Heat Transfer Characteristics of a Continuous, Stretching Surface With Variable Temperature

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Nomenclature

A = constant
B = constant
c = specific heat
F = dimensionless stream function defined by equation (4)
M = Kummer's function defined by equation (9)
Pr = Prandtl number, ν/α
q = local heat transfer rate per unit area, $-k(\partial T/\partial y)$
u = velocity component in the x-direction
v = velocity component in the y-direction
x = coordinate measuring distance in direction of surface motion
y = coordinate measuring distance normal to surface
α = thermal diffusivity
γ = temperature parameter
η = dimensionless similarity variable, $y(B/\nu)^{1/2}$
θ = dimensionless temperature, $T-T_\infty/T_w-T_\infty$
υ = kinematic viscosity
ξ = dimensionless function, $-\Pr\exp(-\eta)$
ψ = stream function

Subscripts

w = continuous surface conditions
∞ = ambient conditions

Introduction

The continuous surface heat transfer problem has many practical applications in industrial manufacturing processes. Since the pioneering work of Sakiadis [1, 2], various aspects of the problem have been investigated by many authors. Most studies have been concerned with constant surface velocity and temperature (see, for example, Tsou et al. [3]) but for many practical applications the surface undergoes stretching and cooling or heating that cause surface velocity and temperature variations. Crane [4], Vleggaar [5], and Gupta and Gupta [6] have analyzed the stretching problem with constant surface temperature while Soundalgekar and Ramana Murty [7] investigated the constant surface velocity case with power law temperature variation. In the present paper the effect of power law surface temperature variation on the heat transfer characteristics of a continuous, linearly stretching surface is analyzed. A series solution to the energy equation in terms of Kummer's functions is developed for the specified problem. Several closed-form analytical solutions are also reported for specific conditions.

Analysis

The laminar velocity and thermal boundary layers on a continuous stretching surface with velocity $u_w = Bx$ and temperature $T_w = T_\infty + Ax^\gamma$ moving axially through a stationary incompressible fluid with constant physical properties and temperature T_∞ may be described using normal boundary layer approximations by the following continuity, momentum, and energy equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the boundary conditions

$u = Bx,\ v = 0,\ T = T_\infty + Ax^\gamma \quad \text{at } y = 0$

$u \to 0,\ T \to T_\infty \quad \text{as } y \to \infty$

The x-axis runs along the continuous surface in the direction of motion, and the y-axis is perpendicular to it; u and v are the velocity components in the direction of x and y, respectively.

The solution of equation (1) may be written in terms of the stream function ψ defined by the relations

$$u = \frac{\partial \psi}{\partial y},\ v = \frac{-\partial \psi}{\partial x}$$

Introducing the usual similarity transformations and dimensionless temperature

$$\eta = y(B/\nu)^{1/2},\ F = \frac{\psi}{x(\nu B)^{1/2}},\ \theta = \frac{T-T_\infty}{T_w - T_\infty} \tag{4}$$

the momentum equation (2) and energy equation (3) can be written as

$$F''' + FF'' - (F')^2 = 0 \tag{5}$$

$$\theta'' + \Pr F\theta' - \gamma \Pr F'\theta = 0 \tag{6}$$

with the boundary conditions

$F(0) = 0,\ F'(0) = 1,\ \theta(0) = 1$

$F'(\infty) \to 0,\ \theta(\infty) \to 0$

where primes denote order of differentiation with respect to η. The solution of equation (5) was shown by Crane [4][3] to be

$$F = 1 - e^{-\eta}$$

Introducing a new variable ξ as

$$\xi = -\Pr e^{-\eta}$$

and substituting the solution for F into equation (6) gives

$$\xi\frac{d^2\theta}{d\xi^2} + (1 - \Pr - \xi)\frac{d\theta}{d\xi} + \gamma\theta = 0 \tag{7}$$

with the appropriate boundary conditions

$\theta(-\Pr) = 1$

$\theta(0) \to 0$

It can be readily demonstrated that the solution of equation (7) in terms of Kummer's functions [8] is

$$\theta = \left(\frac{-\xi}{\Pr}\right)^{\Pr}\frac{M(\Pr - \gamma, \Pr + 1, \xi)}{M(\Pr - \gamma, \Pr - 1, -\Pr)} \tag{8}$$

where

$$M(a,b,z) = 1 + \sum_{n=1}^{\infty}\frac{a_n}{b_n}\frac{z^n}{n!}$$

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[3]Crane's solution to the boundary layer equations also happens to be an exact solution to the Navier–Stokes equations.

Table 1 Temperature and wall gradient expressions for various Prandtl numbers and temperature parameter values

γ	Pr	θ	$\theta'(0)$
-2	Pr	$(1+\mathrm{Pr}-\mathrm{Pr}(e^{-\eta}))\exp(\mathrm{Pr}(1-\eta-e^{-\eta}))$	Pr
-1	Pr	$\exp(\mathrm{Pr}(1-\eta-e^{-\eta}))$	0
0	1	$\frac{e}{1-e}(\exp(-e^{-\eta})-1)$	$\frac{1}{1-e}$
Pr	γ	$e^{-\mathrm{Pr}(\eta)}$	$-\mathrm{Pr}$

Table 2 Wall temperature gradient as a function of Prandtl number and temperature parameter

	$\theta'(0)$					
γ \ Pr	.01	.72	1.0	3.0	10.0	100.0
-3	0.0201	2.2500	4.0000	-12.0000	-5.0000	-4.0816
-2	0.0100	0.7200	1.0000	3.0000	10.0000	100.0000
-1	0.0	0.0	0.0	0.0	0.0	0.0
0	-0.0099	-0.4631	-0.5820	-1.1652	-2.3080	-7.7657
1	-0.0197	-0.8086	-1.0000	-1.9237	-3.7207	-12.2940
2	-0.0294	-1.0885	-1.3333	-2.5097	-4.7969	-15.7120
3	-0.0390	-1.3270	-1.6154	-3.0000	-5.6934	-18.5516

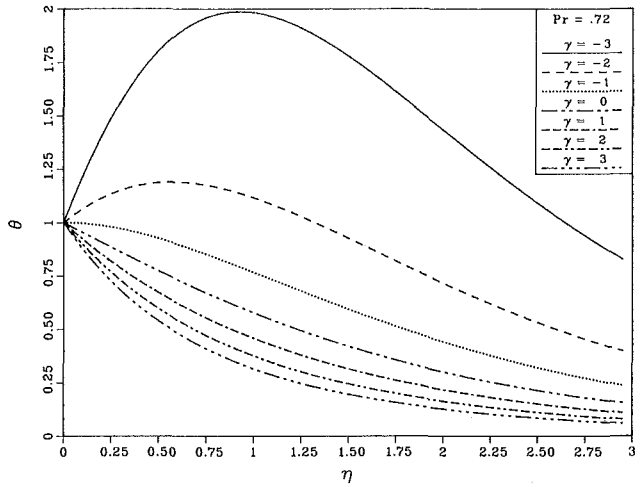

Fig. 1 Temperature profiles as a function of the similarity variable for various values of the temperature parameter at Pr = .72

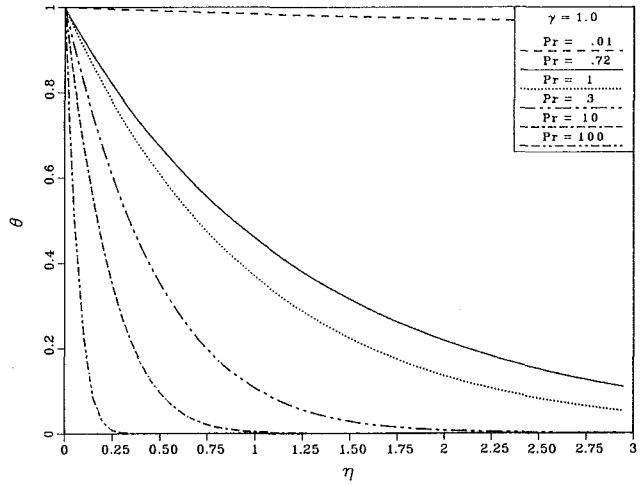

Fig. 2 Temperature profiles as a function of the similarity variable for various values of Prandtl number at γ = 1.0

$$a_n = a(a+1)(a+2)\ldots(a+n-1) \qquad (9)$$
$$b_n = b(b+1)(b+2)\ldots(b+n-1)$$

Rewriting equation (8) in terms of η

$$\theta = e^{-\mathrm{Pr}\cdot\eta}\frac{M(\mathrm{Pr}-\gamma,\mathrm{Pr}+1,-\mathrm{Pr}e^{-\eta})}{M(\mathrm{Pr}-\gamma,\mathrm{Pr}+1,-\mathrm{Pr})} \qquad (10)$$

Closed-form solutions can readily be developed from equation (10) for specific values of γ and Pr. Four such cases are reported in Table 1. The local dimensionless wall temperature gradient corresponding to equation (10) is

$$\theta'(0) = -\mathrm{Pr} + \frac{\mathrm{Pr}(\mathrm{Pr}-\gamma)}{(\mathrm{Pr}+1)}\frac{M(\mathrm{Pr}-\gamma+1,\mathrm{Pr}+2,-\mathrm{Pr})}{M(\mathrm{Pr}\text{-}\gamma,\mathrm{Pr}+1,-\mathrm{Pr})} \qquad (11)$$

and the local wall heat flux can be expressed as

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_w = -kA(B/\nu)^{1/2}\theta'(0)x^{\gamma}$$

Results and Discussion

Equations (10) and (11) were evaluated to determine the temperature distribution and the surface temperature gradient as a function of the temperature parameter γ and Prandtl number Pr. The results were checked by direct numerical integration of equation (6) and comparison with published results. Agreement was found to be excellent, with the exception of the work of Gupta and Gupta [6] where significant differences in wall temperature gradient at Pr not equal to 1 were found. There is some concern about their numerical results due to exhibiting incorrect Pr trends.

Temperature profiles were obtained for Pr of .01, .72, 1, 3, 10, and 100 with γ ranging between -3 and 3. Plots for the various parameter combinations are shown in Figs. 1 and 2. Both parameters are seen to have a significant effect on the temperature profiles. The Pr effects found for this problem are typical. For a given γ value, the smaller the Pr, the larger the thermal boundary layer thickness. To discuss the effect of γ, it is helpful to examine Table 2, which gives a tabulation of the wall temperature gradient. For $\gamma > -1$, the wall temperature gradient is negative, and heat flows from the continuous surface to the ambient. The magnitude of the temperature gradient increases as γ increases. When $\gamma = -1$, there is no heat transfer between the continuous surface and the ambient. For $-2 \le \gamma < -1$, the sign of the temperature gradient changes and heat flows into the continuous surface

from the ambient. The magnitude of the temperature gradient increases with decreasing γ. For $\gamma < -2$ and for Pr greater than certain values, the sign of the temperature gradient changes again and the heat flow is directed from the continuous surface. Temperature distributions for these γ and Pr values are found to have regions of temperature less than ambient. The temperature solutions, though satisfying equation (6), are physically unrealistic and calculations for these γ and Pr combinations are not reported.

The results for $\gamma \leq -1$ appear to be somewhat surprising. However, the studies by Sparrow and Gregg [9] and Levy [10], concerning boundary layer problems with specified power law wall temperature distributions, do indicate that this type of behavior is indeed characteristic. A more detailed discussion of some aspects of this behavior is given by Eckert and Drake [11] in regards to the wedge flow problems studied by Levy.

Conclusions

In this work, the heat transfer from a linearly stretching, continuous surface with a power law temperature distribution has been solved in terms of Kummer's functions. Several closed form temperature solutions for specified conditions are presented.

The heat transfer characteristics for this problem are found to be determined by the temperature parameter γ and Prandtl number Pr. The magnitude of γ affects the direction and quantity of heat flow. For $\gamma = -1$, there is no heat transfer occurring between the continuous surface and the ambient. Heat is transferred from the continuous surface to the fluid for $\gamma > -1$ and to the continuous surface for $-2 \leq \gamma < -1$. For $\gamma < -2$ and certain Pr values, unrealistic temperature distributions are encountered. Typical Pr effects on thermal boundary layer thickness, for a specified γ, are found.

Acknowledgment

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Mixed Free and Forced Convection From a Short Vertical Cylinder Placed in a Laminar Horizontal Flow

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Nomenclature

A = surface area
c_p = specific heat
d = diameter or differential
g = acceleration of gravity
Gr_L = Grashof number based on $L = g\beta(T_w - T_\infty)L^3/\nu^2$
$\bar{h}$ = average convective heat transfer coefficient
k = thermal conductivity
L = height
m = mass
$\bar{\mathrm{Nu}}_d$ = average Nusselt number based on $d = \bar{h}d/k$
$\dot{Q}$ = heat transfer rate
Re_d = Reynolds number based on $d = U_\infty d/\nu$
t = time
T = temperature
U_∞ = free-stream velocity
β = coefficient of thermal expansion
ϵ = emissivity
σ = Stefan–Boltzmann constant
ν = kinematic viscosity

Introduction

Mixed free- and forced-convection external flows arise when free-stream inertial and near-wall buoyant forces have strong effects on the resulting convective heat transport. Mixed convection is a fundamentally important heat transfer problem that occurs in a variety of technological and industrial applications. These applications include electronic devices cooled by fans, heat exchangers placed in low-velocity environments, nuclear reactors cooled during emergency shutdown, and solar central receivers exposed to wind currents.

Analytical studies of orthogonal mixed convection from vertical surfaces have been few, probably because of the complexities with treating a three-dimensional flow field. One of the first three-dimensional studies of laminar mixed convection from a vertically heated plate was conducted by R. J. Young and K. T. Yang [1]. Eichhorn and Hasan [2] reported numerical results for orthogonal mixed convection from vertical and horizontal wedges while Evans and Plumb [3] studied orthogonal laminar mixed convection from an isothermal vertical flat plate. Orthogonal laminar mixed convection from a vertical cylinder was studied by Yao and Chen [4]. Oosthuizen [5] also studied mixed convection from a vertical cylinder placed in a laminar horizontal flow and found that the numerical solution was inaccurate near the thermal leading edge. In a recent study, Yao [6] used a finite difference solution method to investigate the thermal leading edge region. It was found in [6] that buoyancy delays separation and significantly affects the heat transfer in the thermal leading edge region.

Few investigators have measured mixed convection heat transfer coefficients from vertical surfaces placed in a crossflow. Oosthuizen and Leung [7] measured orthogonal

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