SPACETIME MESHING WITH ADAPTIVE COARSENING AND REFINEMENT

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We propose a new algorithm for constructing finite-element meshes suitable for spacetime discontinuous Galerkin solutions of linear hyperbolic PDEs. Our new method is a generalization of the 'Tent Pitcher' algorithms of Üngör and Sheffer [3] and Erickson *et al.* [2]. Given a simplicially-meshed domain Ω in $\mathbb{R}^d$ and a target time value T, our method constructs a mesh of the spacetime domain $\Omega \times [0, T]$ in $\mathbb{R}^{d+1}$ using an advancing front method. Elements are added to the evolving mesh in small patches by moving a vertex of the front forward in time. Spacetime discontinuous Galerkin methods [4] allow the numerical solution within each patch to be computed as soon as the patch is created. This work introduces new mechanisms for adaptively coarsening and refining the front in response to error estimates returned by the numerical code. A change in the front induces a corresponding refinement or coarsening of future elements in the spacetime mesh.

If the numerical code indicates that the estimated solution error within a spacetime element exceeds a given tolerance, our algorithm immediately subdivides the facet of that element on the front. We also immediately subdivide any element of the front if any of it neighbors have been subdivided more than once. When $d = 2$, each triangular element of the front has one marked vertex. We refine such an element by cutting from the marked vertex to the midpoint of the opposite edge. We mark a vertex in each of the two new triangles so that the triangles created by further subdivision fall into a small number of homothetic classes. This process is similar but not identical to Rivara refinement [2]. Our subdivision rules limit both the number of elements in each patch without degrading the quality of the elements on the front. When allowed by the numerical code, our algorithm schedules for coarsening a pair of adjacent elements created as a result of an earlier refinement.

Neither the front nor the resulting spacetime mesh is a conforming mesh—that is, its elements do not necessarily form a cell complex. However, discontinuous Galerkin methods [4] explicitly accommodate discontinuities in the solution fields across element boundaries. Therefore, unlike conventional finite-element methods, they do not require conforming meshes.

References

[1] J. Erickson, D. Guoy, A. Üngör, and J. Sullivan, "Building space-time meshes over arbitrary spatial domains," *Proc. 11th Int. Meshing Roundtable*, pp. 391–402, 2002.
[2] M. C. Rivara, "New mathematical tools and techniques for the refinement and/or improvement of unstructured triangulations," *Proc. 5th Int. Meshing Roundtable*, pp. 77–86, 1996.
[3] A. Üngör and A. Sheffer, "Tent-Pitcher: A meshing algorithm for space-time discontinuous Galerkin methods," *Proc. 9th Int. Meshing Roundtable*, pp. 111–122, 2000.
[4] L. Yin, A. Acharya, N. Sobh, R. Haber, and D. A. Tortorelli, "A space-time discontinuous Galerkin method for elastodynamic analysis," *Discontinuous Galerkin Methods: Theory, Computation, and Applications*, pp. 459–464. Lecture Notes Comput. Sci. Eng. 11, Springer-Verlag, 2000.