Paper 20

THE RATIONAL EFFICIENCY OF POWER PLANTS WITH EXTERNAL COMBUSTION

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The use of availability in the analysis of engineering power plants is extended to the study of plants with 'external combustion', in which heat is transferred from high temperature products of external combustion to an internal closed cycle. A rational definition of plant efficiency is the work output from the internal cycle divided by the maximum work that could be obtained in the change from external reactants to external products at constant (ambient) pressure and temperature.

It is shown that the rational efficiency is dependent upon the ratio of the thermal capacities of internal and external fluid flows, and the specific work of the internal cycle.

INTRODUCTION

THE USE OF AVAILABILITY in assessing the maximum work that can be achieved in a thermodynamic process is well known. Keenan (1)† has shown that in a steady-flow process through a control surface, in which heat may be exchanged with the atmosphere at temperature T_0 and changes in kinetic and potential energy are neglected, the maximum work output‡ for unit mass flowing through the control surface is§

$$b_1 - b_2 \quad . \quad . \quad . \quad . \quad (20.1)$$

where $b = h - T_0 s$ and h, s are the enthalpy and entropy respectively, 1, 2 are the end states of the process. In this paper the quantity b is referred to as the *steady-flow availability function* and the quantity $(b_1 - b_2)$ as the decrease in steady-flow availability.

If there is heat transfer dq from a reservoir at temperature T_{R} during the process then the maximum work output is

$$b_1 - b_2 + \int_1^2 \left(\frac{T_{\text{R}} - T_0}{T_{\text{R}}}\right) \mathrm{d}q \quad . \quad . \quad (20.2)$$

Keenan (2) and Bruges (3) have defined the irreversibility in a process as the difference between this maximum work and the actual work (w),

$$i = b_1 - b_2 + \int_1^2 \left(\frac{T_{\text{R}} - T_0}{T_{\text{R}}}\right) \mathrm{d}q - w \quad . \quad (20.3)$$

Keenan has shown that the irreversibility may be written

$$i = T_0(s_2 - s_1) + \Delta S_{\text{R}} + \Delta S_{\text{a}}$$

where ΔS_{R} and ΔS_{a} are the increase in entropy of the reservoir and the atmosphere respectively. This approach has been used by Fratzscher (4).

With heat transfer to the atmosphere alone the irreversibility becomes

$$i = b_1 - b_2 - w \quad . \quad . \quad . \quad (20.4)$$

and in such a process with no work output

$$i = b_1 - b_2 \quad . \quad . \quad . \quad . \quad (20.5)$$

The steady-flow availability function has been used in the analysis of thermodynamic cycles and power plants by many authors (Keenan, Bruges, Fratzscher, Traupel (5), Meyer, Silvestri and Martin (6)).

In a *closed cycle*, in which heat q_α supplied from an external reservoir produces a change in state from 2 to 1 within the cycle, maximum work can be obtained if the cycle is reversible and if heat rejection takes place at the temperature of the surroundings. This maximum work is $b_1 - b_2$ and the maximum thermal efficiency is

$$\eta_{t_{\max}} = \frac{b_1 - b_2}{q_\alpha} = \frac{b_1 - b_2}{h_1 - h_2} \quad . \quad . \quad (20.6)$$

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† *References are given in Appendix 20.I.*

‡ *Work output means shaft work output in this paper, for only steady flow processes are considered.*

§ *A list of symbols is given in Notation, p. 45.*

Small letters are used for properties per unit mass, and for heat and work transfers per unit mass of fluid flowing through the control surface.

if the heat is supplied to the fluid flowing at low velocity. Such maximum efficiency is attained in the Rankine cycle in which all internal processes are reversible and heat is rejected at T_0.

In an actual closed cycle, the maximum work output (b_1-b_2) is not achieved because of internal irreversibilities in the several processes linking states 1 and 2 and the failure to reject heat q_β at the lowest temperature T_0. Thus

$$b_1-b_2-w = \sum_{\substack{\text{all}\\ \text{processes}}} i+\int_\beta \left(\frac{T_R-T_0}{T_R}\right) dq \quad (20.7)$$

Tabulation of the contribution of each of the processes to this quantity, the decrease in availability less the work done, enables the performance of each process to be assessed. The efficiency of the real cycle is

$$\begin{aligned}\eta_t &= \frac{w}{h_1-h_2}\\ &= \frac{b_1-b_2}{h_1-h_2}-\frac{(b_1-b_2-w)}{(h_1-h_2)}\\ &= \eta_{t\max}-\frac{\sum i+\int_\beta \left(\frac{T_R-T_0}{T_R}\right) dq}{h_1-h_2}. \quad (20.8)\end{aligned}$$

This method of cycle analysis has been used by Meyer, Silverstri and Martin who have drawn up availability balances for steam power plants.

Several other criteria of cycle efficiency have been suggested (2) (4). The most useful of these appears to be what Fratzscher calls the degree of quality, v, the ratio of the work output to the 'work value' of the heat supplied: (q_α) from the reservoir at temperature T_R.

$$\begin{aligned}v &= w\Big/\int_\alpha \left(\frac{T_R-T_0}{T_R}\right) dq\\ &= 1-\left[\left(\int_\beta \left(\frac{T_R-T_0}{T_R}\right) dq+\sum i\right)\Big/\int_\alpha \left(\frac{T_R-T_0}{T_R}\right) dq\right] \quad (20.9)\end{aligned}$$

This degree of quality may also be expressed as the ratio of the actual work output to the maximum possible work output:

$$v = \frac{w}{b_1-b_2} = \frac{w}{\Delta b_\alpha} \quad . \quad . \quad (20.10)$$

if the process of heat supply is fully reversible with $T = T'_R$.

For *power plants with internal combustion*, the definition of thermal efficiency used for closed cycles is not valid. Heat is not supplied in a continuously operating cycle, although in first simple analyses calculations are often made of the thermal efficiency of an equivalent air standard cycle; it is supposed that the air circulates in a closed loop, supplied with heat from an external reservoir. In practice, this energy is supplied from the combustion of fuel within the power plant.

For such an internal-combustion power plant Keenan (1) has suggested a rational definition of efficiency and Haywood (7) has summarized this as follows:

If the reactants enter an internal-combustion work-producing device at the pressure p_0 and temperature T_0 of the environment, and the products leave at a pressure and temperature different from those of the environment, then it is evident that still more work could be obtained by restoring the products to p_0 and T_0 as a result of passing them through a further work-producing device. Hence a proper criterion for maximum possible work will be one which relates to a process in which the products are restored to p_0 and T_0.

This maximum work will be

$$W_{\max} = B_{R_0}-B_{P_0} \quad . \quad . \quad (20.11)$$

where subscripts R and P denote reactants and products and subscript 0 refers to states at pressure p_0 and temperature T_0. Since both states have the common temperature T_0, we may write

$$W_{\max} = G_{R_0}-G_{P_0} \quad . \quad . \quad (20.12)$$

where G is the Gibbs function or Gibbs 'free energy'. The value of G_{P_0} varies with the completeness of the reaction; the minimum value, corresponding very nearly to the state of complete combustion, is taken for the determination of $W_{\max}$. A rational criterion for the efficiency of an internal-combustion power plant is then

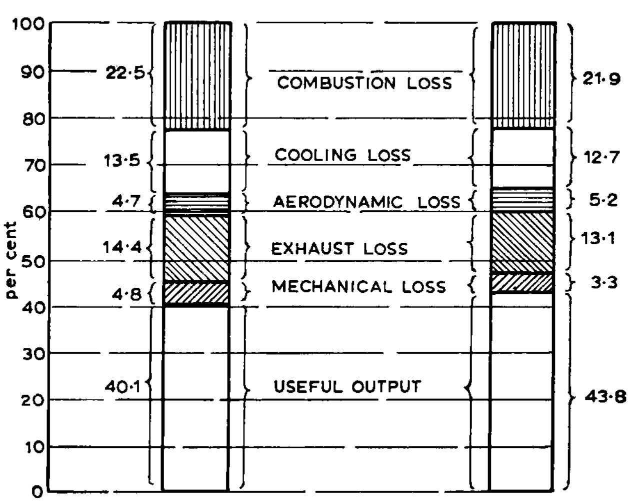
$$\eta_r = W'/W_{\max} = W/(G_{R_0}-G_{P_0}) \quad (20.13)$$

where W' is the actual work delivered by the plant when supplied with fuel and air, both at p_0 and T_0.

The quantity $G_{R_0}-G_{P_0}$ is little different from the calorific value $H_{R_0}-H_{P_0}$, but the difference, $T_0(S_{R_0}-S_{P_0})$, may be positive or negative.

The performance of the various processes in an internal-combustion power plant may be assessed by tabulating the quantity (B_1-B_2-W) for the processes.

Several internal-combustion power plants, the gas turbine, the diesel engine and the supercharged diesel engine, have been compared on this basis by Traupel. His comparison of the last two is reproduced in Fig. 20.1.



(a) Unsupercharged. (b) Supercharged.

After Traupel.

Fig. 20.1. Losses in diesel engines based on availability

The useful work plus the irreversibilities ('losses') make up the maximum work that could be obtained in a change from reactants to products at atmospheric pressure and temperature.

The present paper extends this type of analysis to an *external-combustion (E.C.) power plant*, in which a closed circuit receives heat from hot gases which have undergone combustion externally. For example, in a closed gas-turbine cycle with external combustion, a rational efficiency is the ratio of the work output to the maximum work that could be obtained from the fuel supplied in external combustion,

$$\eta_r = W_{CY}/(G_{R_0}-G_{P_0}) \quad . \quad . \quad (20.14)$$

where W_{CY} is the work delivered from the internal cycle.

This rational efficiency is less than the thermal efficiency, because of the difference between the availability of the gases leaving the heat exchanger (or boiler), and the availability in the base state, $B_{P_0} = G_{P_0}$. The rational efficiency may be related to the thermal efficiency, η_t,

$$\eta_r = \frac{W_{CY}}{q_\alpha}\left(\frac{q_\alpha}{G_{R_0}-G_{P_0}}\right) = \eta_t \frac{q_\alpha}{(G_{R_0}-G_{P_0})} \quad (20.15)$$

or to the degree of quality,

$$\eta_r = \frac{W_{CY}}{(\Delta b)_\alpha}\left(\frac{\Delta b_\alpha}{G_{R_0}-G_{P_0}}\right) = v\frac{\Delta b_\alpha}{G_{R_0}-G_{P_0}} \quad (20.16)$$

The importance of the performance of the heater in overall power-plant efficiency has, of course, been widely recognized, but the optimization of overall efficiency of E.C. power plants using the availability function does not appear to have been attempted.

Notation

b, B	Steady-flow availability function.
c_p	Specific heat at constant pressure.
G	Gibbs function, $H-TS$.
h, H	Enthalpy.
i	Irreversibility.
$M = mc_p$	Thermal capacity.
m	Mass flow rate.
q	Heat supply or rejection.
r	Pressure ratio.
s, S	Entropy.
T	Temperature.
v	Degree of quality.
w, W	Shaft work output.
γ	Ratio of specific heats.
δ	Temperature ratio defined in text.
$\epsilon = r^{(\gamma-1)/\gamma}$	Isentropic temperature ratio of compression or expansion.
η_C	Compressor isentropic efficiency.
η_r	Rational efficiency.
η_T	Turbine isentropic efficiency.
η_t	Thermal efficiency.
λ	Heat exchanger effectiveness.
$\mu = M_g/M_a$	
$\psi = m_g/m_a$	
ϕ, α	Defined in text.

Subscripts and superscripts

a	Air.
CY	Internal cycle of E.C. power plant.
f	Finite.
g	Gas.
i	Irreversible.
max	Maximum.
R	Reservoir.
R, P	Reactants, products.
r	Reversible.
0	Ambient conditions.
α	Heat supply.
β	Heat rejection.
∞	Infinite.
$'$	I.C. power plant

CHANGES OF AVAILABILITY IN SOME THERMODYNAMIC PROCESSES

In order that steady-flow power plants with external combustion may be analysed, the changes in availability in the various flow processes involved are summarized below. The velocity of flow is assumed to be small in each process. Any heat transfers are supposed to take place with the atmosphere, except in heat transfer between external and internal flow in a heat exchanger, where it is assumed that there is no heat loss to the surroundings.

The calorific value process at constant pressure and temperature

If reactants flow steadily into a combustion chamber at pressure p_0, temperature T_0, and combustion takes place at constant pressure, then the heat abstracted (per unit mass of fuel entering the reaction) in order to restore the products of combustion to temperature T_0 is the calorific value of the fuel at pressure p_0, temperature T_0. At low velocity, the calorific value is

$$H_{R_0}-H_{P_0} = \Delta H_0 \quad . \quad . \quad (20.17a)$$

and is the area $(A+B)$ shown on Fig. 20.2.

The maximum work that could be obtained in the change from reactants to products, each at p_0 and T_0, is the change in the steady-flow availability function, which is also the change in the Gibbs function,

$$\Delta G_0 = G_{R_0}-G_{P_0}$$
$$= \Delta H_0 - T_0(S_{R_0}-S_{P_0}) \quad . \quad (20.17b)$$

where the entropies of the individual reactants and products are evaluated at the ambient pressure p_0. This ideal work output is represented by the area $(A+B+C)$ on the T, s chart of Fig. 20.2, and could be achieved in a van't Hoff box.

Adiabatic combustion at constant pressure

In adiabatic combustion at constant pressure, between reactants at p_0, T_0 to products at p_0, T_1 the enthalpy is unchanged:

$$H_{R_0} = H_{P_1} \quad . \quad . \quad . \quad (20.18)$$

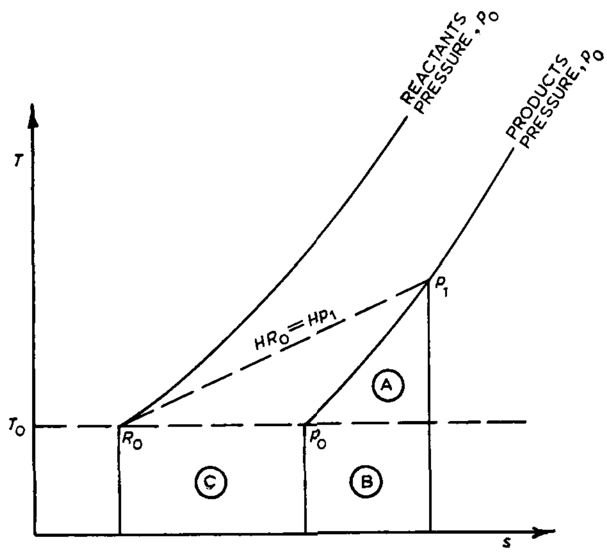


Fig. 20.2. The calorific value process and adiabatic combustion

The change in the steady-flow availability function is, therefore,

$$B_{R0}-B_{P1} = H_{R0}-H_{P1}-T_0(S_{R0}-S_{P1})$$
$$= T_0(S_{P1}-S_{R0}) \quad . \quad . \quad . \quad . \quad (20.19)$$

the area (B+C) on the T, s diagram of Fig. 20.2. Alternatively the change may be expressed as

$$B_{R0}-B_{P1} = T_0(S_{P0}-S_{R0})+T_0(S_{P1}-S_{P0})$$
$$= \Delta G_0-\Delta H_0+T_0 c_p \log_e\left(\frac{T_1}{T_0}\right) \quad . \quad (20.20)$$

if the products are perfect gases with mean specific heat c_p.

The quantities ΔG_0 and ΔH_0 may be determined from tables such as those of Geyer and Bruges. The temperature after combustion (T_1) and the difference in entropy ($S_{P1}-S_{P0}$) between products at T_1 and products at T_0 may also be determined using gas tables.

Adiabatic expansion or compression

In reversible adiabatic (isentropic) expansion or compression between states 1 and 2, $s_{2s} = s_1$ and $b_1-b_{2s} = h_1-h_{2s}$. The change in the steady-flow availability function is the work output or input.

In irreversible adiabatic expansion or compression the work transfer is $h_1-h_2 = w$. The decrease in b in expansion, for example, is $w+T_0(s_2-s_1)$ and the lost work due to irreversibility is $T_0(s_2-s_1)$ which is represented by the area A on the T, s diagram of Fig. 20.3.

Heat transfer to the atmosphere from a fluid flowing at constant pressure

In heat transfer to the environment at temperature T_0 the decrease in the steady-flow availability function is

$$b_1-b_2 = h_1-h_2-T_0(s_1-s_2) \quad . \quad (20.21)$$

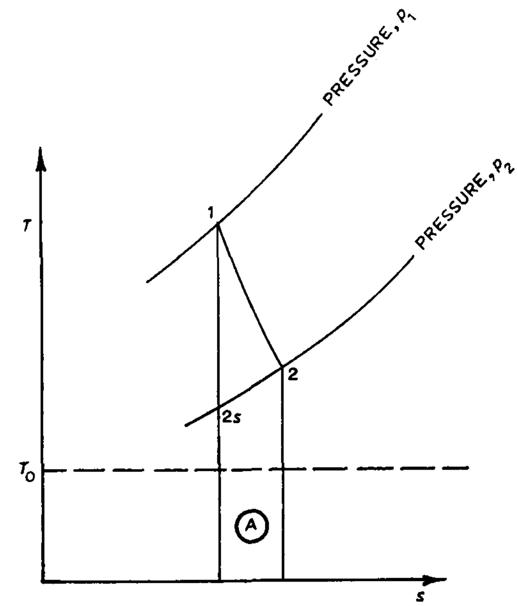


Fig. 20.3. Irreversible adiabatic expansion

and is the area A shown on the T, s chart in Fig. 20.4 if the fluid flow is at constant pressure. If the specific heat of the gas is constant then

$$b_1-b_2 = c_p(T_1-T_2)-T_0 c_p \ln(T_1/T_2) \quad (20.22)$$

Contraflow heat exchanger with internal heat transfer

The flows in a contraflow heat exchanger are diagrammatically shown in Fig. 20.5*a*. Heat is transferred from gas (g) flowing reversibly from temperature T_1 to T_2 at constant pressure to air (a) flowing reversibly from temperature T_3 to T_4 at constant pressure.

The heat transferred is

$$m_g(h_1-h_2) = m_a(h_4-h_3) \quad . \quad (20.23)$$

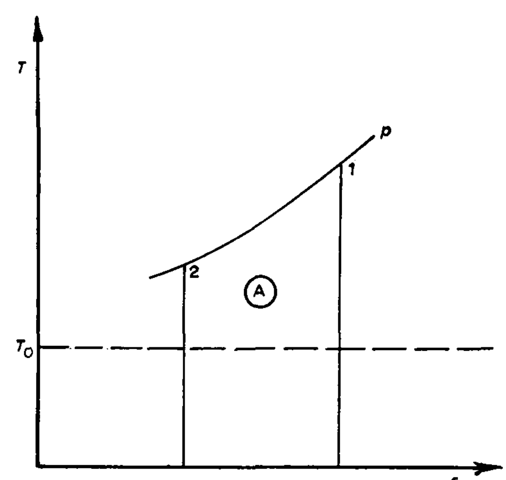
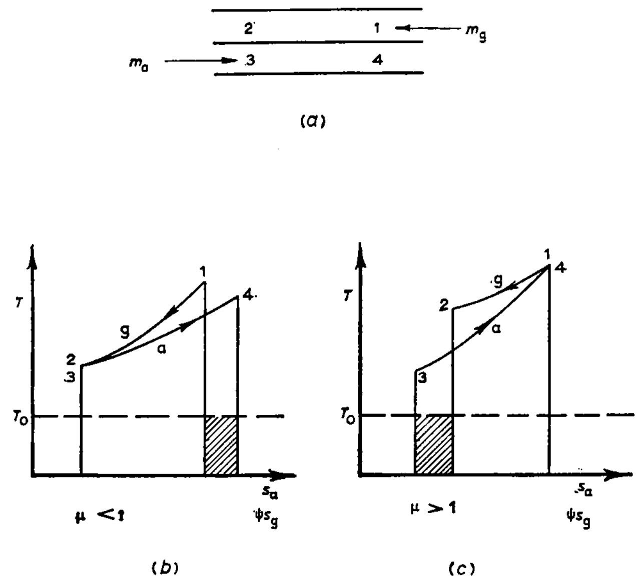


Fig. 20.4. Heat transfer to the atmosphere



$$\psi = m_g/m_a$$

$$\mu = \frac{m_g c_{pg}}{m_a c_{pa}} = \frac{M_g}{M_a}$$

Fig. 20.5. Heat exchanger flows

and the overall decrease in the steady-flow availability of the two streams is

$$\begin{aligned} \Delta B &= T_0[m_a(s_4-s_3)-m_g(s_1-s_2)] \\ &= m_a T_0[(s_4-s_3)-\psi(s_1-s_2)] \qquad (20.24) \end{aligned}$$

where $\psi = m_g/m_a$.

If the overall coefficient of heat transfer between the two streams is U at some section where the surface area is dA, then

$$M_a\, dT_a = U(T_g-T_a)\, dA$$

$$(-M_g)\, dT_g = -U(T_g-T_a)\, dA$$

where M_a, M_g are the thermal capacities of the two streams. Hence

$$\begin{aligned} d(T_g-T_a) &= U(T_g-T_a)\left[\frac{1}{M_g}-\frac{1}{M_a}\right] dA \\ &= \frac{U}{M_a}\left[\frac{1}{\mu}-1\right](T_g-T_a)\, dA \quad . \quad . \quad (20.25) \end{aligned}$$

where $\mu = \dfrac{M_g}{M_a}$ is the ratio of the thermal capacities of the two streams.

Equation (20.25) may be integrated if the factor $\dfrac{U}{M_a}\left(\dfrac{1}{\mu}-1\right)$ is constant.

$$(\ln[T_g-T_a])_{2,3}^{1,4} = \frac{UA\left(\frac{1}{\mu}-1\right)}{M_a} = \phi$$

$$\frac{T_1-T_4}{T_2-T_3} = e^{\phi} \quad . \quad . \quad . \quad (20.26)$$

For a very large heat exchanger $A \to \infty$ and

$$(T_2-T_3) \to 0 \quad \text{if} \quad \mu < 1, M_g < M_a$$

$$(T_1-T_4) \to 0 \quad \text{if} \quad \mu > 1, M_g > M_a$$

These two cases are illustrated in Figs 20.5*b* and *c*. In each figure the entropy scale for air (a) is plotted truly, but that for the external gas (g) is scaled by the factor ψ. The shaded areas represent the overall decrease in steady-flow availability per unit air flow, $T_0[(s_4-s_3)-\psi(s_1-s_2)]$, which is always positive.

The effectiveness of a finite heat exchanger is defined as the ratio of the heat transferred to the air to that which would be transferred (for the same gas and air inlet temperatures and flow rates) in a heat exchanger of infinite surface area. Thus, the effectiveness, λ, is given by

$$\lambda = \frac{M_a(T_4-T_3)}{M_a(T_1-T_3)} = \frac{e^{-\phi}-1}{e^{-\phi}-\frac{1}{\mu}} = \frac{\alpha\, e^{(\mu-1)/\mu}-1}{\alpha\, e^{(\mu-1)/\mu}-\frac{1}{\mu}} \quad (20.27)$$

where $\alpha = e^{UA/M_a}$ and $M_g > M_a$.

Bruges has suggested that a better measure of heat exchanger performance is the 'thermal energy ratio', the ratio of the increase of availability of the cold stream to the maximum possible increase of availability,

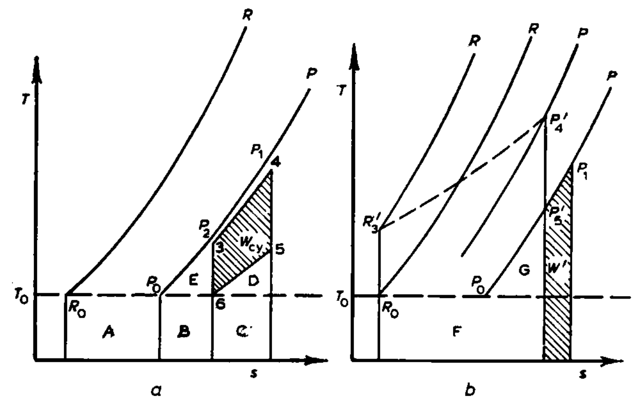
$$\frac{m_a(b_4-b_3)}{m_g(b_1-b_2)} \quad . \quad . \quad . \quad (20.28)$$

A COMPARISON BETWEEN INTERNAL-COMBUSTION AND EXTERNAL-COMBUSTION POWER PLANTS ON THE BASIS OF RATIONAL EFFICIENCY

The idea of rational efficiency for an external-combustion power plant is developed below. It is first of interest to compare an internal-combustion power plant with a corresponding external-combustion power plant, the air/fuel ratio being the same in each combustion process. Availability balances are drawn up for the processes involved in each power plant.

In the E.C. power plant (Fig. 20.6*a*) external combustion takes place at atmospheric pressure (R_0 to P_1) and an internally reversible gas-turbine cycle (6345) receives heat from the hot gases through an ideal heat exchanger. In the corresponding internal-combustion power plant (Fig. 20.6*b*) reactants are compressed isentropically from state R_0 to $R_{3'}$. Adiabatic irreversible combustion then takes place to $P_{4'}$ and the products of combustion expand isentropically to atmospheric pressure ($P_{5'}$). It is assumed that the pressure ratio is the same in each case and that the air/fuel ratios in the internal-combustion and external-combustion chambers are identical. Further, the flows and thermal capacities of the internal and external gas streams are assumed to be the same.

The availability balances are drawn up in Table 20.1 and the items in the balance may be related to areas on the T, s diagram. For the E.C. plant the maximum work is $\Delta G_0 = (A+B+C+D+E+W_{CY})$, Fig. 20.6*a*; the calorific value is $\Delta H_0 = (B+C+D+E+W_{CY})$; the work is the area enclosed by the internal cycle, W_{CY}, the loss in



$$A+B+C+D+E+W_{CY} = \Delta G_0$$

Fig. 20.6a. External-combustion power plant ($\mu = 1$)

$$F+G+W' = \Delta G_0$$

Fig. 20.6b. Internal-combustion power plant ($\mu = 1$)

availability in the cooling of the internal gases is the area D and the loss in the exhausting combustion gases is the area E. The loss in availability in combustion is the area (A+B+C).

For the I.C. plant the maximum work that could be obtained from the reactants is $\Delta G_0 = F+G+W'$ as before. The work output is now

$$W' = (H_{P_4'}-H_{P_5'})-(H_{R_3}-H_{R_0})$$

but $\qquad H_{R_3'} = H_{P_4'},\ H_{R_0} = H_{P_1}$

so that

$$W' = H_{R_3'}-H_{P_5'}-H_{R_3'}+H_{P_1} = H_{P_1}-H_{P_5'}$$

The decrease of the steady-flow availability function in combustion is now the area (F) and the exhaust gas loss is (G).

The rational efficiency of the E.C. plant is

$$\frac{W}{\Delta G_0} = \frac{W_{CY}}{(A+B+C+D+E+W_I)} \qquad (20.29)$$

and that of the I.C. plant is

$$\frac{W}{\Delta G_0} = \frac{W'}{(F+G+W')}$$
$$= [\Delta H_0-(H_{P_5'}-H_{P_0})]/\Delta G_0$$
$$\doteqdot 1-(H_{P_5'}-H_{P_0})/\Delta H_0 \qquad . \quad (20.30)$$

since $\Delta G_0 \doteqdot \Delta H_0$.

This efficiency may be compared with the air standard cycle efficiency of the Joule–Brayton cycle $(1-1/r^{(\gamma-1)/\gamma})$. As the pressure ratio r is increased so $H_{P_5'} \rightarrow H_{P_0}$ and the efficiency increases.

For the E.C. plant, with the quantity ΔG_0 fixed, the maximum efficiency is obtained when the maximum specific work is obtained from the internal cycle. For $\mu = 1$, the maximum temperature, T_4, is specified, but the pressure ratio, r, is variable. The condition for maximum work (when the working fluid is a perfect gas) is well known (Hodge (**9**)), $\epsilon = r^{(\gamma-1)/\gamma} = \sqrt{\frac{T_4}{T_0}}$.

This comparison of rational efficiency is interesting in that the efficiency of the I.C. plant is closely related to that of the air standard cycle (maximum efficiency with infinite pressure ratio), but the maximum efficiency of the

Table 20.1

A. Losses in E.C. plant for $\mu = 1$ (Fig. 20.6a)

	$\Delta B - W$	Area
Combustion	$T_0(S_{P_1}-S_{R_0})$	A+B+C
Exhaust gases	$H_{P_2}-H_{P_0}-T_0(S_{P_2}-S_{P_0})$	E
Cooling of internal gases	$H_5-H_6-T_0(S_5-S_6)$	D
Compressor	0	0
Turbine	0	0
Work output	$(H_4-H_5)-(H_3-H_6)$	W_{CY}
Total $H_4-H_3+H_{P_2}-H_{P_0}+T_0(S_{P_1}-S_{R_0}-S_{P_2}+S_{P_0}-S_5+S_6) = H_{R_0}-H_{P_0}-T_0(S_{R_0}-S_{P_0})$ since $H_4-H_3 = H_{P_1}-H_{P_2}$, $H_{P_1} = H_{R_0}$, $S_{P_1}-S_{P_2} = S_5-S_6$.		A+B+C+D+E+W_{CY}

B. Losses in I.C. plant (Fig. 20.6b)

	$\Delta B - W$	Area
Combustion	$T_0(S_{P_4'}-S_{R_3'})$	F
Exhaust gases	$H_{P_5'}-H_{P_0}-T_0(S_{P_5'}-S_{P_0})$	G
Compressor	0	0
Turbine	0	0
Work output	$(H_{P_4'}-H_{P_5'})-(H_{R_3'}-H_{R_0'})$	W'
Total $H_{P_4}-H_{R_3'}+H_{R_0}-H_{P_0}+T_0(S_{P_4'}-S_{R_3'}-S_{P_5'}+S_{P_0}) = H_{R_0}-H_{P_0}-T_0(S_{R_0}-S_{P_0})$ since $H_{P_4'} = H_{R_3'}$, $S_{P_4'} = S_{R_3'} = S_{R_0}$		F+G+W'

E.C. plant (when $\mu = 1$) is obtained when the specific work of the internal cycle is maximum. Similar but more complex analyses are developed below for the E.C. gas-turbine plant with internal irreversibilities.

Similarly, maximum rational efficiency for an I.C. plant with a constant volume (Otto) internal cycle would be obtained with infinite compression ratio, but maximum rational efficiency for an E.C. plant would be obtained with maximum specific work.

VARIATION OF RATIONAL EFFICIENCY WITH THERMAL CAPACITY RATIO (μ)

A further study of the E.C. gas-turbine plant is now made by considering changes in the ratio of the mass flows (and thermal capacities) in the combustion chamber and in the internal cycle. The loss in availability in the external gases is thereby varied, together with the maximum temperature, and the mean temperature of heat supply in the internal cycle.

The analysis is developed in four stages:

(1) the internally reversible cycle, with a heat exchanger of infinite area ($C_r X_\infty T_r$).

(2) the internally irreversible cycle, with a heat exchanger of infinite area ($C_i X_\infty T_i$).

(3) the internally reversible cycle, with a heat exchanger of finite area ($C_r X_f T_r$),

(4) the internally irreversible cycle with a heat exchanger of finite area ($C_i X_f T_i$).

A notation similar to that conventionally used for gas turbine cycle analysis is adopted. Subscripts r and i denote reversible and irreversible, and ∞ and f denote infinite and finite. No account is taken of heat-exchanger pressure-losses, but allowance for these may be made by adjusting turbine and compressor efficiencies. It is assumed that the products of combustion and the gas within the cycle are perfect gases.

The ($C_r X_\infty T_r$) power plant

Fig. 20.7 shows, on the T, s chart, two ($C_r X_\infty T_r$) processes, one for $\mu < 1$ and the other for $\mu > 1$. The entropy scales are drawn truly for the internal gases, but the external gas entropy scale is scaled by the factor ψ. In Fig. 20.7*a* ($\mu < 1$) entropy and temperature scales for the internal and external gases coincide at 2, 3: in Fig. 20.7*b* ($\mu > 1$) entropy and temperature scales coincide at 1, 4. The areas under lines 12, 34 are equal in each figure, for the internal flows are assumed to be reversible.

The thermal efficiency η_t is $\left(1-\frac{1}{\epsilon}\right)$, where $\epsilon = r^{(\gamma-1)/\gamma}$ and r is the pressure ratio, and this efficiency remains constant independent of the thermal capacity ratio μ.

The rational efficiency of the E.C. plant is

$$\eta_r = \frac{W_{CY}}{\Delta G_0} \quad . \quad . \quad . \quad (20.31)$$

If ΔG_0 is a little different from ΔH_0, then the rational efficiency is approximately the same as the so-called

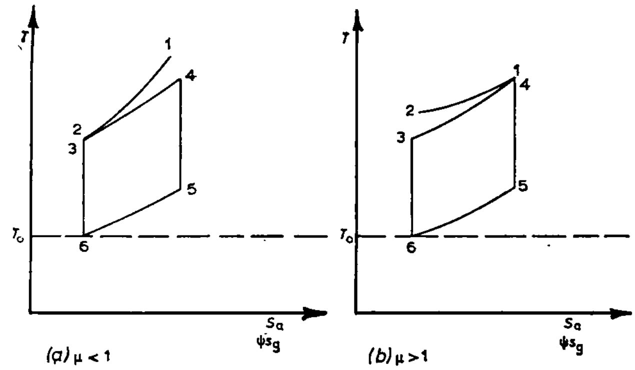


Fig. 20.7. External-combustion power plants

overall efficiency, the ratio of work output to calorific value,

$$\eta_0 = \frac{W_{CY}}{\Delta H_0} \quad . \quad . \quad . \quad (20.32)$$

But

$$\Delta H_0 = M_g(T_1 - T_0) \quad . \quad . \quad (20.33)$$

so that

$$\eta_r \doteq \eta_0 = \frac{1}{\mu}\left[\frac{(T_4-T_5)-(T_3-T_6)}{(T_1-T_0)}\right] \quad (20.34)$$

For the special case of equal thermal capacities $T_2 = T_3$, $T_1 = T_4$. It may then be shown that

$$\eta_{r(\mu=1)} \doteq \frac{(\epsilon-1)\left(\frac{\delta}{\epsilon}-1\right)}{(\delta-1)} \quad . \quad (20.35)$$

and

$$\frac{\eta_{r(\mu=1)}}{\eta_t} = \frac{(\delta-\epsilon)}{(\delta-1)} \quad . \quad . \quad (20.36)$$

where $\delta = \frac{T_4}{T_6} = \frac{T_1}{T_0}$, and $\epsilon = r^{(\gamma-1)/\gamma}$ is the isentropic temperature ratio.

For $\mu < 1$, and an infinite heat exchanger,

$$T_2 = T_3,\ \mu(T_1-T_2) = (T_4-T_3)$$

and

$$\eta_r \doteq \frac{\left(\frac{\delta}{\epsilon}-1\right)(\epsilon-1)}{(\delta-1)}$$
$$= \eta_{r(\mu=1)} \quad . \quad . \quad . \quad . \quad (20.37)$$

where $\delta = \frac{T_1}{T_0}$.

For $\mu > 1$ and an infinite heat exchanger $T_1 = T_4$, and

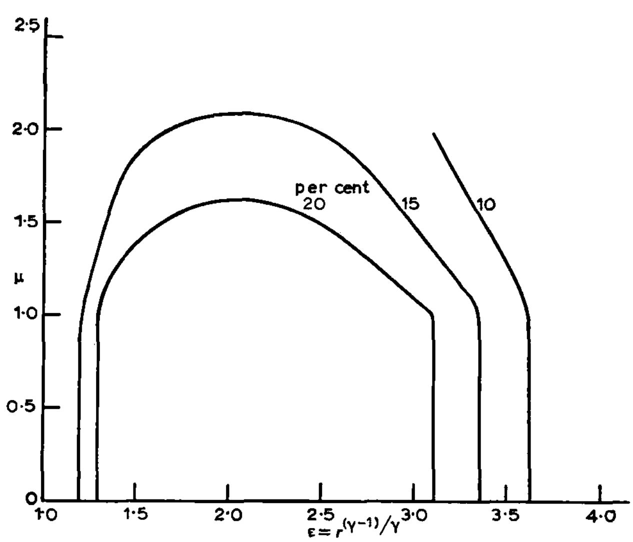
$$\eta_r \doteq \frac{1}{\mu}\left[\frac{(T_4-T_5)-(T_3-T_6)}{(T_1-T_0)}\right] \quad (20.38)$$
$$= \frac{1}{\mu}(\eta_{r(\mu=1)})$$

Thus, for a given ratio of temperature after combustion, T_1, to ambient temperature ($T_0 = T_6$), and a given internal cycle pressure ratio, the rational efficiency, η_r, remains constant as μ is increased to unity and then decreases as μ is increased further. An explanation of this behaviour is as follows:

(1) With $\mu < 1$ the temperature $T_2 = T_3$, and for a given cycle pressure ratio, this temperature remains constant. The heat transferred to the internal cycle is $M_g(T_1-T_2)$ and is, therefore, proportional to μ (if M_a is held constant and M_g varied; T_1-T_2 is constant). The work output from the internal cycle is the product of the constant thermal efficiency and the heat transferred, and is, therefore, also proportional to μ, as is the maximum available work (ΔG_0). The rational efficiency is, therefore, constant; both work output and losses (the loss in availability in the exhaust gases and the loss in the cooling in the internal cycle) are directly proportional to μ.

(2) With $\mu > 1$ the temperature T_4 is constant and equal to T_1, so that the work output is unaltered if M_a is held constant. The maximum available work (ΔG_0) is proportional to μ. The efficiency is, therefore, inversely proportional to μ. As the external mass flow is decreased, so the loss in availability in the exhaust gases is decreased.

A contour plot of η_r against ϵ and μ for a given δ, which would in general be fixed, is shown in Fig. 20.8. Maximum η_r is obtained with $\mu \leqslant 1$ and $\epsilon = \sqrt{\delta} = \sqrt{\frac{T_1}{T_0}}$. It is of interest to note that when $\mu < 1$ maximum η_r is obtained when $\epsilon = \sqrt{\frac{T_1}{T_0}}$ (not $\epsilon = \sqrt{\frac{T_4}{T_0}}$, the pressure ratio for maximum specific work). This point is not of great practical importance, however, for although there would appear to be some attraction in keeping $\mu < 1$, and maintaining the efficiency while lowering the maximum internal temperature T_4, this is not a profitable practice when internal irreversibility is considered, as the next section shows.



$\delta = 4{\cdot}0$, $\eta_T = \eta_C = 1{\cdot}0$.

Fig. 20.8. Contour plot of efficiency for $(C_r X_\infty T_r)$ E.C. power plant

The $(C_i X_\infty T_i)$ power plant

The analysis can be simply modified to include internal irreversibilities.

With an infinite heat exchanger and $\mu > 1$, $T_1 = T_4$. The rational efficiency is then

$$\eta_r \doteqdot \frac{\eta_T \delta\left(1-\frac{1}{\epsilon}\right)-\frac{1}{\eta_C}(\epsilon-1)}{\mu(\delta-1)}$$

$$= \frac{\eta_{r(\mu=1)}}{\mu} \quad . \quad . \quad . \quad . \quad (20.39)$$

where η_T and η_C are the isentropic efficiencies of turbine and compressor, and $\delta = T_1/T_0$.

For $\mu < 1$, it may be shown that

$$\eta_r = \frac{(\epsilon-1)}{(\delta-1)}\left[\eta_T\frac{\delta}{\epsilon}+\eta_T\left(\frac{1}{\epsilon}+\frac{1}{\eta_C}-\frac{1}{\epsilon\eta_C}\right)\left(\frac{1-\mu}{\mu}\right)-\frac{1}{\mu\eta_C}\right] \quad . \quad . \quad . \quad (20.40)$$

A contour plot of η_r against ϵ and μ for given δ now shows that there is no incentive to reduce μ below unity (Fig. 20.9). Maximum efficiency is obtained with $\mu = 1$ and $\epsilon = \sqrt{\frac{\eta_C\eta_T T_1}{T_0}}$, the condition for maximum specific work.

The $(C_r X_f T_r)$ power plant

The introduction of a heat exchanger of finite size complicates the analysis considerably. Since the analysis of the $(C_i X_\infty T_i)$ power plant indicates that there is no point in reducing μ below unity, subsequent analyses are developed for $\mu > 1$.

The effectiveness of the finite heat exchanger may be obtained from equation (20.27). It must now be remembered that although $\mu > 1$ the maximum internal temperature T_4 is now less than the combustion temperature T_1.

The rational efficiency may be written:

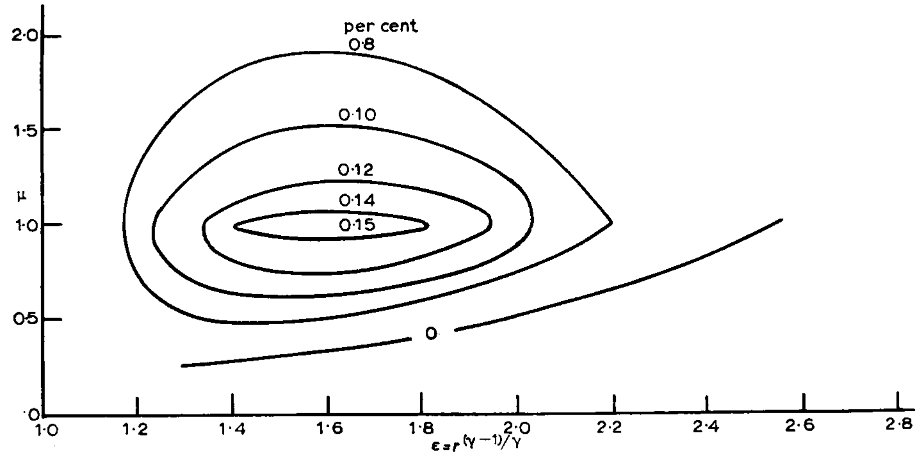
$$\eta_r \doteqdot \frac{1}{\mu}\left[\frac{T_4\left(1-\frac{1}{\epsilon}\right)-T_0(\epsilon-1)}{T_1-T_0}\right] \quad (20.41)$$

where $T_4 = \lambda(T_1-T_3)+T_3 = \lambda T_1+\epsilon T_0(1-\lambda)$ and λ is given by equation (20.27).

Maximum rational efficiency is obtained by putting $\left(\frac{\partial\eta_r}{\partial\mu}\right)_\delta, \left(\frac{\partial\eta_r}{\partial\epsilon}\right)_\mu = 0$. From the first condition, it follows that

$$(\mu-1)^2 = \alpha\, e^{(\mu-1)/\mu}-1 \quad . \quad . \quad (20.42)$$

where $\alpha = e^{UA/M_a}$.



$\delta = 4{\cdot}0,\ \eta_T = \eta_C = 0{\cdot}8.$

Fig. 20.9. Contour plot of efficiency for $(C_1X_\infty T_1)$ E.C. power plant

Since equation (20.42) is independent of ϵ, it follows that maximum efficiency is obtained by satisfying equation (20.42), determining the corresponding T_4, and choosing ϵ so that $\epsilon = \sqrt{\frac{T_4}{T_0}}$.

The $(C_1X_fT_1)$ power plant

For the E.C. gas-turbine plant with external irreversibility, with $\mu > 1$ and a finite heat exchanger, T_4 is again less than T_1 and is a function of μ. The rational efficiency is now

$$\eta_r = \frac{1}{\mu}\frac{\left[\eta_T T_4\left(1-\frac{1}{\epsilon}\right)-\frac{T_0}{\eta_C}(\epsilon-1)\right]}{(T_1-T_0)} \quad (20.43)$$

where $T_4 = \lambda(T_1-T_3)+T_3$

$$= \lambda T_1+(1-\lambda)T_0\left(1+\frac{\epsilon-1}{\eta_C}\right)$$

λ is given by equation (20.29).

Optimization for maximum efficiency requires that $\left(\frac{\partial\eta_r}{\partial\mu}\right)_\epsilon, \left(\frac{\partial\eta_r}{\partial\epsilon}\right)_\mu = 0$. Analysis of this case is complex and leads to the conditions

$$\frac{\mu^2\alpha\, e^{(\mu-1)/\mu}\left((\mu-1)^2-(\alpha\, e^{(\mu-1)/\mu}-1)\right)}{(\mu\alpha\, e^{(\mu-1)/\mu}-1)^2} = \frac{1-\frac{1}{\eta_C}}{\delta-1-\frac{\epsilon-1}{\eta_C}} \quad . \quad . \quad . \quad (20.44)$$

and

$$\left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{1-\lambda}{\eta_C}\right)+\frac{1}{\epsilon^2}\left[\delta+(1-\lambda)+\left(\frac{1-\lambda}{\eta_C}\right)(\epsilon-1)\right] = \frac{1}{\eta_C\eta_T} \quad . \quad . \quad . \quad (20.45)$$

where $\lambda = (\alpha\, e^{(\mu-1)/\mu}-1)\Big/\left(\alpha\, e^{(\mu-1)/\mu}-\frac{1}{\mu}\right)$

but numerical calculation for specific cases is obviously more profitable.

DISCUSSION

For the analysis of the E.C. gas-turbine cycle, it appears that for large heat exchangers with $\delta = T_1/T_0$ fixed and no internal limit on T_4, the maximum rational efficiency is obtained by choosing the value of thermal capacities near to unity and optimizing r to give maximum $\eta_{r(\mu=1)}$. This pressure ratio is that which gives maximum specific work in the internal cycle $\epsilon = \sqrt{\frac{\eta_C\eta_T T_1}{T_0}}$. It is of interest to note that the rational efficiency would not be affected by the introduction of a heat exchanger within the internal cycle; increased efficiency would be balanced by increased losses in the availability of the exhaust gas.

With an internal limit set on maximum temperature ($T_4 = T_{max}$), a value of μ must first be selected to give $T_4 = T_{max}$. The efficiency is then maximized by selecting a pressure ratio to give maximum specific work.

Similar, more complex, numerical analyses may be made of other E.C. power plants. The Rankine power plant obviously has high rational efficiency, because of the low loss in availability in the exhaust gases from the boiler after passage through an economizer, and the small loss in the condensing process in the internal cycle. There will, however, be considerable loss of availability in the heat-transfer process because the constant pressure heating and cooling lines are widely separated, because of their different shapes, on the T, s chart. This means that the irreversibility (area A, Fig. 20.10) must be larger than in the case of a gas-to-gas exchanger where constant pressure lines lie closer together.

Field (**10**) has proposed a power plant in which a Rankine cycle is topped by a Joule–Brayton cycle and the boiler is eliminated. The steam flow at exit from the Joule–Brayton heat exchanger is split, part going through a Rankine turbine condenser and feed pump, and part being used to mix with the water thus produced. The resulting mixture is dry saturated. It is compressed, heated in the internal heat exchanger and an externally

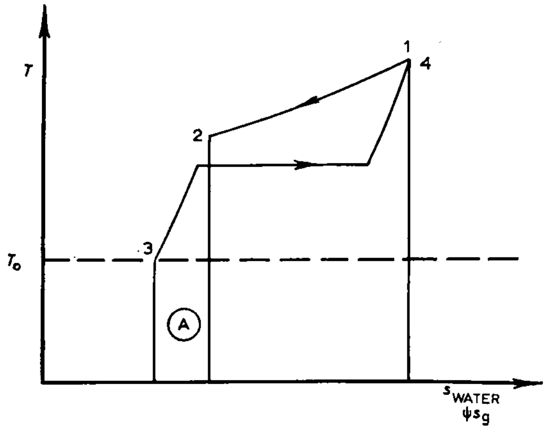


Fig. 20.10. Heat supply in the Rankine cycle

fired superheater, and then enters the Joule–Brayton turbine. The thermal efficiency of this cycle is high, and may be shown to be

$$\eta_{\text{Rankine}}+\eta_{\text{Brayton}}-\eta_{\text{Rankine}}\eta_{\text{Brayton}}$$

(Horlock (11)), where η_{Rankine}, η_{Brayton} are the thermal efficiencies of the basic Rankine and Joule–Brayton cycles. The irreversibility in the superheater may be small, but the loss in availability in the combustion gases may be large unless they are used to preheat the air before combustion. Again, there would appear to be little point in using an internal heat exchanger, except in so far as the superheat in the exhaust from the Rankine turbine will be reduced.

CONCLUSIONS

The value of the availability concept in analysing an external combustion cycle is established. The rational efficiency of such a cycle is dependent upon the ratio of the thermal capacities of the external and internal fluid flows, as well as the specific work of the internal cycle.

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APPENDIX 20.I

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