacking axis is offset from the centroid of the section by the specified distances. If IPRINT $=0$ or 1 , the coordinates are printed. (B5510B5670)
33. The coordinates of 31 points describing the blade edge(s) are determined. For the standard thickness distribution, it is assumed that the wedge angle is zero, so that the radius extends over a complete semicircle, hence the points are distributed at 60 intervals. For the double-circular-arc blade, the 2 nd and 30 th points of the edges are determined at the exact location of the tangency between the particular blade surface and the edge radius, resulting in an angular interval distribution uniform between points 2 and 30 , with irregular first and last intervals. (B5740-B6220)
34. The origin of the m-coordinate system is shifted to the section centroid or centroid-offset as provided by the input data. (B6250-B6290)
35. The m-length corresponding to each of the 100 points describing the camber line is determined. (B6360-B6410)
36. The reference $\varepsilon_{0}$ of Equation (77) is determined. (B6420)
37. The integration of Equation (78) is performed toward the ends of the blade section from the centroid or centroid-offset, giving the $\varepsilon$ coordinate for the camber line. (B6430-B6840)
38. The m-coordinate of each point of the blade surface is determined. (B6870-B6980)
39. The $\varepsilon$ coordinate system is shifted to the leading or trailing edge if the blade is stacked there. (B7000-B7030)
40. The $\varepsilon$ corresponding to each point on the section surface is determined from the $\varepsilon$ of the camber line at that axial location and $\Delta \varepsilon$ calculated from the displacement from the camber line and the streamsurface radius. (B7070-B7260)
41. The location of the stacking axis is given (in the input data) by specifying its axial coordinate. Linear interpolation yields the corresponding streamsurface coordinate from the $x-m$ table created in 7. (A2550)
42. The origin of the m-coordinate shifted in 34. is again shifted, to the stacking axis (determined in 41.). (A2560A2580)
43. The origin of the table of axial coordinates of the streamsurface-computing station intersection points (obtained in 4.) is shifted to the stacking axis. (A2590-A2600)
44. If IFPLOT $=1$ or 3 , the streamsurface blade section plot is produced. (A2620-A2850)
45. A trigger is set if the calculation of quantities for aerodynamic analysis of the blade is specified at any computing station. (A2860-A2890)
46. If the calculations for aerodynamic analysis are specified, the angular location of the camber line with respect to the stack axis, the slope of the camber line on the streamsurface, and the Cartesian coordinates of the camber line at the stream-surface-computing station intersections are determined and stored. (A2910-A2960)
47. The axial coordinate of each of the NPOINT points specifying the streamsurface blade section suction surface is obtained by linear interpolation from the $x-m$ table produced in 42. (A2970-A2990)
48. The streamsurface radius at each axial location derived in 47. is obtained by spline-curve interpolation from the table created in 43. (A3000)
49. Cartesian coordinates for each point on the streamsurface section suction surface are obtained using the ' $\varepsilon$ 's computed in 40. and Equation (79). (A3010-A3060)
50. Steps 47., 48., and 49. are repeated for the pressure surface. (A3070-A3160)
51. Steps 47., 48., and 49. are repeated for the points describing the leading edge. (A3170-A3260)
52. Steps 47., 48., and 49. are repeated for the points describing the trailing edge if ISECN $=2$. (A3280-A3370)
53. If IPRINT $=0$ or 1 , the Cartesian coordinates of each point describing the streamsurface blade section are printed. (A3390-A3910)
54. The loop initiated in 3. for each streamsurface is terminated. Upon completion of this loop, the Cartesian coordinates of all streamsurface sections have now been stored for interpolation on the specified manufacturing planes. (A.3920)
55. Unless IPRINT $=1$, the volume of the blade is calculated, using Equation (106), and printed. (A3950-A4130)
56. If the calculations for aerodynamic analysis are specified, the program prints the appropriate heading. If not, the program moves to step 62. (A4160-A4180)
57. In the original $x-r$ coordinate system, the axial coordinate of the streamsurfaces-computing station intersections and the computing station slope, are determined for each station at which the aerodynamic analysis quantities were specified. (A4260-A4270)
58. The blade blockage of Equation (107) is obtained at each of these points. (A4280-A4400)
59. The blade lean angle of Equation (108) is obtained at each of these points. (A451.0)
60. The mean section angle of Equation (109) is obtained at each of these points. (A4530-A4540)
61. The quantities required for aerodynamic analysis are printed, and produced on cards if IPUNCH $=1$. (A4550-A4570)
62. If IFPLOT $=2$ or 3, the axes are drawn and titled for the superimposed plot of all the manufacturing sections. (A4620A4630)
63. If no output relating to manufacturing sections is specified by either IFPLOT or IPRINT, the remainder of the program is bypassed. Alternatively, if printed details of the manufacturing sections are specified, a heading is printed. (A4640-A4700)
64. The location of each of the manufacturing planes is determined. (A4710-A4760)
65. The (Cartesian) coordinates of each of NPOINT points on the blade surface are obtained by spline-curve interpolation at each of the manufacturing sections. (A4770-A4860)
66. The (Cartesian) coordinates of each of 31 points describing the blade edge(s) are obtained by spline-curve interpolation at each of the manufacturing sections. (A4870-A4990)
67. A loop that is performed for each manufacturing section is initiated. This loop contains the determination of section properties and the output of results for the section. (A5000)
68. The section area and location of the centroid are determined using Equations (98), (99) and (100). (A5010-A5250)
69. The torsional constant is determined using Equations (104) and (105). (A5260-A5420)
70. The second moments of area and product of inertia for the manufacturing section about its centroid are determined using Equations (101), (102), (103), (86), (87), (88), (89), (90) and (91). (A5430-A5650)
71. From the second moments of area and product of inertia, the orientation of the principal axes and hence principal moments of area are determined, using Equations (92), (93) and (94). (A5660-A5690)
72. If IPRINT $=0$ or 2 , section properties and coordinates are printed. (A5720-A6260)
73. If IFPLOT $=2$ or 3, a plot of the manufacturing section is produced. (A6310-A6530)
74. If IFPLOT $=4$, an individual plot of the manufacturing sections is made. The axes are rotated clockwise until the chord line is horizontal. The angle of rotation is indicated as the stagger angle. (A6550-A7070)
75. The loop initiated in Step 67. for each manufacturing
section is terminated. (A7080)
76. If precision plots have been made, the plotting is terminated. (A7090)


Figure 1 Variation of Normalized Second Derivative
of Polynomial Camber Line with Parameter 'p'


Figure $2 \begin{aligned} & \text { Variation of Polynomial Camber Line with } \\ & \text { Parameter }\end{aligned}$


Figure 3 Variation of Polynomial Camber Line with Parameter ' $P$ '


Figure $4 \begin{aligned} & \text { Variation of Polynomial Camber Line with } \\ & \text { Parameter ' } \mathrm{Q} \text { ' }\end{aligned}$


Figure $5 \begin{aligned} & \text { Variation of Polynomial Camber Line with } \\ & \text { Alpha } 2\end{aligned}$

$\begin{aligned} \text { Figure } 6 & \text { Variation of Normalized Second Derivative } \\ & \text { of Exponential Camber Line with Parameters }\end{aligned}$


Figure 7 Variation of Exponential Camber Line with Parameters 'P' and 'Q'




".an. L" Uaristion of Eaponentlal Camber Line with


Figure 11 Some Camber Lines Available Under the Multiple-Circular-Arc Camber Line Option


Figure 12 Variation of Multiple-Circular-Arc Camber Line with Parameter 'S'


Figure 13 Variation of Multiple-Circular-Arc Camber Line with Parameter 'S'

```
YZERO = 0.00250
T = 0.05000
YONE =0.00250
z=0.7000u
```

| YZERO | $=0.00250$ |
| :--- | :--- |
| $T$ | $=0.05000$ |
| YONE | $=0.00250$ |
| $Z$ | $=0.60000$ |


| YZERO | $=0.00250$ |
| :--- | :--- |
| $T$ | $=0.05000$ |
| YONE | $=0.00250$ |
| $Z$ | $=0.50000$ |

Figure 14 Variation of Thickness Distribution with Point of Maximum Thickness


Figure $15 \begin{aligned} & \text { Cartesian and Streamsurface Coordinates } \\ & \text { of a Point }\end{aligned}$

STREAMSURFACE BLADE SECTION


## manufacturing blade section



Figure 16 Calculation of Section Properties



Figure 17 Parameters Describing Blade Sections


Figure 18 Locations of Streamsurfaces and Manufacturing Planes for Example Blade Designs


Streamsurface Sections

Figure $19 \begin{aligned} & \text { Example Blade Design - Polynomial } \\ & \text { Camber Line }\end{aligned}$


Cartesian Sections

Figure 20 Example Blade Design - Polynomial Camber Line


Streamsurface Sections

Figure 21 Example Blade Design - Exponential


Figure 22 Example Blade Design - Exponential Camber Line

Streamsurfac: Sections

Figure 23 Example Blade Design - Double-CircularArc Blade


Cartiation Sectione



Streamsurface Sections

Figure 25 Example Blade Design - Multiple-Circular-Arc Camber Line


Cartesian Sections

Figure 26 Example Blade Design - Multiple-Circular-Arc Camber Line

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[^0]:    2
    FCINT

[^1]:    secticn ccoritinates

[^2]:    $=E . \varepsilon 21 E E E-03$
    $=4.47 E \varepsilon C E-C \Sigma$
    $=-1.4 月 8 C \in E-C E$

[^3]:    $$
    \text { GO TO } 77 \text { J }
    $$

    CALL PLOY (XPLOT,YPLOT, 2)
    GO TO 77 J

