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Virtual slant explains perceived slant, distortion, and motion in pictorial scenes

Casper J Erkelens
Helmholtz Institute, Utrecht University, Padualaan 8, 3584 CH Utrecht, The Netherlands
e-mail: c.j.erkelens@uu.nl, website: http://www.helmholtz.uu.nl
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Abstract. Virtual slants of obliquely viewed figures consisting of skewed grids, computed as a function of depicted slant and slant of the physical surface, were compared with perceived slants in both monocular and binocular viewing conditions. Computations were based on an assumption of parallelism. Perceived slant was well-correlated with virtual slant, even though slants were generally underestimated. The qualitative relationship between virtual slants and physical surface slants was variable, and liable to produce different degrees and directions of apparent rotation as well as of apparent compensation, depending on the depicted slant of the stimulus. Contributions of screen-related cues were small and mainly limited to small slants. Remarkably, binocular disparity did not have any effect on perceived slant. The results imply that many past findings of both transformation and (apparent) compensation in pictorial viewing may in principle be straightforwardly explained as a function of the virtual stimulus.

Keywords: 3-D perception in pictures, linear perspective, apparent motion

1 Introduction

We, members of the Information Age, watch (moving) pictures during many hours a day. After having viewed pictures drawn on walls, paper, and canvas for millennia, we devote much time now to pictures displayed on a screen (television, monitor, laptop, tablet, and smartphone). Despite the fact that pictures are flat, viewing pictures can give us the impression of looking at 3-D objects and scenes. Still, perception of depicted 3-D objects differs from that of real 3-D objects, in particular when the pictures are viewed from oblique directions. Differences in perception may become compelling when the viewer moves along a large picture such as a billboard (Papathomas et al 2010). The viewer may then experience a vivid impression of illusory motion in that the depicted object rotates in the direction of travel as if following the viewer. Apparent rotations have been reported in a number of studies (Gombrich 1972; Goldstein 1979, 1987, 1988; Kubovy 1986; Cutting 1987, 1988; Koenderink et al 2004). Goldstein (1979) reported a differential rotation effect implying that surfaces pointing out of the picture rotate more than surfaces oriented along the picture. Cutting (1988) explained Goldstein's results by affine transformations that shear, compress, and dilate pictorial space. Goldstein (1988) dismissed Cutting's explanation, because of the fact that it did not explain the relatively constant layout of depicted scenes. Koenderink et al (2004) measured the pictorial relief of torsos depicted on slanted screens, and reported that the torsos maintained their orientation relative to the viewer in oblique viewing directions. In contrast, the depicted brick wall that served as the background maintained its orientation relative to the screen. Koenderink et al (2004) explained the different orientations of torso and brick wall by assigning the torso to pictorial space and the brick wall to physical space. In other words, according to Koenderink et al (2004) the perception of the torso reflects looking *into* the picture and that of the brick wall looking *at* the picture.

The "reverspective" paintings of the artist Patrick Hughes (Slyce 1998) have evolved into a new tool for studying picture perception (Wade and Hughes 1999; Cook et al 2002, 2008a, 2008b; Papathomas 2002, 2007; Papathomas and Bono 2004; Rogers and Gyani 2010;

Papathomas et al 2012). Reverspectives, also called reverse perspectives (Wade and Hughes 1999; Cook et al 2002), consist of 3-D truncated pyramids and wedges that protrude out of the background, the faces of which are painted with perspective images. The critical feature of reverspectives is that the perspective gradients depict a 3-D scene whose depth relations are opposite in direction to those of the physical structure (Rogers and Gyani 2010). The perceptual feature that strikes viewers of reverspectives immediately is the concomitant motion of the perceived 3-D structure when the viewer makes side-to-side or up-and-down head movements (Wade and Hughes 1999; Cook et al 2002; Papathomas 2002; Rogers and Gyani 2010). Although researchers disagree about the underlying mechanism (Papathomas 2007; Rogers and Gyani 2010), they unanimously conclude that linear perspective is the most powerful cue contributing to the illusion.

Despite the considerable body of research, there is not yet a generally accepted theory of picture perception. There are many ideas, but these are either too loosely formulated (Papathomas 2007) or too tailored to particular cases (Koenderink et al 2004) to allow general and accurate predictions. In this paper I present a computational model and experiments dealing with the perceived slants of drawings of skewed grids viewed from oblique directions. The choice for grids as a stimulus was inspired by the fact that these are sufficiently simple to allow accurate computations and sufficiently general to allow predictions for perception of more complex pictures. The work on reverspectives has been the motivation to base computations, made to predict the perceived slant of surfaces in pictures, on linear perspective. The computations were based on the following hypothesis: human observers perceive a *skewed* grid depicted on a screen (the *physical* distal stimulus) as if it originates from a *regular* grid (the *virtual* distal stimulus) that produces the same retinal image (the *proximal* stimulus).

The computations were made for a perspective-rich stimulus ie a rectangular grid (15° × 10°) that consisted of 5 horizontal and 5 vertical evenly distributed bars. The images on the screen were projections of the grid slanted about the vertical axis (by a pictorial slant angle of ϕ). Computations were made for 11 values of ϕ ranging from –90° to 90° (where the frontoparallel plane was defined as 0°). The slant of the screen (σ) was treated as a continuous variable that also ranged from –90° to 90°. Figure 1 shows the distal physical and virtual stimuli, the proximal stimulus, and the angles relevant for the computations.

Figure 2 shows the proximal grids for various combinations of σ and ϕ that were used in the experiments. The rows show how the proximal grids are affected by the slant of the screen (σ) for a particular depicted slant (ϕ). For $\phi = 0°$ the effect of σ is strong, whereas it is modest for $\phi = 75°$. The columns show how much proximal grids can vary at a particular slant of the screen. Proximal grids vary modestly for positive ϕs viewed on a screen whose σ is positive (the right side of figure 2), whereas they vary strongly for positive ϕs viewed on a screen whose σ is negative (the left side of figure 2).

The geometric computations showed that for each combination of ϕ and σ there existed a rectangular grid slanted by υ, the virtual stimulus. The virtual slants υ were computed as functions of σ for the 11 values of ϕ (figure 3a). If visual perception is fully based on the linear perspective hypothesis, the computed virtual slants will predict the perceived slant of skewed grids depicted on flat surfaces when these are viewed from oblique directions.

Figure 3a shows that the relationship between υ and σ depends strongly on ϕ. The virtual slant of zero-slanted grids ($\phi = 0°$) is a linear function of σ with a coefficient of 1 [$\upsilon(\sigma) = \sigma$]. In other words, the virtual slant of a zero-slanted grid is identical to the physical slant of the surface on which the grid is depicted. For $\phi = \pm 45°$, $\upsilon(\sigma) = 0.5\sigma + \phi$. For such skewed grids, υ is a linear function of σ with a coefficient of 0.5. Thus for $\phi = \pm 45°$, υ changes by 90° from the frontoparallel plane ($\upsilon = 0°$) to the plane perpendicular to the viewer ($\upsilon = \pm 90°$) over the range of 180° of σ. The relationship between υ and σ is nonlinear for all other ϕs.

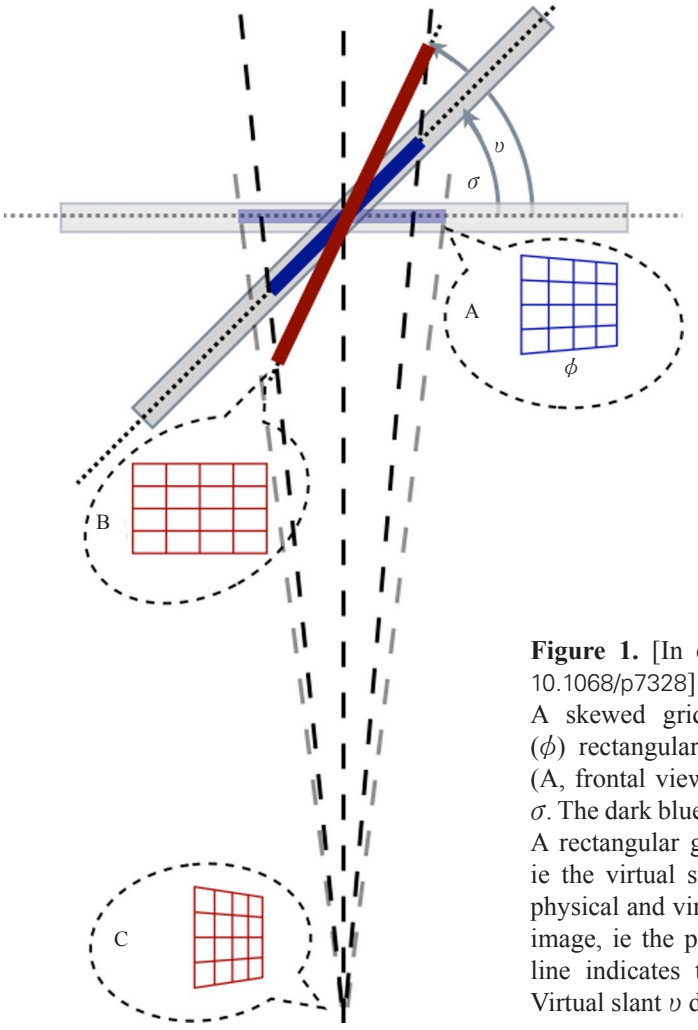

Figure 1. [In colour online, see http://dx.doi.org/10.1068/p7328] Stimulus arrangement (plan view). A skewed grid, ie the projection of a slanted (ϕ) rectangular grid, was depicted on the screen (A, frontal view). The screen itself was slanted by σ. The dark blue line indicates the physical stimulus. A rectangular grid (B, frontal view) slanted by υ, ie the virtual stimulus, was computed so that the physical and virtual grids produced the same retinal image, ie the proximal stimulus (C). The dark red line indicates the virtual stimulus in plan view. Virtual slant υ depends on both ϕ and σ.

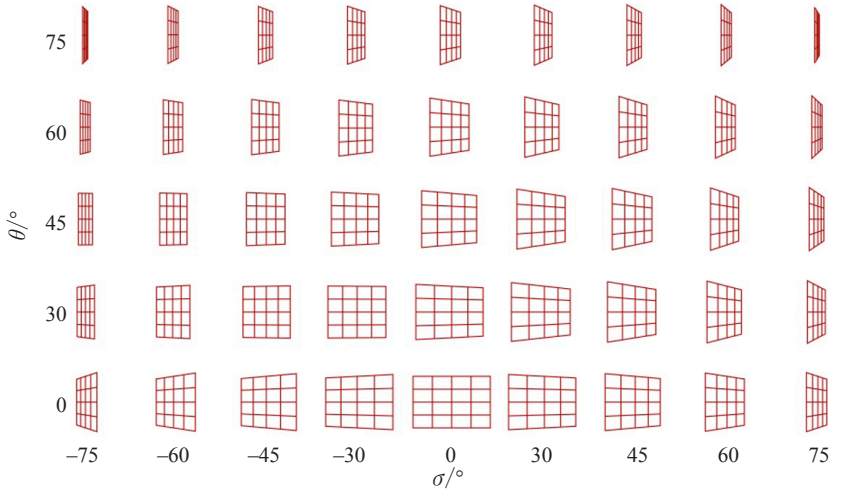

Figure 2. [In colour online.] Computed proximal stimuli. The array of 45 grids shows the proximal stimuli for a range of combinations of pictorial slant ϕ and screen slant σ.

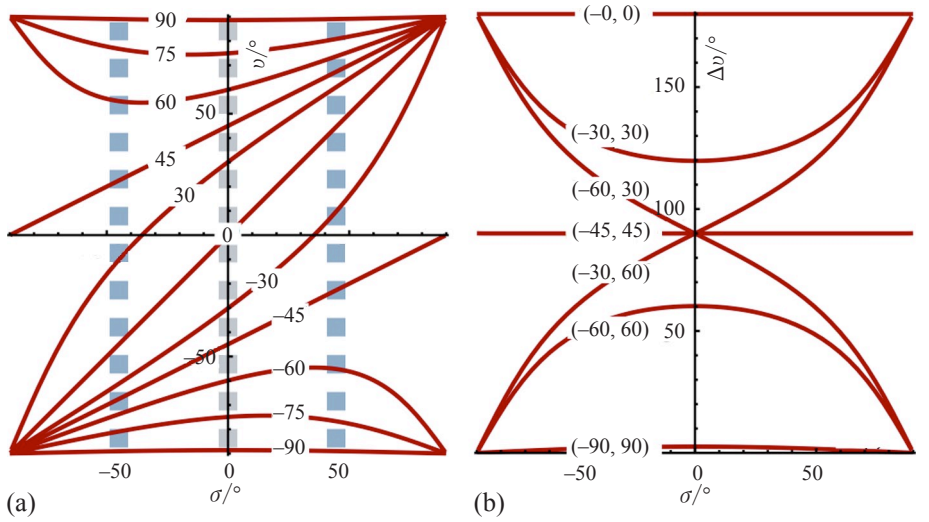

Figure 3. [In colour online.] Computed virtual slants: (a) shows υ as a function of σ for 11 values of ϕ; (b) shows angular differences between virtual slants ($\Delta\upsilon$) as a function of σ for 7 pairs of ϕs. The grey and blue dashed lines are explained in section 4.

For $\phi = \pm 90°$, $\upsilon(\sigma) = \phi$, implying that υ is independent of σ. In other words, the virtual slant of grids that are (almost) perpendicular to the viewer on a frontoparallel screen remains (almost) perpendicular at any slant of the screen (to some extent illustrated by the grids in the top row of figure 2). The nonlinear relationships of υ and σ are qualitatively different for $|\phi| < 45°$ and $|\phi| > 45°$. For $|\phi| < 45°$, the slope of υ is positive for all σs. Thus, increases (decreases) in σ cause increases (decreases) in υ. For $|\phi| > 45°$, the slope of υ is positive for one range of σs and negative for the other. As a consequence the slope of υ is zero, or close to zero at certain slants of σ, implying that υ will hardly change relative to the viewer for variations of σ about those slants.

A consequence of the different relationships between υ and σ for different values of ϕ is that the angular difference in slant between two grids that are depicted on the same surface will generally not be conserved as a function of σ. Figure 3b shows the computed angular differences in slant ($\Delta\upsilon$) of two virtual grids for 7 pairs of depicted slants that are shown in figure 3a. The pairs are denoted by (ϕ_1, ϕ_2), indicating that one grid is slanted by ϕ_1 and the other by ϕ_2. $\Delta\upsilon$ is defined as $\Delta\upsilon = \upsilon_1 - \upsilon_2 + 180°$, where υ_1 is the virtual slant corresponding to ϕ_1 and υ_2 the virtual slant corresponding to ϕ_2. $\Delta\upsilon$ is constant for three pairs of ϕs, namely (ϕ_1, ϕ_2) = (0°, 0°), (−45°, 45°), and (−90°, 90°). $\Delta\upsilon$ changes as a function of σ for all other pairs of ϕs. Obtuse angles in frontal view ($\sigma = 0°$) become more obtuse at oblique slants of the screen for symmetric combinations of slanted grids as is illustrated by example (−30°, 30°) in figure 3b. On the other hand, acute angles at $\sigma = 0°$ become more acute at oblique slants of the screen as is illustrated by the combination (−60°, 60°). The relationship between $\Delta\upsilon$ and σ is different again for two pairs of grids that are slanted perpendicular to each other in frontal view. Figure 3b shows that $\Delta\upsilon$ is acute if $\sigma < 0°$ and obtuse if $\sigma > 0°$ for the pair (−30°, 60°). Acuteness and obtuseness are reversed as a function of σ for the pair (−60°, 30°). $\Delta\upsilon$ changes by as much as 180° as a function of σ for these asymmetric perpendicular pairs of grids.

2 Experiments

If the linear perspective hypothesis is true for picture perception, the computations will give accurate predictions for the perceived slant of perspective-rich surfaces depicted on slanted screens. The computations predict how much perspective-rich surfaces will appear

slanted for each combination of σ and ϕ. To test the validity of the hypothesis, perceived slant was measured for a number of depicted slants (A in figure 1 is just one example) as a function of σ (experiment 2).

A possible error in the assumption that virtual slants will predict perceived slants is that the computations are based on linear perspective alone, and thus ignore the fact that other cues contribute to slant perception. Extensive research has established that the visual system integrates various cues in a linear fashion when different cues indicate similar values of slant (Johnston et al 1993, 1994; Young et al 1993; Landy et al 1995; Jacobs 1999; Knill and Saunders 2003; Hillis et al 2004). Slant perception can become unstable or be dominated by one cue if cues indicate very different slants (Stevens et al 1991; Hill and Bruce 1993; van Ee et al 2002, 2003; Papathomas 2002; van Ee 2005). To estimate the contribution of linear perspective to the perceived slant of the grid stimuli, an experiment was conducted in which the contribution of perspective was segregated from that of other cues without having to compare responses to different types of stimuli (experiment 1).

In experiment 1, sets of physical stimuli (As and Bs in figure 4) were computed such that the corresponding virtual stimuli, and thus the proximal stimuli (Cs in figure 4), were identical and therefore independent of σ. Since the virtual stimuli were purely based on linear perspective, contributions of perspective-related cues to slant were independent of σ, whereas contributions of screen-related cues depended on σ. In other words, by measuring perceived slant as a function of υ for three values of σ (–30°, 0°, and 30°, the latter of which is not shown in figure 4), the contribution of perspective-related cues could be separated from those of screen-related cues (eg binocular disparity, blur, vergence eye movements). Measuring perceived slant during binocular and monocular viewing enabled the separation of binocular disparity from the other screen-related cue contributions.

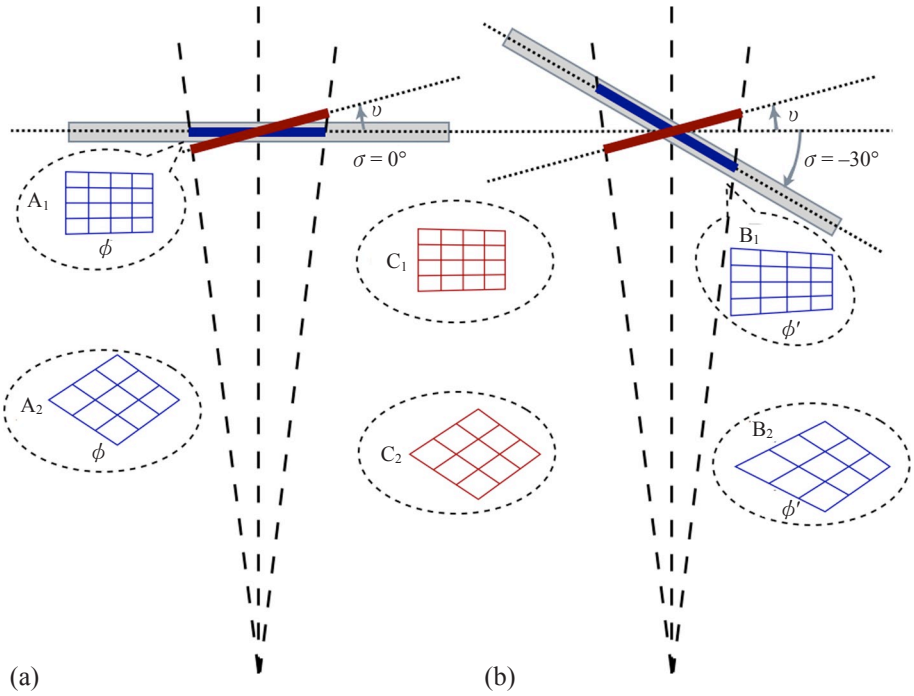

Figure 4. [In colour online.] Stimulus arrangements for $\sigma = 0°$ (a) and $\sigma = -30°$ (b). Grids As and Bs were computed such that they produced identical proximal stimuli (Cs). Thus shapes and sizes of C were independent of σ. The thick blue lines indicate the physical stimuli in plan view. The thick red lines indicate the identical virtual stimuli. Stimuli were rectangular (A_1, B_1, C_1) and rhombic (A_2, B_2, C_2) grids.

2.1 *Experimental setup*

Stimuli were displayed on a TFT monitor (21 inch LaCie 321, 1600 × 1200 pixels, 75 Hz). The display area of the monitor was approximately 43 deg × 32 deg. A chin-rest restricted head movements so that the centre of the forehead (the "cyclopean eye") was positioned at the centre of projection of the stimuli at all times. The setup was placed in a normally lit room because pictures and reverspectives are usually viewed against visible frames and backgrounds. The monitor was centred on a turntable at a distance (= centre of the forehead to the centre of the turntable) of 57 cm. A second turntable, centred within a protractor and carrying a vertical rectangular pad (23 cm × 16 cm), was placed between the centre of the monitor and the chin-rest at a distance of 21 cm from the forehead. Subjects oriented the pad with both hands to indicate the perceived slant of the stimuli. The vertical angle between the bottom of the stimuli and the upper edge of the pad was 38° so that the subjects alternated their fixation between up and far and near and down during the adjustment of the pad.

2.2 *Stimuli*

The physical stimuli were white (93 cd m^{-2}) grids depicted on a black (approximately 2 cd m^{-2} in frontal view) background. Two types of grids were used, rectangular (15° × 10°) and rhombic (22.5° × 15° tip-to-tip). The rectangular grids contained 5 horizontal and 5 vertical lines. The rhombic grids contained equal numbers of parallel but not orthogonal lines (figure 4). Thus, the rectangular grids contained two regularity features (parallelism and orthogonality) whereas the rhombic grids contained just one feature (parallelism). In experiment 1, υ was varied between −75° and 75° in steps of 15°. Screen slant σ was −30°, 0°, and 30°, respectively. For $\sigma = 0°$ we have that $\upsilon = \phi$, implying that the stimuli were projections of the rectangular and rhombic grids as described above, slanted by ϕ. For $\sigma = \pm 30°$, the physical stimuli were computed such that the corresponding virtual stimuli were geometrically identical to the virtual stimuli for $\sigma = 0°$ (figure 4). In other words, the patterns of the proximal stimuli were indistinguishable if viewed from the centre of projection. Relative binocular disparities between the different lines of the stimuli were smaller than 0.6° on the frontoparallel screen ($\sigma = 0°$) and ran up to values of 1.9° on the slanted screens ($\sigma = \pm 30°$). In experiment 2, σ was varied between −75° and 75° in steps of 15°. Depicted slant ϕ was also varied between −75° and 75° in steps of 15°, however excluding $\phi = \pm 15°$. These slants were omitted because, based on the computations, variation in the results was expected to be limited in the ranges between $\phi = 0°$ and $\phi = \pm 30°$ (figure 3a).

2.3 *Procedure*

Eight subjects (seven first-year physics students and the author) indicated the perceived slant of stimuli by slanting the pad on the turntable until they judged that the two slants matched. All but the author were naive with respect to the purpose of the study. The subjects had normal or corrected-to-normal vision. Stereo-acuity was assessed by shape detection in a series of selected random-dot stereograms. All subjects were successful in naming the hidden shapes. In order to familiarise the subjects with the task, they ran 3 pre-trials without a stimulus on the screen. They were asked to match the slant of the screen while they viewed the screen binocularly or with the left or right eye in separate trials. Results of pre-trials were evaluated and the trials repeated if the settings were too inaccurate. The purpose of practising was to ensure that the slant settings reflected perceived slant as much as possible. After some practice all subjects were able to judge the slants of the screen within error margins of 5° of slant and without a noticeable bias. The subjects were instructed to ignore the slant of the screen in the real experiments and to focus on the slant of the stimuli. Measurements in the different viewing conditions (ie left eye, right eye, and binocular viewing) were made in separate blocks. Combinations of ϕ and σ were presented in fully randomised order.

3 Results

3.1 *Experiment 1*

Mean results for the eight observers are presented in figure 5 for rectangular grids. The 9 graphs showing perceived slant as a function of υ are presented twice. The top row shows the graphs arranged by viewing condition, which is helpful in showing an effect of screen slant (σ) on the results. In the bottom rows the graphs are arranged by screen slant. This arrangement should easily reveal an effect of viewing condition.

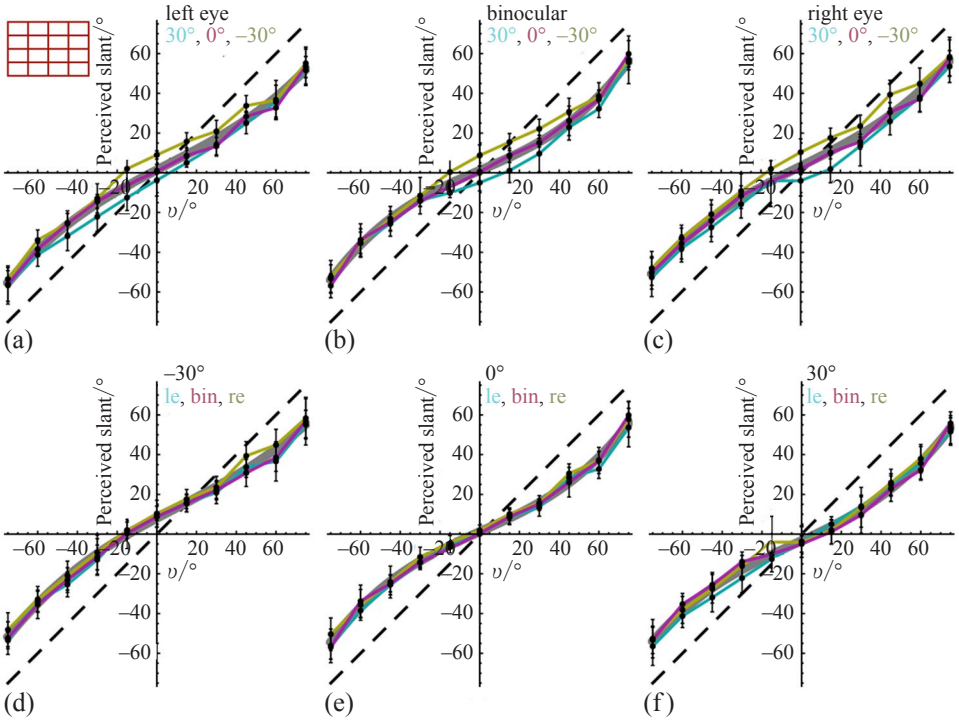

Figure 5. [In colour online.] Perceived slant as a function of υ for rectangular grids. The panels show means and SDs grouped by viewing condition with screen slant as a parameter [(a), (b), (c)] and slant of the screen with viewing condition as a parameter [(d), (e), (f)]. Cubic functions (grey) are fitted to the means of all data of the three graphs. The dashed lines indicate a slope of 1.

Two characteristics of the data stand out. First, perceived slant was considerably smaller than υ for all values of υ. Slopes close to 1 would have been expected if perceived slant was fully based on linear perspective. Perceived slant was well described by cubic relationships to υ in the 9 experimental conditions (figure 5). When averaged over the different viewing and screen-slant conditions, the slopes of the linear components, reflecting the slopes of the responses near $\upsilon = 0°$, were 0.48 ± 0.07 and 0.49 ± 0.09, respectively. Slopes were steeper for $|\upsilon| > 45°$. The obvious explanation for the low values of the slopes is that cues different from linear perspective contributed to slant perception. However, the second striking characteristic of the results is that cues related to screen slant had clear but just mild effects on perceived slant (figures 5a, 5b, 5c). A three-way ANOVA showed that the factor σ was highly significant ($F_{2,69} = 36.5$, $p < 0.001$). The main effects of viewing condition ($F_{2,69} = 2.85$, $p = 0.065$) and subject ($F_{7,64} = 1.04$, $p = 0.41$) did not reach significance. The main effect of binocular versus monocular viewing was not significant either ($F_{1,70} = 0.19$, $p = 0.66$). A main effect of binocular versus monocular viewing would have revealed a significant contribution of disparity to perceived slant. The lack of an effect of disparity is surprising because of the general belief that disparity is the major contributor to perceived depth and slant (Held et al 2012). Effects of σ (figures 5a, 5b, 5c) were most visible for small υs.

Remarkably, the deviations of perceived slant were opposite to screen slant. For instance at $\upsilon = 0°$, slant deviations were positive for the negative screen slant and vice versa. Apparently the subjects inadvertently compensated partially for the slant of the screen.

The results for rhombic grids are presented in figure 6. The results resemble those for rectangular grids (figure 5). Again, perceived slant was well described by cubic relationships to υ. One feature was different, namely σ hardly seemed to affect perceived slant of rhombic grids (figures 6a, 6b, 6c).

When averaged over the different viewing and σ conditions, the slopes of the linear components were 0.35 ± 0.14 and 0.37 ± 0.11, respectively. These slopes were about 20% shallower than the corresponding slopes for rectangles. Main effects of σ ($F_{2,69} = 3.66$, $p = 0.031$) and viewing condition ($F_{2,69} = 2.65$, $p = 0.078$) were not significant at the level of 5%, respectively. Main effects of subject ($F_{7,64} = 1.12$, $p = 0.36$) and of binocular versus monocular viewing ($F_{1,70} = 1.70$, $p = 0.20$) were not significant.

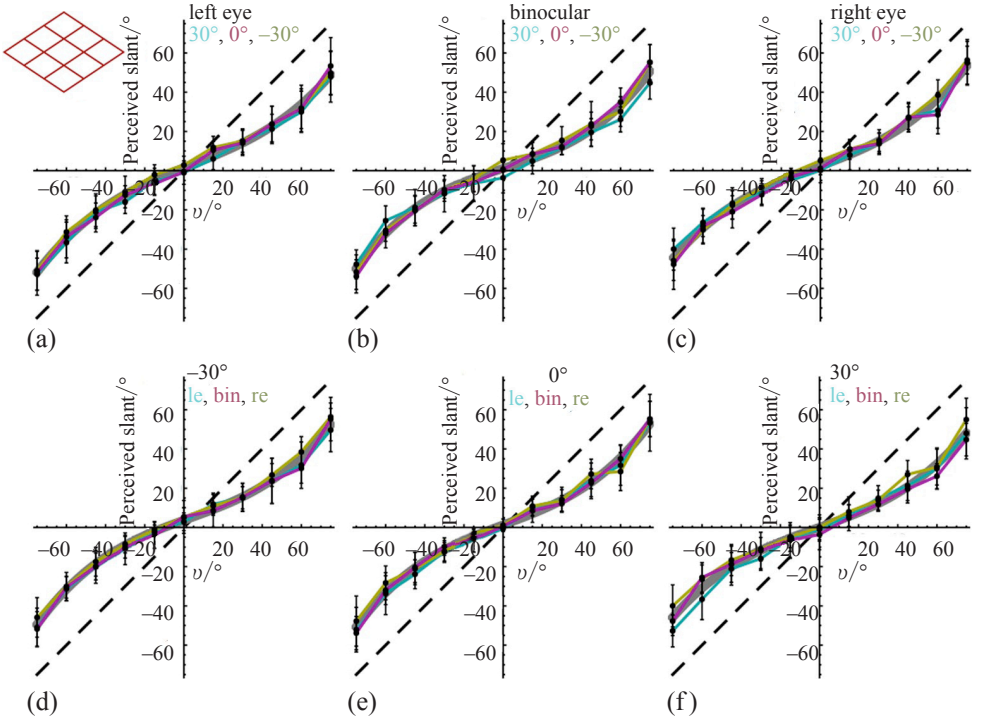

Figure 6. [In colour online.] Perceived slant as a function of υ for rhombic grids. The panels show means and SDs grouped by viewing condition with screen slant as a parameter [(a), (b), (c)] and slant of the screen with viewing condition as a parameter [(d), (e), (f)]. Cubic functions (grey) are fitted to the means of all data presented in the individual panels. The dashed lines indicate a slope of 1.

As a consequence of the nonlinearity of the relationship between perceived slant and υ, the perceived angle between two grids (υ_1 and υ_2), if measured, would have changed as a function of υ. Figure 7 shows the angles (slant differences) as a function of the mean virtual slant [$\upsilon = 0.5\,(\upsilon_1 + \upsilon_2)$] for three different $\Delta\upsilon$s ($\Delta\upsilon = \upsilon_1 - \upsilon_2$). $\Delta\upsilon$ was computed from the means of the υs shown in figures 5 and 6. The graphs show the results for $\Delta\upsilon$ values of 30°, 45°, and 60°. The results show four features that are relevant for the comparison of perceived slant to predictions of the linear perspective hypothesis shown in figure 3. First, most angles would have been strongly underestimated if these had been measured, especially angles between symmetric or closely symmetric slants (small υs). Second, the data were well fitted by quadratic functions, meaning that the underestimation was symmetric about $\upsilon = 0°$.

Third, the underestimation was equally strong in frontal and oblique viewing conditions [see the similar graphs (d), (e), and (f) of figures 5 and 6] showing that the underestimation was independent of σ. Fourth, the underestimation was slightly stronger for rhombic than rectangular grids. Differences between the results and the corresponding dashed lines show that all perceived angles would have been much more obtuse than $\Delta\upsilon$. The underestimation of slant measured in experiment 1 will be relevant for the interpretation of the results of experiment 2.

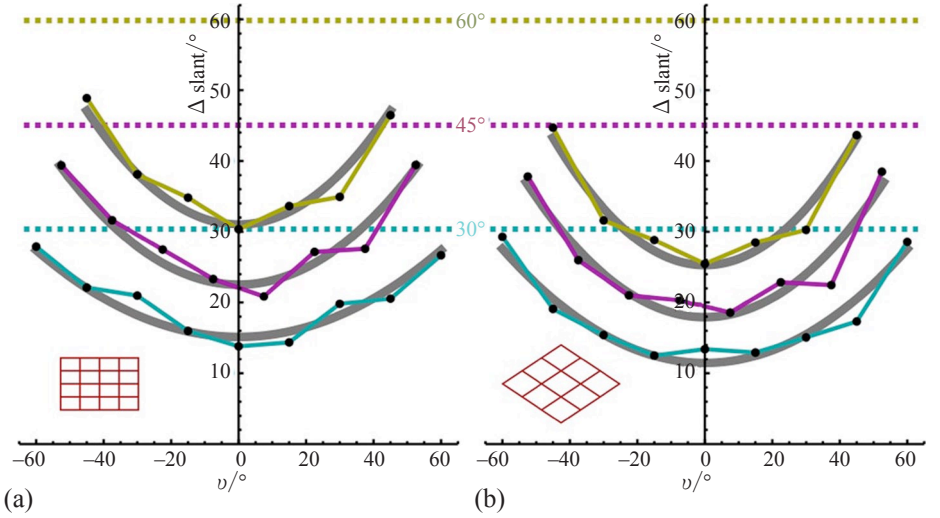

Figure 7. [In colour online.] Slant differences computed between pairs of perceived slants shown in figures 5a and 6b. The υs of pairs differ by 30° (cyan), 45° (magenta), and 60° (yellow). Quadratic functions (grey) are fitted to the data. Dashed lines indicate the theoretical slant differences (= $\Delta\upsilon$).

3.2 *Experiment 2*

Figure 8 presents the results of experiment 2 in which the perceived slant of rectangular grids slanted by ϕ on the screen was measured, while it was binocularly viewed on the screen that was slanted by σ.

The data of six subjects were rather similar (figure 8a) whereas the results of two subjects deviated in some aspects (figure 8b). The pattern of the data of figure 8a resembles that of the computations of figure 3a in that the measured perceived slant as well as the computed υ strongly depended on σ for low values of ϕ and gradually became less dependent on σ for

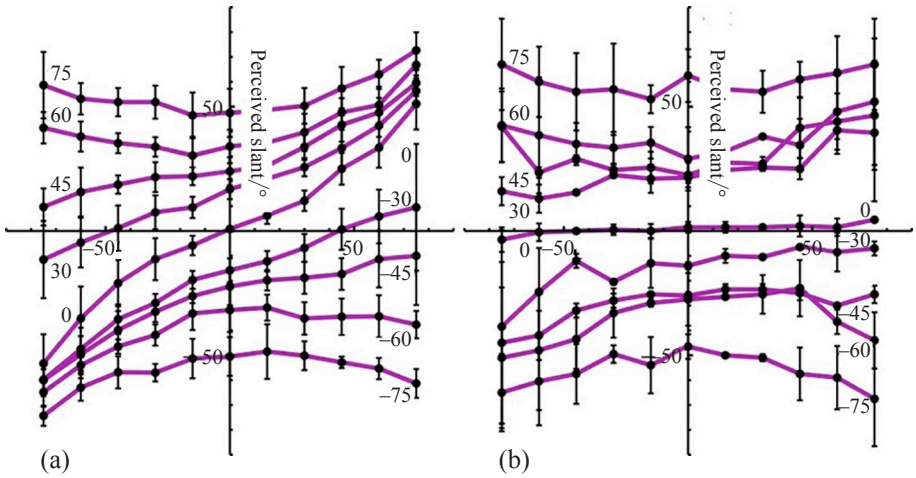

Figure 8. [In colour online.] Perceived slant as a function of σ for 9 values of ϕ: (a) shows means and SDs for six subjects, (b) for two different subjects.

increasing angles of ϕ. The two deviating subjects (figure 8b) showed similar results for the high values of ϕ. However, perceived slant did not show the strong dependence on σ for low values of ϕ. Remarkably, perceived slant of rectangular zero-slanted grids ($\phi = 0°$) was almost independent of σ for these two subjects. This result is not explained by either the linear perspective hypothesis or contributions of screen-related cues. Both explanations would have predicted a linear relationship between perceived slant and σ. The two subjects perceived a rectangular grid on a slanted screen as a frontoparallel grid, whereas they experienced a skewed grid on a frontoparallel screen as a slanted grid (see the results of experiment 1 in figure 5). Apparently, these subjects fully compensated the slant of the grid for the slant of the screen. To check the plausibility of this explanation, the two subjects were asked to repeat experiment 2 in a completely dark room. Under this condition, the results of the two subjects were similar to those of the other six subjects in the lit room condition shown in figure 8a. The different judgments in lit and dark viewing conditions suggest that compensation for screen slant is the likely explanation. An alternative explanation is that the two subjects performed a task different from the one they were instructed to perform. Namely, they indicated ϕ, the frontal view of the grid on the screen, rather than the slant of the grid in space. This would explain why their responses for a given ϕ were nearly constant.

In all subjects and for all ϕ perceived slants were much smaller than υ (compare figures 8 and 3a). Strong underestimation of slant was also measured in experiment 1, where it hardly depended on σ or viewing condition. Underestimation was thus a general feature of slant judgments made in oblique as well as in frontal viewing directions. The subjects underestimated small slants ($|\upsilon| < 30°$) by more than a factor of two (figures 5, 6, and 7). Large slants ($|\upsilon| > 60°$) were only slightly underestimated. Slant underestimation, and particularly its nonlinear character, obscure the resemblance between perceived slant and υ. To single out the oblique viewing effects on perceived slant, the results of experiment 2 were corrected for the underestimation measured in experiment 1. Cubic functions that were fitted to the data of the individual subjects for rectangular grids (similar to those for all subjects shown in figure 5) were used to adjust the individual data of experiment 2.

Figure 9 shows the data of the six subjects shown in figure 8a corrected for the underestimation hypothesis. Comparison of perceived slant (figure 9a) and υ (figure 3a) shows a striking resemblance for most of the values of ϕ as a function of σ. Prominent features are

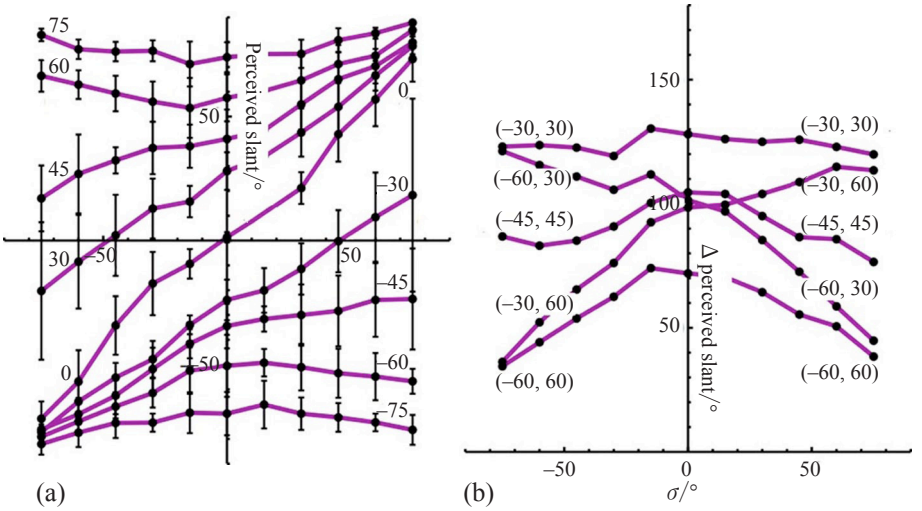

Figure 9. [In colour online.] Perceived slant as a function of σ for 9 values of ϕ. Data are corrected for the underestimation measured in experiment 1: (a) shows means and SDs for the six subjects whose data are shown in figure 8a; (b) shows computed differences between perceived slants as a function of σ for 5 pairs of ϕs.

the similarities of the nearly linear relationships to σ for zero-slanted ($\phi = 0°$) grids, and the almost complete independence of σ for highly slanted grids. The slopes of the $\phi = \pm 30°$ and $\phi = \pm 45°$ lines are somewhat shallower than expected, especially for large σs. The resemblance between perceived slant and computed υ is also visible in the resemblance between the perceived angles (figure 9b) and $\Delta\upsilon$ (figure 3b) for the various pairs of ϕs. Due to the higher sensitivity to noise of the data for the angles, however, the resemblance between the perceived angles and $\Delta\upsilon$ is somewhat less striking that the resemblance between perceived slant and υ.

4 Discussion

4.1 *Main conclusions*

Experiment 1 showed four major results. First, experiment 1 showed that perceived slant was strongly and reproducibly related to the virtual slant. As a result, the perceived slant of the grids was dominated by the linear perspective in frontal as well as oblique views. Second, screen-related cues played a significant but limited role in the perceived slant of the grids. The slant of the screen affected the judgments mainly at the lower slants, although the effects were generally smaller than 5° of slant. Third, the insignificance of viewing condition indicated that eye position did not matter, despite that the vantage point differed by up to about 6.5 cm in the three viewing conditions. Differences in viewing direction held for both the screen and the pad that was used for the slant settings. Apparently, the two effects cancelled each other. Disparity appeared to be totally irrelevant for slant perception, although considerable disparities were present within the stimuli and the within-stimulus disparities were very different for the three slants of the screen. The fourth major result of experiment 1 was that slant was systematically underestimated in both frontal ($\sigma = 0°$) and oblique ($\sigma = \pm 30°$) viewing directions. The level of underestimation was remarkably similar in the different subjects. The similarity of the underestimation in frontal and oblique viewing directions showed that screen-related cues hardly played a role in the underestimation. The level of underestimation depended strongly on υ and just slightly on σ implying that underestimation was related to the slant of the grid rather than the screen.

There are four possible explanations for the systematic underestimations. The first possibility is that the method of slant estimation was inappropriate. Perceived slant was measured by matching the slant of a physical pad to the perceived slant of a slanted grid viewed on a slanted screen. The combination of a perceptual and motor task could in principle produce biases and gains different from unity. The pre-trials, however, showed that the task itself did not produce systematic errors. The second possible explanation is that the linear perspective hypothesis is false. This explanation is highly unlikely because experiment 1 showed that perceived slant and virtual slant were strongly and reproducibly related to each other. The third explanation may be that perspective-induced slant is opposed by slant measured by other cues. This explanation is unlikely too, because experiment 1 showed that disparity and other screen-related cues hardly contributed to the perceived slants of the grids. The fourth possibility is that the effectiveness of perspective information of the grids was suboptimal. This explanation is feasible and supported by the fact that slant underestimation was slightly stronger for rhombic than rectangular grids. Saunders and Backus (2007) reported that both parallelism and orthogonality contribute to slant perception. These two sources of perspective information were differently represented in the rectangular and rhombic grids. Rectangular grids contained parallelism (converging lines in 2-D images representing parallel lines in 3-D space) and orthogonality (skewed angles in 2-D images representing orthogonal angles in 3-D space) as components of the perspective constraint. Rhombic grids contained parallelism only (the angles between the lines were non-orthogonal in frontal view).

Furthermore, the rectangular grids contained 10 lines and the rhombic grids 8. Therefore, the rectangular grids probably contained stronger perspective constraints. Number and length of lines may have been suboptimal in all the stimuli used in the current experiments.

4.2 *Predictions for planar and non-planar surfaces*

If we accept suboptimality of linear perspective information as the valid explanation for slant underestimation, then the results of experiments 1 and 2 strongly support the perspective hypothesis. Earlier Todorovic (2008) found some support for the perspective hypothesis from an experiment in which he confronted compensation with transformation theories related to changes in vantage point. He observed systematic deviations that, without testing, he attributed to cue conflicts and asymmetry of visual angles. In the current study, these potential sources of slant perception were tested and largely invalidated by experiment 1. Perceived slant of perspective-rich surfaces depended mainly on linear perspective and, therefore, on depicted slant as well as slant of the screen. The dependence on both depicted and screen slant caused that differently slanted grids slanted differently in oblique viewing directions. Extremes were on the one hand zero-slanted grids: such surfaces slanted linearly with the screen, and on the other hand highly slanted grids: these surfaces hardly slanted with the screen. The results are in line with findings reported by Goldstein (1979). He found that objects in a picture appeared to change orientation as the observers viewed the picture from different viewing angles. Some objects changed orientation more than others. Goldstein (1979) called this a differential rotation effect. The computations and measurements in the current study provide a mechanistic basis and a quantitative prediction for the differential rotation effect.

Surfaces of objects are generally non-planar and will contain various parts having richer and poorer perspective information. Therefore, it is still a bridge too far to predict the perceived slant of natural objects from the perceived slants of grids. Koenderink et al (2004) measured the perceived orientation of torsos extensively for various combinations of physical and virtual slant. The authors concluded that perceived orientation follows what is indicated by the proximal stimulus. This conclusion is valid but problematic, because innumerable virtual and physical stimuli can produce identical proximal stimuli. The current results show that observers perceive a particular interpretation of the proximal stimulus namely the linear-perspective-hypothesised grid at the virtual slant and not the depicted grid at the slant of the physical surface. Virtual slant dominates visual perception, notwithstanding the fact that physical slant is indicated by numerous cues (such as the width and brightness of the lines, blur, accommodation, vergence, and disparity). Virtual slant explains why a painted zero-slanted brick wall is seen in the plane of the screen, even from oblique viewing directions. The linear perspective hypothesis makes no distinction between pictorial and physical objects and, thus, does not require segregation of the pictorial and physical spaces (Koenderink et al 2004).

4.3 *Deformations during oblique viewing of pictures*

A perspective image accurately reproduces a view of a 3-D scene only when it is viewed from the picture's centre of projection. If viewed from another vantage point, the retinal image of the picture is generally no longer consistent with the depicted scene (Farber and Rosinski 1978; Sedgwick 1980, 1991; Nicholls and Kennedy 1993; Saunders and Backus 2007). The perceived distortion of surfaces has been a topic of considerable debate (eg see Kubovy 1986; Cutting 1987; Koenderink et al 2004). Several studies have presented evidence that the visual system exploits information about the slant of the picture surface itself to compensate for viewing angle (Perkins 1973; Rosinski et al 1980; Wallach and Marshall 1986; Goldstein 1987, 1988; Halloran 1993; Vishwanath et al 2005).

To assess the distortions in the retinal images of the grids as functions of ϕ and σ, horizontal compression and vertical deformation were computed for both the physical and virtual stimuli of experiment 2 (figure 10). Horizontal compression was defined as the angular width of the grid slanted by ϕ viewed on a screen slanted by σ divided by the angular width of the grid slanted by ϕ in frontal view ($\sigma = 0°$). Vertical deformation was defined as the ratio between the angular heights of the right and left sides of the stimulus viewed on a screen slanted by σ divided by the same ratio between the sides of the stimulus in frontal view. Figure 10a shows the relationship between horizontal compression and σ for all physical and corresponding virtual grids (11 values of ϕ) that have been used in all the computations. The relationship appears to be identical for all physical and virtual grids (the small offset between the red and blue lines was purposely included for reasons of clarity). In figure 1, the common angle between the outer black dashed lines subtending the physical and virtual stimuli illustrates the identical width of physical and virtual stimuli for an individual combination of ϕ, σ, and υ. Computations further showed that horizontal compression is independent of the viewing distance, so that horizontal compression depends on σ only. The relationships for vertical deformation are slightly more complicated (figure 10b). The figure shows that the relationship between vertical deformation and σ depends on ϕ. Similar as for horizontal compression, the relationships between vertical deformation and σ are identical for sets of physical and corresponding virtual grids. Manipulation of viewing distance showed that the relationship also depends on viewing distance, vertical deformations becoming smaller (the values deviate less from unity) with increasing viewing distance.

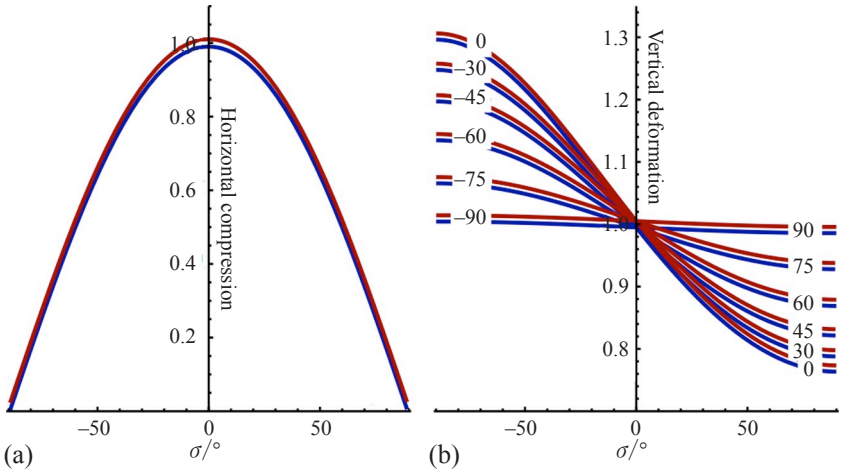

Figure 10. [In colour online.] Computed deformation of virtual and physical grids: (a) shows horizontal compression as a function of σ for 11 values of ϕ; (b) shows vertical deformation as a function of σ for the same ϕs. Results for virtual grids are depicted in red, results for physical grids in blue. The superimposed red and blue graphs are slightly offset for clarity.

An interesting result of the computations is that both horizontal compression and vertical deformation are identical for physical and corresponding virtual grids. The identical compressions and deformations imply that there is no reason to assume compensation mechanisms that are specific for picture perception. Such mechanisms have been proposed to adjust perceived aspect ratios for oblique viewing of pictures (Perkins 1973; Rosinski et al 1980; Wallach and Marshall 1986; Goldstein 1987, 1988; Halloran 1993; Vishwanath et al 2005). Vishwanath et al (2005) measured invariant aspect ratios in some conditions but not in others. The authors, however, averaged over randomly selected depicted slants, a procedure that, as the analyses of this study show, leads to unpredictable results.

The range of slopes for υ (figure 3a) and perceived slant (figures 8a and 9a) show that physical and virtual grids may slant differently in oblique viewing directions, depending on the slant of the depicted grids. It is not horizontal compression, it is perceived slant that makes depicted surfaces stand out from physical surfaces in oblique views. Physical and depicted surfaces are compressed horizontally by equal amounts. If depicted surfaces slant as physical surfaces do, the compression remains unnoticed or is experienced as natural. Certain depicted surfaces, however, slant only partially or not at all when viewed obliquely. Then compression becomes conspicuous and is experienced as unnatural (because slant and width are not well matched).

The horizontal compression of depicted stimuli explains why the perceived layout of a painted scene is almost invariant as a function of viewing direction (Goldstein 1979). Compression implies that the 2-D arrangement of objects in a picture remains fixed for all viewing directions. In contrast to this invariance, the 2-D arrangement of physical objects placed at different depth changes considerably when we view the scene from different angles.

4.4 *Motion in pictures and reverspectives*

The relationships between perceived, virtual, and screen slant (figures 3a and 9a) explain the "following" behaviour of fingers pointing out of the picture (Koenderink et al 2004). Highly slanted surfaces such as the left and right sides of a finger, are perceived as almost independent of the slant of the screen. Thus, such surfaces are pointing to the viewer irrespective of whether the screen is slanted relative to the viewer or the viewer rotates relative to the screen. In the latter case, the pointing surface will appear to move with the viewer.

The perspective hypothesis also explains the compelling motion perceived in reverspectives (Wade and Hughes 1999; Cook et al 2002; Papathomas 2002; Rogers and Gyani 2010). Figure 3a contains three dashed vertical lines. The grey line in the middle indicates virtual slants of grids depicted on a screen or canvas in frontal view. The two outer blue lines indicate virtual slants of grids painted on the side surfaces of a reverspective in frontal view. The horizontal offsets of the blue lines represent the slants of the physical surfaces of the reverspective. The slanted panels are painted with scenes such that their linear perspective opposes the physical slant (perspective- and disparity-defined slants have opposite signs). In reverspectives, the painted 45° panels portray highly slanted surfaces. Figure 3a shows that for highly slanted surfaces (with positive perspective slants at negative screen slants and vice versa) the virtual slant curves have slopes near zero. The near-zero slopes imply that the slant of the scene will hardly change during rotation of the reverspective relative to a static viewer. Or, if the reverspective is static and the viewer moves sideways, the scenes will retain their orientation relative to the viewer, which will be experienced as "following". The "following" behaviour would also be experienced if a flat physical object would retain its orientation relative to the viewer and does not require a conflict between perspective and motion parallax signals (Papathomas 2007; Cook et al 2008a, 2008b). Cook et al (2008a, 2008b) suggested that the illusory motion results from a conflict between the 2-D pictorial cues and the 3-D motion parallax. Papathomas (2007) argued that the illusory motion is the consequence of the misperceived depths relationships that give rise to a discrepancy between expected and actual optic flows. As Rogers and Gyani (2010) argued, there is no conflict between perspective as the cue for perceived *slant* and motion parallax as the cue for perceived *change in slant*. The argument goes as follows. One should realise that motion parallax is a cue for perceived depth as well as perceived motion, and that the two cues are not independent. It is a cue for perceived depth under the assumption that the viewed objects are stable and rigid. Motion parallax acting as a cue for perceived motion requires the calibration by depth that is specified by other cues. In reverspectives, linear perspective provides the perceived depth and slant and motion parallax the accompanying apparent motion. Thus, changes in the virtual slants of the depicted grids are sufficient to explain the apparent motion.

Figure 3a shows that apparent motion is also expected in highly perspective scenes painted on frontoparallel surfaces ($\sigma = 0°$), because here the slopes for large values of ϕ differ strongly from unity too. For equally large ϕs, however, deviations from unity are smaller for frontoparallel surfaces than for reverspectives, so that the apparent motion will be less compelling. Indeed, the apparent motion that has been reported during moving in front of billboards (Papathomas et al 2010) is less striking than that in front of reverspectives (Wade and Hughes 1999; Cook et al 2002; Papathomas 2002; Rogers and Gyani 2010).

The centre of projection is an important parameter of perspective-rich pictures, which, however, is not usually specified in studies of picture perception. Figure 3 shows virtual slants of the grids if properly viewed from the centre of projection. Centre of projection and viewing position coincided in the experiments of this study. Figure 11 shows computed virtual slants if the viewing position does not coincide with the centre of projection. In figure 11a the viewing distance is 4 times as long as the distance of the centre of projection to the screen and in figure 11b it is 4 times as short. Comparison with figure 3a shows that virtual slants increase with increasing viewing distances and become progressively more independent of physical slants. Even depicted grids having just a shallow slant get almost perpendicular virtual slants and thus will be perceived as attached to the viewer rather than the screen (with the exception of $\phi = 0°$). Virtual slants decrease with decreasing viewing distances. Virtual slants resemble the physical slant of the screen or the reverspective at very short viewing distances. As a result the "following" behaviour will be reduced with shorter viewing distances. This analysis predicts that there is a minimum viewing distance at which the "following" effect will vanish. In studies of reverspectives the viewing distance has been measured at which perception switches from being consistent with perspective information to being consistent with the actual 3-D structure of the reverspective (Cook et al 2002, 2008a, 2008b; Papathomas 2002, 2007; Papathomas and Bono 2004; Rogers and Gyani 2010). The various studies reported switch distances ranging from about 50 cm to 250 cm.

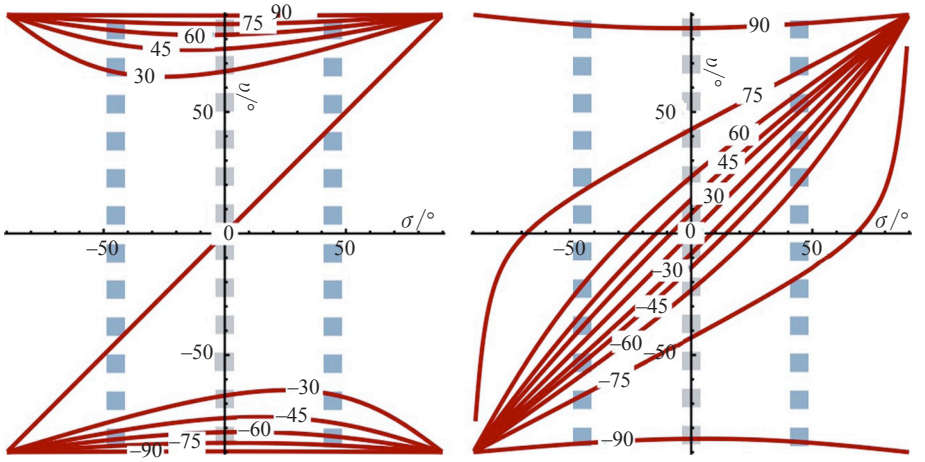

Figure 11. [In colour online.] Computed virtual slants. The viewing distance is four times (a) and one-fourth of (b) the distance between the centre of projection and the screen. See figure 2 for further explanation of the details.

Papathomas (2002) attributed the perceptual switch to changing weights of perspective and disparity-driven mechanisms caused by changing viewing distances. Rogers and Gyani (2010) argued that the switches were caused by inconsistencies in the perspective signals themselves. The present analysis of virtual slant supports the view of Rogers and Gyani (2010). Furthermore, the results of experiment 1 showed that disparity did not contribute to

the perceived slant of the grids. Analysis of disparity thresholds at the distance of perceptual switches in the experiments of Papathomas (2002) and Rogers and Gyani (2010) also shows that it is very unlikely that disparity was involved in the switches. Papathomas (2002) and Rogers and Gyani (2010) used stimuli of very different sizes of which the depth is relevant for the disparity present at the distance of perceptual switches. The reverspectives of Papathomas (2002) had a depth of 3.1 cm while those of Rogers and Gyani (2010) had a depth of 17 cm. As a result disparity differed by a factor of 5 in the two experiments. Yet, both studies reported similar switch distances ranging from 80 cm to 200 cm. It seems unlikely that disparity played a decisive role in the perceptual switches. Furthermore, disparities were as small as 2 s (at 200 cm) at switch distances in the experiments of Papathomas (2002). It should be mentioned, however, that several studies reported that switches occur at longer distances during binocular viewing (Papathomas 2002; Cook et al 2008a; Rogers and Gyani 2010). Why this occurs remains to be investigated in more detail than has been done until now.

4.5 *A thought on the linear perspective constraint and binocular disparity*

In the present computational analysis the linear perspective constraint was implemented in a computer algorithm solving the geometric equations that transform convergent lines in the proximal stimulus into parallel lines in the physical world. The experimental results show that the visual system and the computer algorithm reach similar solutions, although it is evident that the visual system may implement the constraint in a different yet unknown way. A direct consequence of the linear perspective constraint is that we cannot see convergent lines unless convergence is reinforced by other information in the visual image. Binocular disparity would be one of the obvious sources for such information. However, binocular disparity did not affect slant judgments in the present experiments. As a result, linear perspective was in binocular vision as powerful as it was in monocular vision. The absence of binocular disparity effects demonstrated again that the role of binocular disparity as a cue for depth perception is still poorly understood. Perceptual effects of binocular disparity are prominently present in certain experimental paradigms [depth from random-dot stereograms (Julesz 1960), optimal cue-combination (Knill and Saunders 2003), binocular slant rivalry (van Ee et al 2002)], less influential in others [depth from reverspectives (Papathomas 2002), hollow-mask illusion (Hill and Bruce 1993)], and absent in still others [perceptual ambiguity in Necker cubes (Erkelens 2012), Schroeder staircase (Beer 1990), slant in perspective-rich stimuli (this study)]. Generally, binocular disparity is considered to be a cue to depth similar to the monocular pictorial cues. Conceptually, however, there is a difference that is usually overlooked. Pictorial cues are 2-D measurements that the visual system uses to infer depth on the basis of prior experience. Stereopsis is a measurement in 3-D of which binocular disparity specifies depth. Depth signaled by disparity may be noisy but not, like depth from prior experience, completely false. Cue combination models that average the weights of cues based on their reliability in single-cue conditions may be inappropriate to predict perceived depth and slant in pictures containing perspective-rich stimuli. In fact, several authors have proposed alternative decision rules for conditions in which the different cues indicate very different depths or slants (van Ee et al 2003; Tyler 2004; Knill 2007). Recently, I proposed a new framework of binocular vision that explained the bi-stability of random-dot Necker cubes as well as the cooperation between disparity and monocular cues in the 3-D perception of pictures and random-dot stereograms (Erkelens 2012). In this proposition, depth derived from disparity is not weighted against depth inferred from individual pictorial cues but is a depth signal that may or may not be added to the 3-D percept obtained by monocular vision. The results of the present study suggest that depth from disparity is not included if the monocular percept is dominated by linear perspective. Why binocular disparity is included in some cases and not in others is still a mystery.

References

Beer J, 1990 "Facilitation effects with the order of presentation of the Necker cube and Schroeder staircase" *Perceptual & Motor Skills* **71** 1265–1266

Cook N D, Hayashi T, Amemiya T, Suzuki K, Leumann L, 2002 "Effects of visual-field inversions on the reverse-perspective illusion" *Perception* **31** 1147–1151

Cook N D, Yutsudo A, Fujimoto N, Murata M, 2008a, "Factors contributing to depth perception: Behavioural studies on the reverse perspective illusion" *Spatial Vision* **21** 397–405

Cook N D, Yutsudo A, Fujimoto N, Murata M, 2008b "On the visual cues contributing to pictorial depth perception" *Empirical Studies of the Arts* **26** 69–92

Cutting J E, 1987 "Rigidity in cinema seen from the front row, side aisle" *Journal of Experimental Psychology: Human Perception and Performance* **13** 323–334

Cutting J E, 1988 "Affine distortions of pictorial space: Some predictions for Goldstein (1987) that la Gournerie, 1859 might have made" *Journal of Experimental Psychology: Human Perception and Performance* **14** 305–311

Erkelens C J, 2012 "Contribution of disparity to the perception of 3D shape as revealed by bistability of stereoscopic Necker cubes" *Seeing and Perceiving* **25** 561–567

Farber J, Rosinski R R, 1978 "Geometric transformations of pictured space" *Perception* **7** 269–282

Goldstein E, 1979 "Rotation of objects in pictures viewed at an angle" *Journal of Experimental Psychology: Human Perception and Performance* **5** 78–87

Goldstein E, 1987 "Spatial layout, orientation relative to the observer, and perceived projection in pictures viewed at an angle" *Journal of Experimental Psychology: Human Perception and Performance* **13** 256–266

Goldstein E, 1988 "Geometry or not geometry—perceived orientation and spatial layout in pictures viewed at an angle" *Journal of Experimental Psychology: Human Perception and Performance* **14** 312–314

Gombrich E, 1972 "The visual image" *Scientific American* **227**(3) 82–96

Halloran T O, 1993 "The frame turns also: Factors in differential rotation in pictures" *Perception & Psychophysics* **54** 496–508

Held R T, Cooper E A, Banks M S, 2012 "Blur and disparity are complementary cues to depth" *Current Biology* **22** 426–431

Hill H, Bruce V, 1993 "Independent effects of lighting, orientation, and stereopsis on the hollow-face illusion" *Perception* **22** 887–897

Hillis J M, Watt S J, Landy M S, Banks M S, 2004 "Slant from texture and disparity cues: Optimal cue combination" *Journal of Vision* **4**(12):1, 967–992

Jacobs R A, 1999 "Optimal integration of texture and motion cues to depth" *Vision Research* **39** 3621–3629

Johnston E B, Cumming B G, Landy M S, 1994 "Integration of stereopsis and motion shape cues" *Vision Research* **34** 2259–2275

Johnston E B, Cumming B G, Parker A J, 1993 "Integration of depth modules—stereopsis and texture" *Vision Research* **33** 813–826

Julesz B, 1960 "Binocular depth perception of computer generated patterns" *Bell Systems Technical Journal* **39** 1125–1162

Knill D C, 2007 "Robust cue integration: A Bayesian model and evidence from cue-conflict studies with stereoscopic and figure cues to slant" *Journal of Vision* **7**(7):5, 1–24

Knill D C, Saunders J A, 2003 "Do humans optimally integrate stereo and texture information for judgments of surface slant? *Vision Research* **43** 2539–2558

Koenderink J J, van Doorn A J, Kappers A M L, Todd J T, 2004 "Pointing out of the picture" *Perception* **33** 513–530

Kubovy M, 1986 *The Psychology of Perspective and Renaissance Art* (Cambridge: Cambridge University Press)

Landy M S, Maloney L T, Johnston E B, Young M, 1995 "Measurement and modeling of depth cue combination: In defense of weak fusion" *Vision Research* **35** 389–412

Nicholls A L, Kennedy J M, 1993 "Angular subtense effects on perception of polar and parallel projections of cubes" *Perception & Psychophysics* **54** 763–772

Papathomas T V, 2002 "Experiments on the role of painted cues in Hughes's reverspectives" *Perception* **31** 521–530

Papathomas T V, 2007 "Art pieces that 'move' in our minds: an explanation of illusory motion based on depth reversal" *Spatial Vision* **21** 79–95

Papathomas T V, Baker N, Yeshua A S, Zhuang X, Ng A, 2012 "The ingenious Mr. Hughes: Combining forced, flat, and reverse perspective all in one art piece to pit objects against surfaces" *i-Perception* **3** 182–187

Papathomas T V, Bono L M, 2004 "Experiments with a hollow mask and reverspective: Top–down influences in the inversion effect for 3-D stimuli" *Perception* **33** 1129–1138

Papathomas T V, Kourtzi Z, Welchman A E, 2010 "Perspective-based illusory movement in a flat billboard—an explanation" *Perception* **39** 1086–1093

Perkins D N, 1973 "Compensating for distortion in viewing pictures obliquely" *Perception & Psychophysics* **14** 13–18

Rogers B, Gyani A, 2010 "Binocular disparities, motion parallax, and geometric perspective in Patrick Hughes's 'reverspectives': Theoretical analysis and empirical findings" *Perception* **39** 330–348

Rosinski R R, Mulholland T, Degelman D, Farber J, 1980 "Picture perception: An analysis of visual compensation" *Perception & Psychophysics* **28** 521–526

Saunders J A, Backus B T, 2007 "Both parallelism and orthogonality are used to perceive 3D slant of rectangles from 2D images" *Journal of Vision* **7**(6):7, 1–11

Sedgwick H A, 1980 "The geometry of spatial layout in pictorial representation", in *The Perception of Pictures* Ed. M A Hagen (volume 1) (New York: Academic Press) pp 33–90

Sedgwick H A, 1991 "The effect of viewpoint on the virtual space of pictures", in *Pictorial Communication in Virtual and Real Environments* Eds S R Ellis, M K Kaiser, A C Grunwald (London: Taylor & Francis) pp 460–469

Slyce J, 1998 *Patrick Hughes: Perverspective* (London: Momentum)

Stevens K A, Lees M, Brookes A, 1991 "Combining binocular and monocular curvature features" *Perception* **20** 425–440

Todorovic D, 2008 "Is pictorial perception robust? The effect of the observer vantage point on the perceived depth structure of linear-perspective images" *Perception* **37** 106–125

Tyler C W, 2004 "Theory of texture discrimination based on higher-order perturbations in individual texture samples" *Vision Research* **44** 2179–2186

van Ee R, 2005 "Dynamics of perceptual bi-stability for stereoscopic slant rivalry and a comparison with grating, house-face, and Necker-cube rivalry" *Vision Research* **45** 29–40

van Ee R, Adams W J, Mamassian P, 2003 "Bayesian modelling of perceived slant in bistable stereoscopic perception" *Journal of the Optical Society of America* **20** 1398–1406

van Ee R, van Dam L C, Erkelens C J, 2002 "Bi-stability in perceived slant when binocular disparity and monocular perspective specify different slants" *Journal of Vision* **2**(9):2, 597–607

Vishwanath D, Girshick A R, Banks M S, 2005 "Why pictures look right when viewed from the wrong place" *Nature Neuroscience* **8** 1401–1410

Wade N J, Hughes P, 1999 "Fooling the eyes: *trompe l'oeil* and reverse perspective" *Perception* **28** 1115–1119

Wallach H, Marshall F J, 1986 "Shape constancy in pictorial representation" *Perception & Psychophysics* **39** 233–235

Young M J, Landy M S, Maloney L T, 1993 "A perturbation analysis of depth perception from combinations of texture and motion cues" *Vision Research* **33** 2685–2695