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A Fitting Method for Establishing an Aeroengine's State Variable Model Based on Process Identification

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Abstract. The bias derivative method for aeroengine's State Variable Model(SVM) doesn't have a satisfying accuracy. This method usually needs a linear modification to achieve a higher accuracy. In order to obtain a SVM with a good accuracy, this paper proposes a process identification based method. In this method, the coefficient matrices of the SVM are identified based on the input and output of the nonlinear model, according to the principle that the step responses of the SVM and the nonlinear model should be consistent. Then, the SVMs of small perturbation about steady operating points of an engine are established. Simulation results show that the SVMs established by the process identification based method have a good fidelity both in terms of steady and dynamic performance.

Introduction

The linear control theory plays an important role in modern aeroengine control. In linear control theory, the controller parameters are designed in allusion to the State Variable Model(SVM). Traditionally, the SVMs of small perturbation about steady operating points of an engine are established by means of bias derivative method. This method usually needs a modification to achieve a satisfying accuracy[1,2], especially for the dynamic process. However, it is not convenient to modify the dynamic response of the SVM[3,4].

This paper makes a research on establishing the SVMs of small perturbation about steady operating points of an aeroengine. In order to obtain a SVM of an aeroengine with a high fidelity, this paper proposes a process identification based method. In this method, the coefficient matrices of the SVM are identified based on the input and output of the nonlinear model, according to the principle that the responses to a step signal of the SVM and the nonlinear model should be consistent. Then, the SVM is verified based on its step response at 85% operating state. The simulation result shows that the SVM established by process identification based method is of desirable fidelity, in terms of steady output and dynamic behavior.

This paper is organized as follows. First, we introduce the process identification based method in Section 2, including the theory and solving algorithm. Then, an example is illustrated in Section 3, including the simulation result of the SVM. Finally, according to the simulation result, the conclusion is made in Section 4.

Process Identification based Method

Process Identification is a theory to establish the model of producing process, by means of the input and output data of the research object. It comprises non-parametric model identification and parametric model identification. As for establishing aeroengine's SVM, we can consider it as a parametric model identification problem. The pending SVM can be regarded as a known structure model with unknown parameters. The unknown parameters can be confirmed in manner of minimizing the error's rule function of the process and the model.

To acquire the SVMs of small perturbation about steady operating points, the State Variable Model is to match the step response process of the nonlinear model.

State Variable Model. In this case, the second-order SVM of a civil turbofan engine is described as follows,

$$\begin{bmatrix} \Delta\dot{N}_H \\ \Delta\dot{N}_L \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} [\Delta W_F] \tag{1}$$

$$\begin{bmatrix} \Delta N_H \\ \Delta N_L \\ \Delta P_{T3} \\ \Delta P_{T5} \\ \Delta T_{T5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ c_{31} & c_{32} \\ c_{41} & c_{42} \\ c_{51} & c_{52} \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ d_{31} \\ d_{41} \\ d_{51} \end{bmatrix} [\Delta W_F] \tag{2}$$

Where N_H, N_L respectively denote the high pressure shaft speed and the low pressure shaft speed. P_{T3}, P_{T5} and T_{T5} respectively denote the exit total pressure of the high pressure compressor, the exit total pressure of the low pressure turbine and the exit total temperature of the low pressure turbine. W_F denotes the main fuel flow.

SVM's Step Response. In this section, we deduce the SVM's solutions of step response in time domain. The characteristic equation of state equation (1) is $|\lambda I - A| = 0$, and the characteristic value λ_1, λ_2 satisfy the conditions below,

$$\lambda_1 + \lambda_2 = a_{11} + a_{22} \tag{3}$$

$$\lambda_1 \lambda_2 = a_{11}a_{22} - a_{12}a_{21} \tag{4}$$

Set the control variable ΔW_F to be a step signal as Equation (5),

$$\Delta W_f(t) = \begin{cases} \Delta f, t > 0 \\ 0, t < 0 \end{cases} \tag{5}$$

The step response of output in time domain can be described as follows,

$$\begin{bmatrix} \Delta N_H(t) \\ \Delta N_L(t) \\ \Delta P_{T3}(t) \\ \Delta P_{T5}(t) \\ \Delta T_{T5}(t) \end{bmatrix} = \begin{bmatrix} \Delta f\left[\frac{a_{21}b_{11}-a_{11}b_{21}}{\lambda_1\lambda_2} + \frac{(\lambda_1-a_{11})b_{21}+a_{21}b_{11}}{\lambda_1(\lambda_1-\lambda_2)}e^{\lambda_1 t} + \frac{(\lambda_2-a_{11})b_{21}+a_{21}b_{11}}{\lambda_2(\lambda_2-\lambda_1)}e^{\lambda_2 t}\right] \\ \Delta f\left[\frac{a_{12}b_{21}-a_{22}b_{11}}{\lambda_1\lambda_2} + \frac{(\lambda_1-a_{22})b_{11}+a_{12}b_{21}}{\lambda_1(\lambda_1-\lambda_2)}e^{\lambda_1 t} + \frac{(\lambda_2-a_{22})b_{11}+a_{12}b_{21}}{\lambda_2(\lambda_2-\lambda_1)}e^{\lambda_2 t}\right] \\ \Delta f\left[d_{31} - \frac{b_{11}(c_{31}a_{22}-c_{32}a_{21})+b_{21}(c_{32}a_{11}-c_{31}a_{12})}{\lambda_1\lambda_2} + \frac{b_{11}[c_{31}(s_1-a_{22})+c_{32}a_{21}]+b_{21}[c_{32}(s_1-a_{11})+c_{31}a_{12}]}{\lambda_1(\lambda_1-\lambda_2)}e^{\lambda_1 t} + \frac{b_{11}[c_{31}(s_2-a_{22})+c_{32}a_{21}]+b_{21}[c_{32}(s_2-a_{11})+c_{31}a_{12}]}{\lambda_2(\lambda_2-\lambda_1)}e^{\lambda_2 t}\right] \\ \Delta f\left[d_{41} - \frac{b_{11}(c_{41}a_{22}-c_{42}a_{21})+b_{21}(c_{42}a_{11}-c_{41}a_{12})}{\lambda_1\lambda_2} + \frac{b_{11}[c_{41}(s_1-a_{22})+c_{42}a_{21}]+b_{21}[c_{42}(s_1-a_{11})+c_{41}a_{12}]}{\lambda_1(\lambda_1-\lambda_2)}e^{\lambda_1 t} + \frac{b_{11}[c_{41}(s_2-a_{22})+c_{42}a_{21}]+b_{21}[c_{42}(s_2-a_{11})+c_{41}a_{12}]}{\lambda_2(\lambda_2-\lambda_1)}e^{\lambda_2 t}\right] \\ \Delta f\left[d_{51} - \frac{b_{11}(c_{51}a_{22}-c_{52}a_{21})+b_{21}(c_{52}a_{11}-c_{51}a_{12})}{\lambda_1\lambda_2} + \frac{b_{11}[c_{51}(s_1-a_{22})+c_{52}a_{21}]+b_{21}[c_{52}(s_1-a_{11})+c_{51}a_{12}]}{\lambda_1(\lambda_1-\lambda_2)}e^{\lambda_1 t} + \frac{b_{11}[c_{51}(s_2-a_{22})+c_{52}a_{21}]+b_{21}[c_{52}(s_2-a_{11})+c_{51}a_{12}]}{\lambda_2(\lambda_2-\lambda_1)}e^{\lambda_2 t}\right] \end{bmatrix}$$

Nonlinear Model's Step Response. Input the step signal (5) to the nonlinear model at the steady operating point, we obtain the step response of output in relative increment.

$$\begin{bmatrix} \Delta N_H(t_i) \\ \Delta N_L(t_i) \\ \Delta P_{T3}(t_i) \\ \Delta P_{T5}(t_i) \\ \Delta T_{T5}(t_i) \end{bmatrix}_{nonlinear} = \begin{bmatrix} N_H(t_i) - N_H(0) \\ N_L(t_i) - N_L(0) \\ P_{T3}(t_i) - P_{T3}(0) \\ P_{T5}(t_i) - P_{T5}(0) \\ T_{T5}(t_i) - T_{T5}(0) \end{bmatrix}_{nonlinear} \tag{6}$$

Where t_i is the sampling time, $t_i = [0, T, 2T..NT]$, T denotes the sampling period. $N_H(0)$, $N_L(0)$, $P_{T3}(0)$, $P_{T5}(0)$, and $T_{T5}(0)$ respectively denote the parameters of steady operating point.

Least Square Solutions. After we acquire the step response expression of the SVM and the experiment data of the nonlinear model's step response, we first construct the step response error equations of the SVM and nonlinear model. The principle of the parameter identification is to minimize the response error, to achieve this purpose, we solve the error equations making use of the Least Square Method. The errors between the SVM and nonlinear model is as follow,

$$\begin{bmatrix} Err_{\Delta N_H}(t_k) \\ Err_{\Delta N_L}(t_k) \\ Err_{\Delta P_{T3}}(t_k) \\ Err_{\Delta P_{T5}}(t_k) \\ Err_{\Delta T_{T5}}(t_k) \end{bmatrix} = \begin{bmatrix} \dfrac{[\Delta N_H(t_k)]_{SVM} - [N_H(t_k) - N_H(0)]_{nonlinear}}{N_H(t_k)_{nonlinear}} \\ \dfrac{[\Delta N_L(t_k)]_{SVM} - [N_L(t_k) - N_L(0)]_{nonlinear}}{N_L(t_k)_{nonlinear}} \\ \dfrac{[\Delta P_{T3}(t_k)]_{SVM} - [P_{T3}(t_k) - P_{T3}(0)]_{nonlinear}}{P_{T3}(t_k)_{nonlinear}} \\ \dfrac{[\Delta P_{T5}(t_k)]_{SVM} - [P_{T5}(t_k) - P_{T5}(0)]_{nonlinear}}{P_{T5}(t_k)_{nonlinear}} \\ \dfrac{[\Delta T_{T5}(t_k)]_{SVM} - [T_{T5}(t_k) - T_{T5}(0)]_{nonlinear}}{T_{T5}(t_k)_{nonlinear}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

Where $k = [1,2,3 \ldots K]$. Thus, we can choose several character points during the step response process. And we can deduce the characteristic value λ_1, λ_2 equations from Equation(3) and Equation(4),

$$Err_{SUM(\lambda)} = \frac{\lambda_1 + \lambda_2 - (a_{11} + a_{22})}{a_{11} + a_{22}} = 0 \tag{8}$$

$$Err_{PRODUCT(\lambda)} = \frac{\lambda_1 \lambda_2 - a_{11}a_{22} + a_{12}a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = 0 \tag{9}$$

In order to identify the unknown parameters, we can solve the simultaneous equations (7),(8)and (9), totally including $5K + 2$ equations. In this case, the Least Square Method is utilized to achieve the unknown parameters' approximate solutions.

Linear Modification for SVM

It is hard to avoid producing error during the linearization process of nonlinear model. In this section, a modification approach is proposed to modify the static error of the SVM. The matrices A and C reflect the dynamic performance of the step response. Otherwise, the matrices B and D reflect the static performance of the step response. The SVM obtained by the identification approach has a good performance in dynamic process. So, we only modify the matrices B and D to eliminate the static error.

The principle of the modification is that the matrices B and D are modified according to the step response of the nonlinear model. The static output of the SVM ought to be consistent with that of the nonlinear model. For this case, the ultimate static value of the nonlinear model's step response is used to modify the SVM.

After the static state of the step response, the state equation with modification can be described as follows,

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \end{bmatrix} + \begin{bmatrix} k_{11}b_{11} \\ k_{21}b_{21} \end{bmatrix} [\Delta W_F]$$

Where $\Delta N_H, \Delta N_L$ denote static output of the nonlinear model's step response. k_{11}, k_{21} denote the modification coefficients of matrix B.

To solve k_{11}, k_{21} as unknown variable, we can get the expression for k_{11} and k_{21}.

$$k_{11} = -\frac{a_{11} \cdot \Delta N_H + a_{12} \cdot \Delta N_L}{b_{11} \cdot \Delta W_F}$$

$$k_{21} = -\frac{a_{21} \cdot \Delta N_H + a_{22} \cdot \Delta N_L}{b_{21} \cdot \Delta W_F}$$

In addition, the output equation with modification can be described as follows,

$$\begin{bmatrix}\Delta N_H\\ \Delta N_L\\ \Delta P_{T3}\\ \Delta P_{T5}\\ \Delta T_{T5}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\\ c_{31} & c_{32}\\ c_{41} & c_{42}\\ c_{51} & c_{52}\end{bmatrix}\begin{bmatrix}\Delta N_H\\ \Delta N_L\end{bmatrix}+\begin{bmatrix}0\\ 0\\ m_{31}d_{31}\\ m_{41}d_{41}\\ m_{51}d_{51}\end{bmatrix}\left[\Delta W_F\right]$$

Where the vector $\left[\Delta N_H \quad \Delta N_L \quad \Delta P_3 \quad \Delta P_5 \quad \Delta T_5\right]^T$ denotes the static output of the nonlinear model's step response. m_{31}, m_{41} and m_{51} denote the modification coefficients of matrix D.

To solve m_{31}, m_{41} and m_{51} as unknown variable, we can get the expression for m_{31}, m_{41} and m_{51}.

$$m_{31}=\frac{\Delta P_{T3}-c_{31}\cdot\Delta N_H-c_{32}\cdot\Delta N_L}{d_{31}\cdot\Delta W_F}$$

$$m_{41}=\frac{\Delta P_{T5}-c_{41}\cdot\Delta N_H-c_{42}\cdot\Delta N_L}{d_{41}\cdot\Delta W_F}$$

$$m_{51}=\frac{\Delta T_{T5}-c_{51}\cdot\Delta N_H-c_{52}\cdot\Delta N_L}{\cdot d_{51}\cdot\Delta W_F}$$

Thus, the SVM is modified according to the static output of nonlinear model.

Example and Simulation

In this section, a civil turbofan aeroengine's SVMs are established over the operating states. And several SVMs of specified steady points are verified. To obtain the SVMs, a small 2% step perturbation of main fuel is added to the nonlinear model at the steady operating points. The simulation time is 2 seconds and the fundamental sample time is 20 milliseconds. As the Section 2 described, select 10 characteristic data during the process of step response($K=10$), the sampling time $t_i=\left[0.1s,0.2s,0.4s,0.5s,0.6s,0.7s,0.8s,1.2s,1.6s,2.0s\right]$. As a result, totally 52 error equations are established. Solve the error equations using the function lsqnonlin(•) in Matlab, the pending parameters of A,B,C,D can be obtained.

Under the sea level standard-day condition, we obtain the matrices of the SVM at the 85% steady operating point,

$$A=\begin{bmatrix}-1.2316 & -0.5008\\ 2.1455 & -11.2358\end{bmatrix}\quad B=\begin{bmatrix}2675.4623\\ 5934.2869\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0\\ 0 & 1\\ 417.1219 & -53.5425\\ 26.0546 & -75.1677\\ -0.0833 & -0.0463\end{bmatrix}\quad D=\begin{bmatrix}0\\ 0\\ 408488.9608\\ 38339.8943\\ 301.9755\end{bmatrix}$$

After modification, the coefficient matrices B and D are changed,

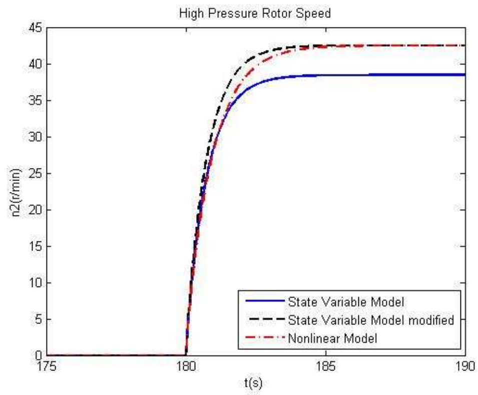
$$B=\begin{bmatrix}2915.2906\\ 5602.4717\end{bmatrix}\quad D=\begin{bmatrix}0\\ 0\\ 350897.098\ \ 7\\ 32701.0417\\ 307.7666\end{bmatrix}$$

For the sake of verification, the step response of the SVM is compared with that of the nonlinear model. The simulation result is shown in Fig.1. Fig.1 illustrates that the step response of the SVM is well consistent with the nonlinear model's output, both in terms of the dynamic process and steady value. The simulation result shows that after modification, the static performance of the SVM is dramatically improved.

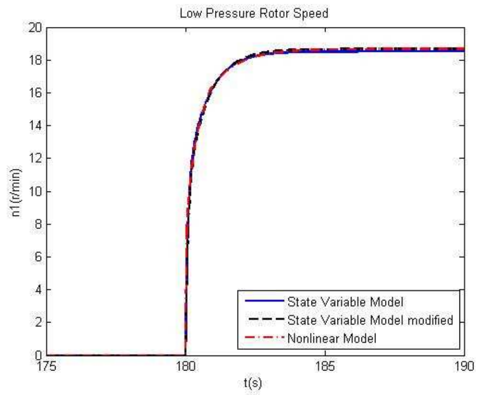
Conclusion

This paper proposes a process identification based method to establish the aeroengine's SVM. First of all, the modeling theory is described detailedly in Section 2. Furthermore, an civil turbofan aeroengine's SVM is established over the operating states. Finally, the SVM is verified by the nonlinear model based on their step response. The simulation result shows that the SVM established by process identification based method is of desirable fidelity, in terms of steady output and dynamic behavior.

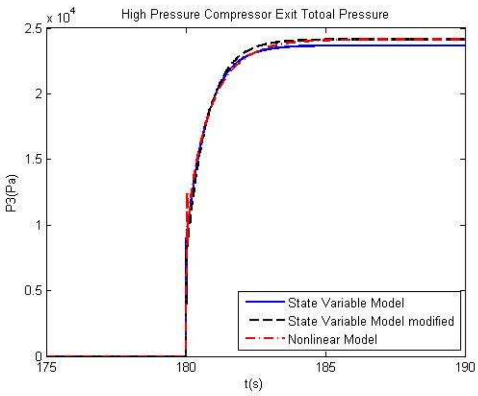
APPENDIX: Figures and Tables



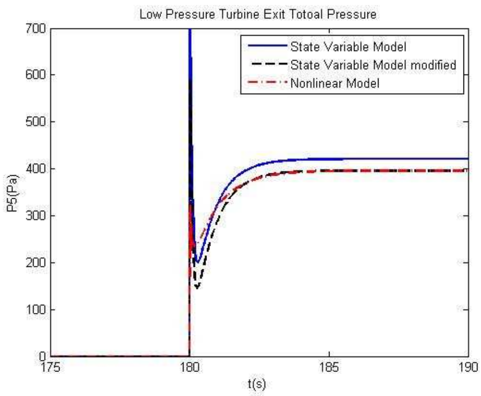
(a) Step Response of N_H



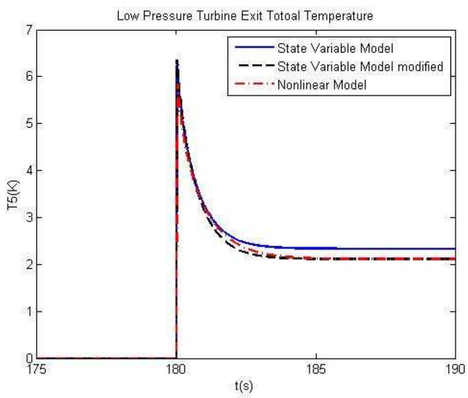
(b) Step Response of N_L



(c) Step Response of P3



(d) Step Response of P5



(e) Step Response of T5

Fig.1. N_H%=0.85, step response comparison between SVM and nonlinear model

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