Performance Analysis of Cellular CDMA with Handoff under Signal-to-Interference Ratio based Admission Control

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Abstract

Admission control policies in Code Division Multiple Access(CDMA) systems can be based upon code availability or interference measurements. We have considered voice only cellular CDMA systems with admission control based on interference measurements. Probability of outage is considered as performance measure. In the past, analysis for admission control based on interference measurements have been considered in the literature. However, these studies have not analyzed the system under 'handoff scenarios'. In this paper, we derive an expression for the probability of outage for CDMA system under signal to interference (SIR) based admission control, considering the effect of both hard handoff and soft handoff. The analytical results are compared with the results obtained from actual simulation of the CDMA system and we demonstrate that the analytical and simulation results exhibit a reasonably good match.

Index Terms

Code Division Multiple Access (CDMA), Signal to Interference Ratio (SIR), Quality of Service (QoS), Handoff, Outage.

I. INTRODUCTION

Code Division Multiple Access (CDMA) has been acclaimed as the predominant medium access control (MAC) technique for next generation wireless systems for its effectiveness in achieving high capacity for a given wireless spectrum allocation. The capacity of a CDMA air interface is determined by the capacity of the reverse link which is typically interference limited. Since the reverse link of CDMA is interference limited, the power transmitted by a mobile station (MS) is controlled by the base station (BS) using sophisticated power control algorithms [1]. If the signal to interference ratio (SIR) seen by the BS for a MS falls below a certain threshold, the Quality of Service (QoS) requirements for the MS may not be met leading to poor voice quality or erroneous packet transmission. This condition is called outage in CDMA.

In the past, quite a few attempts have been made to analyse the outage and capacity performance of CDMA. Some of this literature has been summarized in text-book material [1], [2]. In [2], an analytical expression for probability of outage, taking multi-class users into consideration for a single cell scenario has been discussed. In [1], an analytical expression for probability of outage taking multiple cell interference into consideration has been derived. The effect of multiple cell interference has been approximated by using the other-cell interference factor which gives an upper bound on the interference due to other-cell users. In [3], the voice call outage

probability for a multiple cell system has been derived in the presence of data traffic. The analysis of outage and capacity for voice only CDMA system have been evaluated in [4], [5]. In [6], the authors have used probability of outage as a function of fade margins, to compare hard handoff with soft handoff.

Whenever SIR falls below the threshold required for QoS guarantees, new incoming users in a cell are blocked. Admission control policy in a CDMA system is designed to reduce the probability of outage in the system. The earlier research [7], [8] in CDMA admission control, however, was concentrated on Code Availability (CA) based admission control. CA based admission control policy adds a user if a spreading code is available in the systems. In [7], the authors propose an admission control scheme for delay sensitive voice and delay insensitive data. They have considered a queuing model for integrated service and also derived an expression of blocking probability.

Our interest in this paper, however, is on SIR based admission control and outage performance analysis. SIR based admission control admits a user if the SIR after admitting the new user is above a threshold. Recently, SIR based admission control have been considered in [9], [10], [11]. In [9], the authors have derived outage probability with SIR based admission control. In [10], the authors have given an expression for blocking probability under SIR based admission control. More recently, in [11], a comprehensive performance analysis of voice and data CDMA systems has been considered. This analysis has evaluated the outage probability for voice traffic and the average system throughput and mean delay for data traffic. The analysis has also considered multiple cell scenario.

In all the above studies, while the effects of multiple cell interference has been taken into consideration, the analysis have not considered the effect of handoff. The authors of [12] have analyzed the CDMA systems under SIR based admission control in soft handoff scenarios but blocking probabilities has been taken as the performance measure. Our primary contribution of the paper is to evaluate the outage performance for voice only CDMA system with multiple cell interference and handoff with SIR based admission control. We have considered both hard handoff and soft handoff. We have extended the approach of [11] for computing the outage probability.

The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, we extend the analysis in [11] considering a system with hard handoff. In Sec-

tion IV, CDMA system with soft handoff is considered. We compare the results obtained from the analysis to the results obtained by actual simulation of the system. Concluding remarks are presented in Section V.

II. System Model

Consider a CDMA cellular system with voice calls only. We analyze the performance of the system with hard and soft handoff under SIR-based admission control policy. The performance measure being considered is the outage probability. We follow the same notation as in [11].

According to SIR-based admission control policy considered, a voice call is admitted in the system only if:

- User is admissible according to CA based admission control policy.
- The SIR at the base station after admitting the user is greater than the desired threshold.

An incoming call is blocked if it does not satisfy the above given conditions.

The following assumptions are made about the system considered:

- The signal undergoes distance attenuation, shadow loss and multi-path Rayleigh fading. The path loss exponent is taken to be 4. The shadow loss is assumed to be log-normally distributed of the form $10^{-\frac{\psi}{10}}$, where $\psi \sim \mathcal{N}(0, \sigma^2)$. For voice traffic, the multi-path Rayleigh fading is assumed to be averaged out due to long holding times of voice calls.
- Perfect power control is assumed for voice calls, i.e., a base station receives unit power from all the voice users power controlled by it, irrespective of the position of users.
- The voice call arrival in each cell has Poisson distribution with mean arrival rate, λ_v. The voice call holding times are exponentially distributed with mean μ_v^{-1} seconds. Mean voice traffic load, ρ=λ_v/μ_v Erlangs/cell.
- Handoff of calls occur uniformly across the cells with rate λ_H for system with hard handoff.
- Interference is only due to the users from the cell in consideration and first tier surrounding it. The threshold for ratio of interference to signal power noise density (I/S), for admitting voice calls is ε.
- The neighboring cells are the first tier of cells surrounding the cell of interest, i.e the six surrounding cells.

1) Simulation Model: The following system parameter values are used in all the analytical computations and simulations:

- Maximum of 64 users permitted in a cell,
- $\mu_v^{-1} = 100$ seconds,
- $\sigma = 8$ dB,
- $\varepsilon = 14$ dB,
- $\lambda_H = 0.2\lambda_v$ and $\lambda_H = 0.3\lambda_v$.

III. CDMA System with Hard Handoff

Handoff of calls is a very important feature in cellular systems. Handoff of calls from one cell to another in CDMA system alters power control of users, which in turn affects the interference at the base station. In this section, we obtain an analytical expression for the probability of outage considering hard handoff. In hard handoff, we assume that the handoff occurs at the fixed cell boundary as assumed in [1]. This assumption is somewhat restrictive. In practice, handoff algorithms may be based on received signal strength measurements. However, this assumption simplifies our analysis. We later consider more practical soft handoff scenario.

Let λ_H be the rate of arrival of a call due to handoff. Total call arrival rate is $\lambda = \lambda_v + \lambda_H$. The rate of arrival of a call from a boundary can be calculated as $\lambda_h = \lambda_H/6$. The mean traffic load, ρ, is given by

$$\rho = \frac{\lambda_v + \lambda_H}{\mu_v}. \tag{1}$$

Let k denote the reference cell for our analysis. Let $\mathbf{N_k}$ denote the region of neighboring cells of k. The number of users power-controlled by a particular cell i is denoted by $\triangle_i$ and the number of users in the first tier surrounding cell i is given by $\triangle_{0_i}$.

A. Analysis of Outage Probability

In cellular CDMA, the interference at the base station in a given cell is due to all the other users. To simplify our analysis, we assume that the interference is due to users inside cell k, i.e., in-cell users and due to the users in neighboring cells of cell k, i.e. other-cell users. The total number of interfering users seen by cell k is given by

$$\triangle_{T_k} = \triangle_k + \triangle_{O_k}. \tag{2}$$

We assume that the users are in perfect power control by the base station of the cell to which they belong. The power received at its base station due to any mobile is assumed to be unity. Thus the total interference at the base station of cell k, denoted as $I(\triangle_k, \triangle_{O_k})$, is given by

$$I(\triangle_k, \triangle_{O_k}) = \triangle_k + I(\triangle_{O_k}) \tag{3}$$

where $\triangle_k$ is the interference due to in-cell users (equal to the number of in-cell users under power control) and $I(\triangle_{O_k})$ is the interference due to other-cell users given by

$$I(\triangle_{O_k}) = \sum_{i \in \mathbf{N}_k} \sum_{j=1}^{\triangle_i} \frac{D_{ji}^4 10^{-\frac{\psi_{ji}}{10}}}{D_{jk}^4 10^{-\frac{\psi_{jk}}{10}}} \tag{4}$$

Here, $\triangle_{O_k} = \sum_{i \in \mathbf{N}_k} \triangle_i$. Also, $\psi_{jk}, \psi_{ji} \sim \mathcal{N}(0, \sigma^2)$ corresponds to the shadow loss from j^{th} user in cell i to the base station in cell k and i respectively. D_{jk} is the distance between the j^{th} voice interferer in cell i and k^{th} base station and D_{ji} is the distance between the j^{th} voice interferer in cell i and i^{th} base station. There are four cases of arrival of a call in cell $i \in \mathbf{N_k}$ which can cause outage in cell k:

1) **A new call arrival:** A new call arrival in cell i will increase the interference at cell k and it may cause outage in the cell. Note that a new call arrival in cell k will be blocked if it is likely to cause outage in cell k, as per the SIR-based admission control policy.
2) **Handoff arrival from first tier of neighboring cells of cell k:** Handoff of calls from one neighboring cell of cell k to another neighboring cell of cell k can also cause outage in the cell. Handoff of calls causes power control to change from one base station to another base station. This change in power control may require the user (whose call is in handoff) to transmit at a higher power now, which may increase the interference at the base station of cell k. This increase in interference can cause outage in cell k. There are two such cells in the first tier of cell k, cell 2 and 3 as illustrated in Fig (2), hence, the rate of of arrival of such calls will be $2\lambda_h$
3) **Handoff arrival from cell k:** After the handoff of call from cell k to one of its neighboring cell i, the user will no longer be power controlled by cell k. It will be power controlled by cell i. Earlier the interference caused by that user at cell k was unity. After the handoff, the user may be transmitting at higher power levels which may increase the interference

at cell k [1]. The rate of arrival of such calls will be λ_h.

4) **Handoff arrival from second tier of neighboring cells:** Since, initially the interference from users in second tier of neighboring cells is assumed to be negligible, their handoff from second tier to first tier can be modeled as a new arrival only. We have three such cells, cell 4, 5 and 6 in Fig. (2), hence the new call arrival rate will be $\lambda_v + 3\lambda_h$.

Thus, there are three cases only that we discuss below.

1) Outage due to New Arrival: Due to a new arrival in cell i, the number of users in cell i increases by 1 and is given by $\triangle_i^{'} = \triangle_i + 1$ and the number of other-cell users becomes $\triangle_{O_k}$ from $\triangle_{O_k} - 1$. Interference seen at the BS of cell k due to other-cell interferers after the new arrival is given by

$$I(\triangle_{O_k}) = I(\triangle_{0_k} - 1) + X_{ik}^{j}, \tag{5}$$

where X_{ik}^{j} denotes the interference at the base station of cell k due to a newly arrived call j in one of the neighboring cell $i \in \mathbf{N_k}$. Hence,

$$X_{ik}^{j} = \frac{D_{ji}^{4} 10^{-\frac{\psi_{ji}}{10}}}{D_{jk}^{4} 10^{-\frac{\psi_{jk}}{10}}}.$$

The probability of outage is given by

$$p_{oa}(\triangle_k, \triangle_{O_k}, \triangle_i, \triangle_{O_i}, \varepsilon) = Pr\Big\{ I(\triangle_k, \triangle_{O_k}) > \varepsilon \mid I(\triangle_k, \triangle_{O_k} - 1) \leq \varepsilon, I(\triangle_i, \triangle_{O_i}) \leq \varepsilon \Big\}. \tag{6}$$

The above equation gives the probability that a newly admitted call in cell i which does not cause outage in cell i (since $I(\triangle_i, \triangle_{O_i}) \leq \varepsilon$), causes an outage in cell k (since $I(\triangle_k, \triangle_{O_k}) > \varepsilon$), given that there was no outage in cell k before admission of the new call (since $I(\triangle_k, \triangle_{O_k} - 1) \leq \varepsilon$). We assume that $I(\triangle_i, \triangle_{O_i})$ is independent of $I(\triangle_k, \triangle_{O_k} - 1)$ to simplify the analysis as done in [11]. Hence, $p_{oa}(\triangle_k, \triangle_{O_k}, \triangle_i, \triangle_{O_i}, \varepsilon)$ becomes independent of the cell i in which the call arrives and can be denoted by $p_{oa}(\triangle_k, \triangle_{O_k}, \varepsilon)$, which can be written as-

$$p_{oa}(\triangle_{O_k}, \varepsilon^{'}) = Pr\Big\{ I(\triangle_{O_k}) > \varepsilon^{'} \mid I(\triangle_{O_k} - 1) \leq \varepsilon^{'} \Big\}, \tag{7}$$

[1] Note that this condition is possible since we are considering hard handoff where the handoff occurs at the fixed cell boundaries. In actual practice, however this is not possible. We consider the more practical case of soft handoff later.

where $\varepsilon^{'} = \varepsilon - \triangle_k$. Averaging over $\triangle_{O_k}$, the outage probability conditioned on $\triangle_k$, $p_{oa}(\varepsilon^{'})$, is given by

$$p_{oa}(\varepsilon^{'}) = 1 - \prod_{i \in \mathbf{N}_k} \left[1 - \sum_{m_i} p_o(m_i, \varepsilon^{'}) Pr\{\triangle_{O_i} = m_i\} \right]. \tag{8}$$

Here we model the other cell interfering users $\triangle_{O_k}$ as queue length process of an $M/G/\infty$ queue as assumed in [1]. Thus $\triangle_{O_k}$ is a Poisson random variable with mean $\hat{\rho} = n_k \rho$, where n_k is the number of neighboring cells of cell k (i.e n_k=6) and ρ is given in Eqn. (1). The probability mass function of $\triangle_{O_k}$ is given by

$$Pr\{\triangle_{O_k} = m\} = \frac{e^{-\hat{\rho}} \hat{\rho}^m}{m!}. \tag{9}$$

The outage probability averaged over $\triangle_k$, is then given by

$$p_{oa} = \sum_{m_k=0}^{\infty} p_o(\varepsilon^{'}) Pr\{\triangle_k = m_k\}. \tag{10}$$

$\triangle_k$ is a Poisson random variable with mean ρ. The probability mass function of $\triangle_k$ is given by

$$Pr\{\triangle_k = m_k\} = \frac{e^{-\rho} \rho^{m_k}}{m_k!}. \tag{11}$$

The conditional probability in Eqn. (7) can be computed by computing the marginal probability and joint probability terms given by p_M and p_J as:

$$p_M = Pr\{I(\triangle_{0_k} - 1) \leq \varepsilon^{'}\} \tag{12}$$

$$p_J = Pr\left\{ I(\triangle_{0_k}) > \varepsilon^{'}, I(\triangle_{0_k} - 1) \leq \varepsilon^{'} \right\} \tag{13}$$

To evaluate these probabilities, consider Eqn. (4). In Eqn. (4), each term in the summation is a log-normal random variable and their sum can be approximated to be a normal distribution using the Central Limit Theorem (CLT) as done in [11], [4].

a) Approximation of Marginal Probability: Using CLT, we approximate $I(\triangle_{O_k} - 1)$ by a normal random variable with mean $\mu_{\triangle_{O_k}-1}$ and variance $\sigma^2_{\triangle_{O_k}-1}$, which are given by

$$\mu_{\triangle_{O_k}-1} = \sum_{i \in \mathbf{N}_k} \sum_{j=1}^{\triangle_i} e^{\left(a\mu_{ik}^j + a^2\sigma^2\right)} \tag{14}$$

and

$$\sigma^2_{\triangle_{O_k}-1} = \sum_{i \in \mathbf{N}_k} \sum_{j=1}^{\triangle_i} e^{\left(2a\mu_{ik}^j + 2a^2\sigma^2\right)} \left(e^{2a^2\sigma^2} - 1\right) \tag{15}$$

where $a = \frac{\ln(10)}{10}$, $\sum_{i \in \mathbf{N}_k} \triangle_i = \triangle_{O_k} - 1$, and

$$\mu_{ik}^j = 10 \log_{10} \frac{D_{ji}^4}{D_{jk}^4}. \tag{16}$$

Here, μ_{ik}^j denotes the interference at cell k due to the distance losses from the j^{th} user in the i^{th} cell. The outage probability is conditioned on the position of users and therefore has to be averaged out over the location of interferers. We assume that mean μ_{ik}^j are i.i.d. random variables, which is valid due to symmetry of the system. Hence μ_{ik}^j will now be represented as μ_k (averaged over the area covered by $\mathbf{N_k}$). Eqns. (14) and (15) can then be written as

$$\mu_{\triangle_{O_k}-1} = (\triangle_{O_k} - 1) e^{\left(a\mu_k + a^2\sigma^2\right)} \tag{17}$$

and

$$\sigma^2_{\triangle_{O_k}-1} = (\triangle_{O_k} - 1) e^{\left(2a\mu_k + 2a^2\sigma^2\right)} (e^{2a^2\sigma^2} - 1). \tag{18}$$

$f_k(x)$, the marginal pdf of $I(\triangle_{O_k} - 1)$ is given as

$$f_k(x) = \frac{1}{(2\pi\sigma^2_{\triangle_{O_k}-1})^{1/2}} \exp\left(- \frac{\left(x - \mu_{\triangle_{O_k}-1}\right)^2}{2\sigma^2_{\triangle_{O_k}-1}} \right). \tag{19}$$

The marginal probability in Eqn. (12) can be evaluated as

$$Pr\{I(\triangle_{O_k} - 1) \leq \varepsilon`\} = \int_{-\infty}^{\varepsilon'} f_k(x) dx \tag{20}$$

b) Approximation of Joint Probability: In order to evaluate the joint probability in Eqn. (12), we need to evaluate the cumulative distribution function of $I(\triangle_{O_k} - 1)$ and $I(\triangle_{O_k})$. Assuming both $I(\triangle_{O_k} - 1)$ and $I(\triangle_{O_k})$ to have normal distribution, we can write the joint distribution as

$$\tilde{f}_{\triangle_{O_k}}(x, y) = \frac{1}{2\pi\sigma_1\sigma_2(1 - r^2)^{1/2}} \exp\left(- \frac{1}{2(1 - r^2)} \left(\frac{\hat{x}^2}{\sigma_1^2} - 2\frac{\hat{x}\hat{y}}{\sigma_1\sigma_2} r + \frac{\hat{y}^2}{\sigma_2^2} \right) \right)$$

where $\hat{x} = x - \mu_{\triangle_{O_k}}$ and $\hat{y} = y - \mu_{\triangle_{O_k}-1}$, $\sigma_1 = \sigma_{\triangle_{O_k}}$ and $\sigma_2 = \sigma_{\triangle_{O_k}-1}$ and the correlation coefficient r, is given by

$$r = \frac{\mathbf{E}\left[I(\triangle_{O_k}) I(\triangle_{O_k} - 1)\right] - \mu_{\triangle_{O_k}-1}\mu_{\triangle_{O_k}}}{\sigma_{\triangle_{O_k}-1}\sigma_{\triangle_{O_k}}}. \tag{21}$$

To evaluate the expectation $\mathbf{E}\left[I(\triangle_{O_k}) I(\triangle_{O_k} - 1)\right]$, we rewrite $I(\triangle_{O_k})$ as

$$I(\triangle_{O_k}) = I(\triangle_{O_k} - 1) + X_{ik}^j \tag{22}$$

where

$$X_{ik}^{j} = \frac{D_{ji}^{4} 10^{-\frac{\psi_{ji}}{10}}}{D_{jk}^{4} 10^{-\frac{\psi_{jk}}{10}}}.$$

Here, X_{ik}^{j} denotes the additional interference due to newly admitted call in cell i which is independent of the interference in the system before it arrived. Hence,

$$\mathbf{E}[I(\triangle_{O_k})I(\triangle_{O_k}-1)] = \mathbf{E}[I(\triangle_{O_k}-1)^2] + \mathbf{E}[X_{ik}^{j}]\mathbf{E}[I(\triangle_{O_k}-1)].$$

The joint probability p_J can then be obtained as,

$$p_J = \int_{\varepsilon'}^{\infty} \int_{-\infty}^{\varepsilon'} \tilde{f}_{\triangle_{O_k}}(x,y)dydx. \tag{23}$$

From Eqn. (23), we can obtain the joint probability as in Eqn. (13). Using the joint probability and marginal probability terms (Eqns. (12)-(13) we can get the conditional probability as in Eqn. (7). Using Eqns. (7)-(11), we get the analytical expression for probability of outage.

2) Outage due to Call Handoff in Neighboring Cells: Due to handoff of a call from cell i_2 to cell i_1 the number of users in cell i_1 increases by 1 and the number of users in cell i_2 decreases by 1. New interference due to other-cell users at the base station of cell k can be written as

$$I'(\triangle_{O_k}) = I(\triangle_{i_1}+1) + I(\triangle_{i_2}-1) + \sum_{i \in \mathbf{N_k}-\{i_1,i_2\}} I(\triangle_i) \tag{24}$$

which can also be written as

$$I'(\triangle_{O_k}) = I(\triangle_{O_k}) + X_{i_1 i_2}^{j} \tag{25}$$

where $X_{i_1 i_2}^{j}$ is the change in the interference at the BS of cell k due to handoff of a call from cell i_2 to cell i_1, hence $X_{i_1 i_2}^{j}$ can be written as,

$$X_{i_1 i_2}^{j} = \frac{D_{ji_1}^{4}}{D_{jk}^{4}} \frac{10^{-\frac{\psi_{ji_1}}{10}}}{10^{-\frac{\psi_{jk}}{10}}} - \frac{D_{ji_2}^{4}}{D_{jk}^{4}} \frac{10^{-\frac{\psi_{ji_2}}{10}}}{10^{-\frac{\psi_{jk}}{10}}} \tag{26}$$

where ψ_{ji_1}, ψ_{ji_2} and ψ_{jk} correspond to the shadowing loss from j^{th} user to i_1^{th} cell, i_2^{th} cell and k^{th} cell respectively. It is assumed that the hard handoff takes place at the cell boundary. We assume that during handoff, distance between base station of cell i_1 and that of cell i_2 is same, therefore, $D_{ji_1} = D_{ji_2}$. From Eqn. (26), $X_{i_1 i_2}^{j}$ can be written as

$$X_{i_1 i_2}^{=} \frac{D_{ji_1}^{4}}{D_{jk}^{4}} \left[10^{-\frac{\psi_{ji_1}-\psi_{jk}}{10}} - 10^{-\frac{\psi_{ji_2}-\psi_{jk}}{10}} \right] \tag{27}$$

where $10^{-\frac{\psi_{ji_1}-\psi_{jk}}{10}}$ and $10^{-\frac{\psi_{ji_2}-\psi_{jk}}{10}}$ are log-normal variables represented by $10^{-\frac{\psi_1}{10}}$ and $10^{-\frac{\psi_2}{10}}$, and $\psi_1, \psi_2 \sim \mathcal{N}(0, 2\sigma^2)$. Eqn. (27) can then be written as

$$X^j_{i_1 i_2} = 10^{-\frac{\psi_1}{10}} - 10^{-\frac{\psi_2}{10}}. \tag{28}$$

Mean and variance of log-normal random variable $10^{-\frac{\psi_1}{10}}$ is given by

$$\mu_1 = e^{a\mu^j_{i_1 k}+a^2\sigma^2} \text{ and } var_1 = e^{2a\mu^j_{i_1 k}+2a^2\sigma^2}(e^{2a^2\sigma^2}-1).$$

Similarly, mean and variance of log-normal random variable $10^{-\frac{\psi_2}{10}}$ is given by

$$\mu_2 = e^{a\mu^j_{i_2 k}+a^2\sigma^2} \text{ and } var_2 = e^{2a\mu^j_{i_2 k}+2a^2\sigma^2}(e^{2a^2\sigma^2}-1).$$

Averaging $\mu^j_{i_1 k}$ and $\mu^j_{i_2 k}$ over the boundary of cells i_1 and i_2 we write $\mu^j_{i_1 k}$, $\mu^j_{i_2 k} \sim \mu_{i_1 i_2}$ Therefore, mean and variance of the random variable $10^{-\frac{\psi_1}{10}} - 10^{-\frac{\psi_2}{10}}$ is given by

$$\mu_f = \mu_1 - \mu_2 = 0 \tag{29}$$

and

$$\sigma_f^2 = var_1 + var_2 = 2e^{2a\mu_{i_1 i_2}+2a^2\sigma^2}(e^{2a^2\sigma^2}-1). \tag{30}$$

The probability of outage due to handoff from cell i_2 to i_1 can be written as

$$p_{oh}(\triangle_{O_k}, \varepsilon^{'}) = Pr\{I^{'}(\triangle_{O_k}) > \varepsilon^{'} \mid I(\triangle_{O_k}) \leq \varepsilon^{'}\}, \tag{31}$$

i.e., the probability that a handoff arrival from cell i_2 to i_1 ($i_1, i_2 \in \mathbf{N}_k$) causes outage at cell k, since $I^{'}(\triangle_{O_k}) > \varepsilon^{'}$, given that there was no outage in cell k before handoff, since $I(\triangle_{O_k}) \leq \varepsilon^{'}$.

Averaging Eqn. (31) over $\triangle_{O_k}$ we get

$$p_{oh}(\varepsilon^{`}) = 1 - \prod_{i \in \mathbf{N_k}} \Big[1 - \sum_{m_i} p_{oh}(m_i, \varepsilon^{'}) Pr\{\triangle_{O_i} = m_i\}\Big]$$

and then averaging Eqn. (32) over $\triangle_k$ we get

$$p_{oh} = \sum_{m_k=0}^{\infty} p_{oh}(\varepsilon^{`}) Pr\{\triangle_k = m_k\}. \tag{32}$$

Eqn. (31) can be computed in a way similar to that discussed in Section III-A.1.b, by replacing X^j_{ik} in Section III-A.1.b with $X^j_{i_1 i_2}$.

3) Outage due to Handoff from Cell k to Neighborhood Cell: After a call-handoff from cell k to say cell i_1, the number of users in cell i_1 increases by 1 and the number of users in cell k decrease by 1 , i.e.,

$$\begin{aligned} \triangle_{i_1}^{'} &= \triangle_{i_1} + 1 \\ \triangle_{k}^{'} &= \triangle_{k} - 1 \end{aligned}$$

Therefore, the altered interference due to other-cell users of cell k is given by

$$I(\triangle_{O_k}) = I(\triangle_{i_1} + 1) + \sum_{i \in \mathbf{N}_k - \{i_1\}} I(\triangle_i).$$

The probability of outage is given by

$$p_{oi}(\triangle_{0_k}, \varepsilon^{'}, \varepsilon^{''}) = Pr\{I(\triangle_{O_k}) > \varepsilon^{''} \mid I(\triangle_{O_k} - 1) \leq \varepsilon^{'}\} \tag{33}$$

where $\varepsilon^{''} = \varepsilon - (\triangle_k - 1)$. The above equation gives the probability that a handoff call from cell k to one of its neighboring cell i_1 causes outage in cell k, since $I(\triangle_{O_k}) > \varepsilon^{''}$, given that there was no outage in cell k before, since $I(\triangle_{O_k} - 1) \leq \varepsilon^{'}$. Eqn. (33) can be averaged over $\triangle_{O_k}$ and $\triangle_k$ in the same way as in Section III-A.1 to get p_{oi}, the probability of outage due to handoff from cell k to the neighboring cell. The marginal probability

$$p_M = Pr\{I(\triangle_{O_k}) \leq \varepsilon^{'}\} \tag{34}$$

can be computed using CLT as in Section III-A.1.a. The joint probability, is given by the equation

$$p_J = Pr\{I(\triangle_{O_k}) > \varepsilon^{''}, I(\triangle_{O_k} - 1) \leq \varepsilon^{'}\}. \tag{35}$$

This equation can also be computed as done in Section III-A.1.b. But here the integration of the joint probability distribution function $\tilde{f}_{\triangle_{O_k}}$ will take place from $\varepsilon^{''}$ to ∞, i.e the joint probability in Eqn. (35) will be given by

$$p_J^i = \int_{\varepsilon^{''}}^{\infty} \int_{-\infty}^{\varepsilon^{'}} \tilde{f}_{\triangle_{O_k}}(x, y) dy dx. \tag{36}$$

After the computation of p_{oa}, p_{oh} and p_{oi} in the last three sections, the average outage probability in a CDMA system having hard handoff can be computed as

$$P_o = \frac{(\lambda_v + 3\lambda_h)p_{oa} + 2\lambda_h p_{oh} + \lambda_h p_{oi}}{\lambda_v + 6\lambda_h}. \tag{37}$$

Fig. (3) shows the simulation and analysis results for the probability of outage when there are no handoff arrivals in the system. In Section III-A.1 we have modeled the handoff arrival from the second tier of the neighboring cell of the reference cell, as a new arrival and obtained an expression for the probability of outage. The expression obtained, gives us the probability of outage when there are no handoff arrivals in cell i from cell k and the first tier surrounding it (cell 2 and 3 in Fig. (2)), and when the rate of arrival of new calls in the system is $\lambda_v + 3\lambda_h$. Fig. (3) gives the probability of outage for the special case in which we assume $\lambda_h = 0$. In Fig. (4) and Fig. (5), analysis and simulation results of the total probability of outage in a system having hard handoff with $\lambda_H = 0.2\lambda_v$ and $\lambda_H = 0.3\lambda_v$ respectively, have been compared. The analysis results have been obtained by evaluating Eqn. (37). It is observed that the results obtained through analysis matches with the simulation results. The probability of outage for a given system load is found to be greater, when compared to the case with no handoff arrivals. Also, the percentage increase in the probability of outage averaged over system load is seen to be equal to 43% for $\lambda_H = 0.2\lambda_v$. On increasing the rate of arrival of the handoff calls from $\lambda_H = 0.2\lambda_v$ to $\lambda_H = 0.3\lambda_v$ we see an increase in the average probability of outage.

In this section, we have derived an analytical method for computation of probability of outage in a CDMA system with hard handoff. The simulation and analysis results of probability of outage have been compared and it has been observed that analysis results matches with the simulation results. Practical CDMA system have soft handoff instead of hard handoff. Thus, the analysis in this section is further extended considering a system with soft handoff.

IV. CDMA System with Soft Handoff

In a CDMA system with soft handoff, a mobile is always in soft-connection with two of the nearest base stations and is power controlled by the best base station, i.e., the base station to which it requires minimum power to transmit. Due to this feature of soft handoff, interference at the base-station of a cell is minimum. In this section, we analyze a CDMA system with soft handoff and obtain an analytical expression for probability of outage in a cell for such a system. A user lying in cell k may be power controlled by a neighboring cell of cell k, say cell i. Similarly, a user lying in cell i may be power controlled by cell k. Therefore, the effective region of cell k where all the users power controlled by cell k lies, will include the hexagonal cell k and some part of all the neighboring cells where cell k is one of the nearest base stations. In this

system, the cell considered is not a hexagonal shaped cell but a star shaped cell (as explained in [1]), where all stars have some common overlapped area. As shown in Fig. (6), the shadowed area is the region of cell k. The user j being in soft-connection with cells i and k is denoted as n_{ik}^{j} and the total number of users in soft-connection with cells i and k is denoted by $\triangle_{ik}$, where $\triangle_{ik} = \triangle_{ki}$. Our cell of interest is cell k, i.e., we will be considering outage in cell k. The total number of users in soft-connection with cell k is denoted by $\triangle_k^s$, where $\triangle_k^s$ is

$$\triangle_k^s = \sum_{i \in \mathbf{N}_k} \triangle_{ik}.$$

The total number of users outside the region of cell k, i.e., not in soft-connection with cell k is denoted by $\triangle_k^o$. The mean traffic load ρ is given by $\rho = \lambda_v/\mu_v$. The probability of number of users in soft-connection with cell k as in [1] is given by

$$Pr\{\triangle_k^s = m\} = \frac{e^{-\rho}\rho^m}{m!}. \tag{38}$$

A. Analysis of Outage Probability

Consider interference at the base station of cell k due to all the interferers. Total interference at the base station of cell k is given by

$$\begin{aligned} I(\triangle_k^s, \triangle_k^o) &= I(\triangle_k^s) + I(\triangle_k^o) \\ &= I^p(\triangle_k^s) + I^{np}(\triangle_k^s, \triangle_k^o) \end{aligned} \tag{39}$$

where, $I^p(\triangle_k^s)$ denotes the interference at the BS of cell k due to the users power controlled by cell k and $I^{np}(\triangle_k^s, \triangle_k^o)$ denotes the interference at the BS of cell k due to the users lying in the region covered by cell k and first tier surrounding it and power controlled by one of its neighboring cell $i \in \mathbf{N}_k$.

The interference due to the users power controlled by cell k, i.e., $I^p(\triangle_k^s)$, is given by

$$I^p(\triangle_k^s) = \sum_{i \in \mathbf{N}_k} \sum_{j=1}^{\triangle_{ki}} \mathbf{1}_{\{\phi_{ki}^j = 1\}} \tag{40}$$

where $\mathbf{1}_{\{x\}}$ is the indicator function and

$$\phi_{ki}^j = \begin{cases} 1 & \text{if } \frac{D_{jk}^4}{D_{ji}^4} 10^{-\frac{\psi_{jk} - \psi_{ji}}{10}} \leq 1 \\ 0 & \text{if } \frac{D_{jk}^4}{D_{ji}^4} 10^{-\frac{\psi_{jk} - \psi_{ji}}{10}} > 1 \\ 0 & \text{if } j \text{ is not in soft-connection with cells } k \text{ and } i \end{cases}$$

ϕ^j_{ki} is defined such that, for a user j, ϕ^j_{ki} will be 1 if user j is power controlled by cell k and is also in connection with cell i. ϕ^j_{ki} will be 0 if the user j is either not power controlled by cell k or not in soft-connection with cell k and cell i. Now, considering the analysis of $I^p(\triangle^s_k)$, the total number of users in soft-connection with cell k is $\triangle^s_k$, out of which some will be power controlled by cell k and some by the neighboring cells. Consider a neighboring cell of cell k, say cell i. Number of users in connection with cell i and k is $\triangle_{ki}$. Out of these users, some will be power controlled by cell k and some by cell i. Interference at the base station of cell k will be 1 due to all the users power controlled by cell k. For all those users ϕ^j_{ki} will be 1. Therefore interference due to all the users in connection with cell k and i and power controlled by cell k is given by $\sum_{j=1}^{\triangle_{ki}} \mathbf{1}_{\{\phi^j_{ki}=1\}}$. Summing it for all the neighboring cells we get Eqn. (40). The interference due to the users not power controlled by cell k, i.e., $I^{np}(\triangle^s_k, \triangle^o_k)$ is given by

$$I^{np}(\triangle^s_k, \triangle^o_k) = \sum_{i_1 \in \mathbf{N}_k} \sum_{i_2 \in \mathbf{N}_{i_1}} \sum_{j=1}^{\triangle_{i_1 i_2}} \frac{D^4_{ji_1}}{D^4_{jk}} 10^{-\frac{\psi_{ji_1} - \psi_{jk}}{10}} \mathbf{1}_{\{\phi^j_{i_1 i_2}=1\}} \tag{41}$$

Consider the analysis of $I^{np}(\triangle^s_k, \triangle^o_k)$. All the users which are not power controlled by cell k and belong to $\triangle^o_k$ lie in the region of all the neighboring cells. Consider a cell i_1 which is one of the neighboring cells of cell k. We will consider all the users which are in connection with cell i_1. Consider another cell i_2 which belongs to the region $\mathbf{N}_{i_1}$. $\triangle_{i_1 i_2}$ is the number of users in connection with cell i_1 and cell i_2. For all users power controlled by cell i_1 among $\triangle_{i_1 i_2}$, we compute the interference at cell k. We do this for all the cells neighboring to cell i_1. Thus, we compute the interference at cell k due to all the users power controlled by cell i_1. This is done for all i_1 belonging to the neighborhood of cell k. In this way, we compute the interference at the base station of cell k, due to all the users not power controlled by cell k and belonging to $\triangle^o_k$, i.e., power controlled by one of the neighboring cells.

To approximate the interference equations, we first need to analyze ϕ^j_{ki}. We know that $\phi^j_{ki} = 1$, when $\frac{D^4_{jk}}{D^4_{ji}} 10^{-\frac{\psi_{jk} - \psi_{ji}}{10}} \leq 1$, hence, $Pr(\phi^j_{ki} = 1)$ is given by $Q(\mu^j_{ki}/\sqrt{2\sigma^2})$, where $\mu^j_{ki} = 10 \log_{10}\left(\frac{D^4_{jk}}{D^4_{ji}}\right)$ and $2\sigma^2$ is variance of $\psi_{ji} - \psi_{jk}$. So, $\mathbf{E}[\phi^j_{ki}]$ is given by

$$\begin{aligned} \mathbf{E}[\phi^j_{ki}] &= \mu_\phi \\ &= P(\phi^j_{ki} = 0)[0] + P(\phi^j_{ki} = 1)[1] = P(\phi^j_{ki} = 1) \\ &= Q\left[\frac{\mu^j_{ki}}{\sqrt{2\sigma^2}}\right] \end{aligned} \tag{42}$$

and

$$var[\phi_{ki}^{j}] = \sigma_{\phi}^{2} = \mathbf{E}[{\phi_{ki}^{j}}^{2}] - (\mathbf{E}[\phi_{ki}^{j}])^{2}$$
$$= Q\left[\frac{\mu_{ki}^{j}}{\sqrt{2\sigma^{2}}}\right]\left(1 - Q\left[\frac{\mu_{ki}^{j}}{\sqrt{2\sigma^{2}}}\right]\right)$$

Analyzing $\frac{D_{ji_1}^4}{D_{jk}^4}$ in Eqn. (41), two cases will have to be considered.

- Case 1: When user j is in soft-connection with both cell i_1 and cell k.
- Case 2: When user j is not in soft-connection with cell k.

For Case 1, all the users are closer to cell k as compared to the Case 2. The range of D_{jk} will be smaller as compared to the range for Case 2. Therefore, the mean of $10\log_{10}\left(\frac{D_{ji_1}^4}{D_{jk}^4}\right)$ will be different for both the cases. For Case 1 mean is denoted as $\mu_{i_1k}^{(1)}$, whereas for Case 2 it is denoted as $\mu_{i_1k}^{(2)}$. Outage in cell k can be caused by a new call arrival in cell i which is not in connection with cell k. A new call call arriving and in connection with cell k which cause outage in cell will be blocked. Also, the net interference after handoff arrivals in any cell will decrease the interference seen at the base station of cell k.

1) Outage due to New Arrival: Due to a new arrival of a user in soft-connection with cell i_1 and i_2, the number of users in soft-connection with cells i_1 and i_2 has increased by 1, i.e., $\triangle_{i_1i_2}^{'} = \triangle_{i_1i_2} + 1$ and the number of other-cell users become $\triangle_k^o$ from $\triangle_k^o - 1$. Increased interference at cell k is given by

$$I^{np}(\triangle_k^s, \triangle_k^o) = I^{np}(\triangle_k^s, \triangle_k^o - 1) + X_{i_1i_2}^{j} \tag{43}$$

where $X_{i_1i_2}^{j}$ is the interference at the BS of the reference cell k, due to the newly arrived call j which is in soft connection with cell i_1 and i_2, thus $X_{i_1i_2}^{j}$ is given by

$$X_{i_1i_2}^{j} = \frac{D_{ji_1}^4}{D_{jk}^4}10^{-\frac{\psi_{ji_1}-\psi_{jk}}{10}}\mathbf{1}_{\{\phi_{i_1i_2}^{j}=1\}} + \frac{D_{ji_2}^4}{D_{jk}^4}10^{-\frac{\psi_{ji_2}-\psi_{jk}}{10}}\mathbf{1}_{\{\phi_{i_2i_1}^{j}=1\}}. \tag{44}$$

Hence, we have

$$\mathbf{E}[X_{i_1i_2}^{j}] = e^{a\mu_{ki_1}^{(2)}+a^2\sigma^2} \text{ and } \mathbf{E}[(X_{i_1i_2}^{j})^2] = e^{2a\mu_{ki_1}^{(2)}+4a^2\sigma^2}.$$

The probability of outage due to new arrival in soft-connection with cell i_1 and i_2 is given by

$$p_o(\triangle_k^s, \triangle_k^o, \varepsilon) = Pr\{I(\triangle_k^s, \triangle_k^o) > \varepsilon \mid I(\triangle_k^s, \triangle_k^o - 1) \leq \varepsilon\}. \tag{45}$$

Averaging over $\triangle_k^o$, $p_o(\triangle_k^s, \varepsilon)$ is given by

$$p_o(\triangle_k^s, \varepsilon) = 1 - \prod_{i \in \mathbf{N}_k} \left[1 - \sum_{m_i} p_o(\triangle_j^s, m_i, \varepsilon) Pr\{\triangle_i^o = m_i\} \right].$$

Averaging over $\triangle_k^s$, $p_o(\varepsilon)$ is given by

$$p_o(\varepsilon) = \sum_{m_k} p_o(m_k, \varepsilon) Pr\{\triangle_k^s = m_k\}. \tag{46}$$

The main step in computation of probability of outage is computation of

$$p_o(\triangle_k^s, \triangle_k^o, \varepsilon) = Pr\{I(\triangle_k^s, \triangle_k^o) > \varepsilon \mid I(\triangle_k^s, \triangle_k^o - 1) \leq \varepsilon\} \tag{47}$$

which can be evaluated by determining the joint probability

$$p_J(\triangle_k^s, \triangle_k^o, \varepsilon) = Pr\{I(\triangle_k^s, \triangle_k^o) > \varepsilon, I(\triangle_k^s, \triangle_k^o - 1) \leq \varepsilon\} \tag{48}$$

and the marginal probability

$$p_M(\triangle_k^s, \triangle_k^o, \varepsilon) = Pr\{I(\triangle_k^s, \triangle_k^o - 1) \leq \varepsilon\}. \tag{49}$$

a) Approximation of Marginal Probability: The interference $I(\triangle_k^s, \triangle_k^o - 1)$ is given by

$$I(\triangle_k^s, \triangle_k^o - 1) = \sum_{i \in \mathbf{N}_k} \sum_{j=1}^{\triangle_{ki}} \mathbf{1}_{\{\phi_{ki}^j = 1\}} + \sum_{i_1 \in \mathbf{N}_k} \sum_{i_2 \in \mathbf{N}_{i_1}} \sum_{j=1}^{\triangle_{i_1 i_2}} \frac{D_{ji_1}^4}{D_{jk}^4} 10^{-\frac{\psi_{ji_1} - \psi_{jk}}{10}} \mathbf{1}_{\{\phi_{i_1 i_2}^j = 1\}}.$$

Using CLT, the interference equation can be approximated to be a normal random variable with mean $\mu_{(\triangle_k^s, \triangle_k^o)}$ and variance $\sigma^2_{(\triangle_k^s, \triangle_k^o)}$. To compute mean and variance of $I(\triangle_k^s, \triangle_k^o - 1)$ we first need to compute mean and variance of $\frac{D_{ji_1}^4}{D_{jk}^4} 10^{-\frac{\psi_{ji_1} - \psi_{jk}}{10}} \mathbf{1}_{\{\phi_{i_1 i_2}^j = 1\}}$. The mean and variance of $\frac{D_{ji_1}^4}{D_{jk}^4} 10^{-\frac{\psi_{ji_1} - \psi_{jk}}{10}} \mathbf{1}_{\{\phi_{i_1 i_2}^j = 1\}}$ for the case when user j is in soft-connection with both cell i_1 and cell k is given by

$$\mu_{(1)} = e^{(a\mu_{ki_1}^{(1)} + a^2\sigma^2)} \mu_\phi \text{ and } \sigma^2_{(1)} = e^{(2a\mu_{ki_1}^{(1)} + 2a^2\sigma^2)} \mu_\phi (e^{2a^2\sigma^2} - \mu_\phi).$$

Similarly, the mean and variance of $\frac{D_{ji_1}^4}{D_{jk}^4} 10^{-\frac{\psi_{ji_1} - \psi_{jk}}{10}} \mathbf{1}_{\{\phi_{i_1 i_2}^j = 1\}}$ for the case when user j is not in soft-connection with cell k is given by

$$\mu_{(2)} = e^{(a\mu_{ki_1}^{(2)} + a^2\sigma^2)} \mu_\phi \text{ and } \sigma^2_{(2)} = e^{(2a\mu_{ki_1}^{(2)} + 2a^2\sigma^2)} \mu_\phi (e^{2a^2\sigma^2} - \mu_\phi).$$

Therefore the mean $\mu_{(\triangle_k^s,\triangle_k^o-1)}$ is given by

$$\mu_{(\triangle_k^s,\triangle_k^o)} = \sum_{i_1\in\mathbf{N}_k}\left[(\triangle_{i_1}^s - \triangle_{i_1k})e^{(a\mu_{ki_1}^{(2)}+a^2\sigma^2)}\mu_\phi + \triangle_{i_1k}e^{(a\mu_{ki_1}^{(1)}+a^2\sigma^2)}\mu_\phi\right] + \sum_{i\in\mathbf{N}_k}\triangle_{ki}\mu_\phi$$

and the variance $\sigma^2_{(\triangle_k^s,\triangle_k^o-1)}$ is given by

$$\begin{aligned}\sigma^2_{(\triangle_k^s,\triangle_k^o-1)} &= \sum_{i_1\in\mathbf{N}_k}\Big[(\triangle_{i_1}^s - \triangle_{ki_1})e^{(2a\mu_{ki_1}^{(2)}+2a^2\sigma^2)}\mu_\phi(e^{2a^2\sigma^2}-\mu_\phi) + \\ &\quad \triangle_{ki_1}e^{(2a\mu_{ki_1}^{(1)}+2a^2\sigma^2)}\mu_\phi(e^{2a^2\sigma^2}-\mu_\phi)\Big] + \sum_{i\in\mathbf{N}_k}\triangle_{ki}\mu_\phi(1-\mu_\phi).\end{aligned}$$

So the marginal pdf of $I(\triangle_k^s,\triangle_k^o-1)$ is given by

$$f_{(\triangle_k^s,\triangle_k^o-1)}(x) = \frac{1}{\sqrt{2\pi\sigma^2_{(\triangle_k^s,\triangle_k^o-1)}}}e^{-\frac{(x-\mu_{(\triangle_k^s,\triangle_k^o-1)})^2}{2\sigma^2_{(\triangle_k^s,\triangle_k^o-1)}}}.$$

Marginal probability in Eqn. (49) can thus be evaluated as

$$\begin{aligned}p_M &= Pr\{I(\triangle_k^s,\triangle_k^o-1)\le\varepsilon\} \\ &= \int_{-\infty}^{\varepsilon} f_{(\triangle_k^s,\triangle_k^o-1)}(x)dx. \end{aligned} \tag{50}$$

b) Analysis of Joint Probability: To evaluate the joint probability in Eqn. (48) we need to evaluate the joint mean of $I(\triangle_k^s,\triangle_k^o)$ and $I(\triangle_k^s,\triangle_k^o-1)$. The correlation coefficient, r_s of $I(\triangle_k^s,\triangle_k^o-1)$ and $I(\triangle_k^s,\triangle_k^o)$ is given by

$$r_s = \frac{\mathbf{E}[I(\triangle_k^s,\triangle_k^o-1)I(\triangle_k^s,\triangle_k^o)] - \mu_{(\triangle_k^s,\triangle_k^o-1)}\mu_{(\triangle_k^s,\triangle_k^o)}}{\sigma_{(\triangle_k^s,\triangle_k^o-1)}\sigma_{(\triangle_k^s,\triangle_k^o)}}. \tag{51}$$

From Eqns. (43) and (44), we know that

$$I(\triangle_k^s,\triangle_k^o) = I(\triangle_k^s,\triangle_k^o-1) + \frac{D^4_{ji_1}}{D^4_{jk}}10^{-\frac{\psi_{ji_1}-\psi_{jk}}{10}}\mathbf{1}_{\{\phi^j_{i_1i_2}=1\}} + \frac{D^4_{ji_2}}{D^4_{jk}}10^{-\frac{\psi_{ji_2}-\psi_{jk}}{10}}\mathbf{1}_{\{\phi^j_{i_2i_1}=1\}}. \tag{52}$$

From the above equation, we obtain

$$\mathbf{E}[I(\triangle_k^s,\triangle_k^o-1)I(\triangle_k^s,\triangle_k^o)] = \mathbf{E}[I(\triangle_k^s,\triangle_k^o-1)^2] + \mathbf{E}[I(\triangle_k^s,\triangle_k^o-1)X^j_{i_1i_2}]. \tag{53}$$

Note that

$$\mathbf{E}[I(\triangle_k^s,\triangle_k^o-1)X^j_{i_1i_2}] = \mathbf{E}[I(\triangle_k^s,\triangle_k^o-1)]\mathbf{E}[X^j_{i_1i_2}] = \mu_{(\triangle_k^s,\triangle_k^o)}e^{a\mu^{(2)}_{kj_1}+a^2\sigma^2} \tag{54}$$

as $I(\triangle_k^s, \triangle_k^o - 1)$ and $X_{i_1 i_2}^j$ are assumed to be independent.

Substituting Eqns. (53) and (54) in Eqn. (51), we get the correlation coefficient r_s. Joint distribution function of $I(\triangle_k^s, \triangle_k^o - 1)$ and $I(\triangle_k^s, \triangle_k^o)$ as

$$f_{(\triangle_k^s, \triangle_k^o)}(x, y) = \frac{1}{\sqrt{2\pi\sigma_{(\triangle_k^s, \triangle_k^o - 1)}\sigma_{(\triangle_k^s, \triangle_k^o)}(1 - r_s^2)}} \tag{55}$$

$$e^{-\frac{1}{2(1-r_s^2)}\left[\frac{(x-\mu_{(\triangle_k^s, \triangle_k^o-1)})^2}{\sigma^2_{(\triangle_k^s, \triangle_k^o-1)}} - 2\frac{(x-\mu_{(\triangle_k^s, \triangle_k^o-1)})(y-\mu_{(\triangle_k^s, \triangle_k^o)})}{\sigma_{(\triangle_k^s, \triangle_k^o-1)}\sigma_{(\triangle_k^s, \triangle_k^o)}} r_s + \frac{(y-\mu_{(\triangle_k^s, \triangle_k^o)})^2}{\sigma^2_{(\triangle_k^s, \triangle_k^o)}}\right]} \tag{56}$$

where correlation coefficient, r_s is as in Eqn. (51). Hence joint probability in Eqn. (48) is obtained as

$$p_J = \int_{\varepsilon}^{\infty} \int_{-\infty}^{\varepsilon} f_{(\triangle_k^s, \triangle_k^o)}(x, y) dy dx. \tag{57}$$

Here the integration is done over the whole area to average out the joint probability. Eqn. (57) and (50) can be used to compute the conditional probability in Eqn. (45). The conditional probability equation can averaged using Eqns. (46) and (46) to compute the probability of outage for CDMA system with soft handoff as:

$$P_o = p_o(\varepsilon).$$

Fig. (7) shows the simulation and analysis results for the probability of outage in a CDMA system with soft handoff. From Fig. (3) and Fig. (7) we observe that, under SIR based admission control, soft handoff with a system load of 14.5 Erlangs/cell gives the same probability of outage as given by a system load of 6 Erlangs/cell in the no handoff scenario. This shows that the performance of the systems improves in the soft handoff scenarios. Also, in the soft handoff case we observe that the average probability of outage improves drastically when compared to no handoff case under SIR based admission control.

V. Conclusions

In this paper we have analyzed a CDMA system under SIR based admission control for the probability of outage under hard handoff and soft handoff scenarios. In hard handoff, we assume that the handoff occurs at the boundary of the cells. We have analyzed the system for different rate of arrivals for the hard handoff calls. We have also considered more practical case of soft

handoff. The expressions that we have obtained for probability of outage under SIR based admission control for both hard handoff and soft handoff cases can be used for planning CDMA cellular systems. The analytical and simulation results have been shown to exhibit a reasonably good match. We have observed that the probability of outage is more with hard handoff for the same system load as compared to the no handoff and soft handoff case. In hard handoff, we have observed that on increasing the rate of arrival of the handoff calls from $\lambda_H = 0.2\lambda_v$ to $\lambda_H = 0.3\lambda_v$, there is an increase in average probability of outage. We have also observed that for soft handoff, with a system load of 14.5 Erlangs/cell we get the same probability of outage as given by a system load of 6 Erlangs/cell in the no handoff scenario. This shows that the performance of the systems improves in the soft handoff scenarios as expected. This improvement has been observed to be quite significant under SIR based admission control compared to other admission control measures [6].

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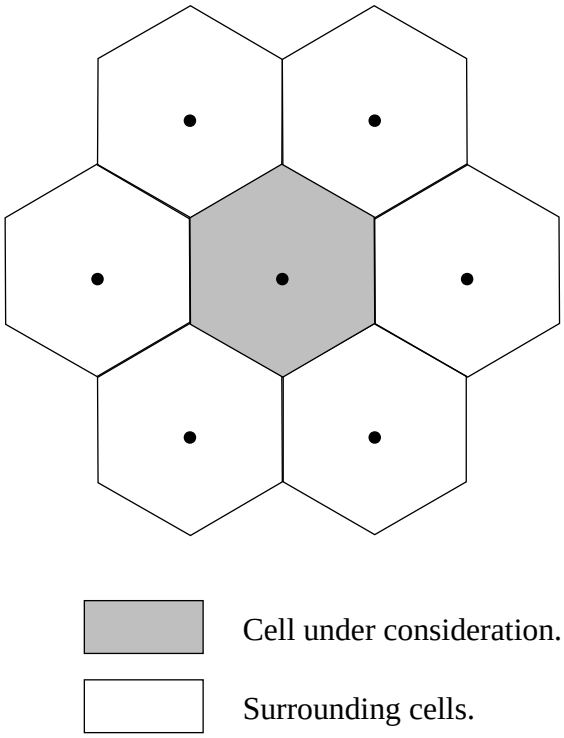

Fig. 1. Cell k and its neighboring cells

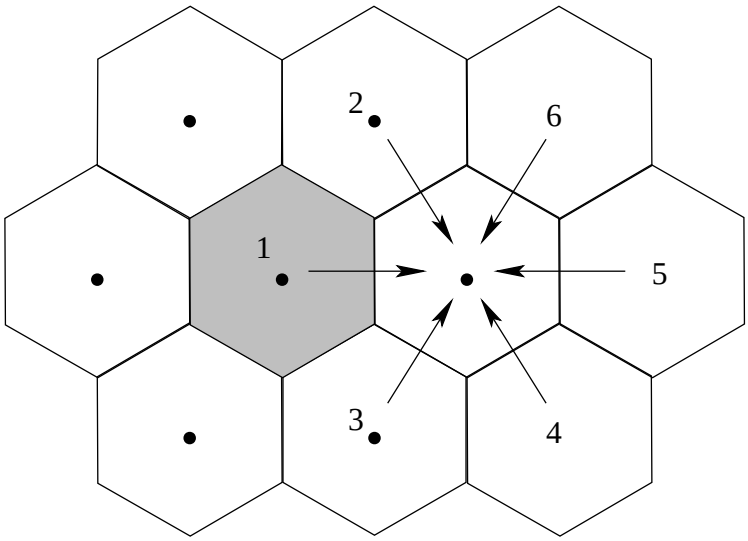

Fig. 2. Call arrivals in a cell

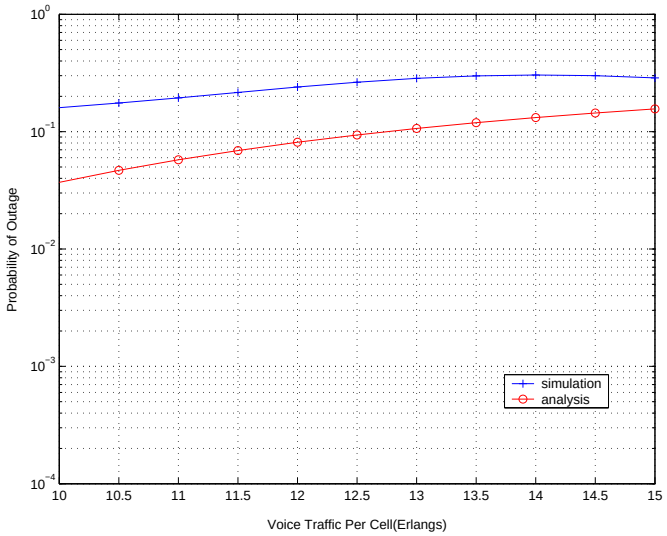

Fig. 3. Probability of outage in a CDMA system with no handoffs

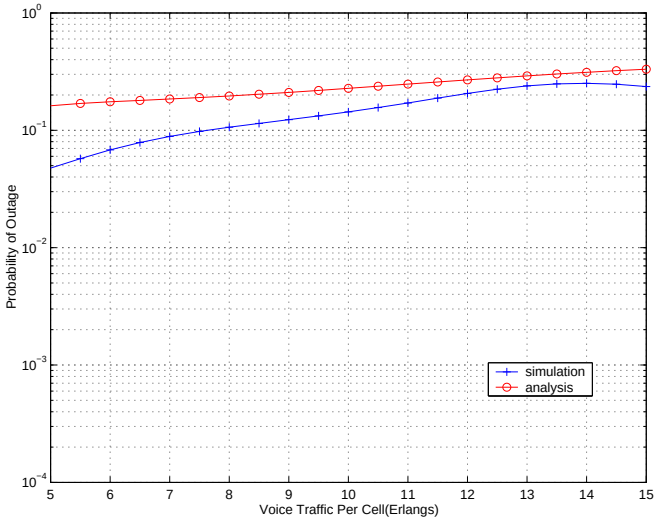

Fig. 4. Probability of outage in a CDMA system having hard handoff($\lambda_H = 0.2\lambda_v$)

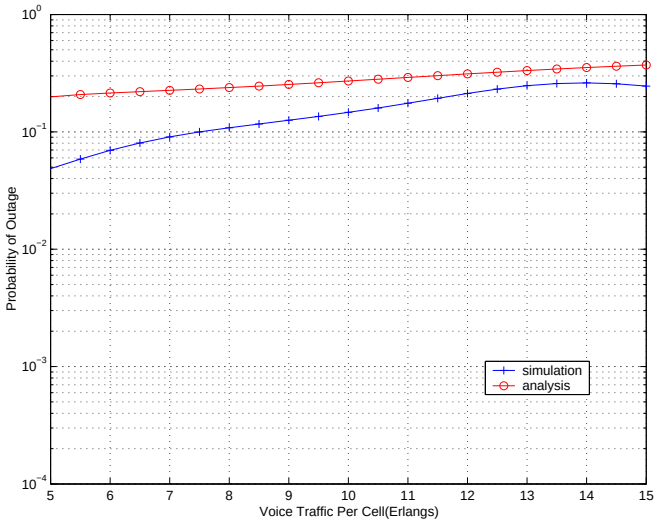

Fig. 5. Probability of outage in a CDMA system having hard handoff($\lambda_H = 0.3\lambda_v$)

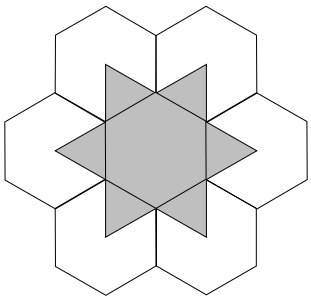

Fig. 6. Cell in a system with soft-connections

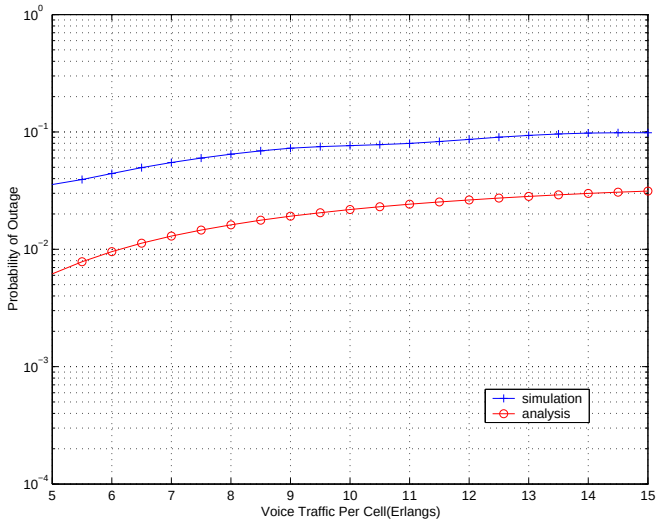

Fig. 7. Probability of outage in a CDMA system with soft handoff