Progress of Theoretical Physics, Vol. 47, No. 5, May 1972

Inclusive Reactions in the Regge Pole Model and Exotic Resonances

Yasuo HARA

Department of Physics, Tokyo University of Education, Tokyo

(Received November 22, 1971)

The Regge pole model and duality are applied to processes of the type $a+b\to c+$anything. The exotic and nonexotic three-particle channels are defined. The existence of resonances in the $K^+\pi^+\pi^0$, $p\bar{n}\pi^+$, $\bar{K}^0\pi^+p$ and $K^-\pi^-n$ channels are predicted in the model.

§ 1. Introduction

Mueller has shown[1] that the differential cross section of the particle c produced in the process

$$a+b\to c+\text{anything} \tag{1}$$

is expressed in terms of a three-particle $(ab\bar{c})\to$three-particle $(ab\bar{c})$ amplitude, expanded the amplitude into $O(2,1)$ representations, and obtained the existence of limiting fragmentation and pionization if the Pomeron with $\alpha_P(0)=1$ is the dominant $O(2,1)$ singularity at high energy;

$$k_0\frac{d^3\sigma}{d\boldsymbol{k}}\xrightarrow[s\to\infty]{}f(k_\parallel{}^L, k_\perp) \tag{2·a}$$

for target fragmentation and

$$k_0\frac{d^3\sigma}{d\boldsymbol{k}}\xrightarrow[s\to\infty]{}f(k_\parallel{}^P, k_\perp) \tag{2·b}$$

for projectile fragmentation, where $k_\perp$ is the transverse momentum of the produced particle c and $k_\parallel{}^L$ and $k_\parallel{}^P$ are the longitudinal momentum of c in the laboratory and projectile frame, respectively.

If we also assume that forward elastic three-body amplitudes are dominated by the usual Regge singularities, the approximately exchange-degenerate meson trajectories $(\rho, f, \omega, A2)$ with $\alpha(0)\approx 0.5$ as well as the Pomeron, a definite prediction for the manner in which the limit (2) is approached is possible.[2]

By assuming duality and the hypothesis of the nonexistence of exotic resonances, Chan et al. have proposed[2] that the cross section for the process $a+b\to c+$anything is virtually independent of energy when $(ab\bar{c})$ is an exotic channel. Furthermore, it has been shown[3] that some of the cross sections of the processes for which $(ab\bar{c})$ are not exotic are virtually independent of energy if we use

the factorization property of the residues of the Regge poles.

It is well known[4)~6)] that we encounter difficulties if we apply duality and the hypothesis of the nonexistence of exotic resonances to baryon-antibaryon scattering and to all the processes where pseudoscalar-meson exchange can make an appreciable contribution. For example, $m_\pi = m_\eta$ is required from the analysis of $V_9P_8 \to P_8V_9$ reactions. Therefore, it is expected that the amplitudes in some of exotic $P_8P_8P_8$ channels which can communicate with P_8V_9 channels and the amplitudes in some of exotic $B_8\bar{B}_8P_8$ channels are dependent on energy in general.

The purpose of this article is (i) to study the consequences of the Regge pole model and duality on the cross sections (2), (ii) to derive relations among the differential cross sections and (iii) to obtain information about exotic resonances by comparing these relations with experimental results.

In § 2 we explain our assumptions. In § 3 we study the projectile fragmentation of mesons and in § 4 we study the fragmentation of baryons. Possible exotic resonances in three particle channels are discussed in connection with the quark model in § 5. A summary is given in § 6.

Some of the results of this article have been reported in our previous letter.[7)]

§ 2. Assumptions

At first we mention our assumptions.

(1) Regge pole model. We assume that the imaginary part of forward scattering amplitude is dominated by the usual Regge singularities, (i) the Pomeron with $\alpha_P(0)=1$ and (ii) the approximately exchange-degenerate meson trajectories $(f, \omega, A2, \rho)$ with $\alpha(0)\approx 0.5$ and (f', ϕ) with $\alpha(0)\approx 0$. We write the pole residues of these trajectories with a particle a or with a two-particle state $(a\bar{c})$ at $t=0$ as P_a, F_a, O_a, A_a, R_a, F_a' and Φ_a or $P(a\bar{c})$, $F(a\bar{c})$, $O(a\bar{c})$, $A(a\bar{c})$, $R(a\bar{c})$, $F'(a\bar{c})$ and $\Phi(a\bar{c})$, respectively. Then, the total cross section of the ab collision and that of the collision of b and $(a\bar{c})$ are expressed as

$$\sigma_T(ab) = P_aP_b + (F_aF_b - O_aO_b + A_aA_b - R_aR_b)s^{-1/2} + (F_a'F_b' - \Phi_a\Phi_b)s^{-1} \tag{3}$$

and[2)]

$$\begin{aligned}\sigma_T((a\bar{c})b) = {} & P(a\bar{c})P_b \\ & + [F(a\bar{c})F_b - O(a\bar{c})O_b + A(a\bar{c})A_b - R(a\bar{c})R_b]s^{-1/2} \\ & + [F'(a\bar{c})F_b' - \Phi(a\bar{c})\Phi_b]s^{-1},\end{aligned} \tag{4}$$

respectively. There are the following relations,

$$\begin{aligned}& P_c = P_{\bar{c}}, \quad F_c = F_{\bar{c}}, \quad O_c = -O_{\bar{c}}, \quad A_c = A_{\bar{c}}, \quad R_c = -R_{\bar{c}}, \\ & F_c' = F_{\bar{c}}', \quad \Phi_c = -\Phi_{\bar{c}}, \quad P(a\bar{c}) = P(\bar{a}c), \quad F(a\bar{c}) = F(\bar{a}c), \\ & O(a\bar{c}) = -O(\bar{a}c), \quad A(a\bar{c}) = A(\bar{a}c), \quad R(a\bar{c}) = -R(\bar{a}c), \\ & F'(a\bar{c}) = F'(\bar{a}c) \quad \text{and} \quad \Phi(a\bar{c}) = -\Phi(\bar{a}c).\end{aligned} \tag{5}$$

However, there are no relations between $P(ab)$ and $P(ba)$ unless the helicity independence of the Pomeron residues holds.[8]

(2) Duality and nonexistence of exotic resonances. For reactions in which the direct channel have exotic quantum numbers, these assumptions imply that the imaginary part of the forward amplitude is due entirely to the Pomeron exchange, which in turn imply that cross sections for such processes are virtually independent of energy. Only unitary singlets and octets have been found as resonances of two pseudoscalar-mesons and only unitary singlets, octets and decimets have been found as baryon resonances. Therefore, from the following expressions of the total cross sections of exotic channels (i.e., channels in which two-body resonances have not been found),

$$\begin{aligned}
&\sigma_T(\pi^+\pi^+) = P_\pi^{\,2} + (F_\pi^{\,2} - R_{\pi^+}^2)\, s^{-1/2},\\
&\sigma_T(K^+\pi^+) = P_K P_\pi + (F_K F_\pi - R_{K^+} R_{\pi^+})\, s^{-1/2},\\
&\sigma_T(K^+K^{+,0}) = P_K^{\,2} + (F_K^{\,2} - O_K^{\,2} \pm A_{K^+}^2 \mp R_{K^+}^2)\, s^{-1/2} + (F_{K'}^{\,2} - \Phi_K^{\,2})\, s^{-1}
\end{aligned} \tag{6}$$

and

$$\sigma_T(K^{+,0}p) = P_K P_N + (F_K F_N - O_K O_N \pm A_{K^+} A_p \mp R_{K^+} R_p)\, s^{-1/2},$$

we find

$$\begin{aligned}
&F_\pi = R_{\pi^+} \equiv P,\\
&F_K = O_K = A_{K^+} = R_{K^+} \equiv K,\\
&F_{K'} = -\Phi_K \equiv K'
\end{aligned} \tag{7}$$

and

$$F_N = O_N \equiv F, \qquad A_p = R_p = -A_n = -R_n \equiv A,$$

where we have defined that F_N, O_N, A_p, R_p, $F_{K'}$ and $-\Phi_K$ are all positive and have made use of the experimental results,

$$\sigma_T(\pi^-p) > \sigma_T(\pi^+p) > \sigma_T^{\infty}(\pi N) \tag{8}$$

and

$$\sigma_T(K^-p) > \sigma_T(K^-n) > \sigma_T(K^+p).$$

According to a recent analysis,[9] the total cross sections are expressed as follows,

$$\begin{aligned}
&\sigma_T(\pi^-p) = 21.3 + 17.6\nu^{-1/2}, \ \text{(mb)}\\
&\sigma_T(\pi^+p) = 21.3 + 11.2\nu^{-1/2},\\
&\sigma_T(K^-p) = 17.1 + 17.1\nu^{-1/2},\\
&\sigma_T(K^-n) = 17.1 + 11.45\nu^{-1/2},\\
&\sigma_T(K^+p) = \sigma_T(K^+n) = 17.1,
\end{aligned}$$

$$\sigma_T(\bar{p}p) = 37.4 + 50.7\nu^{-1/2}$$

and (9)

$$\sigma_T(pp) = 37.4 + 7.4\nu^{-1/2}$$

in the Regge pole model, where ν is the laboratory energy in GeV.

Since

$$\sigma_T(K^-N^{+,0}) = P_K P_N + 2K(F \pm A)$$

and (10)

$$\sigma_T(\pi^{\mp}p) = P_\pi P_N + P(F \pm A),$$

we find

$$P \approx 2K > 0 \tag{11}$$

and

$$A/F \approx 1/5 \tag{12}$$

from (9). In the following we assume that

$$P = 2K. \tag{13}$$

If we also assume that the amplitudes in two-baryon channels are virtually independent of energy, we can derive relations among pole residues of other baryons.[5] The residues of the Regge trajectories with various single-particles thus obtained are tabulated in Appendix A.

(3) Possible exotic resonances. If we substitute the pole residues given in Appendix A into the expression (3) for baryon-antibaryon total cross sections, we find that total cross sections of some of the so-called exotic channels are energy dependent. The dependence of the high energy behavior of the total cross sections on the quantum numbers Y and Q are shown in Fig. 1. From duality and the energy dependence of the $B_8\bar{B}_8$ total cross sections, the existence of meson resonances which belong to the 27 (and 10 $+10^*$) dimensional representation of the $SU(3)$ sym-

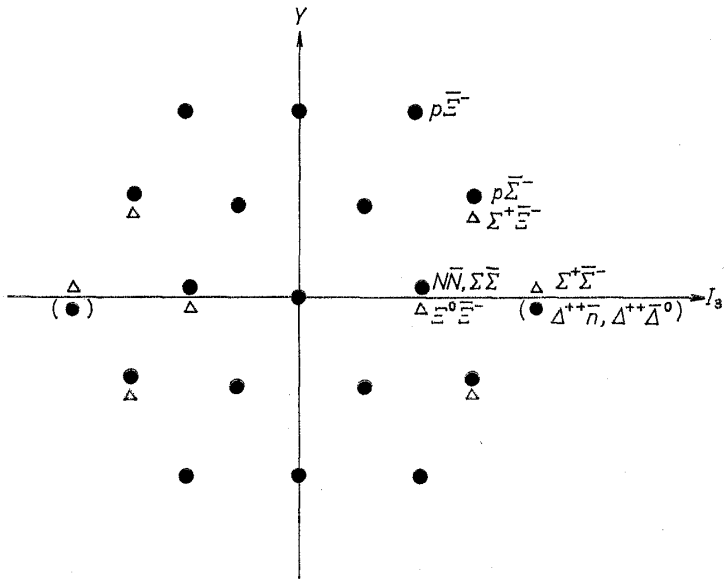

Fig. 1. Energy dependence of $B_8\bar{B}_8$ total cross sections.
●; $\sigma_T(a\bar{b}) = P_a P_b + O(s^{-1/2})$,
△; $\sigma_T(a\bar{b}) = P_a P_b + O(s^{-1})$.

metry is required. If these resonances have unnatural parity ($P=-(-1)^J$), they cannot decay into two pseudoscalar mesons. The existence of the exotic resonances with unnatural parity will solve the difficulties associated with the exchange of pseudoscalar mesons.

Therefore, in this article we assume the existence of 27-plets (and $10+10^*$-plets) of meson resonances with unnatural parity and assume that the amplitudes in the corresponding channels are dependent on energy.

§ 3. Projectile fragmentation

If the laboratory momentum of the particle c produced in the process $a+b \to c+$anything is not large, the particle c is called a target-fragment.[10] In this case the asymptotic behavior of the single particle distribution is expressed as

$$k_0\frac{d^3\sigma}{d\boldsymbol{k}}=P_aP(b\bar{c})+A(a(b\bar{c}))s^{-1/2}+B(a(b\bar{c}))s^{-1}, \tag{14}$$

where

$$A(a(b\bar{c}))=F_aF(b\bar{c})-O_aO(b\bar{c})+A_aA(b\bar{c})-R_aR(b\bar{c}) \tag{15}$$

and

$$B(a(b\bar{c}))=F_a'F'(b\bar{c})-\Phi_a\Phi(b\bar{c}) \tag{16}$$

in our model. The $P(b\bar{c})$, etc., are functions of $k_{\parallel}^L$ and $k_{\perp}$ where k^L is the laboratory momentum of the particle c.

If the momentum of the produced particle c is not large in the projectile frame, c is called a projectile-fragment. In this case the asymptotic behavior of the single particle distribution is expressed as

$$k_0\frac{d^3\sigma}{d\boldsymbol{k}}=P_bP(a\bar{c})+A(b(a\bar{c}))s^{-1/2}+B(b(a\bar{c}))s^{-1}, \tag{17}$$

where

$$A(b(a\bar{c}))=F_bF(a\bar{c})-O_bO(a\bar{c})+A_bA(a\bar{c})-R_bR(a\bar{c}) \tag{18}$$

and

$$B(b(a\bar{c}))=F_b'F'(a\bar{c})-\Phi_b\Phi(a\bar{c}) \tag{19}$$

in our model. The $P(a\bar{c})$, etc., are functions of $k_{\parallel}^P$ and $k_{\perp}$ where k^P is the momentum of the particle c in the projectile frame.

In this section we consider projectile fragmentation where both the projectile a and the produced particle c are pseudoscalar-mesons. As was mentioned at the end of the previous section, we assume the existence of 27-plets of meson resonances and nonexistence of higher multiplets which can communicate with $P_8P_8P_8$ channels.

Therefore, the amplitudes in the KKK, $K^+K^+\pi^+$, $K^+\pi^+\pi^+$ and $\pi^+\pi^+\pi^+$ chan-

nels are virtually independent of energy. By requiring that $A(b(a\bar{c}))$ and $B(b(a\bar{c}))$ for these channels are zero, we obtain the relations among $F(a\bar{c})$, etc., tabulated in Appendix B as case A.

Case A

In this case, for example, we obtain the following relations,

$$\begin{aligned} &F(\pi^+\pi^+)=R(\pi^+\pi^+)=F(\pi^-\pi^-)=-R(\pi^-\pi^-)\equiv f_1\,,\\ &F(K^+\pi^+)=R(K^+\pi^+)=F(K^-\pi^-)=-R(K^-\pi^-)\equiv f_8\,,\\ &O(K^+\pi^+)=A(K^+\pi^+)=-O(K^-\pi^-)=A(K^-\pi^-)\equiv a_8 \end{aligned}$$

and (20)

$$F'(K^+\pi^+)=-\Phi(K^+\pi^+)=F'(K^-\pi^-)=\Phi(K^-\pi^-)\equiv f_8'\,,$$

since we have assumed that there are no resonances in the $\pi^+\pi^+\pi^+$, $K^+\pi^+\pi^+$ and $K^+K^+\pi^+$ channels. If we introduce the notation,

$$f^P(a\xrightarrow{b}c)\equiv k_0\frac{d^3\sigma}{d\boldsymbol{k}}\,,\qquad (a+b\to c+\text{anything}), \tag{21}$$

for projectile fragmentation, we find, for example, the following relations,

$$f^P(\pi^+\xrightarrow{p}\pi^-)=P_NP(\pi^+\pi^+)+(F-A)f_1s^{-1/2} \tag{22}$$

and

$$f^P(\pi^-\xrightarrow{p}\pi^+)=P_NP(\pi^+\pi^+)+(F+A)f_1s^{-1/2}. \tag{23}$$

A table of $A(b(a\bar{c}))$ and $B(b(a\bar{c}))$ for other processes are given in Appendix C. Since F_π, F_K and F_N are all positive, f_1 is probably positive. Then, we find an inequality,

$$f^P(\pi^-\xrightarrow{p}\pi^+)>f^P(\pi^+\xrightarrow{p}\pi^-). \tag{24}$$

At present, experimental results[11] indicate that $f^P(\pi^-\xrightarrow{p}\pi^+)$ is equal to $f^P(\pi^+\xrightarrow{p}\pi^-)$ within experimental errors. If this result is confirmed by future experiments, we have to conclude that $f_1=0$ and we have to find another restrictions.

Until now we have not considered the energy dependence of the amplitudes in the two-pseudoscalar-meson-baryon ($P_8P_8B_8$) channels. If we substitute the $F(P_8P_8)$ etc. obtained above into the expression (18) and (19) for $A(B_8(P_8P_8))$ and $B(B_8(P_8P_8))$, we find that the amplitudes for 35 dimensional representation of $SU(3)$ symmetry are dependent on energy.

Case B

Since the $\pi^+\pi^+p$ resonance has not yet been found experimentally, let us assume that the amplitudes in the channels which can communicate with B_1, B_8, B_{10} and B_{27} are dependent on energy, but that there are no B_{35} resonances which can communicate with the $B_8P_8P_8$ channels. As will be shown below, the ex-

perimental results of the projectile fragmentations lead to the energy dependent amplitudes in the $\bar{K}^0\pi^+p$, $\bar{K}^0\pi^+Y^0$, $K^-\pi^-n$ and $K^-\pi^-Y^0$ channels in our model and we cannot assume that the amplitudes in the channels which can communicate with B_{27} are independent of energy.

Since the amplitudes in the $\pi^+\pi^+p$, $K^+\pi^+p$, and $K^+\pi^+\Sigma^+$ channels are independent of energy in this case, we obtain the following additional relations as well as the relations (20),

$$f_1=f_3'=0 \tag{25}$$

and

$$f_3=a_3\,. \tag{26}$$

Therefore, we find the following relations for projectile fragmentations in this case.

$$f^P(\pi^+\xrightarrow{p}\pi^-)=f^P(\pi^-\xrightarrow{p}\pi^+)=P_NP(\pi^+\pi^+), \tag{27}$$

$$f^P(K^+\xrightarrow{p}\pi^-)=P_NP(K^+\pi^+) \tag{28}$$

and

$$f^P(K^-\xrightarrow{p}\pi^+)-f^P(K^+\xrightarrow{p}\pi^-)=2(F+A)f_3s^{-1/2}. \tag{29}$$

At present we can test the relations (27) and (29). As was mentioned before, the relation (27) is consistent with experimental results. By a recent experiment[12] $f^P(K^-\xrightarrow{p}\pi^+)$ has been found to be bigger than $f^P(K^+\xrightarrow{p}\pi^-)$, i.e., it is found experimentally that

$$f_3=F(K^+\pi^+)=O(K^+\pi^+)=A(K^+\pi^+)=R(K^+\pi^+)>0 \tag{30}$$

in this case.

Our model requires that the amplitude in the exotic $K^+\pi^+\pi^0$ channel is dependent on energy since

$$A(\pi^0(K^+\pi^+))=Pf_3>0\,. \tag{31}$$

Since

$$A(p(\bar{K}^0\pi^+))=A(n(K^-\pi^-))$$
$$=2(F-A)f_3>0\,, \tag{32}$$

our model also requires that the amplitudes in the exotic

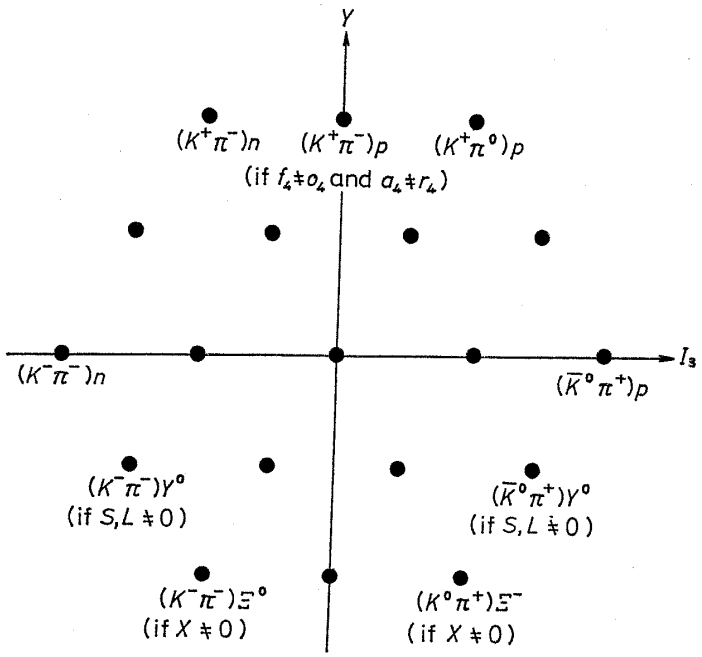

Fig. 2. $A(B_8(P_8P_8))$ in case B.
●; $A(B_8(P_8P_8))\neq 0$.

$\bar{K}^0\pi^+p$ and $K^-\pi^-n$ channels are dependent on energy in our model. The search of the $K^+\pi^+\pi^0$, $\bar{K}^0\pi^+p$ and $K^-\pi^-n$ resonances is waited.

If it happens that $F(K^+\pi^-)-O(K^+\pi^-)=A(K^+\pi^-)-R(K^+\pi^-)=L=S=X=0$, the 27-plets of baryon resonances are incomplete multiplets (see, Fig. 2).

Conclusions obtained from the study of projectile fragmentations

Experimental results[11),12)] seem to require the existence of 27-plets and/or $20(10+10^*)$-plets of meson resonances which communicate with the $P_8P_8P_8$ channels and the existence of 27-plets of baryon resonances which communicate with the $B_8P_8P_8$ channels, but do not seem to require the existence of 35-plets of baryon resonances which communicate with the $B_8P_8P_8$ channels.

§ 4. Fragmentations of baryons

In this section we discuss fragmentations of baryons, i.e., the target fragmentation, the projectile fragmentation of the proton and that of the antiproton. For this purpose we need information about the energy dependence of the amplitudes in the $B_8B_8P_8$ channels.

Let us assume that there are no resonances in the $B_8B_8P_8$ channels and (i) that there are 35-plets of baryon resonances which communicate with the $B_8P_8P_8$ channels (case A) *or* (ii) that there are no such 35-plets, but there are 27-plets of baryon resonances which communicate with the $B_8P_8P_8$ channels (case B). Once we assume these assumptions, we can derive relations among the functions of $k_{\parallel}^{P}$ and $k_{\perp}$, $F(b\bar{c})$, etc. These relations are tabulated in Appendix B.

We use the notation

$$f^L(b\overset{a}{\to}c)\equiv k_0\frac{d^3\sigma}{d\boldsymbol{k}} \tag{33}$$

for the differential cross section of the target-fragment of the process $a+b\to c+$anything. In our model

$$f^L(b\overset{a}{\to}c)=P_aP(b\bar{c})+A(a(b\bar{c}))s^{-1/2}+B(a(b\bar{c}))s^{-1}, \tag{34}$$

where

$$A(a(b\bar{c}))=F_aF(b\bar{c})-O_aO(b\bar{c})+A_aA(b\bar{c})-R_aR(b\bar{c}), \tag{35}$$

and

$$B(a(b\bar{c}))=F_a'F'(b\bar{c})-\Phi_a\Phi(b\bar{c}). \tag{36}$$

By making use of the $F(b\bar{c})$, etc., given in Appendix B, we can calculate $A(a(b\bar{c}))$ and $B(a(b\bar{c}))$. They are tabulated in Appendix D.

In this section we consider only the cases in which the target b is the proton and the target-fragment c is the charged pion. Since we have assumed that there are no resonances in the $Np\pi^{\pm}$ channels, we obtain the following relations,

$$F(p\pi^+)=O(p\pi^+)\equiv F_1\,,\qquad A(p\pi^+)=R(p\pi^+)\equiv A_1\,, \tag{37}$$

$$F(p\pi^-)=O(p\pi^-)\equiv F_2 \quad\text{and}\quad A(p\pi^-)=R(p\pi^-)\equiv A_2\,. \tag{38}$$

Case A

Since all the $p\pi^+P_8$ channels belong to the 35 dimensional representations of $SU(3)$ group, we have no additional relations in this case. By making use of the relations (37) and (38), we can derive the following relations among the differential cross sections.

$$[\sigma_T(pp)]^{-1}f^L(p\xrightarrow{p}\pi^-)=[\sigma_T(K^+p)]^{-1}f^L(p\xrightarrow{K^+}\pi^-)=P_N^{-1}P(p\pi^+)\equiv f_1(k_\parallel{}^L,k_\perp), \tag{39}$$

$$[\sigma_T^{\infty}(\pi N)]^{-1}f^L(p\xrightarrow{\pi^+}\pi^-)-P_N^{-1}P(p\pi^+)=[\sigma_T^{\infty}(\pi N)]^{-1}(F_1-A_1)Ps^{-1/2}, \tag{40}$$

$$\begin{aligned}&f^L(p\xrightarrow{\pi^-}\pi^-)-\sigma_T^{\infty}(\pi N)P_N^{-1}P(p\pi^+)\\&\quad=f^L(p\xrightarrow{K^-}\pi^-)-\sigma_T(K^+p)P_N^{-1}P(p\pi^+)=(F_1+A_1)Ps^{-1/2}\equiv g_1(k_\parallel{}^L,k_\perp)s^{-1/2},\end{aligned} \tag{41}$$

$$f^P(p\xrightarrow{\bar p}\pi^-)=f^P(\bar p\xrightarrow{p}\pi^+), \tag{42}$$

$$[\sigma_T(pp)]^{-1}f^L(p\xrightarrow{p}\pi^+)=[\sigma_T(K^+p)]^{-1}f^L(p\xrightarrow{K^+}\pi^+)=P_N^{-1}P(p\pi^-)\equiv f_2(k_\parallel{}^L,k_\perp), \tag{43}$$

$$[\sigma_T^{\infty}(\pi N)]^{-1}f^L(p\xrightarrow{\pi^+}\pi^+)-P_N^{-1}P(p\pi^-)=[\sigma_T^{\infty}(\pi N)]^{-1}(F_2-A_2)Ps^{-1/2}, \tag{44}$$

$$\begin{aligned}&f^L(p\xrightarrow{\pi^-}\pi^+)-\sigma_T^{\infty}(\pi N)P_N^{-1}P(p\pi^-)\\&\quad=f^L(p\xrightarrow{K^-}\pi^+)-\sigma_T(K^+p)P_N^{-1}P(p\pi^-)=(F_2+A_2)Ps^{-1/2}\equiv g_2(k_\parallel{}^L,k_\perp)\end{aligned} \tag{45}$$

and

$$f^P(p\xrightarrow{\bar p}\pi^+)=f^P(\bar p\xrightarrow{p}\pi^-), \tag{46}$$

where the experimental result, $P=2K$, has been used in deriving the relations (41) and (45).

At present we can test the relations[11),12)] (39), (40) and (41).

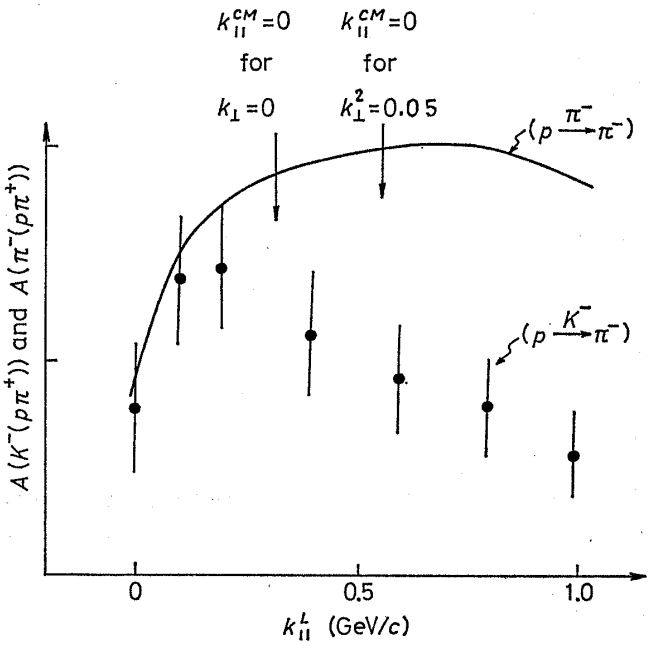

Fig. 3. $k_\parallel{}^L$(GeV/c) vs. $A(K^-(p\pi^+))$ and $A(\pi^-(p\pi^+))$ (arbitrary scale)—A test of the relation (41). The error bars on $A(K^-(p\pi^+))$ indicate uncertainties due only to systematic and statistical errors of $f^L(p\xrightarrow{K^-}\pi^-)$. We have used the results reported in Refs. 11) and 12), and assumed that $A(p(p\pi^+))=0$.

The relations (39) and (41) are consistent with the experimental results within the experimental errors as is shown in Fig. 3, and it has also been found that[12]

$$F_1+A_1>0\,. \tag{47}$$

As to the relation (40), experimental results seem to indicate[11] that

$$F_1\approx A_1\ (\neq 0)\,. \tag{48}$$

Case B

If we assume that there are no 35-plets of baryon resonances which communicate with the $B_8P_8P_8$ channels, the amplitude in the $\pi^+\pi^+p$ channel is independent of energy and we find the relation,

$$F_1=A_1\,. \tag{49}$$

Thus, we obtain an additional relation,

$$[\sigma_T{}^{\infty}(\pi N)]^{-1}f^L(p\xrightarrow{\pi^+}\pi^-)=[\sigma_T(K^+p)]^{-1}f^L(p\xrightarrow{K^+}\pi^-)=[\sigma_T(pp)]^{-1}f^L(p\xrightarrow{p}\pi^-) \tag{50}$$

as well as the relations (39) and (41)$\sim$(46) in this case. As was mentioned before, the relations (49) and (50) are consistent with experimental results.

In this case the amplitude in the exotic $\pi^+p\bar{n}$ channel is dependent on energy since

$$A(\bar{n}(p\pi^+))=2(F-A)F_1>0\,. \tag{51}$$

Conclusions of this section

At present the 35-plets of baryon resonances which communicate with the $B_8P_8P_8$ channels is not necessary. Hence, it seems that the case B, in which there exist B_1, B_8, B_{10} and B_{27} which communicate with the $B_8P_8P_8$ channels, but in which the amplitude in the $p\pi^+\pi^+$ channel is independent of energy, is favored by experimental results at present. As can easily be understood from Appendix D, the cases A and B can be distinguished only by the energy dependence or independence of the differential cross section, $f^L(p\xrightarrow{\pi^+}\pi^-)$.

§ 5. Discussion

The so-called duality diagram[13)~15)] has been useful in correlating quantum numbers of a channel and the energy dependence of the total cross section of the channel (Fig. 4). Let us draw similar diagrams for three-particle→three particle processes (Fig. 5). These diagrams suggest that the amplitudes in the following dimensional representations of the $SU(3)$ group are dependent on energy,

$P_8P_8P_8$ channels; 1, 8, 10+10* and 27,

$P_8P_8B_8$ channels; 1, 8, 10, 10*, 27 and 35,

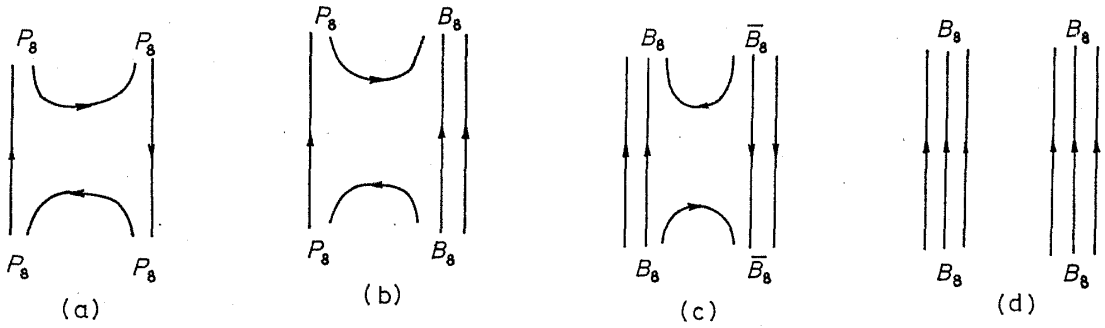

Fig. 4. Two-particle→two-particle processes.

(a) $P_8P_8 \to P_8P_8$, $3\times3^*=1,\ 8.$
(b) $P_8B_8 \to P_8B_8$, $3\times3\times3=1,\ 8,\ 10.$
(c) $B_8\bar{B}_8 \to B_8\bar{B}_8$, $3\times3\times3^*\times3^*=1,\ 8,\ 10+10^*,\ 27.$
(d) $B_8B_8 \to B_8B_8$, ——

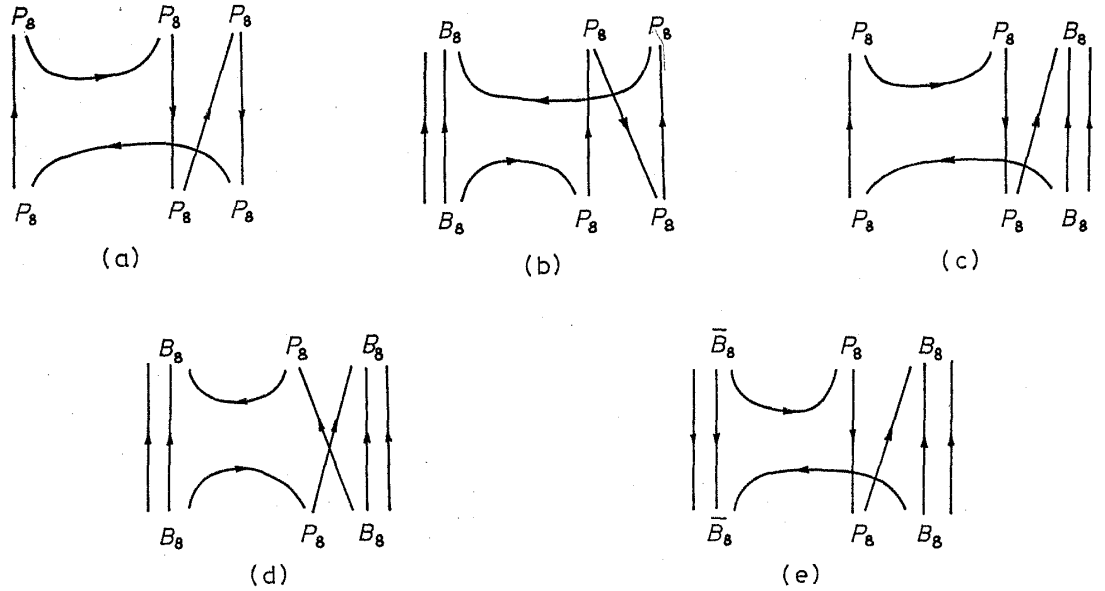

Fig. 5. Three-particle→three-particle processes.

(a) $P_8(P_8P_8) \to P_8(P_8P_8)$, $3\times3\times3^*\times3^*=1,\ 8,\ 10+10^*,\ 27.$
(b) $B_8(P_8P_8) \to B_8(P_8P_8)$, $3\times3\times3\times3\times3^*=1,\ 8,\ 10,\ 10^*,\ 27,\ 35.$
(c) $P_8(P_8B_8) \to P_8(P_8B_8)$, $3\times3\times3\times3\times3^*=1,\ 8,\ 10,\ 10^*,\ 27,\ 35.$
(d) $B_8(P_8B_8) \to B_8(P_8B_8)$, $3\times3\times3\times3\times3\times3=1,\ 8,\ 10,\ 10^*,\ 27,\ 35,\ 28.$
(e) $\bar{B}_8(P_8B_8) \to \bar{B}_8(P_8B_8)$, $3\times3\times3\times3^*\times3^*\times3^*=1,\ 8,\ 10+10^*,\ 27,\ 35+35^*,\ 64.$

$P_8B_8B_8$ channels; 1, 8, 10, 10*, 27, 35 and 28.

Let us call these channels the nonexotic three-particle channels.

If we only assume that the amplitudes in all the exotic three-particle channels are independent of energy, all the amplitudes in the nonexotic three-particle channels are dependent on energy in general. This is a solution to a puzzle raised in Ref. 3). Of course, the energy dependence of the amplitudes in some

of the nonexotic channels may be slight. Experiments to study the s-dependence of the differential cross sections of the fragments from various processes of the type $a+b\to c+$anything and search for various three-particle resonances are extremely important.

In this article we have assumed that there are no resonances in the $P_8B_8B_8$ channels. However, the relations, $F(p\pi^+)=O(p\pi^+)$ and $A(p\pi^+)=R(p\pi^+)$, can be derived even if there exist resonances which belong to 1, 8, 10, 10* and 27 dimensional representations and which communicate with the $B_8B_8P_8$ channels, and the relations, $F(p\pi^-)=O(p\pi^-)$ and $A(p\pi^-)=R(p\pi^-)$ can be derived even if there exist resonances which belong to 1, 8 and 10 or 10* representations and which communicate with the $B_8B_8P_8$ channels.

In the same way we can define the exotic and nonexotic n-particle channels $(n=4, 5, \cdots)$. The nonexotic representation with the highest quantum numbers for the nP_8 channels is n^3 dimensional $(n-1, n-1)$ and that for $(n-1)P_8\cdot B_8$ channels are $[(2n+1)(n+2)(n-1)/2]$ dimensional $(n+1, n-2)$. It is interesting to remember that the same tower of baryon resonances were proposed by Abers, Balazs and the present author[16] based on a static bootstrap model.

§ 6. Summary

In this article we have studied the differential cross sections of the projectile- and target-fragments of the $a+b\to c+$anything process.

We have assumed the following assumptions.

(1) The Regge pole model with the exchange of the usual Regge poles, α_P, $\alpha_f=\alpha_\omega=\alpha_{A2}=\alpha_\rho$, $\alpha_{f'}=\alpha_\phi$ is a good approximation.

(2) Duality.

(3) Nonexistence of resonances in the exotic channels.

In § 5 we have defined the exotic and nonexotic three-particle channels. If we assume that the amplitudes in all the nonexotic three-particle channels are dependent on energy, we cannot derive any relations among these amplitudes. Experiments to measure the s-dependence of the amplitudes in the nonexotic three-particle channels which are exotic for two-particle states are important.

Some of our results on projectile fragmentations are as follows.

(a) If $f_1\neq 0$, existence of M_{27} and B_{35} is required.
(b) If $f_8\neq 0$, existence of M_{20} and/or M_{27} is required.
(c) If $f_8\neq a_8$, existence of B_{35} is required.
(d) If $f_1=f_8-a_8=0$ and if $A(p(K^+\pi^-))\neq 0$, existence of B_{27} is required.
(e) If $f_8=a_8\neq 0$, existence of B_{27} is required.

At present experimental results are consistent with $f_1=0$ and $f_8=a_8\neq 0$. If these relations hold, the existence of B_{27} and M_{20} and/or M_{27} is required, but the existence of B_{35} which communicates with $P_8P_8B_8$ channels are not required.

Some of our results on target fragmentations are written below. Here, we use the symbol D for resonances with baryon number two ($B=2$).

(f) If $A(p(pK^-))=0$, existence of D_8, D_{10}, D_{10*}, D_{27}, D_{35} and D_{28} are not required.

(g) If $A(p(pK_s^0))=0$, existence of D_{10}, D_{27}, D_{35} and D_{28} are not required.

(h) If $A(p(p\pi^+))\neq 0$ and/or if $A(K^+(p\pi^+))\neq 0$, existence of B_{35}, and D_{35} and/or D_{28} is required.

(i) If $A(p(p\pi^-))\neq 0$ and/or if $A(K^+(p\pi^-))\neq 0$, existence of at least one of D_{10*}, D_{27}, D_{35} and D_{28} and existence of at least one of B_{10*}, B_{27} and B_{35} are required.

(j) If $A(\pi^-(p\pi^+))\neq A(K^-(p\pi^+))$ and/or if $A(\pi^-(p\pi^-))\neq A(K^-(p\pi^-))$, there are considerable contributions from secondary vector and tensor trajectories.

(k) If $A(\pi^-(p\pi^+))=A(K^-(p\pi^+))$, existence of D_{28} and D_{35} is not required.

Since experimental results are consistent with $A(p(p\pi^+))=A(K^+(p\pi^+))=0$ and with $A(\pi^-(p\pi^+))=A(K^-(p\pi^+))$, the existence of D_{28}, D_{35} which communicate with three-particle channels is not necessary.

Finally we would like to emphasize that many of the results obtained in this article are not valid if the contribution from the exchange of Regge cuts is important.

The author thanks Professors K. Kawarabayashi and K. Takahashi for discussions.

Appendix A

Residues of Regge poles with various particles

a	P_a	F_a	O_a	A_a	R_a	F_a'	Φ_a
π^+	P_π	P	0	0	P	0	0
π^0	P_π	P	0	0	0	0	0
π^-	P_π	P	0	0	$-P$	0	0
η	P_η	E	0	0	0	E'	0
K^+	P_K	K	K	K	K	K'	$-K'$
K^0	P_K	K	K	$-K$	$-K$	K'	$-K'$
$\bar{K}^0$	P_K	K	$-K$	$-K$	K	K'	K'
K^-	P_K	K	$-K$	K	$-K$	K'	K'
p	P_N	F	F	A	A	0	0
n	P_N	F	F	$-A$	$-A$	0	0
$\bar{n}$	P_N	F	$-F$	$-A$	A	0	0
$\bar{p}$	P_N	F	$-F$	A	$-A$	0	0
Σ^+	P_Σ	S	S	S	S	S'	S'
Σ^0	P_Σ	S	S	0	0	S'	S'
Σ^-	P_Σ	S	S	$-S$	$-S$	S'	S'

a	P_a	F_a	O_a	A_a	R_a	F_a'	Φ_a
Λ	P_Λ	L	L	0	0	L'	L'
Ξ^0	P_Ξ	X	X	X	X	X'	X'
Ξ^-	P_Ξ	X	X	$-X$	$-X$	X'	X'
Δ^{++}	P_Δ	D	D	D	D	0	0
Δ^+	P_Δ	D	D	D'	D'	0	0
Δ^0	P_Δ	D	D	$-D'$	$-D'$	0	0
Δ^-	P_Δ	D	D	$-D$	$-D$	0	0
$Y_1{}^{*+}$	P_Y	Y	Y	Y	Y	Y'	Y'
$Y_1{}^{*0}$	P_Y	Y	Y	0	0	Y'	Y'
$Y_1{}^{*-}$	P_Y	Y	Y	$-Y$	$-Y$	Y'	Y'
Ξ^{*0}	P_{Ξ^*}	X^*	X^*	X^*	X^*	$X^{*\prime}$	$X^{*\prime}$
Ξ^{*-}	P_{Ξ^*}	X^*	X^*	$-X^*$	$-X^*$	$X^{*\prime}$	$X^{*\prime}$
Ω	P_Ω	0	0	0	0	O'	O'

P_a, F_a, O_a, A_a, R_a, F_a' and Φ_a are the residues of α_P, α_f, α_ω, α_{A2}, α_ρ, $\alpha_{f'}$, and α_ϕ with a particle a at $t=0$, respectively.

Appendix B

Residues of Regge poles with various two-particle states

For projectile fragmentation we consider the following two cases.

Case A. We assume the possible existence of $(P_8P_8P_8)_{1,8,(20),27}$ and $(P_8P_8B_8)_{1,8,10,(10^*,27),35}$ resonances and nonexistence of other resonances which can communicate with three-particle states.

Case B. We assume the possible existence of $(P_8P_8P_8)_{1,8,(20),27}$ and $(P_8P_8B_8)_{1,8,10,(10^*),27}$ resonances and nonexistence of other resonances which can communicate with three-particle states.

$a\bar{c}$	Case A						Case B					
	F	O	A	R	F'	Φ	F	O	A	R	F'	Φ
$\pi^+\pi^+$	f_1	0	0	f_1	0	0	0	0	0	0	0	0
$\pi^-\pi^-$	f_1	0	0	$-f_1$	0	0	0	0	0	0	0	0
$\pi^+\pi^-$	f_2	0	0	r_2	0	0	f_2	0	0	r_2	0	0
$\pi^-\pi^+$	f_2	0	0	$-r_2$	0	0	f_2	0	0	$-r_2$	0	0
$K^+\pi^+$	f_3	a_3	a_3	f_3	f_3'	$-f_3'$	f_3	f_3	f_3	f_3	0	0
$K^-\pi^-$	f_3	$-a_3$	a_3	$-f_3$	f_3'	f_3'	f_3	$-f_3$	f_3	$-f_3$	0	0
$K^+\pi^-$	f_4	o_4	a_4	r_4	f_4'	ϕ_4	f_4	o_4	a_4	r_4	f_4'	ϕ_4
$K^-\pi^+$	f_4	$-o_4$	a_4	$-r_4$	f_4'	$-\phi_4$	f_4	$-o_4$	a_4	$-r_4$	f_4'	$-\phi_4$

The F, O, A, R, F' and Φ in the table are the residues of α_f, α_ω, α_{A2}, α_ρ, $\alpha_{f'}$ and α_ϕ with a two-particle state $(a\bar{c})$ at $t=0$, respectively.

For fragmentations from baryons, we assume an additional assumption stated below as well as the assumptions mentioned at the beginning of this appendix. The additional assumption: There are no resonances which can communicate with $B_8B_8P_8$ channels.

$a\bar{c}$	Case A						Case B					
	F	O	A	R	F'	ϕ	F	O	A	R	F'	ϕ
$p\pi^+$	F_1	F_1	A_1	A_1	0	0	F_1	F_1	F_1	F_1	0	0
$n\pi^-$	F_1	F_1	$-A_1$	$-A_1$	0	0	F_1	F_1	$-F_1$	$-F_1$	0	0
$\bar{p}\pi^-$	F_1	$-F_1$	A_1	$-A_1$	0	0	F_1	$-F_1$	F_1	$-F_1$	0	0
$p\pi^-$	F_2	F_2	A_2	A_2	0	0	F_2	F_2	A_2	A_2	0	0
$n\pi^+$	F_2	F_2	$-A_2$	$-A_2$	0	0	F_2	F_2	$-A_2$	$-A_2$	0	0
$\bar{p}\pi^+$	F_2	$-F_2$	A_2	$-A_2$	0	0	F_2	$-F_2$	A_2	$-A_2$	0	0
pK^+	F_3	F_3	A_3	A_3	0	0	F_3	F_3	F_3	F_3	0	0
nK^0	F_3	F_3	$-A_3$	$-A_3$	0	0	F_3	F_3	$-F_3$	$-F_3$	0	0
$\bar{p}K^-$	F_3	$-F_3$	A_3	$-A_3$	0	0	F_3	$-F_3$	F_3	$-F_3$	0	0
pK^0	F_4	F_4	A_4	A_4	0	0	F_4	F_4	A_4	A_4	0	0
nK^+	F_4	F_4	$-A_4$	$-A_4$	0	0	F_4	F_4	$-A_4$	$-A_4$	0	0
$\bar{p}\bar{K}^0$	F_4	$-F_4$	A_4	$-A_4$	0	0	F_4	$-F_4$	A_4	$-A_4$	0	0
pK^-	F_5	F_5	A_5	A_5	F_5'	$-F_5'$	F_5	F_5	A_5	A_5	F_5'	$-F_5'$
$n\bar{K}^0$	F_5	F_5	$-A_5$	$-A_5$	F_5'	$-F_5'$	F_5	F_5	$-A_5$	$-A_5$	F_5	$-F_5'$
$\bar{p}K^+$	F_5	$-F_5$	A_5	$-A_5$	F_5'	F_5'	F_5	$-F_5$	A_5	$-A_5$	F_5'	F_5'
$p\bar{K}^0$	F_6	F_6	A_6	A_6	F_6'	$-F_6'$	F_6	F_6	A_6	A_6	F_6'	$-F_6'$
nK^-	F_6	F_6	$-A_6$	$-A_6$	F_6'	$-F_6'$	F_6	F_6	$-A_6$	$-A_6$	F_6'	$-F_6'$
$\bar{p}K^0$	F_6	$-F_6$	A_6	$-A_6$	F_6'	F_6'	F_6	$-F_6$	A_6	$-A_6$	F_6'	F_6'

$a\bar{c}$	Case A						Case B					
	F	O	A	R	F'	ϕ	F	O	A	R	F'	ϕ
$p\bar{p}$	F_7	O_7	A_7	R_7	0	0	F_7	O_7	A_7	R_7	0	0
$n\bar{p}$	F_8	O_8	A_8	R_8	0	0	F_8	O_8	A_8	R_8	0	0
pp	0	0	0	0	0	0	0	0	0	0	0	0
np	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{p}\bar{p}$	0	0	0	0	0	0	0	0	0	0	0	0
$p\bar{\Lambda}$	F_9	O_9	A_9	R_9	F_9'	Φ_9	F_9	O_9	A_9	R_9	F_9'	Φ_9
$N\Lambda$	0	0	0	0	0	0	0	0	0	0	0	0

In deriving the above relations, we have assumed that the amplitudes in the $N\bar{p}\pi$, $N\bar{p}K$, $N\bar{\Lambda}\pi$ and $N\bar{\Lambda}K$ channels are dependent on energy.

Appendix C

The manner in which the limiting-fragmentation is approached. I

Projectile Fragmentations

In our model the differential cross sections of the projectile-fragments of the process $a+b\to c+$anything is

$$f^P(a\overset{b}{\to}c)=P_bP(a\bar{c})+A(b(a\bar{c}))s^{-1/2},$$

where

$$A(b(a\bar{c}))=F_bF(a\bar{c})-O_bO(a\bar{c})+A_bA(a\bar{c})-R_bR(a\bar{c}).$$

By making use of the $F(a\bar{c})$, etc., tabulated in Appendix B, we obtain $A(b(a\bar{c}))$ tabulated in the following table.

$a\to c$	b	Case A $(PPP)_{27}$ and $(PPB)_{35}$	Case B $(PPP)_{27}$ and $(PPB)_{27}$
		$A(b(a\bar{c}))$	
$\pi^+\to\pi^-$	p	$f_1(F-A)$	0
$\pi^-\to\pi^+$	p	$f_1(F+A)$	0
$\pi^+\to\pi^+$	p	f_2F-r_2A	
$\pi^-\to\pi^-$	p	f_2F+r_2A	
$K^+\to\pi^-$	p	$(f_3-a_3)(F-A)$	0
$K^-\to\pi^+$	p	$(f_3+a_3)(F+A)$	$2f_3(F+A)$
$K^+\to\pi^+$	p	$(f_4-o_4)F+(a_4-r_4)A$	
$K^-\to\pi^-$	p	$(f_4+o_4)F+(a_4+r_4)A$	

For projectile fragmentation when the neutron is the target, replace A by $-A$ in the above table.

Appendix D

The manner in which the limiting-fragmentation is approached. II

Fragmentations of baryons

In our model the differential cross sections of the target-fragments of the process $a+b\to c+$anything is

$$f^L(b\overset{a}{\to}c)=P_aP(b\bar{c})+A(a(b\bar{c}))s^{-1/2}+B(a(b\bar{c}))s^{-1},$$

where

$$A(a(b\bar{c}))=F_aF(b\bar{c})-O_aO(b\bar{c})+A_aA(b\bar{c})-R_aR(b\bar{c})$$

and

$$B(a(b\bar{c})) = F_a'F'(b\bar{c}) - \Phi_a\Phi(b\bar{c}).$$

By making use of the $F(b\bar{c})$ etc., tabulated in Appendix B, we obtain $A(a(b\bar{c}))$ and $B(a(b\bar{c}))$ tabulated in the following table.

$b\to c$	a	Case A	Case B
		$A(a(b\bar{c}))+B(a(b\bar{c}))s^{-1/2}$	
$p\to\pi^-$	N	0	0
	π^+	$(F_1-A_1)P$	0
	π^-	$(F_1+A_1)P$	$2F_1P$
	K^+	0	0
	K^-	$(F_1+A_1)P$	$2F_1P$
	$\bar{p}$	$2F_1F+2A_1A$	$2F_1(F+A)$
$\bar{p}\to\pi^+$	p	$2F_1F+2A_1A$	$2F_1(F+A)$
$p\to\pi^+$	N	0	
	π^+	$(F_2-A_2)P$	
	π^-	$(F_2+A_2)P$	
	K^+	0	
	K^-	$(F_2+A_2)P$	
	$\bar{p}$	$2F_2F+2A_2A$	
$\bar{p}\to\pi^-$	p	$2F_2F+2A_2A$	
$p\to K^-$	N	0	0
	π^+	$(F_3-A_3)P$	0
	π^-	$(F_3+A_3)P$	$2F_3P$
	K^+	0	0
	K^-	$(F_3+A_3)P$	$2F_3P$
	$\bar{p}$	$2F_3F+2A_3A$	$2F_3(F+A)$
$p\to\bar{K}^0$	p	0	
	π^+	$(F_4-A_4)P$	
	π^-	$(F_4+A_4)P$	
	K^+	0	
	K^-	$(F_4+A_4)P$	
	$\bar{p}$	$2F_4F+2A_4A$	
$p\to K^+$	N	0	
	π^+	$(F_5-A_5)P$	
	π^-	$(F_5+A_5)P$	
	K^+	$2F_5'K's^{-1/2}$	
	K^-	$(F_5+A_5)P$	
	$\bar{p}$	$2F_5F+2A_5A$	

$b\to c$	a	Case A / Case B
		$A(a(b\bar{c}))+B(a(b\bar{c}))s^{-1/2}$
$p\to K^0$	N	0
	π^+	$(F_6-A_6)P$
	π^-	$(F_6+A_6)P$
	K^+	$2F_6'K's^{-1/2}$
	K^-	$(F_6+A_6)P$
	$\bar{p}$	$2F_6F+2A_6A$
$p\to\bar{p}$, $\bar{p}\to p$, $n\to\bar{p}$, $N\to\bar{A}$	N	0
	$\pi^\pm$	0
	$K^\pm$	0
	$\bar{p}$	0
$p\to p$	p	$(F_7-O_7)F+(A_7-R_7)A$
	$\pi^\pm$	$(F_7\mp R_7)P$
	$K^\pm$	$(F_7\mp O_7+A_7\mp R_7)K$
	$\bar{p}$	$(F_7+O_7)F+(A_7+R_7)A$
$n\to p$	p	$(F_8-O_8)F+(A_8-R_8)A$
	$\pi^\pm$	$(F_8\mp R_8)P$
	$K^\pm$	$(F_8\mp O_8+A_8\mp R_8)K$
	$\bar{p}$	$(F_8+O_8)F+(A_8+R_8)A$
$p\to\Lambda$	p	$(F_9-O_9)F+(A_9-R_9)A$
	$\pi^\pm$	$(F_9\mp R_9)P$
	$K^\pm$	$(F_9\mp O_9+A_9\mp R_9)K+(F_9'\mp\Phi_9)K's^{-1/2}$
	$\bar{p}$	$(F_9+O_9)F+(A_9+R_9)A$

In deriving the above relations we have made use of the experimental result, $P=2K$.

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