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The Unitarity Condition and High Energy Scattering

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The effects of unitarization procedure using the K matrix are investigated in the case of 2 channel model. Two kinds of effects are discussed, one is the self-damping effect and the other is the absorption effect, when the exchanged particle has no spin.

Also the self-damping effect is investigated in the high energy limit in the case of the exchanged particle with spin, and the amplitude obtained by this procedure is compared with that obtained by assuming the Regge hypothesis.

§ 1. Introduction

One of the most characteristic features of the reactions of the strongly interacting particles at high energies is that the differential cross sections of the reactions exhibit a peak in the forward direction.[1] Qualitatively, these features seem to suggest that the peripheral mechanism, or the single particle exchange mechanism gives the main contribution in these energy region, explaining the predominance of the small momentum transfers. We can see this also from the experimental fact that the double charge exchange process does not exhibit the forward peak, since there are no particles with double charge.[2]

Quantitatively, however, the single particle exchange mechanism cannot reproduce the experimental data well either in the shape or in the absolute value. The observed differential cross section of the reaction is smaller in the absolute value and is more peripheral than that predicted from the single particle exchange model. Phenomenologically the correction should be made to suppress the low partial wave amplitude, which reduces the absolute cross section and gives a narrower forward peak. A modification of the single particle exchange model is to introduce the form factors phenomenologically, as Feinberg and Chernavskii[3] or Chan and Liu[4] did. However, the shape of the form factor can be determined only from experiment and not from theory. Dar and others[5] proposed to cut out the low partial waves from the Born amplitude by taking into account the absorption in the inner region, which is called the strong absorption model. Later the strong absorption model was interpreted as the absorption effect in the initial and final states,[6] which is analogous to the distorted-wave Born approximation in low energy nuclear physics. The model includes the absorption strength as the adjustable parameter, since in many cases one of the particles in the final state is unstable and the final state interactions are not completely determined by experiment.[*] Many attempts have been made to justify the strong absorption model from the theoretical point of view.[8]

On the other hand the modification of the single particle exchange model should be made from the following theoretical requirement.

(i) The amplitude of the single particle exchange diagram should be corrected so as to satisfy the unitarity condition.

(ii) The effect of other channels must be taken into account.

The effect of the requirement (i) can be called "self-damping" since it does work even if there is no coupled channel. The effect of the requirement (ii), which we name the absorption effect, works if the coupling to other channels is large. Many authors have investigated the unitarity correction using the K matrix; Bialas et al. did in the high energy limit[9] and Kikugawa et al. in the low energy region in relation to the OBEC model.[10] They did not make a distinction, however, between the effect of the self-damping and that of the absorption.

In this paper we are to investigate the unitarization effect in the case of the two-channel reaction of spinless particles with spinless particle exchange in order to see what effects the self-damping and the absorption have on the unmodified amplitude without using the high energy limiting procedure. It is shown that the self-damping effect always reduces the low partial wave part of the unmodified amplitude but that the absorption effect does not always reduce the low partial wave amplitude and it even amplifies the low partial wave part. (§ 2) Further the results in § 2 will be a guide to compare the method of unitarization procedure with the other ones and to clarify the limit of these methods. We discuss this point in § 4. Section 3 deals with the case when the particles have spin, and the amplitude obtained by unitarization procedure is compared with that obtained by assuming Regge hypothesis

§ 2. The unitarization effect

Following the discussion of reference 10), we will investigate the unitarization effect in the two channel case:

$$\begin{tikzcd} A+A \arrow[r] \arrow[dr] & A+A \\ B+B \arrow[r] \arrow[ur] & B+B \end{tikzcd}$$

where A and B are spinless baryons with equal masses. The unmodified amplitude is given by exchanging a neutral scalar meson with mass μ. In order

*) In a process such as the charge exchange one, $\pi^- + p \to \pi^0 + n$, there are no adjustable parameters and the absorption parameters can be determined completely from the experimental data of pion-nucleon elastic scattering. The analysis is made by Barger and Ebel[7] and they concluded that the absorption model is quite inadequate.

to study the effect of the self-damping and the absorption, the coupling constants of the Yukawa interaction, g_A, g_B and g_{AB} as defined in Fig. 1 remain as variable quantities. Then the unmodified amplitude is expressed as

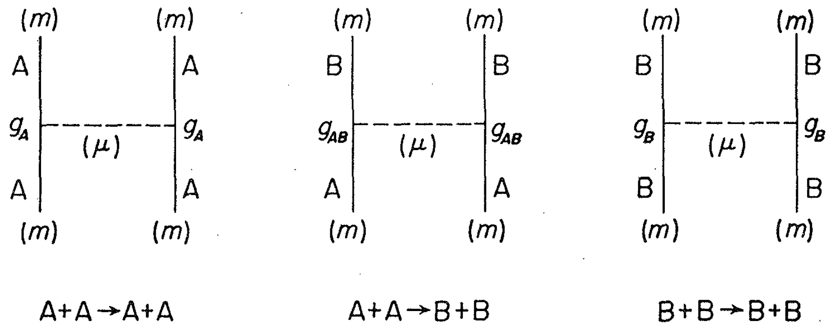

Fig. 1. The schematical diagrams for the 2 channel reaction via a scalar meson exchange.

$$\boldsymbol{T}^0=\begin{pmatrix} g_A{}^2, & g_{AB}^2 \\ g_{AB}^2, & g_B{}^2 \end{pmatrix}\frac{1}{4E}\cdot\frac{1}{\mu^2+t}, \tag{2·1}$$

where t is the squared four momentum transfer and E is the energy of the baryon in the center of mass system. The S matrix is expressed in terms of the T matrix as

$$\boldsymbol{S}=\mathbf{1}+i\boldsymbol{T}, \tag{2·2}$$

where $\mathbf{1}$ is the unit matrix. The partial wave component of the T matrix is readily obtained:

$$\boldsymbol{T}_l{}^0=\begin{pmatrix} g_A{}^2, & g_{AB}^2 \\ g_{AB}^2, & g_B{}^2 \end{pmatrix}\frac{1}{4Ep}Q_l(x_0), \tag{2·3}$$

where p is the momentum of the baryon in the c.m. system, $Q_l(x_0)$ is the Legendre function of the second kind and x_0 is given by

$$x_0=1+\frac{\mu^2}{2p^2}. \tag{2·4}$$

The unitarity condition of the S matrix is satisfied if $\boldsymbol{S}$ is expressed as

$$\boldsymbol{S}_l=\frac{1+i\boldsymbol{K}_l}{1-i\boldsymbol{K}_l}, \quad \boldsymbol{K}^\dagger=\boldsymbol{K}. \tag{2·5}$$

The unitarized amplitude of the $\boldsymbol{T}_l^0$ is obtained by inserting

$$\boldsymbol{K}_l=\frac{1}{2}\boldsymbol{T}_l^0. \tag{2·6}$$

Then the unitarized amplitude is given as follows:

$$
\left\{
\begin{aligned}
T_l^{11} &= \frac{1}{\Delta^2}\left\{g_A{}^2\frac{Q_l(x_0)}{4Ep}\left(1-\frac{i}{2}\cdot g_B{}^2\frac{Q_l(x_0)}{4Ep}\right)+\frac{i}{2}\cdot\left(g_{AB}^2\frac{Q_l(x_0)}{4Ep}\right)\right\}^2,\\
T_l^{12} &= T_l^{21} = \frac{1}{\Delta^2}g_{AB}^2\frac{Q_l(x_0)}{4Ep},\\
T_l^{22} &= \frac{1}{\Delta^2}\left\{g_B{}^2\frac{Q_l(x_0)}{4Ep}\left(1-\frac{i}{2}\cdot\frac{Q_l(x_0)}{4Ep}\right)+\frac{i}{2}\cdot\left(g_{AB}^2\frac{Q_l(x_0)}{4Ep}\right)\right\}^2,
\end{aligned}
\right.
$$

$$
\Delta^2 = \left(1-\frac{i}{2}\cdot g_A{}^2\frac{Q_l(x_0)}{4Ep}\right)\left(1-\frac{i}{2}\cdot g_B{}^2\frac{Q_l(x_0)}{4Ep}\right)+\frac{1}{4}\cdot\left(g_{AB}^2\frac{Q_l(x_0)}{4Ep}\right)^2. \quad (2\cdot7)
$$

The unitarized matrix (2·7) is different from that of Eq. (11) in reference 10); in the latter the diagonal and non-diagonal matrix elements have the same value due to the equal coupling constants, $g_A=g_B=g_{AB}$.

(a) *The self-damping effect*

Let us consider the matrix element T_l^{11}. If we set $g_{AB}=g_B=0$ which is equivalent to the single channel problem, the ratio of the unitarized amplitude to the unmodified one is expressed as the effect of the self-damping. In the case of scalar meson exchange process, the unmodified amplitude is asymptotically $\log E/Ep$, and the ratio of the unitarized amplitude to the original one is equal to unity if the energy is large enough.

The angular distribution resulting from the unitarized and the original T matrix is expressed as

$$
\begin{aligned}
\sigma^{011}(\theta) &= |\sum_l(2l+1)T_l^{011}P_l(z)|^2 = \frac{g_A{}^2}{16E^2p^2(x_0-z)^2},\\
\sigma^{11}(\theta) &= |\sum_l(2l+1)T_l^{11}P_l(z)|^2, \qquad (2\cdot8)
\end{aligned}
$$

respectively, where $z=\cos\theta$.

To see the effect of unitarization procedure more clearly, we also calculate the angular distribution with two other types of modifications for the T matrix. One type is the maximum unitarity condition, that is, the partial wave amplitudes which exceed the unitarity limit are replaced by the maximum value allowed by the unitarity condition. The other one is the strong absorption, that is, they are replaced by zero. Taking the nucleon and pion mass as m and μ respectively, we calculate T_l^{11} in these three cases in the case of, as an example, $g_A{}^2=10$ and $p_L=0.4\,\mathrm{BeV}/c$, where p_L is the incident momentum in the laboratory system. The results are shown in Fig. 2. It is seen that in all cases the reduction of the cross section occurs and the forward peaks become sharper than those in the unmodified case, although the shapes of the angular distributions differ according to the models.

Next we examine the self-damping effect of the matrix element T_l^{12}. In

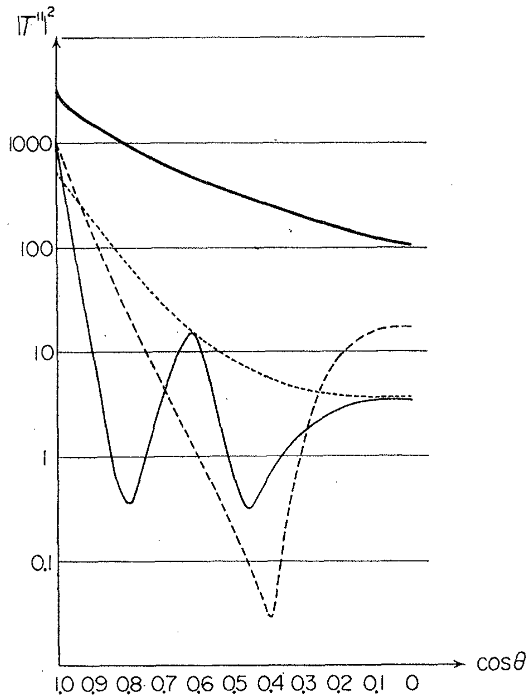

Fig. 2. The angular distributions of the original Born approximation and the modified ones. The modified ones are the self-damping unitarized with the K matrix (the dotted line), with the maximum unitarity condition where the partial wave amplitudes which exceed the unitarity limit are replaced by the maximum value allowed by the unitarity condition (the broken line) and the strong absorption where they are replaced by 0 (the solid line).

——— unmodified Born
·········· unitarized with K matrix
– – – – maximum unitarity
———— strong absorption

$g_A{}^2=10$ $p_L=0.4$Gev/c
$g_B{}^2=g^2{}_{AB}=0$

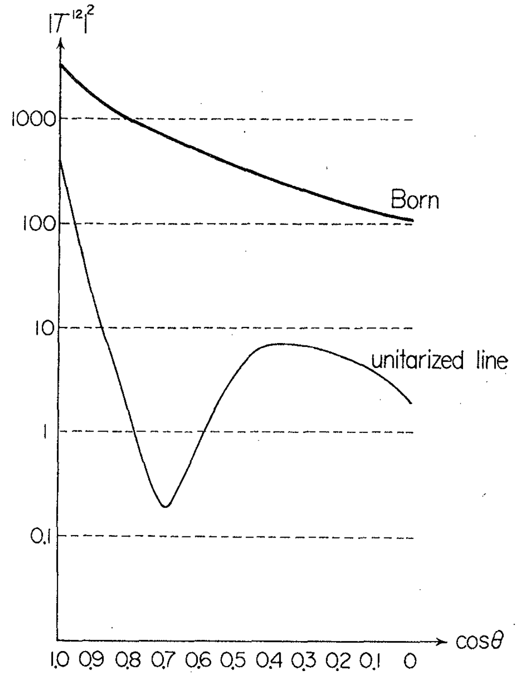

Fig. 3. The angular distribution is shown as an example of the self-damping effect on the T^{12}, together with the original Born one.

$p_L=0.4$ Gev/c
$g^2{}_{AB}=10$
$g_A{}^2=g_B{}^2=0$

this case the unitarity correction reduces the low partial wave part more strongly than in the elastic case; the unitarized amplitude becomes approximately as follows:

$$T_l^{12}\to\frac{4}{g^2_{AB}\dfrac{Q_l(x_0)}{Ep}},\qquad(2\cdot9)$$

where $g^2_{AB}(Q_l(x_0)/4Ep)\gg1$ and $g^2_{AB}\gg g_A{}^2,\ g_B{}^2$, while in the elastic case it becomes

$$T_l^{11} \longrightarrow 2i\,, \tag{2·10}$$

when $g_A^2 \cdot Q_l(x_0)/4Ep \gg 1$. The angular distributions are shown in Fig. 3.

(b) *The effect of the absorption*

Let us examine the absorption effect for the elastic amplitude T_l^{11} due to the coupled channels, changing the coupling constant, g_{AB}. In this case, of course the unitarity correction does not always reduce the low partial wave part especially for the case of larger coupling constant g_{AB}; this we can naturally understand, since in this case the elastic scattering is mainly the shadow of the inelastic process. The angular distribution, in this case, is not sharpened for a large value of the coupling, g_{AB}^2, as shown in Fig. 4. On the other hand, the unitarization effect on the scattering ampling amplitude T_l^{12} always reduces the low partial wave part, especially for the case of the strong coupling constant, g_A^2 or g_B^2. It is instructive that the formula obtained from the distorted-wave Born approximation (DWBA) is reproduced as a limiting case of the unitarization procedure, that is, when the scattering amplitude T_l^{012} is negligible compared to $|\{1-(i/2)T_l^{011}\}\{1-(i/2)T_l^{022}\}|$. In the case of DWBA, the experimental data (or the complex phase shift) is used as the unmodified elastic amplitude, and the effect on the inelastic amplitude appears as the reduction of the low partial wave parts of the inelastic amplitude. In our case, the situation is of the same tendency. The angular distribution is shown in Fig. 5.

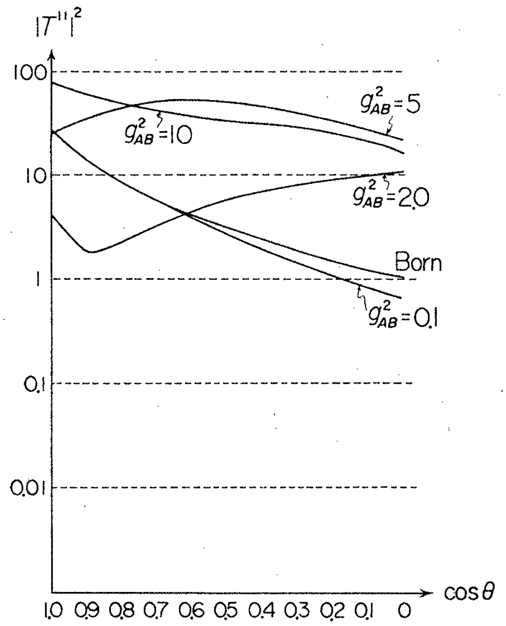

Fig. 4. The angular distributions using the modified T^{11}, together with the original Born one.
$g_A^2=1.0$
$p_L=0.4$ Gev/c

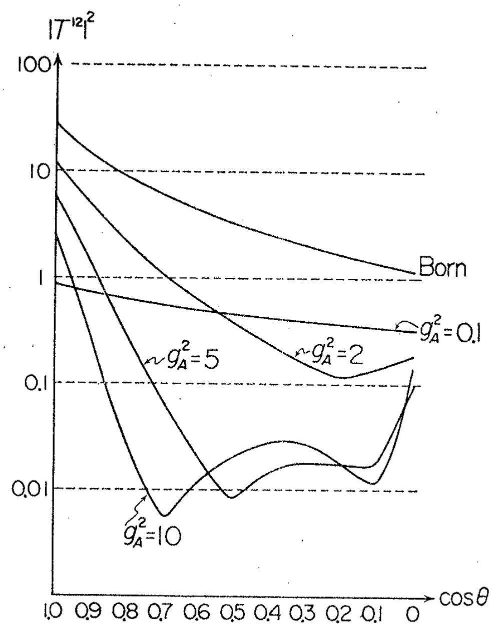

Fig. 5. The angular distributions using the modified T^{12}, together with the original Born one.
$g^2_{AB}=1$
$p_L=0.4$ Gev/c

§ 3. The case of the meson exchanged with spin

We have investigated in detail the unitarization effect in the case of scalar particle exchange process. In the reactions of exchanging the meson with spin, the discussion leads to some complications. The behaviour of the amplitude for the one-meson-exchange process with spin α will have the pole term of the following form:

$$T^0=\frac{g^2P_\alpha(\cos\theta_t)}{t-t_B}\cdot\frac{1}{2}\sqrt{\frac{s-4m^2}{s}}, \tag{3·1}$$

where $\cos\theta_t=1+2s/(t-4m^2)$, $\sqrt{t_B}$ is the mass of the exchanged meson, s is the squared total energy in the c.m. system of the s channel, t is the squared four momentum transfer and m is the mass of scattering particles. The transition amplitude is expressed by s and $\cos\theta_s\equiv x$,

$$T^0=\sqrt{\frac{s-4m^2}{s}}\cdot\frac{1}{s-4m^2}\cdot\frac{g^2}{x-x_1}\cdot P_\alpha\left(1+\frac{4s}{s-4m^2}\cdot\frac{1}{x-x_2}\right), \tag{3·2}$$

where

$$\left\{\begin{aligned} x&\equiv\cos\theta_s=1+\frac{2t}{s-4m^2},\\ x_1&=1+\frac{2t_B}{s-4m^2},\\ x_2&=1+\frac{8m^2}{s-4m^2}.\end{aligned}\right. \tag{3·3}$$

Expanding $P_\alpha(1+4s/(s-4m^2)\cdot(x-x_2))$ in the power series of the $(x-x_2)^{-m}$,

$$T^0=c_1(s,\alpha)\frac{g^2}{x-x_1}-\sum_{n=0}^{\alpha}\sum_{i=1}^{n}c_0^{n,i}(s,\alpha)\frac{g^2}{(x-x_2)^{n-i+1}},$$

$$\left\{\begin{aligned} c_1(s,\alpha)&=\sqrt{\frac{s-4m^2}{s}}\cdot\sum_{n=0}^{\alpha}\frac{(2s)^n}{s-4m^2}\frac{1}{(t_B-4m^2)^n}\cdot\frac{(\alpha-n+1)(\alpha-n+2)\cdots(\alpha+n)}{n!},\\ c_0^{n,i}(s,\alpha)&=\sqrt{\frac{s-4m^2}{s}}\cdot\frac{(2s)^n}{s-4m^2}\cdot\frac{1}{(t_B-4m^2)^n}\cdot\frac{(\alpha-n+1)(\alpha-n+2)\cdots(\alpha+n)}{n!}.\end{aligned}\right. \tag{3·4}$$

The partial wave expansion of the term $(x-x_2)^{-q}$ is expressed in the form

$$\frac{1}{(x-x_2)^q}=\sum_l(2l+1)\frac{2}{(l-1)!}\cdot\frac{\partial^{(q-1)}}{\partial x_2^{(q-1)}}\cdot Q_l(x_2)P_l(x). \tag{3·5}$$

Now the partial wave expansion of the scattering amplitude is obtained:

$$T^0=g^2\sum_{l=0}^{\infty}(2l+1)T_l^0(s,\alpha)P_l(x),$$

$$\begin{cases} T_l^0(s,\alpha)=c_1(s,\alpha)Q_l(x_1)-\sum\limits_{n=0}^{\alpha}\sum\limits_{i=1}^{n}c_0^{n,i}(s,\alpha)Q_l^{(n-i)}(x_2), \\ Q_l^{(n-i)}(x_2)=\dfrac{\partial^{(n-i)}}{\partial x_2^{(n-i)}}\cdot Q_l(x_2). \end{cases} \tag{3·6}$$

In the s channel, the behaviour of the partial wave amplitude $T_l^0(s,\alpha)$ for large s and fixed x is given:

$$T_l^0(s,\alpha)\underset{s\to\infty}{\sim}\frac{s^\alpha}{s-4m^2}\frac{1}{(t_B-4m^2)^\alpha}(Q_l(x_1)-Q_l(x_2)) \tag{3·7}$$

where we have used the following relations:

$$\begin{aligned} c_1(s,\alpha)&\underset{s\to\infty}{\sim}\frac{s^\alpha}{s-4m^2}\cdot\frac{1}{(t_B-4m^2)^\alpha}, \\ \sum_{n=0}^{\alpha}\sum_{i=1}^{n}c_0^{n,i}(s,\alpha)Q_l^{(n-i)}(x_2)&\underset{s\to\infty}{\sim}c_0^{\alpha,\alpha}(s,\alpha)Q_l(x_2) \\ &\sim\frac{s^\alpha}{s-4m^2}\frac{Q_l(x_2)}{(t_B-4m^2)^\alpha}. \end{aligned} \tag{3·8}$$

In contrast to the case of the scalar meson exchange process, the partial wave amplitude becomes infinitely large for high energies in this case.

Let us introduce the Fourier-Bessel component or impact parameter representation[12] in order to see the effect of the unitarization procedure in the high energy limit. The Fourier-Bessel component of the scattering amplitude denoted by $H(b^2,s)$ is defined as follows:

$$T(s,t)=\int_0^\infty bdbJ_0(b(-t)^{1/2})H(b^2,s), \tag{3·9}$$

where $J_0(x)$ is the zeroth order Bessel function and b is the impact parameter.

For the unitarized amplitudes, H_u^{12} and H_u^{11} are obtained using the relations

$$\begin{cases} \lim\limits_{l\to\infty}Q_l\left(1+\dfrac{x^2}{2l^2}\right)=K_0(x), \\ \lim\limits_{l\to\infty}P_l\left(1-\dfrac{y^2}{2l^2}\right)=J_0(y), \end{cases} \tag{3·10}$$

where $K_0(x)$ is the zeroth order modified Bessel function of the second kind. Then the Fourier-Bessel component of the unitarized amplitude is expressed as

$$\begin{cases} (1/s)\cdot H_u^{11}(s,b^2)\sim\dfrac{g(s,b)}{1-(i/2)\cdot g(s,b)}, \\ (1/s)\cdot H_u^{11}(s,b^2)\sim\dfrac{g(s,b)}{1+(1/4)\cdot g^2(s,b)}, \end{cases} \tag{3·11}$$

where

$$g(s, b) = \frac{g^2 s^{\alpha-1}}{(t_B - 4m^2)^\alpha}[K_0(\sqrt{t_B}\, b) - K_0(2mb)]. \tag{3·12}$$

On the other hand it is pointed out by several authors that the basic mechanism should involve the exchange of a Regge pole since the amplitude of the process exchanging an elementary particle with spin larger than 1/2 violates the unitarity condition at high energies.

These two methods, unitarization procedure and Regge hypothesis, both satisfy the unitarity condition.

We compare these two methods. The Regge hypothesis[11] gives a high energy behavior of the scattering amplitude with the form

$$\begin{aligned} T_R(s, t) &\sim \beta(t) \frac{P_{\alpha(t)}(\cos\theta_t) \pm P_{\alpha(t)}(-\cos\theta_t)}{\sin \pi\alpha(t)} \\ &\sim \beta(t) \frac{1 \pm e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)} \cdot s^{\alpha(t)}, \quad (s \to \infty) \end{aligned} \tag{3·13}$$

where s is the total energy in the c.m. system and each Regge trajectory $\alpha(t)$ is associated with a definite set of conserved quantum numbers (except spin) in the t channel with a definite signature ± 1 and $\beta(t)$ is slowly varying function of t. The bahavior of $\alpha(t)$ is not known and we can only expand $\alpha(t)$ in the small t region,

$$\begin{aligned} &\alpha(t) = \alpha(0) + \alpha'(0)t + \cdots, \\ &(\alpha(0) \leq 1). \end{aligned} \tag{3·14}$$

Then in this region the scattering amplitude behaves as

$$T_R(s, t) \sim f(t) \exp[\{\alpha(0) + \alpha'(0)t\} \ln s], \tag{3·15}$$

where $f(t)$ is a slowly varying function of t (at most a polynomial function of t). The Fourier-Bessel component of this type was obtained by Baker and Blankenbecler,[13]

$$H_R(s, b^2) \sim s^{\alpha(0)} \exp[\alpha'(0) b^2 \ln s]. \tag{3·16}$$

Now let us compare s- and t-dependence of the unitarized amplitudes with that obtained by Regge hypothesis.

(a) *The case of* $\alpha=1$ (*vector meson exchange processes*)

The s dependence of the unitarized amplitude can easily be obtained, since the function $g(s, b)$ is independent of s for large s,

$$\begin{aligned} T_u^{11}(s, t) &= s\int_0^\infty h_u^{11}(b^2) J_0(b\sqrt{-t})\, bdb, \\ T_u^{12}(s, t) &= s\int_0^\infty h_u^{12}(b^2) J_0(b\sqrt{-t})\, bdb, \end{aligned} \tag{3·17}$$

where

$$h_u^{11}(b^2) = \frac{g(b)}{1-(i/2)g(b)},$$
$$h_u^{12}(b^2) = \frac{g(b)}{1+(1/4)g^2(b)}, \tag{3·18}$$
$$g(b) = \frac{g^2}{t_B-4m^2}\cdot[K_0(\sqrt{t_B}\,b)-K_0(2mb)].$$

The unitarized amplitudes in Eq. (3·17) have the same s-dependence as the unmodified ones. As for the t-dependence, the b-integral cannot be carried out analytically and so we shall examine the behavior of $h_u(b^2)$ as a function of b. For small b the function $h_u^{11}(b^2)$ is of the form

$$h_u^{11}(b^2) \underset{b\to 0}{\sim} \frac{g^2\ln\left(\frac{2m}{\sqrt{t_B}}\right)\Big/(t_B-4m^2)}{1-(i/2)\frac{g^2}{t_B-4m^2}\cdot\ln\left(\frac{2m}{\sqrt{t_B}}\right)},$$
$$h_u^{12}(b^2) \underset{b\to 0}{\sim} \frac{g^2\ln\left(\frac{2m}{\sqrt{t_B}}\right)\Big/(t_B-2m^2)}{1+(1/4)\left[\left(\frac{g^2}{t_B-4m^2}\ln\left(\frac{2m}{\sqrt{t_B}}\right)\right]^2}, \tag{3·19}$$

and for large b

$$h_u^{11}(b^2)\sim h_u^{12}(b^2) \underset{b\to 0}{\sim} \frac{g^2}{t_B-4m^2}\cdot\left[\left(\frac{\pi}{2\sqrt{t_B}\,b}\right)^{1/2}e^{-\sqrt{t_B}b}-\left(\frac{\pi}{4mb}\right)e^{-2mb}\right], \tag{3·20}$$

where we have used the following properties of the function $K_0(x)$:

$$K_0(x)\sim\begin{cases} -\ln(\gamma x/2), & (x\to 0)\\ \left(\frac{\pi}{2x}\right)^{1/2}e^{-x}, & (x\to\infty)\end{cases} \tag{3·21}$$

where $\ln\gamma$ is the Euler constant.

(b) *The case of* $\alpha>1$

The Fourier-Bessel component of the unitarized elastic amplitude behaves for large s as follows:

$$(1/s)H_u^{11}(s, b^2)\sim\begin{cases} 2i & (b\ll R)\\ 0 & (b\gg R),\end{cases} \tag{3·22}$$

where R is determined so that $g(s, R)$ is of order 1,

$$R \sim \begin{cases} \dfrac{(\alpha-1)}{\sqrt{t_B}} \ln s & (\text{when } \sqrt{t_B} < 2m) \\[2ex] \dfrac{(\alpha-1)}{2m} \ln s & (\text{when } 2m > \sqrt{t_B}) . \end{cases} \tag{3·23}$$

The b-integral can easily be carried out for large s,

$$\begin{aligned} T_u^{11}(s,t) &= \int_0^\infty H_u^{11}(s,b^2) J_0(b\sqrt{-t})\, bdb \\ &= 2si \int_0^R J_0(b\sqrt{-t})\, bdb \\ &= i\frac{2sR}{\sqrt{-t}} \cdot J_1(R\sqrt{-t}) , \end{aligned} \tag{3·24}$$

which has the same t-dependence as the amplitude for the scattering by an absorptive sphere whose radius is R. We must remark here that the radius of the interaction R is logarithmically expanding in this case in contrast to the classical optics and that the radius R depends on the spin of the exchanged meson $(R \propto (\alpha-1)\ln s)$.

In the Regge theory, though the differential cross section is exponentially damping in t, the width of the main foward peak is determined by $\alpha'(0)\ln s$ as seen from Eq. (3·15), which plays a role similar to R.

As for the Fourier-Bessel component of the unitarized inelastic amplitude, we obtain

$$(1/s) H_u^{12}(s,b^2) \sim \begin{cases} \dfrac{1}{s^{\alpha-1}} \cdot \dfrac{(t_B-4m^2)^\alpha}{g^2[K_0(\sqrt{t_B}b)-K_0(2mb)]} , & (R \gg b) \\[2ex] \dfrac{g^2 s^{\alpha-1}}{(t_B-4m^2)^\alpha} [K_0(\sqrt{t_B}b)-K_0(2mb)] , & (R \ll b) \end{cases} \tag{3·25}$$

where R is defined in Eq. (3·23). The b integral cannot be carried out analytically in this case, and therefore we see only the b-dependence of $H_u^{12}(s,b^2)$ in the limiting case,

$$\begin{aligned} (1/s) H_u^{12}(s,b^2) &\underset{b\to 0}{\sim} \frac{1}{s^{\alpha-1}} \cdot \frac{(t_B-4m^2)^\alpha}{g^2 \ln\left(\dfrac{2m}{\sqrt{t_B}}\right)} , \\ &\underset{b\to 0}{\sim} \frac{g^2 s^{\alpha-1}}{(t_B-4m^2)^\alpha} \left[\left(\frac{\pi}{2\sqrt{t_B}\, b}\right)^{1/2} e^{-\sqrt{t_B}b} - \left(\frac{\pi}{4mb}\right)^{1/2} e^{-2mb} \right] . \end{aligned} \tag{3·26}$$

It is seen from the equations derived above that the self-damping effect cannot reduce the low partial wave amplitude (or the amplitude for the small impact parameter) enough to reproduce the differential cross section whose t dependence is exponentially damping as in the case of Regge hypothesis, and as for the s dependence of the amplitude, these two methods of unitarization give a very different behaviour in the high energy limit.

§ 4. Discussion

The unitarization procedure investigated in the previous sections assumes that for a high partial wave part the scattering amplitude is correctly represented by the one meson-exchange process in the case of the scalar meson.*) In this case the behaviour of the scattering amplitude in the high energy limit is precisely given by the original Born approximation if the channels of the inelastic process does not open. (This is analogous to the case of the potential scattering.[14]) In the case of the scalar particle exchange process, therefore, the self-damping effect works when the energy is low and the coupling constant is large enough, and then the self-damping effect always reduces the low partial wave part of the scattering amplitude and in general the differential cross section of the inelastic scattering is a function of t much steeper than that of elastic scattering in the strong coupling limit. Now the reactions at high energies include many channels, so that we cannot discuss these processes as two channel problem, and the unitarization procedure is one of the ways of taking into account the effect of other coupling channels as the absorption effect. The results in § 2 suggest that the absorption effect works especially for the inelastic process and for the small coupling constant g_{AB}. In this case the unitarization procedure is equivalent to the distorted wave Born approximation,[6] though the former takes the Born term as $T^{0,11}(s, t)$ and the latter uses the experimental data phenomenologially. We must remark here, however, that the unitarization procedure cannot reproduce the differential cross section whose t dependence is exponentically damping either in the case of scalar meson exchange process or in the case of the exchanged particle with spin as far as we take the Born term as the original amplitude.

The introduction of the form factor is another method of modification of

*) By the analysis of various choices for the original function, Bialas et al.[9] concludes that the results are insensitive to a change in the dynamical model assumed for the K matrix. But they takes for the original partial wave amplitude as the following three types:

(i) $H^0(b)=K_0(\mu b)$,

(ii) $H^0(b)=3/2\cdot\exp(-\mu b)$,

(iii) $H^0(b)=\begin{cases}\infty & \text{for} \quad b<R \\ K_0(\mu b) & \text{for} \quad b\geqq R\,.\end{cases}$

In any case, the behaviour for the large impact parameter is that of the single meson exchange process.

the single particle exchange model. Ferrari and Selleri[15] determined the form factors phenomenologically from the proton-proton reaction in Sub-GeV region as follows:

$$F_{FS}(t) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + t}. \tag{4·1}$$

Chan and Liu[4] examined the process, $\pi^+ + p \to \Sigma^+ + K^*$ at the incident energy 2.7 GeV/c, and showed that the form factor of the type

$$F_{CL}(t) \sim \exp[\gamma(t+\mu^2)] \tag{4·2}$$

reproduces the experimental data well.

The form factor obtained by Chan and Liu has the same form as that of Ferrari and Selleri[15] for $|t| \ll 1$:

$$F_{CL}(t) \sim \frac{(1/\gamma)^2 - \mu^2}{(1/\gamma)^2 + t}, \tag{4·3}$$

where $(1/\gamma)$ corresponds to Λ of Ferrari and Selleri.

On the other hand the form factor determined by Feinberg and Chernavskii[3] to fit the high energy behaviour is given by

$$F_{FC}(s,t) \approx \mu^2 \gamma \ln\left(\frac{s}{2M^2}\right) \cdot \exp\left[\gamma t \ln\left(\frac{s}{2M^2}\right)\right], \tag{4·4}$$

which has the same form as $F_{CL}(t)$ for the fixed s. Such form factors as $F_{FC}(s, t)$ or $F_{CL}(t)$ reduce the scattering amplitude for large t drastically.

The situation seems to suggest that the t dependence of the scattering amplitude is of the type of exponentially damping. As already described, the unitarization procedure cannot reproduce this type of the behaviour correctly if we take the Born amplitude as the original one.

In the low energy region where $|t|$ is not so large, the type of the function introduced in the original amplitude is not essential and we may explain the experimental data fairly well using the unitarization procedure.

For a higher energy region, however, we must introduce something like form factors in order to reproduce the experimental data. The distorted wave Born approximation is one of the ways of obtaining such types of the amplitude since we put the experimental value in $T^{0,11}(s,t)$.

Moreover none of the theories except Regge hypothesis can explain the s-dependence of the scattering amplitude of the vector meson exchange process. We can only expect that absorption effect could explain these processes by assuming suitable s-dependence of the number of the coupled channels.

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