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The Analysis of Stiffness Condition in Rolling Thin-Walled Ring

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Abstract. The thin-walled ring rolling, a new rolling process, can produce the thin-walled rings that the traditional methods are difficult to process. It has the advantages of simple equipment, low investment and high productive. According to the mechanics analysis of the multiple rollers rolling, it established a mechanical model of small deflection bending compression rod. After solved the mechanics equations, it obtained the thin-walled ring's rolling stiffness condition. From the theoretical formula, it could find that the increase of friction coefficient, ring radius, roller radius and feed per rotation were disadvantage for the thin-walled ring rolling stiffness condition. However, the increase of the ring cross sectional inertia was advantage for the stiffness condition. The theoretical formula could provide theoretical basis and guidance for the quantity design of roller pairs in rolling machine, design of ring cross section shape and selection of ring raw material.

Introduction

Thin-walled ring is a ring that has small ratio of thickness to radius. Generally, the ratio is less than 1:20, and some rings are even less than 1:1000. With the development of industrial technology, in the field of electrical machinery[1], aviation[2], automobile[3], engineering machinery[4], mining machinery[5] and others often need some rings with thin thickness, high strength, high precision and shaped cross section, such as the metal belt group in automotive CVT[3]. Because of its soft stiffness, the thin-walled ring was difficulty to be processed. At present, there are many different ways of manufacturing thin-walled ring, such as mechanical processing, electromagnetic forming, spinning, ultrasonic vibration machining and high-speed cutting. These methods had complexity technique process, long processing time, high accurate fixture and equipment, which increased the production cost. Especially when the production batch is small, the high cost of fixture would make the ring expensive.

The traditional ring rolling is widely used to rolling various shaped section rings, however it is only suitable for the large wall thickness ring. As for thin-walled ring, a new technology with multiple pairs of driven rollers and idle rollers could shape its section easily. According to the finite element simulation[6], a 12 pairs of driven rollers and idle rollers machine was manufactured[7]. The work principle of ring rolling was shown in Fig. 1. The quantity of rollers pairs is the critical parameter of machine design. However, there is no research report for the stiffness condition of the new rolling technology. The research on thin-walled ring rolling stiffness condition is significant to machine design, ring cross section design and ring material selection.

Thin-walled ring rolling mechanics model

Based on the thin-walled ring rolling work principle, its stiffness condition can be simplified as shown in Fig. 2. Known from the graph, there is no guide roller acted pressure on the ring which compared with tradition ring rolling. Ring, driven by the friction force of driven roller, continuously passed among the multiple gaps which constructed by pairs of driven roller and idle roller. However, during the rolling process, the ring elastically accumulated at the roll gap entrance and expanded at the

exit. When the ring moved from the former gap to the next gap, because the idle roller had a feed to the driven roller, which resulting in a smaller roll gap, all the above factors could impede the rotation of ring. If the stiffness of the ring itself were insufficient, it would cause plastic instability, distortions and defects such as folding and pull shrinking among roll gaps. Moments generated by these forces are far less than that by the traditional ring rolling, thus the overall thin-walled ring rolling stiffness condition has greatly improved.

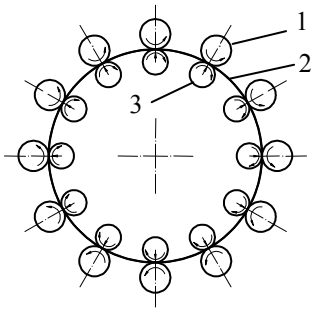

Fig. 1 The work theory of thin-walled ring rolling
1-Driven rollers 2-Ring 3-Idle rollers

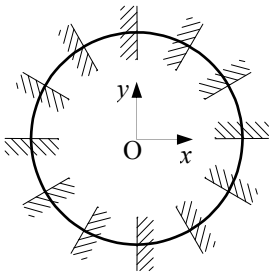

Fig. 2 The stiffness condition model of thin-walled ring rlling

According to the thin-walled ring rolling stiffness condition model, the mechanical model was established from analysis of ring rolling process force, as shown in Fig. 3. The driven roller acted force F_2 on the ring, and then it produced the friction force F_{s2}. Similarly, the idle roller acted force F_1 on the ring, and then it introduced friction force F_{s1}. Supposed the driven roller and the ring contact friction in line with Coulomb's law, so $F_{s1} = \mu F_1$, $F_{s2} = \mu F_2$, where μ is the contact surface friction coefficient. Because the idle roller is idle, it can't withstand the friction torque, and the coefficient of friction between the ring and idle roller can be regarded as 0, so $F_{s2} = 0$.

This force structure is a statically indeterminate structure. Because of the extra constrain, the yield of one longitudinal cross section will produce plastic hinge which will impede the structure yield immediately and withstand the continuously increased loads. When the load increased continuously, the points, occurred plastic hinge firstly, could still bear the cross section moments and remain unchanged, but result in a rotation. The cross section that had no plastic hinge undertakes the increased load until the geometric structure distortion occurred. The rolling stiffness condition model shows that the ring and roller contact point was suffered largest moment, so that plastic hinges occur in contact point. For 12 pairs of the driven roller and idle roller thin-walled ring rolling machine, one-twelfth ring was intercepted as the analysis object. One end is a fixed end and the other acted by force. The force is the division of F_1 and F_{s2} that projected in the two gaps center line, which caused the thin-walled ring stiffness instability. Therefore, the mechanical model can be regarded as a small deflection bending compression bar mechanical model, as shown in Fig. 4.

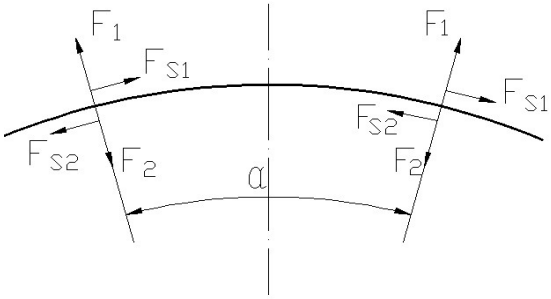

Fig 3 Thin-walled ring mechanical analysis

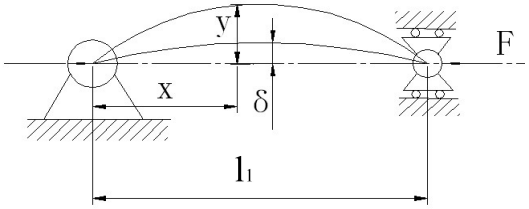

Fig 4 Schematic diagram of thin-walled ring rolling

Rod end pressure F is：

$$F = F_{s2}\cos(\alpha/2) + F_2\sin(\alpha/2) = \mu F_2\cos(\alpha/2) + F_2\sin(\alpha/2) \quad (1)$$

Where, α is the roll center angle, $\alpha = 2\pi / N$, and N is the pairs of driven roller and idle roller.

Because the thin-walled ring rolling only shapes cross section and has no thickness deformation, it can introdue cylindrical deep drawing force formula to calculate the single pair of rollers forming force:

$$F_2 = [1.1\sigma_s \ln\frac{w}{W}(1+1.6\mu) + \frac{\sigma_b}{2r_d / t + 1}]bl \tag{2}$$

Where, w is the ring width after rolling, W is the ring width before rolling, r_d is the section groove radius of idle roller, t is the thickness of the ring, b is the section groove width and l is the contact arc length between ring and idle roller. From Reference [8], the contact arc length $l \approx \sqrt{\frac{2\Delta h}{1/R_1 + 1/R_2}}$, where Δh is the rolling feed per rotation, R_1 is the driven roller radius and R_2 is the idle roller radius. Substituted above parameters into Eq. 1, so that the force F leaded the ring to stiffness instability is:

$$F = (\mu\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2})[1.1\sigma_s \ln\frac{w}{W}(1+1.6\mu) + \frac{\sigma_b}{2r_d / t + 1}]b\sqrt{\frac{2\Delta h}{1/R_1 + 1/R_2}} \tag{3}$$

Within the nonlinear elastic range, when the compress force of ring is less than the proportional limit σ_p, the deflection curve differential equation is：

$$EI\frac{d^2y}{dx^2} = M(x) = -F(y + x\tan\alpha) \tag{4}$$

Where, E is the elastic modulus of the material and I is the moment of inertia.

Because both ends are fixed, allowing the rod occur slightly curved deformation at any longitudinal plane, the slightly curved bend of rod should occur in the vertical plane with the smallest bending resistance ability. Therefore, I in Eq. 4 should be the smallest moment inertia in cross section. Here introduces mark $k^2 = F / EI$, so Eq. 4 can be written as:

$$\frac{d^2y}{dx^2} + k^2 y = -k^2 x\tan\alpha \tag{5}$$

As for non-homogeneous second-order ordinary differential equation, its general solution is:

$$y = c_1\sin kx + c_2\cos kx + cx \tag{6}$$

Substituted both ends of globe hinge boundary conditions into the general solution, the critical pressure of both ends of globe hinge could be obtained by the Euler critical load formula.

$$F_{cr} = \pi^2 EI / l_1^2 \tag{7}$$

Where, $l_1 = R\alpha$, R .is the ring radius.

As for forming thin-walled cross section in terms of rolling, the cross section moment of inertia will increase with the rolling process. Therefore, as long as the rolling process meets the stiffness condition at the rolling beginning, it also could meet the stiffness condition in later.

According to the work principle of thin-walled ring rolling and actual work situation, stiffness condition should be the mechanical model of both ends of globe hinge, therefore, if ring maintain stiffness condition in the rolling process, it needs to meet:

$$F \le NF_{cr} \tag{8}$$

Substituted Eq. 3 and Eq. 7 into Eq. 8, and simplified:

$$\frac{\mu\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{N^3} \le \frac{EI}{[1.1\sigma_s \ln\frac{w}{W}(1+1.6\mu) + \frac{\sigma_b}{2r_d / t + 1}]bR^2\sqrt{\frac{2\Delta h}{1/R_1 + 1/R_2}}} \tag{9}$$

Analysis and discussion

(1) If the friction coefficient μ increases, the value on the right of the Eq. 9 will be decreased. On the left of the Eq. 9, it results to increase N in order to keep the balance of the equation. But the increase of N is disadvantage for machine manufacture. That is to say, it needs more pairs of rollers to ensure smooth rolling. However, decreasing coefficient friction μ is disadvantage for biting into the rolling gap, therefore, it is necessary to reasonably select the friction coefficient to ensure the ring rolling bite conditions and stiffness conditions.

(2)If the thin-walled ring cross sectional moment of inertia I increases, the value on the right of the Eq. 9 will be increased. On the left of the Eq. 9, even if N reduced to some extent, it also can ensure that the left less than the right. That is to say, increasing I could decrease the quantity N of roller pairs in some extent that also can keep the rolling process stiffness condition.

(3)If the ring radius R increases, the value on the right of the Eq. 9 will be reduced. On the left of Eq. 9, its value is unchanged. Therefore, it need increase N to ensure that the left is less than the right, which means that need to increase the quantity N of roller pairs to keep the rolling process.

(4)If the driven roller radius R_1 and idle roller radius R_2 increase, the value on the right of the Eq. 9 will be reduced. On the left of Eq. 9, its value remains unchanged. Therefore, it need increase N to ensure that the left is less than the right, which means that need to increase the quantity of roller pairs to keep the rolling process. But it can also be known from the formula, the radius of driven roller and idle rollers have little effect on ring rolling stiffness condition. In the design of roller radius, the layout of machine should consider the position of rollers.

(5)If the feed per rotation Δh increases, the value on the right of the Eq. 9will be reduced. On the left of Eq. 9, its value remains unchanged. Therefore, it need increase N to ensure that the left is less than the right, which means that need to increase the quantity of roller pairs to keep the rolling process. However, too small feed is disadvantage for rolling and biting into the plastic forming conditions, thus the feed per rotation can not be selected too small to ensure the rolling bite condition.

Summary

In general, If the friction coefficient μ, the ring radius R the feed per rotation Δh, the driven roller radius R_1 and the idle roller radius R_2 increased, it could cause the thin-walled ring rolling stiffness condition variation, which lead to the increase of roller pairs quantity N. Only the increase of section inertia I could improve the stiffness condition, so it can decrease appropriately the roller pairs quantity N. This also proved that as long as the rolling process met the stiffness condition at the rolling beginning, it also could meet the stiffness condition in later.

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References

[1] R.Q. Yao. Machine Tool & Hydraulics, Vol. 35 (2007), p. 250 (Chiness)
[2] M.H. Ji. Tool Technology, Vol. 2 (2006), p.44 (Chiness)
[3] S. Nilabh and H. Imtiaz. Journal of Computational and Nonlinear Dynamics,Vol. 1 (2007), p.86
[4] H.L.Song. Mechanical Engineers,V ol. 5 (2001), p.17 (Chiness)
[5] Y.P. Han, M.J. Wang and B.W. Wang. Coal Technology,Vol. 23 (2004), p.11 (Chiness)
[6] Z.J. Zuo and Y. Li. 2010 International Conference on Technology Management and Innovation, 2010. 7, p.1
[7] Z.J. Zuo, X. Tan and Y. Li.Applied Mechanics and Materials, Vol.44-47(2011), p 436
[8] L. Hua and Z.Z. Zhao. Journal of Materials Processing Technology, Vol. 69 (1997) p. 273