

# METHODS FOR THE EVALUATION OF THE GO-KART VEHICLE DYNAMIC PERFORMANCE BY THE INTEGRATION OF CAD/CAE TECHNIQUES.

MUZZUPAPPA Maurizio, MATRANGOLO Giuseppe, VENA Gianpiero

University of Calabria, Italy  
Mechanical Engineering Department  
e-mail: muzzupappa@unical.it; g.matrangolo@unical.it; gianpiero.vena@unical.it

## ABSTRACT

This work is about the definition of a numerical design and testing methodology of Go-Kart vehicles, particularly oriented to the evaluation of their dynamic behavior. The resulting mathematical model of the vehicle, undergoes to a validation process aimed at the verification of the numerical FEM models and of the level of their integration in a multi body system. The multi body numerical analysis of a Go-Kart is focused on the evaluation of the vehicle dynamic performance sensibility with respect to the stiffness of its main structural components. The structural behavior of these elements has great importance in the study of the vehicle performance; so, the capability to predict this behavior gives the opportunity to better design all the vehicle subsystems.

The study has started with the definition of the design methodology and of its validation process; a numerical FEM model has been created, and then validated through specific experimental analyses. Subsequently, through the integration of deformable bodies in multi body environment, different types of tubular structures was analyzed, characterized by a specific rate of deformation. At the end, a standard vehicle dynamic test was conducted to evaluate the influence that this parameters have on the Go-Kart dynamic performance.

**Key words:** Go-Kart, Vehicle Dynamics, Numerical Analysis, Design Methodologies, FEM, Multi body.

## 1. Introduction

It's well known that Go-Kart vehicles are characterized by the absence of differential and suspensions systems [1,2]. For this reason, the cornering behavior and the entire performance of the vehicle are strongly influenced by the structural characteristics of the frame. In this way the structural analysis of the tubular frame recovers a fundamental role inside the process of design and valuation of Go-Kart vehicles. Nowadays, the need of scientific research about this kind of vehicles is too often underestimated. Today, in fact, design and testing processes are mainly accompanied by experimental test not supported by theoretical and simulation studies.

The research work is focused on the definition of a detailed methodology of virtual design and testing, able to evaluate the dynamic performances of Go-Kart vehicles, studying with particular attention the structural behavior of the tubular frame.

So, the primary topics of this work are:

- The definition of the different phases of the analysis process
- The planned integration of the different software systems
- The validation of the analysis process
- The evaluation of the results obtained by the dynamic analyses

## 2. Design and testing methodology

The research work, as previously said, foresees the definition of a planned methodology of virtual design and prototyping of Go-Kart vehicles, able to be applied both in the design process of a new vehicle and in the tuning process of an existing one.

This means that we must take into account all the different phases of design and tuning processes, to evaluate every kind of necessity. Looking at the common processes of mechanical design (FIG.1), we can clearly distinguish their single phases by means of:

- Purposes
- Methodologies
- Tools of aided design

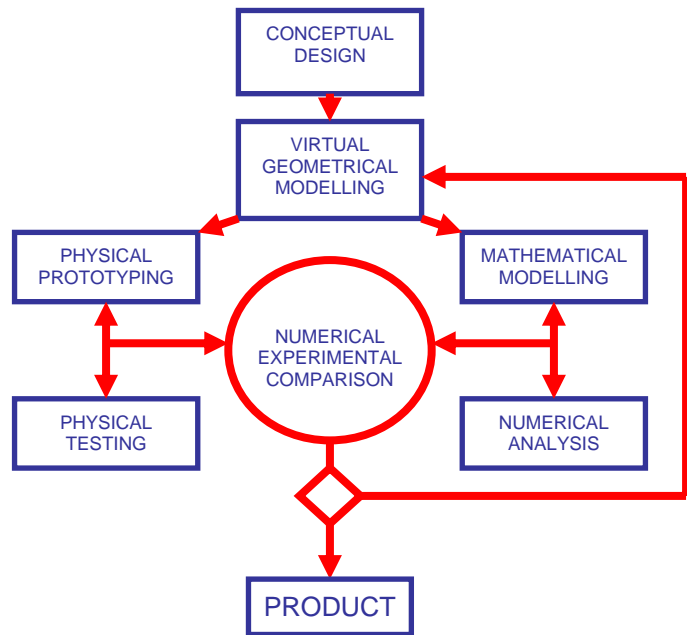


FIG. 1: *The standard design methodology.*

We can see how the process of mathematical modeling and numerical analysis of the mechanical systems proceeds in a parallel way respect to the physical process of prototyping and testing: the interaction of these ones speeds up the entire process of testing and design, optimizing performances and reducing time. Talking about racing competition vehicles like Go-Kart, we can easily understand how the process of numerical analysis of the dynamic performances and its target values are the fundamental guide for the entire design process. So, the methodology previously described can be revisited and better defined to achieve a new process structure with new logical links among the different phases. The definition of the target dynamic performances of the vehicle, become the first and more important phase of the entire process.

Here the guidelines and purposes of the other stages are defined. Looking with particular attention at the main characteristics of Go-Kart vehicles, we can precisely identify the different subsystems and their influence on the dynamic behavior. Tubular frame, engine, steering and break system are the fundamental subsystems of the Go-Kart. For each of these ones we can identify a development process like that shown in FIG.1.

The dynamic behavior of the vehicle is strongly influenced by the structural characteristic of the tubular frame; in fact, since the kart has not differential and suspension systems, its turning behavior is strongly influenced by the torsional deformation.

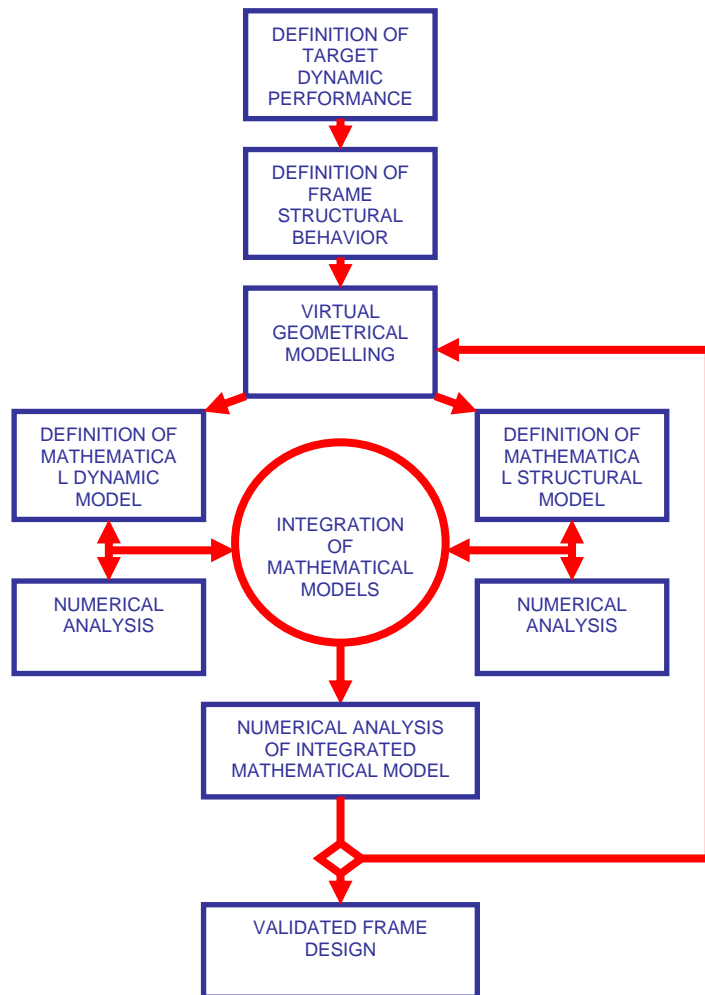


FIG. 2: *The proposed methodology of virtual design and testing of the vehicle.*

All the numerical activities of analysis and modeling must be validated, to develop a structural-dynamic integrated mathematical model. The mathematical model so defined is ready to be used in the future research works for a numerical-experimental comparison between the dynamic numerical analysis and the physical test conducted on track.

### 1.1 The validation process

To carry on the comparison between numerical and physical model, two different ways must be followed. The first step is a complete Reverse Engineering process aimed to analyze the physical, geometrical and structural characteristics of the reference test case, with particular reference to the tubular frame.

So, this work focuses its attention on the tubular frame: the definition and validation of a mathematical model representative of its structural behavior, and the integration of this one inside a dynamic mathematical model is our main purpose.

We can easily define the workflow of the frame numerical design process, by the precise definition of its main activities (FIG.2). The methodology is entirely characterized by numerical processes of geometrical, structural and dynamic modeling: in this way it can speed up the entire process of vehicle design, ensuring high flexibility and accuracy in the evaluation of dynamic performances and subsystems correlation.

To validate the proposed methodology a reference physical test case must be analyzed, comparing the output results with the experimental test output data. So, a racing 125cc Go-Kart vehicle belonging to the Mechanical Engineering Dept. of the University of Calabria, has been studied in depth by:

- Structural numerical analysis
- Structural and dynamic numerical model integration

This process is carried on with the help of a CMM machine and a complete extensimetric set-up for the structural analysis. The second step is the vehicle virtual modeling process, aimed to define a mathematical model of the physical tubular frame able to represent its structural characteristics. This process is carried on with the help of some of the more efficient CAD and FEM systems. The validation of the numerical model is done by the comparison of the results supplied by these two different processes.

At the end, the model is integrated inside a multi-body system of the vehicle for the simulation of its dynamic performance. The multi-body systems are adopted for their main characteristics:

- extremely good capabilities of integration with CAE systems of virtual prototyping
- systematic way of defining and resolving the differential equations of motion

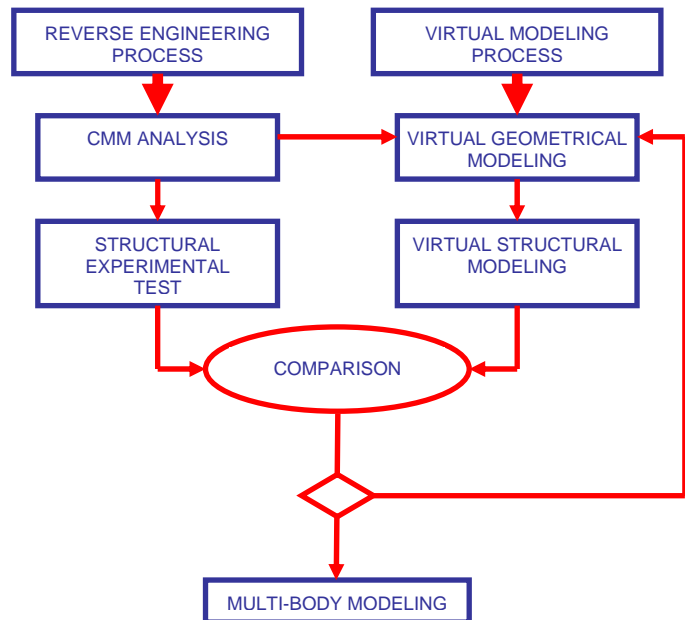


FIG. 3 – Analysis process for the validation of the mathematical model

## 2. Process of geometrical reverse engineering and virtual modeling.

The geometrical RE process foresees the measurement and the acquisition of the fundamental geometries that characterize the physical frame. This process is essential to reconstruct a precise geometrical model of the vehicle frame that can be easily adopted in the subsequent structural and dynamic virtual analyses. The whole process has been carried on using a CMM Coordinate Measuring Machine *Ares Coord 3* (FIG.4) to gather all the information needed to reconstruct the tubular frame geometry [3-5].



FIG. 4: The CMM Machine



FIG. 5: Go-Kart frame ready to be measured

To analyze the frame geometry, the vehicle has been completely disassembled; then, the frame has been accurately polished to eliminate every possible source of measurement “noise”. After the preliminary calibration of the sensor, the frame has been mounted on the machine workbench (FIG.5). So, a rectified parallelepipedon with plane faces was firmly attached in the middle of the frame, using it as the reference coordinate system for the analysis.



FIG. 6: *Two examples of non-cylindrical elements to be measured with reverse*

The frame is composed by tubular elements and some other components welded to them, whose function is that of supporting the rear axle, the seat, the front wheels, the steering system, etc. Each tubular element is characterized by a tridimensional geometry formed by straight and curved elements.

To correctly reconstruct each straight piece, eight measurement points was taken, equally distributed along two different sections of the tube at a small distance one from the other. The informations obtained as output from the machine software were the spatial coordinates of the measured points, together with the cosine directors of the cylinder axis and the evaluation of its external diameter; besides, to gather the curved elements it was necessary to measure also the curvature radius. Once completed the cylinders measurement, the other elements were analyzed (FIG.6): for the planar surfaces, three misaligned points were taken upon them, while for the holes another three points were taken on their inner surface. Where the measure was not possible with these tools, a caliber and a goniometer were adopted to complete the reverse engineering process. The CMM analysis give us the possibility to obtain a measurement error smaller than required. Besides, the output data (points, curves, plane and tridimensional elementary forms) are supplied in an universal exchange format automatically manageable by the geometrical 3D modeler.

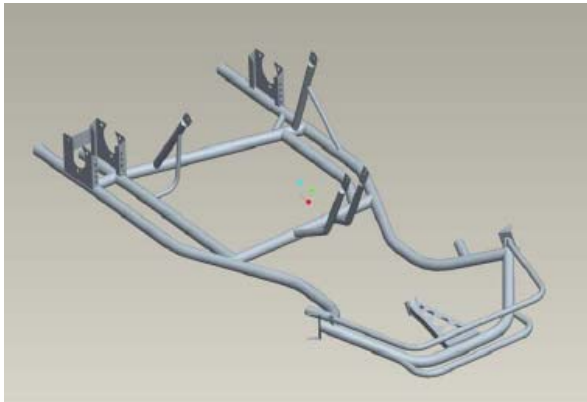


FIG 7: *The 3D geometrical model of the Go-Kart frame*

The set of data obtained in the measurement stage is used to generate the virtual geometrical model of the frame. So, in this phase the 3D parametric modeler Pro/Engineer *Wildfire* v2.0 has been used; starting from the geometrical entities gathered by the CMM machine a complete model of the tubular frame was created. The fundamental geometric elements like points, curves, reference axis, planes and coordinate systems were used to create a surface model of the frame. The choice of surface elements is due to structural numerical analysis necessities. In fact, the virtual geometrical model so created needs to be imported in the FEM environment for using it as reference model. The virtual geometrical model was created with the help of the more advanced technique of parametric modeling; this ensures a more flexible process of geometrical modification. The result of this process is a complete and accurate 3d CAD model of the frame (FIG. 7). The error range is the same ensured by the CMM machine used in the RE process. This kind of model is a fundamental requisite for the correct evaluation of the frame structural behavior in FEM environments.

### 3. Structural analysis process

To investigate the frame structural behavior, specific experimental tests have to be carried on, for the objective evaluation of its stiffness properties. First of all we need to understand what are the fundamental frame characteristics that need to be considered in this analysis, and that have more influence on the vehicle dynamic behavior. A lot of studies [6-11] demonstrate that torsional stiffness of the Go-Kart frame is the main responsible of vehicle global performance. In fact, the frame deformation compensates the lack of a differential system, allowing the vehicle to travel along curve trajectories without significant tire slip. So the frame rate of torsional deformation could be considered as the main parameter influencing vehicle dynamic behavior: other variables such as bending stiffness of the frame and of the rear axle will be neglected, since their effect on the vehicle dynamic is not so important like that of the tubular frame. In



FIG. 8: *The experimental set up adopted to analyze the frame level of deformation under static loads.*

order to determine the level of stress and strain induced to the frame by torsional loads, a specific experimental set-up (FIG. 8) has been created in the Mechanical Department laboratories: this allowed to simulate the deformation of the main structure under typical load conditions, and to estimate with extreme accuracy the flexibility of the frame. First of all, the auxiliary parts that slightly influence frame structural behavior were removed, leaving the main structure completely “naked”. To simulate frame deformation under real working conditions a particular load and constraint system has to be built up [10].

So, two of the rear cylindrical longitudinal elements were firmly mounted on a structure that could be considered infinitely stiff if compared to the frame, in order to simulate a perfect fixed joint. The front end of the frame has been positioned on a spherical head, right in the middle of the front transverse tube: in this way all degrees of freedom are allowed but the displacement in vertical direction, and so the shear stress caused by external loads can be easily balanced.



FIG. 9: *To apply the loads, a circular plate mounted on the left front tire support of the frame was used.*

To apply the loads, a circular plate with an adjustable spherical joint was attached to the right front tire support of the frame; once regulated the plate on an horizontal spatial position, the joint has been blocked to ensure its correct stability (FIG.9). Slight translations of the plate center of mass under load conditions can be neglected, since the deformation of the frame are usually quite small in these kind of experimental test.

The amount of load needed for the experiment has been determined by simple considerations on the vehicle dynamic loads during typical turning maneuvers [12]; so a series of six identical weights of 5 Kg were provided for the test, for a total applicable load of about 300 N. This set-up allows to apply to the frame an almost pure momentum along the longitudinal axis of the structure, in the way to simulate a real working condition. To measure the frame level of deformation, a set of strain gauges has been used to investigate frame local strain. Since the

front side of the structure is the more excited from these kind of load and constraint set [6-11] five extensometer rosette were applied in the more representative points of the frame front side, whose approximated position has been determined both by literature and preliminary numerical analyses. (FIG.12) All these measurement tools have been connected to a main data acquisition unit directly manageable from a Personal Computer by a specific software environment. Together with the strain gauges, a comparator was mounted to the left front tire support of the frame, in order to capture a punctual displacement measure, very useful both for the virtual model validation and for the torsional stiffness determination.

Load (kg)	Displacement Load Cycle (mm)	Displacement Unload Cycle (mm)
0	0	0
5	0,36	0,37
10	0,72	0,73
15	1,10	1,12
20	1,49	1,50
25	1,86	1,88
30	2,24	2,24

TAB. 1: *The displacement values registered by the comparator.*

After a preliminary phase needed to reach an environmental steady-state condition, the experiment started and a complete load-unload cycle was applied to the structure, during which each weight has been applied for the time needed to reach a stationary condition in terms of strain measured by the gauges. Once reached the maximum level of applicable load, the weights have been removed one by one, respecting the same application time conditions. The whole test has been carried on in a time as short as possible, to avoid the natural amplifier drift. During the experiment all the strain signals measured by the gauges has been gathered, together with the displacements registered by the comparator. (TAB.1) The maximum displacement measured at the reference point was about 2,24 mm. Moreover, the frame behavior under

crescent applied load seems to be quite linear, with slight difference between displacement measured in the load and unload cycles.

After the completion of the physical test, it is possible to build a numerical model through Finite Element Method software systems, and validate it on the basis of the experimental data. This allows the creation of a virtual prototype of the vehicle frame capable to reproduce its structural behavior within, for example, the numerical optimization process of the vehicle dynamic performance. At first it's important to determine if it's possible to do some idealization on the numerical model, in order to simplify the analysis in terms of computational costs and preprocessing time.

A Go-Kart frame is a typical tubular structure, where each element is characterized by two dominant dimensions with respect to the third one; so it's possible to neglect the stress variation along the depth of the cylinders, assuming it constant. This assumption gives the possibility to consider in the numerical analysis only the mid-surface of the cylinders, modeling them with shell elements. A step further in this idealization process, is that of considering the tubular elements as beams of known section: this allows a further reduction in computational costs with an acceptable rate of approximation; even if it is a valid idealization level for this kind of analysis it will not be adopted, mainly to validate the numerical model on the basis of the local level of deformation measured in the experimental test.

So the geometrical model already built in the reverse engineering process has been imported into the pre-processing environment MSC Patran v.2005, that will be also used for post-processing together with MSC Nastran v.2005 as the solver. To correctly set-up the numerical analysis, frame structural and geometrical properties have to be defined. The frame structure is composed by cylindrical elements of hollow section characterized by an inner and an outer diameter of known values.

The reference material for this Go-Kart model is a **25CrMo4** steel, whose properties can be summarized in the following table.

<b>Elastic Modulus</b>	210000 N/mm <sup>2</sup>
<b>Poisson Coefficient</b>	0,27
<b>Density</b>	7,801 e-06 Kg/mm <sup>3</sup>

Constraint and loads has been applied to the numerical model, trying to simulate as near as possible the conditions imposed during laboratory test. So a fixed link has been applied to the end section of the main longitudinal tubes, while a total load has been imposed on the center of the bottom hole located at the right front tire support of the frame.

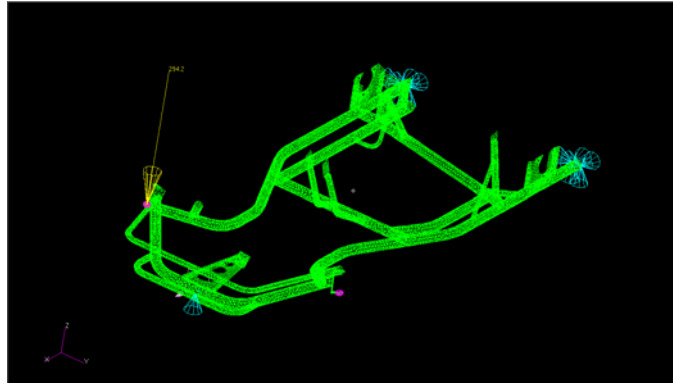


FIG. 10: *FEM model*

In addition, a special constrain was applied in the center of the front transverse element, to prevent vertical displacements, just like the spherical head did in the experimental analysis. (FIG.10)

After the pre processing of the shell model, a series of numerical test has been carried on, with different load level. The comparison between the numerical model and the experimental test show an excellent correspondence in terms of displacements measured at the reference point.

<b>Weight (Kg)</b>	<b>Measured (mm)</b>	<b>Calculated (mm)</b>	<b>Accuracy %</b>
5	0,36	0,385	93,5
10	0,72	0,770	93,5
15	1,10	1,16	94,8
20	1,49	1,54	96,7
25	1,86	1,93	96,3
30	2,24	2,31	96,9

Even the local deformation level is quite similar, with some errors mainly due to the inner properties of the FEM model used to represent the vehicle frame. Having compared even the local deformations and the punctual displacements, the numerical model can be considered quite validated and certainly able to represent the frame structural behavior. So with this model it's possible to evaluate the influence that material stiffness has on frame torsional flexibility; a series of test have been carried on using materials with different values of the modulus of elasticity.

In these conditions, the displacement of the reference point has been calculated and compared for each material.

<b>Modulus of Elasticity (MPa)</b>	<b>Displacement (mm)</b>
190	2,55
220	2,21
250	1,94



As expected, the increasing of the material stiffness brings to a reduction of the calculated displacement and of the frame torsional compliance.

#### 4. Multi body analysis

Multi body numerical analysis is a powerful tool to evaluate the global vehicle performance; with the help of sophisticated vehicle-oriented software environments, based on this kind of solution codes, it's possible to investigate the dynamic behavior of mechanical systems. The prediction of vehicle performance can improve its design process, without the need to build a lot of hardware prototypes to evaluate how the variation in a parameter influences the dynamic behavior. The primary task was the evaluation of the level of integration between FEM and Multi Body environments, together with the study of the accuracy that characterizes the flexible mathematical model once included in the multi body system. For the purposes of this work, the MSC ADAMS v2005 software system has been adopted, especially for the great capabilities that it offers in vehicle dynamic analyses. Two kind of numerical test has been conducted: a preliminary torsional excitation test on the frame, and a subsequent dynamic event on the entire vehicle model: a step steer maneuver. Even without the possibility to do experimental test to compare vehicle behavior with the real one, some considerations can be done on the importance of frame torsional deformation during a curve.

The torsional analysis has been done to evaluate at first the difference between the finite element model and the flexible frame imported in the multi body environment and integrated into it by a modified Craig-Bampton reduction method [13].

A pre processing phase is necessary to arrange the model, during which the set of boundary degrees of freedom have to be declared, in order to correctly define loads and constraints in the multi body analysis. In addition, particular attention has to be paid to the number of deformation modes that have to be included in the numerical model, trying to reach a good compromise between solution accuracy and computational cost. After the FEM model has been correctly "translated", it was possible to set up the torsional analysis with the same load and constraint condition adopted above. The displacement of the reference point has been measured once the model has reached a stationary condition, and its value compared with that measured in the FEM environment. The results of these analyses showed an accuracy of about 95% between them with all the materials considered, confirming the good quality of the mathematical model. In addition, the calculated natural frequencies of the model were analyzed both in the FEM and in the Multi Body environment, with a reference material.

Modulus of Elasticity (MPa)	Displacement (mm)
190	2,4356
210	2,203
220	2,1027
250	1,8501

TAB.2: *The displacement measured within the multi body environment, during a torsional excitation test.*

Mode	Nat. Frequencies of FEM model (Hz)	Nat. Frequencies of Multi body model (Hz)
1	34,258	34,324
2	38,762	39,109
3	50,728	50,731
4	73,384	73,553
5	88,988	89,317
6	99,740	99,715
7	103,48	103,51
8	115,31	118,19
9	118,70	118,65
10	138,61	138,63

As it can be seen, there are only small differences between the two models even comparing their natural frequencies correspondent to their vibrational modes. Subsequently, a go-kart model has been built like a set of interconnected rigid and deformable bodies. The subsystems modeled were the front steering mechanism, frame, front and rear tires. These last ones were modeled using the equations developed by Fiala [14]. At this stage, engine and driver subsystems were modeled as point mass, but in the future a more accurate model will be prepared to carry on more numerical analyses.

The rear axle has been modeled as rigid, to consider exclusively only the frame deformation rate in the dynamic maneuvers. The virtual prototype has been subjected to a Step Steer Analysis, to understand the vehicle dynamic behavior and the importance of the frame torsional stiffness to reach the maximum level of performance. In this maneuver the steering angle is varied from 0 to 30 degrees in a time of 0.5 seconds, with an initial speed of about 80 Km/h. (FIG.12)

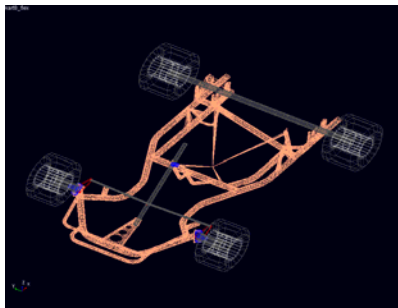


FIG. 11: *The final multi body model*

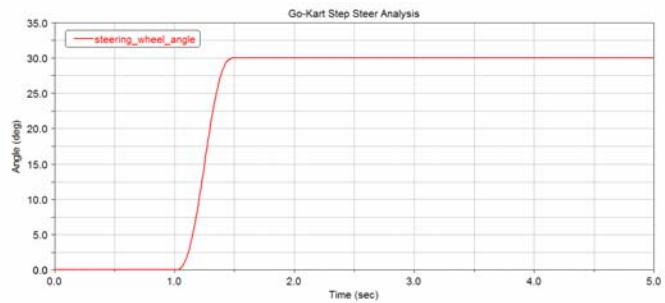
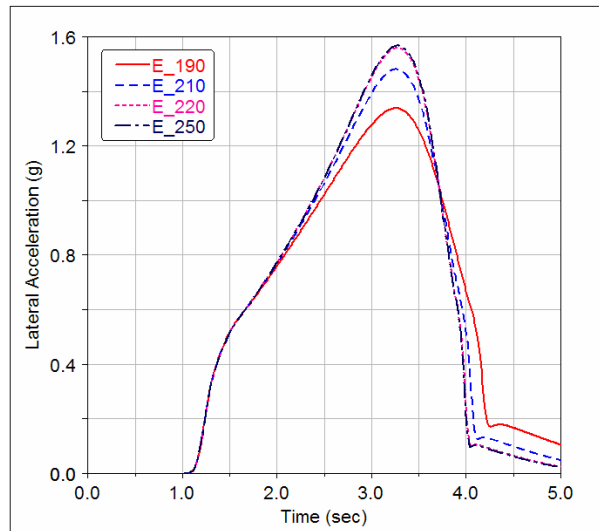
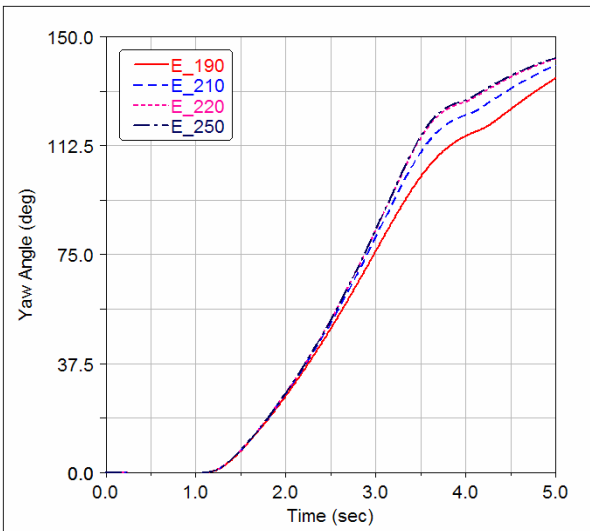


FIG. 12: *Steering wheel angle during step steer maneuver*

The velocity is not maintained constant during the maneuver, due to the absence of the engine subsystem: this does not correspond exactly to the maneuver definition[15], but can still give us some important informations regarding vehicle behavior. We can look at two important vehicle dynamic parameters: the yaw angle and the lateral acceleration:



It's possible to notice that the stiffer is the frame the higher will be the yaw angle and the lateral acceleration: this demonstrate how it can give a more direct response to the steering wheel input, also developing an higher level of lateral acceleration that has to be sustained by the tires. These dynamic

simulations of the vehicle aren't really indicative of its real behavior, because the multi body model lacks of some important subsystems. The implementation of these components will be done in future developments of the research work, that was instead focused on the integration level of FEM and multi body tools. In this way, great importance covered the comparison of results accuracy showed by them in the study of deformable bodies behavior. These activities allowed the definition of a dynamical model of a Go-Kart vehicle with integrated deformable bodies, completely validated from a structural point of view; it is now ready to be used for future vehicle dynamic analyses that will be conducted with numerical and experimental test, for a complete validation of the mathematical model.

## **Conclusion**

In this work a detailed methodology of virtual design and testing has been presented, to evaluate the dynamic performances of Go-Kart vehicles. Particular attention was paid to the understanding of the structural behaviour of the tubular frame, that is the most important element in the determination of vehicle turning performance. The aroused methodology is made of numerical processes of geometrical modelling, structural and dynamic analysis. To validate its accuracy, a validation process made of experimental activities of geometric and structural reverse engineering was carried on. Each of these activity was deeply analyzed to ensure a high level of performance; in fact the results accuracy and the operational flexibility are the fundamental parameter of evaluation. To validate and optimize this methodology a real Go-Kart vehicle has been taken as reference *physical* test case: it has been subjected to experimental test for the objective evaluation of the frame deformation properties under static load. The result of these analyses allowed to build an accurate mathematical FEM model that, once validated, has shown an excellent capability to reproduce the frame structural behaviour. The validated mathematical model has been then integrated within a multi body environment to simulate the vehicle response during a step steer manoeuvre: this allowed to do some considerations about the influence that the frame flexibility has on the global vehicle performance.

Even if the entire process of design and testing proposed has shown interesting results, part of the methodology must be still validated through dynamic experimental test. This will allow the creation of a mathematical model completely defined and validated, giving the basis for the future developments regarding the optimisation process of the vehicle performance.

## **Acknowledgements**

A special thanks goes to Prof. Ing. Leonardo Pagnotta for his precious advices and to Mr. Francesco Pulice for its technical support.

## **References**

- [1] FACCHINELLI, F.L. Kart, messa a punto teorica e pratica. Editrice Motor Books Tech, 2000.
- [2] NATOLI, Marco. Il manuale del kart. Archimede Editore, Marzo 1999.

- [3] MUZZUPAPPA, M. - MATRANGOLO, G. - VENA, G. Reverse Engineering of a Go-Kart Tubular Frame. XVII INGEGRAF - XV ADM. Seville, June 2005
- [4] BOSCH, John A. Coordinates Measuring Machines. Edited by John A. Bosch, Giddings & Lewis. Dayton, Ohio 1995
- [5] BALSAMO, Alessandro. Macchine di misura tridimensionali. E.M.I.T. Laboratori di automatica e strumentazione
- [6] BAUDILLE, R – BIANCOLINI, M.E. – BRUTTI, C. – RECCIA, L. Analisi integrata multi-body FEM del comportamento dinamico di un Kart. Atti del Convegno AIAS. Alghero, Settembre 2001.
- [7] BAUDILLE, R. – BIANCOLINI, M.E. – RECCIA, L. Integrated multi-body/FEM analysis of vehicle dynamic behaviour. Fisita Congress, Giugno 2002.
- [8] GUGLIELMINO, E. - GUGLIELMINO I.D. – MIRONE, G. – RISITANO, A. Multi-body analysis and frame stiffness effect on the behavior of a go-kart in steering pad tests. - Dipartimento di Ingegneria Meccanica, Università degli Studi di Messina e di Catania.
- [9] GUGLIELMINO, E. - GUGLIELMINO I.D. – MIRONE, G. Caratterizzazione numerica e sperimentale di un go-kart da competizione. AIAS 2000. Lucca. Settembre 2000.
- [10] SOLAZZI, L. – MATTEAZZI, S. Analisi e sviluppi strutturali di un telaio per kart da competizione. AIAS 2002. Parma. Settembre 2002.
- [11] MIRONE, Giuseppe. Mulibody modelisation of a go-Kart with flexible frame: simulation of the dynamic behaviour and experimental validation. SAE International, 2003.
- [12] GIGLIO, M. - PECCHIO, A. – RAVASI, P. L'utilizzo del FEM per l'ottimizzazione di un telaio di Go-Kart -ATA Ingegneria Automobilistica, 2000.
- [13] CRAIG, Roy R. Jr – BAMPTON, Mervin C.C. Coupling of substructures for dynamic analyses. AIAA Journal, Vol 6, No 7. July 1968.
- [14] FIALA, E. Ingegneria automobilistica, in Dubbel, "Manuale di ingegneria meccanica ". Edizione di scienza e tecnica. Milano, 1985.
- [15] ISO/CD 7401. Road vehicles – Lateral transient response test method. 1995
- [16] MSC ADAMS v2005 User's Guide
- [17] MSC Patran v2005r2 User's Guide
- [18] MSC Nastran v2005r2 User's Guide
- [19] PTC Pro Engineer Wildfire 2.0 User's Guide