Superposition Turbo TCM for Multirate Broadcast

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***Abstract*—Bergmans and Cover identified the capacity region of the Gaussian degraded broadcast channel, where different receivers observe the transmitted signal with different signal-to-noise ratios. This letter presents a superposition turbo-coding scheme that performs within 1 dB of the capacity region boundary of the degraded broadcast channel at a bit-error rate of 10^{-5}.**

***Index Terms*—Degraded broadcast channel, superposition, time division, trellis-coded modulation, turbo coding, unequal error protection.**

I. Introduction

In MANY applications, different receivers experience different signal-to-noise ratios (SNRs). In satellite television broadcasting, for example, receivers in rain-fades have low SNRs, while receivers under clear sky have high SNRs. With progressive source coding, a lower resolution video stream may be embedded in the overall video stream. It is desirable that the lower resolution video stream be recovered, even by receivers with low SNRs.

Consider the Gaussian broadcast channel with one sender transmitting X and two receivers observing Y_1 and Y_2, respectively, with $Y_1 = X + Z_1$ and $Y_2 = X + Z_2$, where $Z_1 \sim \mathcal{N}(0, N_1)$ and $Z_2 \sim \mathcal{N}(0, N_2)$ with $N_2 > N_1$. This broadcast channel can be recharacterized as the statistically equivalent "degraded" broadcast channel $Y_1 = X + Z_1$ and $Y_2 = Y_1 + Z_2'$, where $Z_2' \sim \mathcal{N}(0, N_2 - N_1)$. The capacity region for the two-dimensional memoryless Gaussian broadcast channel with signal power constraint P is given by

$$R_1 < \log_2\left(1 + \frac{\alpha P}{N_1}\right) \quad (1)$$

$$R_2 < \log_2\left(1 + \frac{(1-\alpha)P}{\alpha P + N_2}\right), \quad \text{for} \quad 0 \le \alpha \le 1. \quad (2)$$

This region (bounded by the dashed lines of Fig. 1) is theoretically achieved by the superposition coding scheme described by Cover [1]. Bergmans [2], [3] proved the converse. See also [4] for an excellent survey of broadcast channel information theory.

Time-division coded modulation (TDCM) is theoretically inferior to superposition-coded modulation (SCM). However, Wei [5] considered multirate trellis-coded modulation (TCM) using both TDCM and SCM approaches, and found TDCM to be superior. Gadkari and Rose [6] explained this behavior by showing that TDCM outperforms SCM for some rate regions when the channel code performance is sufficiently worse than capacity achieving. Recently, Wang and Orchard [7] carefully designed a TCM-based SCM scheme that does outperform TDCM in a rate region of interest. Other multirate schemes include [8] and [9].

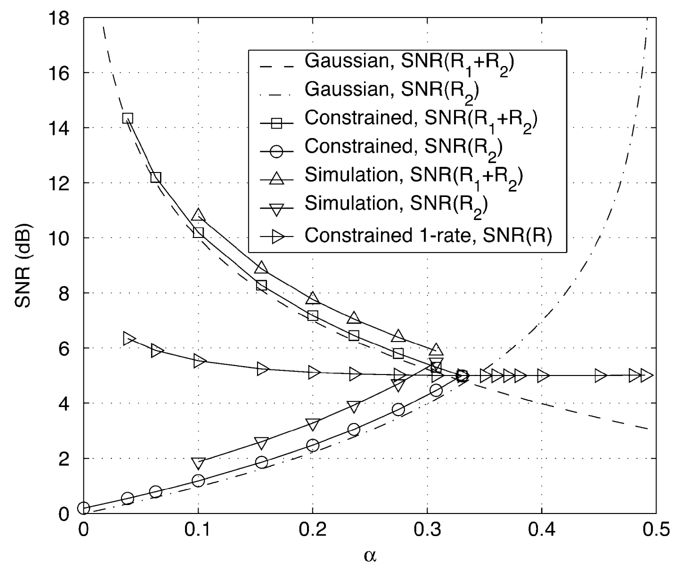

Fig. 1. Required SNR for high-rate and low-rate receivers versus α; interleaver length $= 8\ 192$; number of iterations $= 15$; BER $= 10^{-5}$ for simulations; $R_1 = R_2 = 1$ b/s/Hz; 16-point square constellation.

In this letter, we use turbo codes to design an SCM. Since turbo codes closely approach capacity-achieving performance, it is not surprising that our SCM closely approaches the theoretical performance limits. In all of our examples, the turbo-coded SCM architecture at a bit-error rate (BER) of 10^{-5} performs within 1 dB of the capacity region boundary of the degraded broadcast channel, over the entire useful range of the power-allocation parameter α.

Section II discusses the power-allocation parameter α, and shows that α may be chosen to produce a single-rate multilevel code (MLC) as described in [10]. Section III presents our SCM turbo-TCM architecture. Section IV presents simulation results for two-rate and three-rate SCMs using turbo TCM. Section V concludes the letter.

II. Reconfigurability Using α

The α in (1)–(2) controls the ratio of power allocation to the two data streams. For practical constellations, it controls the placement of constellation points. For a fixed-rate two-rate structure with $R_1 = R_2 = 1, \alpha$ can take a value from 0 to 0.5,

Paper approved by C. Schlegel, the Editor for Coding Theory and Techniques of the IEEE Communications Society. Manuscript received February 3, 2003; revised July 18, 2003. This work was supported in part by the University of California and Conexant under UC MICRO Grant 01-096, and in part by the National Science Foundation under Grant CCR-0209110. This paper was presented in part at the International Conference on Communications, Anchorage, AK, May 11–15, 2003.

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Digital Object Identifier 10.1109/TCOMM.2004.823646

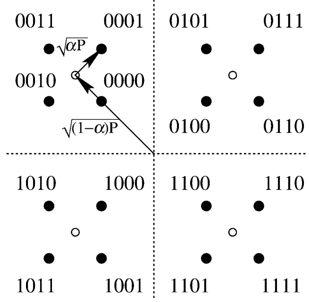

Fig. 2. Example for constellations used in simulation for $\alpha = 0.1$.

according to (1)–(2). However, with two-stage decoding that requires the R_2 stream to be decoded before decoding the R_1 stream, only α's between 0 and $1/3$ are of practical interest. This is shown by the intersection of the dashed curves in Fig. 1. By choosing α within $(0, 1/3)$, we configure the system to support two rates with two corresponding operating SNRs.

When $\alpha = 1/3$, the two-rate structure becomes a single-rate MLC [10]. MLC is a special case of the multirate degraded broadcast channel, where all receivers have the same SNR. Consider the Gaussian broadcast channel with one sender and k receivers. We have $Y_i = X + Z_i$, where $Z_i \sim \mathcal{N}(0, N_i)$. The achievable rate region for $1 \leq i \leq k$ is

$$R_i < \log_2\left(1 + \frac{\alpha_i P}{\sum_{j=1}^{i-1} \alpha_j P + N_i}\right) \quad (3)$$

where $\sum_{i=1}^{k} \alpha_i = 1$. If $N_1 = N_2 = \cdots = N$, summing $R_1, \ldots, R_k$ yields

$$\sum_{i=1}^{k} R_i < \log_2\left(1 + \frac{P}{N}\right) \quad (4)$$

which is the channel capacity of the single-rate additive white Gaussian noise channel. Thus, multirate codes theoretically may perform optimally when configured as single-rate MLCs. We have found this to be the case in practice, as well.

Fig. 2 shows the 16-point square constellation resulting from $\alpha = 0.1$ with a symmetric ultracomposite Gray labeling [11]. Points in the same quadrant have the same two most significant bits (MSBs). The four central points have the same two least significant bits (LSBs). Fig. 2 also shows how α allocates power and places constellation points. The 16-point square constellation can be considered as one quaternary phase-shift keying (QPSK) constellation added to another QPSK constellation. Two bits of rate-R_2 data are modulated as a QPSK signal, which can be mapped to one of four empty dots with amplitude of $\sqrt{(1-\alpha)P}$. Then, two bits of rate-R_1 data are modulated as another QPSK signal, which is represented by a filled dot with amplitude of $\sqrt{\alpha P}$, using one of four empty dots as its origin.

III. Superposition Turbo TCM

For the SCM architecture of this paper, we use the symbol-interleaved parallel-concatenated turbo-TCM (PCTCM) structure of Fragouli and Wesel [12], [13]. As presented in [14] and [15], we also implemented SCM using the Robertson and Wörz [16] approach and saw very similar performance. The bit-interleaved turbo-TCM scheme of Benedetto *et al.* [17] is a possible alternative, but bit interleaving further complicates the initial and subsequent soft-information computations required to support multiple rates.

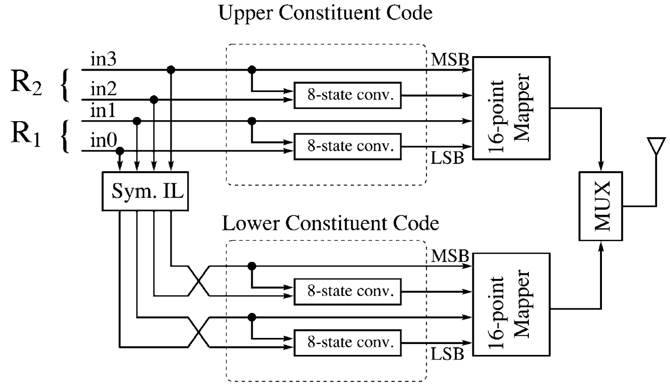

Fig. 3. Superposition turbo-TCM encoder for two rates and a 16-point square constellation.

Fig. 3 shows the structure of a superposition turbo-TCM encoder based on the structure of [12] and [13] for the family of 16-point square constellations illustrated by the example in Fig. 2. It is a PCTCM with constituent encoders of rate-$4/4$. Each constituent code produces both R_1 and R_2 data streams, using two identical recursive convolutional codes of rate-$2/2$. The rate-$2/2$, eight-state encoder within the constituent encoders is the result of an exhaustive computer search to optimize normalized effective free Euclidean distance [13]. The eight-state recursive convolutional code we found is described by the matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

in the encoder state-space description given below

$$[s_1 \quad s_2 \quad s_3]_{j+1} = [s_1 \quad s_2 \quad s_3]_j \cdot A + [u_1 \quad u_2]_j \cdot B \quad (6)$$
$$[y_1 \quad y_2]_j = [s_1 \quad s_2 \quad s_3]_j \cdot C + [u_1 \quad u_2]_j \cdot D. \quad (7)$$

We used this rate-$2/2$ encoder as the basic building block of all the multirate systems described in this letter. To specify the extended-spread symbol interleavers [12] used in our simulations, we give the constraint parameters (defined in [12]) (S, T, X), where $S_1 = S_2 = S, T_1 = T_2 = T$, and $X_1 = X_2 = X$. The interleavers with lengths of 8192, 4096, and 2048 have constraint parameters $(S = 54, T = 6, X = 1), (S = 37, T = 5, X = 1)$, and $(S = 26, T = 4, X = 0)$, respectively.

We also implemented a two-rate system using an eight-point circular constellation and a three-rate system using a four-point square constellation. For the eight-point circular constellation, a binary phase-shift keying (BPSK) data stream (i.e., the LSB) is added to a QPSK data stream. Since we use a rate-$1/2$ code for the LSB, we encode two eight-point circular constellation symbols at the same time. Our encoder for the 64-point square constellation is a direct extension of the encoder in Fig. 3. Three QPSK data streams are summed together to generate the transmitted data stream.

We used two-stage and three-stage multistage decoders with successive cancellation for the two-rate and three-rate systems, respectively. Multistage decoding decodes the low-SNR information, and then subtracts this codeword stream before decoding the higher SNR stream(s). Multistage decoding [10], [18] is typical for MLC and SCM. We also simulated a full joint decoding of the overall encoder trellis. Multistage decoding had the same performance as that of the full joint decoding. Moreover, the complexity of the joint decoding algorithm is 16 times that of the two-stage decoding scheme.

IV. Simulation Results

This section discusses simulation results for our multirate superposition turbo-TCM structures with a 16-point square constellation, an eight-point circular constellation, and 64-QAM.

A. Two-Rate Superposition With A 16-Point Constellation

Fig. 1 shows the SNR required for BER $= 10^{-5}$ versus α for the two-rate encoder using a 16-point square constellation. For the various α's from 0.1 to 0.3077 that we simulated, the high-SNR performance at a BER of 10^{-5} is within 0.6 dB of the constrained capacity curve, while the low-SNR performance gap at a BER of 10^{-5} ranges from 0.7 dB from the constrained capacity curve at $\alpha = 0.1$ to 1 dB at $\alpha = 0.3077$. The block length was 8192.

The first two curves listed in the legend of Fig. 1 are the theoretical curves of SNR versus α (1)–(2) with fixed rates $R_1 = R_2 = 1$. Both the transmitted signal and the noise are assumed to be Gaussian for these two curves. The third and fourth curves are numerically calculated theoretical error-free SNRs, in which the noise is still Gaussian, but the transmitted signals are 16 equiprobable constellation points placed according to α. Curves five and six are the simulation results with various α. The last curve is the numerically calculated theoretical error-free SNR for a single-rate scheme using 16 constellation points placed according to α.

Taking into account that the transmitted signal is not Gaussian, when $\alpha = 0.330\,15$ (instead of $1/3$), the two-rate and the single-rate schemes should, theoretically, perform the same. Our simulation results for $R_1 + R_2$ at an α of 0.3077 are as good as that of typical single-rate turbo TCM [12]. Typical single-rate turbo TCM uses standard 16-QAM ($\alpha = 0.2$), but the theoretical performance of the single-rate scheme is not sensitive to α for $\alpha \geq 0.2$.

B. Two-Rate Superposition With an Eight-Point Constellation

Fig. 4 shows SNR required for BER $= 10^{-5}$ versus α for a two-rate structure using an eight-point circular constellation. The theoretical range for α is from 0 to $(\sqrt{2}-1/2\sqrt{2}-1)$ (i.e., 0.2265), under the assumption that both the transmitted signal and the noise have Gaussian distributions.

Including the performance loss due to an eight-point circular signaling constellation, the practical range of α is around

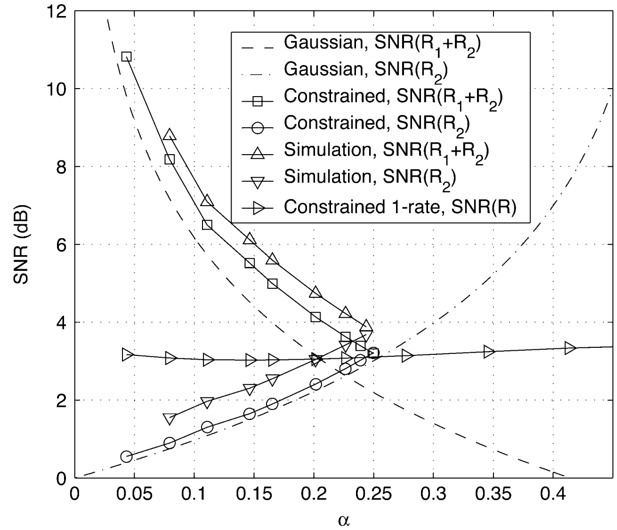

Fig. 4. Required SNR for high-rate and low-rate receivers versus α; interleaver length $=$ 8 192; number of iterations $= 15$; BER $=$ 10^{-5} for simulations; $R_1 = 0.5$ b/s/Hz; $R_2 = 1$ b/s/Hz; eight-point circular constellation.

0–0.25. For the various α's from 0.0794 to 0.25 that we simulated, at high and low SNR, the performance at a BER of 10^{-5} is within 0.6 and 0.7 dB of constrained capacity, respectively. The block length was 8192 in this simulation.

C. Three-Rate Superposition With 64-QAM

We simulated a three-rate 64-QAM structure $(\alpha_1 = 1/21, \alpha_2 = 4/21)$, where α_1 and α_2 are the power-allocation parameters in (3). The constrained capacities are 3.0913, 8.7095, and 13.4094 dB for low, middle, and high SNR ranges, respectively. For standard 64-QAM at a BER of 10^{-5}, the performance for a block length of 8192 is within 0.6, 0.8, and 0.9 dB of constrained capacity at high, middle, and low SNR, respectively.

In all three systems discussed above, we also studied performance with smaller block lengths. For all systems, we found that at BER $= 10^{-5}$ the difference between interleaver lengths of 8192 and 4096 is about 0.1–0.15 dB, and the difference between interleaver lengths of 4096 and 2048 is about 0.2 dB.

V. Conclusion

With turbo codes, superposition coding with multistage decoding performs quite well, always within 1 dB of the achievable region boundary for our example. In the context of the work of Cover and Bergmans [2], and the well-known power of turbo codes, this is not that surprising. The two-stage decoding of the separate eight-state trellises for each substream presented in this letter achieves the same performance of joint decoding of the 64-state overall trellis, while the latter is 16 times more complicated than the former.

Multilevel coding is a special case of multirate superposition coding with α selected so that all rates may decoded at the same SNR. The proposed schemes offer the same complexity and performance as that of the typical single-rate turbo TCM, but provides the flexibility of controlling unequal error protection through the choice of α.

ACKNOWLEDGMENT

The first author thanks C. Jones for the valuable discussions about simulation and Robertson–Wörz turbo-TCM structure.

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