Achieving Sharp Deliveries in Supply Chains through Variance Pool Allocation

D. Garg[*], Y. Narahari[†], N. Viswanadham [‡]

Abstract

Variability reduction and business process synchronization are acknowledged as key to achieving sharp and timely deliveries in supply chain networks. In this paper, we develop an innovative approach that facilitates variability reduction and business process synchronization for supply chains in a highly cost effective way. The approach developed is founded on (1) an analogy between mechanical design tolerancing and supply chain lead time compression, (2) interesting use of process capability indices C_p, C_{pk}, and C_{pm} to measure supply chain delivery performance, and (3) a generalized notion of Motorola six sigma quality. We first illustrate a motivating example to develop an analogy between mechanical assemblies and supply chain networks. Motivated by this analogy, we define, using process capability indices, a new index of delivery performance called *delivery sharpness* which, when used with classical performance index *delivery probability*, measures the accuracy as well as the precision with which products are delivered to the customers. Next we solve the following specific problem: how to compute the allowable variability in lead time for individual stages of the supply chain so that a specified levels of the delivery sharpness and delivery probability are achieved in a cost-effective way. We call this the variance pool allocation (VPA) problem. We suggest an efficient heuristic approach for solving the VPA problem and also show that a variety of important supply chain design problems can be posed as instances of the VPA problem. One such problem, which is addressed in this paper, is the supply chain partner selection problem. We formulate and solve the VPA problem for a plastics industry supply chain and demonstrate how the solution can be used to choose the best mix of supply chain partners.

Keywords

Supply chain management, lead time reduction, variability reduction, process capability indices, statistical tolerancing, variance pool allocation (VPA).

1 Introduction

Businesses today operate in a very tough environment that is constantly in flux [1, 2]. Customers have become increasingly demanding looking for better and innovative goods and services that are specifically

[*]Computer Science and Automation, Indian Institute of Science, Bangalore - 560 012, India. E-mail:dgarg@csa.iisc.ernet.in

[†]Computer Science and Automation, Indian Institute of Science, Bangalore - 560 012, India. E-mail: hari@csa.iisc.ernet.in. (corresponding author)

[‡]Logistics Institute Asia Pacific, National University of Singapore, Singapore - 119 260. E-mail: mpenv@nus.edu.sg

customized to meet their unique needs. There is also an implicit requirement on the accuracy, timeliness, convenience, responsiveness, quality and reliability of the service offered to them. And all of this is desired at ever-lower prices. Simultaneously, the rapid pace of innovation has resulted in shorter product and technology cycles, leading to uncertainties in supply and demand. These trends are clearly evident in the PC industry, where new models are introduced every 6 to 9 months under intense competitive pressure on cost, functionality and service. All these challenges ultimately result in managers asking for an outstanding design of the underlying supply chain which happens to be the backbone of any business company.

The supply chain process is a complex, composite business process comprising a hierarchy of different levels of value-delivering business processes. Designing a high performance supply chain is a very challenging task due to the complex structure of the supply chains and the ever changing business. Our view in this regard is that the key issue in supply chain design, facing companies today, is no longer the location of manufacturing and distribution facilities for their vertically integrated operations, but rather the strategic selection of partners for each stage of their outsourced value chain, in the face of uncertainties of various kinds [3, 4]. This selection needs to take into account the synchronization of schedules for suppliers, manufacturers, and logistics providers in order to streamline processes throughout the supply chain. Variability is a major issue in this process. Variability reduction and business process synchronization are therefore acknowledged as key to achieving superior levels of performance in supply chain networks. Today, when business are operating under intense competitive pressure on rapid deliveries at lower cost, achieving superior delivery performance is the primary objective of any industry supply chain. In this paper, we develop an approach to address this design need by using the idea of variability reduction which has been popularly employed in design of machining processes.

1.1 Contributions

The contributions of this paper can be summarized as follows.

1. We first illustrate the motivating example to develop an analogy between mechanical assemblies and supply chain networks. The example shows that the variation in end-to-end lead time of a supply chain can be viewed as the variation in the dimension of the parts produced by a machining process.

2. The above example motivates us to investigate the use of standard design tolerancing techniques (such as process capability indices), that are popularly used for quantifying and reducing the defective assemblies produced by a machining process, for the purpose of quantifying the delivery performance of the supply chain. In particular, we provide a new perspective of process capability indices C_p, C_{pk} and C_{pm} where we show that these can be used to measure and control the variability in lead time of the supply chain in the same way as they are used for the controlling the defective parts produced by a machining process.

3. Next we describe the delivery performance of a supply chain in terms of two metrics. The first is a traditional metric, *delivery probability* (DP), which is the probability that a typical customer order is delivered during a customer-specified window. We show in the paper that the process capability indices, C_p and C_{pk} [5, 6] provide an appropriate vehicle for computing the delivery probability. The second metric is a new one that we propose, which we call *delivery sharpness* (DS), which is a measure of how close to the target (most desired) delivery date a customer order is actually delivered. DS is defined based on the process capability index C_{pm}. The motivation

behind these two indices comes from the Motorola six sigma program [7, 8] and Taguchi method [9, 10, 11]. We also show that Taguchi method is a complement to the Motorola six sigma program and combining them together results in what we call as *generalized notion of Motorola six sigma quality*. The generalized notion of Motorola six sigma quality takes into account delivery sharpness in addition to delivery probability.

4. All the contributions mentioned so far prepare the ground for formulating the design optimization problem for supply chains and that is the climax of the paper. In this paper, we formulate the following design optimization problem:

   Given a supply chain and the mean and standard deviation of the end-to-end lead time for a certain product mix, the design problem seeks to optimally distribute the pool of variance among individual business processes so as to minimize the cost and achieve outstanding delivery performance.

   We call this problem as the variance pool allocation (VPA) problem. We come up with a five stage approach for solving the VPA problem. We then look at linear or pipelined supply chains and solve the VPA problem through the Lagrange multiplier method.

5. Finally, we show that a rich variety of supply chain design problems, in particular, the supply chain partner selection problem, can be cast as a VPA problem. We show that the optimal variance of each stage obtained by solving the VPA problem cab be used in selecting the best partner mix out of a given set of alternatives for each stage of the supply chain. To substantiate this, we consider a six stage supply chain in the plastics industry, for which we formulate and solve the VPA problem. We show how a manager can use the solution of the VPA problem, inorder to select the best combination of supply chain partners: supplier, manufacturer, inbound logistics provider, assembler and outbound logistics providers out of a given set of choices, so as to ensure timely delivery of finished products to customer destinations.

In a companion article [12, 13], we use the framework (variance pool allocation) developed in this paper to develop a methodology for design of what we call *six sigma supply chains*. We apply the the methodology to an inventory allocation problem in a multi-echelon supply chain, to determine the optimal inventory level for a given stage of the supply chain.

In this current paper, we propose the VPA problem, develop a solution framework for the VPA problem, and apply it to a partner selection problem in the context of a typical multi-stage supply chain.

1.2 Related Work

The subject matter of this paper falls in the intersection of following areas of current interest: (1) variability reduction and lead time compression techniques for business processes, (2) statistical design tolerancing, and in particular, theory of process capability indices, (3) the Motorola six sigma program, and (4)Taguchi methods.

Lead time compression in business processes is the subject matter of a large number of papers in the last decade. See for example, the papers by Hopp, Spearman, and Woodruff [14]; Adler, Mandelbaum, Nguyen, and Schwerer [15]; Narahari, Viswanadham, and Kiran Kumar [16]; and Jackson S. Chao and Stephen C. Graves [17]. Variability reduction is a key strategy used in the above papers and other related papers. Hopp and Spearman, in their book [18], have brought out this key role played by variability reduction. Lead time compression in supply chains is the subject of several recent papers, see for example, Narahari, Viswanadham, and Rajarshi [19]; Garg, Narahari, and Viswanadham [20].

Statistical design tolerancing is a mature subject in the design community. The key ideas in statistical design tolerancing which provide the core inputs to this paper are: (1) theory of process capability indices [5, 21, 6, 22]; (2) tolerance analysis and tolerance synthesis techniques [23, 24, 25]; (3) Motorola six sigma program [8, 7]; (4) Taguchi methods [26, 9]; and (5) design for tolerancing [16, 27, 28].

The present paper innovatively uses the key ideas and notions in the area of statistical design tolerancing in achieving variability reduction and synchronization in the supply chain process, leading to quicker and sharper deliveries. A preliminary version of this paper [20] contains some of these above ideas. A companion paper [12] explores these ideas in a different direction and applies the ideas to inventory optimization in multi-stage supply chains.

2 An Analogy between Mechanical Assemblies and Supply Chain Networks

A typical value delivery process in an enterprise starts with an order from the customer and ends with customer satisfaction. This process consists of series of activities, each performed by various subsystems of enterprise. Formally, we can define a business process as an end to end set of activities that ultimately delivers the value to the customer. Thus a business process consists of several sub processes which in turn consists of work processes whereby work process we mean a value adding activity with well defined inputs and outputs with someone responsible for it. Examples of the work processes include machining at a machine center, transportation from one location to another by a truck, etc. Thus we see that the business process or supply chain process is just a notion or concept but the work process or machining process is a concrete physical entity giving rise to this notion. Also a 'machining process' is one of the building blocks of a 'business process'. Moreover, as a special case, a single machining process can be viewed as a business process. This motivates us to deduce an analogy between the structure of complex mechanical assemblies and the structure of the supply chain process. A supply chain is like a complex mechanical assembly; it is a conglomeration of numerous business processes just like a complex assembly is an arrangement of numerous subassemblies. Figure 1 and 2 show a serial and converging-diverging supply chain respectively together with an analogous mechanical assembly. The analogy for the particular mechanical assembly shown in these figures comes from the fact that we are interested in analyzing end-to-end delivery performance of the supply chain.

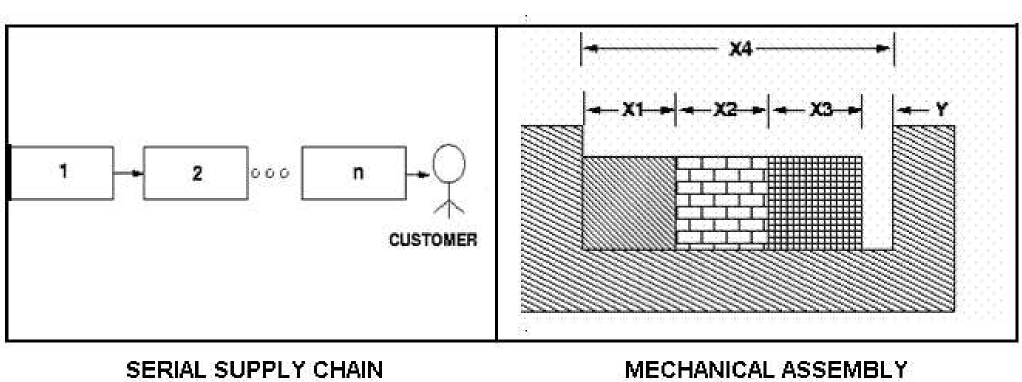

Figure 1: An analogous mechanical assembly for a serial supply chain

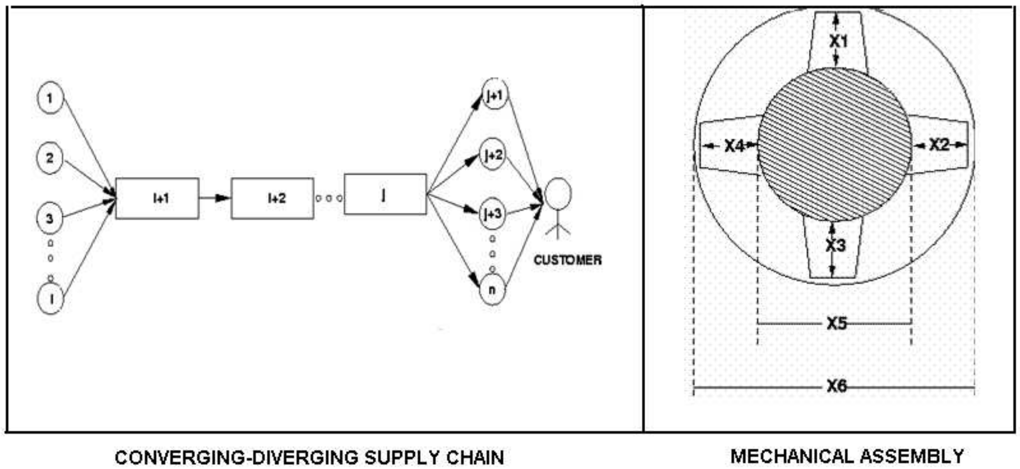

Figure 2: An analogous mechanical assembly for a converging-diverging supply chain

2.1 End-to-End Delivery Performance of a Supply Chain

Apart from the fundamental differences between machining process and business process described above there are subtle commonalities between these two. For example, the performance of a machining process is measured through variations in processing time as well as the dimensions of the parts produced by it. Whereas, business process is amenable to measurement in a variety of dimensions for its performance evaluation: cycle time, defects, variability, variety, cost etc. All these measurements ultimately boil down to the corresponding measurements of the constituent machining processes and work processes.

Among the performance measures of a business process, the end-to-end lead time is perhaps the most important and is the major theme of this paper. The lead time performance of any business process depends not only on how long does it takes to provide a service to the customer but also how much does it varies from one customer to other. Lead time and its variation for individual work processes are key determinants of end-to-end delivery performance of a supply chain networks. When the number of resources, operations, and organizations in a supply chain increases, variability destroys synchronization among the individual processes, leading to poor delivery performance. On the other hand, by reducing variability all along the supply chain in an intelligent way, proper synchronization can be achieved among the constituent processes.

To illustrate the above fact, let us consider a linear supply chain as shown in Figure 1 where material flows through n different processes before it is delivered as finished product to the end customer. Assuming that no inventory is maintained at any intermediate stage of the supply chain, the end-to-end lead time, say Y, of an end customer order becomes equal to the sum of processing time (lead time) of individual processes, say X_i . That is

$$Y = \sum_{i=1}^{n} X_i$$

Note that depending upon requirement one can decompose each business process into a hierarchy of low level processes. In that situation, the supply chain will look like a complex network of business processes and the end-to-end lead time Y will depend on individual process lead time in a more complex way. An instance of such complex supply chain network is shown in Figure 2 where we are considering a very simple converging-diverging supply chain network. For this network end-to-end lead time Y of an end customer order can be given by following expression (assuming no inventory is maintained at any of the stage):

$$Y = \max(X_1, \ldots, X_i) + \sum_{p=i+1}^{j} X_p + \min(X_{j+1}, \ldots, X_n)$$

Note that if other factors like inventory replenishment policy, demand uncertainty, etc. are also taken into account then Y will become even more complex function of the system parameters.

To understand the analogy between structure of mechanical assembly and structure of supply chain network, let us consider a mechanical assembly shown in Figure 1 where the objective is to control the gap (target dimension) Y. This dimension is dependent on the dimension of the other parts as well as the configuration of the assembly in the same way as the end-to-end lead time Y was dependent on the lead time of individual business processes and other system parameters. It is easy to see that gap Y can be expressed in terms of dimensions of its subassemblies in following manner:

$$Y = X_4 - \sum_{i=1}^{3} X_i$$

The objective for mechanical assembly presented in Figure 2 is also to control the gap, denoted by Y, between circular casing and blades of the fan. Like the previous case, this dimension is dependent on the dimension of the other parts as well as configuration of the assembly. Because the configuration of this assembly is more complex than previous one, the target dimension Y is dependent on parts dimensions in following complex way:

$$Y = X_6 - X_5 + \max(X_1, \ldots, X_4)$$

It is well known fact that during the machining process, the dimension of individual parts keep varying from parts to parts which ultimately results in variation in target dimension Y of the assembly. The same is the case with supply chain lead time. In the context of supply chain networks, the lead time of individual business processes are variable in nature and so is Y. The control on variability in Y is the key to achieving outstanding delivery performance.

From above discussion it is clear that variability in lead time of individual business process play a key role in deciding the delivery performance of the supply chain. For one this observation may not be something new and exciting because there exists a plethora of literature [29, 30, 31, 32, 33, 34, 35] about studying the effect of lead time variability and other system parameters (e.g. inventory) on delivery performance of the supply chain. The novel twist which we have given to the classical approach is following. The problem of controlling the variation in end-to-end lead time of a supply chain seems to be analogous to the problem of controlling the variation in target dimension of a mechanical assembly. In other words we have shown that there exists an intriguing connection between the problem of measuring and controlling the variations in two seemingly different settings: target dimension of mechanical assemblies and supply chain end-to-end lead time. This analogy motivates us to seek the usages of standard design tolerancing techniques for measuring and controlling the lead time variations in the supply chain. In the next section we provide a new perspective of process capability indices C_p, C_{pk} and C_{pm} where we show that these can be used to measure and control the variability in lead time of the supply chain in the same way as they are used for the controlling the defective parts produced by a machining process.

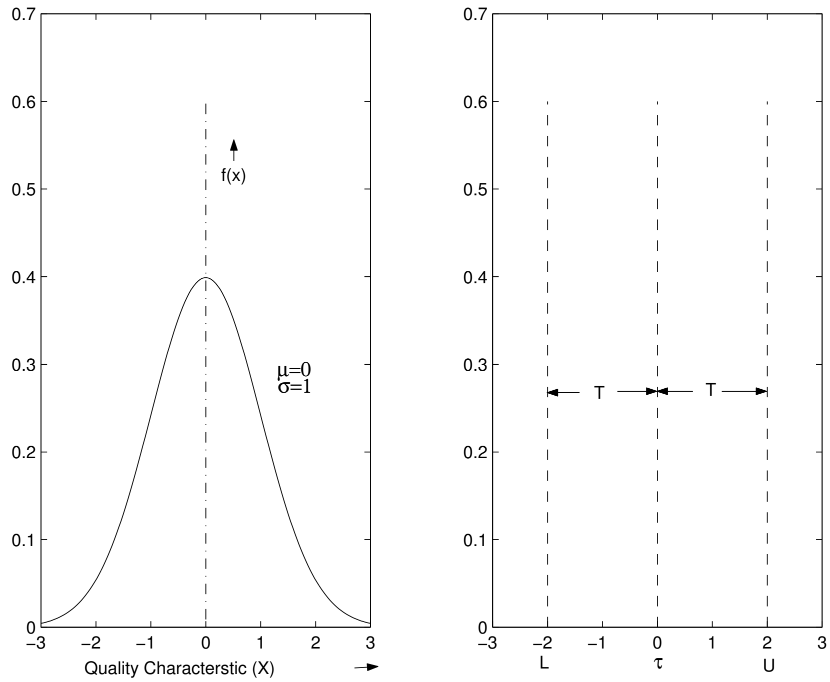

Figure 3: Process variability and customer delivery window

3 Supply Chain Process Capability Indices

3.1 Introduction

The process capability indices C_p, C_{pk}, and C_{pm} [5] are popular in the areas of design tolerancing and statistical process control. Let us consider the situation depicted by Figure 3 in order to describe the idea of how capability of a process, where variability is an inherent effect, can be measured. Below is the list of symbols used in this figure.

$$
\begin{aligned}
X &= \text{Lead time or any general quality characteristic } X \\
\mu &= \text{Mean of } X \\
\sigma &= \text{Standard deviation of } X \\
L &= \text{Lower specification limit of customer delivery window} \\
U &= \text{Upper specification limit of customer delivery window} \\
\tau &= \text{Target value for } X\text{, specified by customer} \\
T &= \text{Tolerance for } X\text{, specified by customer} \\
b &= \text{Bias}|\tau - \mu| \\
d &= \min(|U - \mu|, |\mu - L|)
\end{aligned}
$$

In this figure, variability of the process is characterized by the probability density of the quality characteristic X produced by the process, and customer specifications are characterized by a delivery window which consists of tolerance T and target value τ. Normal distribution is a popular and common choice for X because of its fundamental role in the theory of process capability indices. The target value τ can be any value between L and U but we have assumed it as the mid point of two limits for the sake of convenience, that is $\tau = \frac{U-L}{2}$.

In what follows is the definitions and summary of the allied results for three process capability indices C_p, C_{pk}, and C_{pm}- see [36, 6, 12, 22, 37] for a rigorous discussion.

3.2 The indices C_p, C_{pk}, C_{pm}

The three indices C_p, C_{pk}, and C_{pm} are defined in following way:

$$C_p = \frac{U-L}{6\sigma} = \frac{T}{3\sigma} \tag{1}$$

$$C_{pk} = \frac{\min(U-\mu, \mu-L)}{3\sigma} = \left(\frac{d}{3\sigma}\right) \tag{2}$$

$$C_{pm} = \frac{U-L}{6\xi} = \frac{T}{3\sqrt{\sigma^2+b^2}} \tag{3}$$

A few remarks are in order.

- Index C_p does not reflect the impact that shifting the process mean or target value has on a process's ability to produce a product within specification [21]. For this reason, the C_{pk} index was developed.

- Fraction defective is an indicator for process precision and it does not take accuracy of the process into account. Accuracy of the process is something which analyzes the pattern in which value of X is distributed within specification limits. In order to include the notion of accuracy along with precision, Hasiang and Taguchi defined independently the index C_{pm} [9]. Index C_{pm} indicates whether more parts or less parts are having an X value nearer to the target.

- The following relations can easily be derived [37] among all the three indices: C_p, C_{pk}, and C_{pm}.

$$C_p \geq C_{pk} \geq 0 \,;\, C_p \geq C_{pm} \geq 0 \tag{4}$$

$$C_{pk} = C_p(1-k) \text{ where } k = \frac{b}{T} \tag{5}$$

$$\frac{1}{9C_{pm}^2} = \frac{1}{9C_p^2} + \left(1 - \frac{C_{pk}}{C_p}\right)^2 \tag{6}$$

Two important quantities: *potential* and *actual yield* of the process will play a critical role in the the development of optimization problem. We, therefore, prefer to define these quantities in following manner.

Actual Yield: The probability of producing a part within specification limits.

Potential: The probability of producing a part within specification limits, if process distribution is centered at the target value i.e. $\mu = \tau$.

It is easy to verify [37] the following relations:

$$\text{Potential} = 2\Phi\left(3C_p\right) - 1 \tag{7}$$

$$\text{Actual Yield} = \Phi\left(3C_{pk}\right) + \Phi\left(6C_p - 3C_{pk}\right) - 1 \tag{8}$$

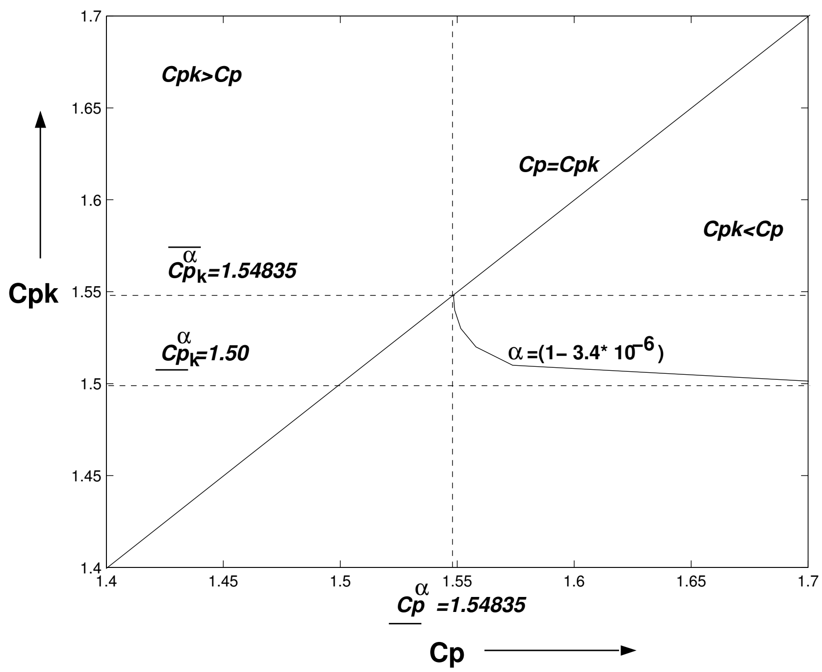

Figure 4: Typical variation of C_{pk} with respect to C_p for constant actual yield

where $\Phi(.)$ is the cumulative distribution function of standard normal distribution. The another set of interesting results [37] show that for a given value of actual yield α (say), one can get the following lower and upper bounds for the values C_p and C_{pk}.

$$\begin{aligned} \underline{C_p^\alpha} &= \frac{1}{3}\left(\Phi^{-1}\left(\frac{1+\alpha}{2}\right)\right) \\ \underline{C_{pk}^\alpha} &= \frac{1}{3}\left(\Phi^{-1}(\alpha)\right) \\ \overline{C_{pk}^\alpha} &= \frac{1}{3}\left(\Phi^{-1}\left(\frac{1+\alpha}{2}\right)\right) \\ \overline{C_p^\alpha} &= \infty \end{aligned}$$

Following is a crisp idea behind the intent of these bounds.

- If process's C_p (C_{pk}) is less than $\underline{C_p^\alpha}$ $\left(\underline{C_{pk}^\alpha}\right)$ then its actual yield cannot be equal to α, no matter how large is the value of $C_{pk}(C_p)$.
- If process's C_{pk} is greater than or equal to $\overline{C_{pk}^\alpha}$ then its actual yield cannot be less than α, no matter how small is the value of C_p .
- The case with $\overline{C_p^\alpha}$ is slightly different. For any value of C_p greater than or equal to $\overline{C_p^\alpha}$ it is possible to find a corresponding C_{pk} such that the actual yield of the process is α.

Figure 4 shows a typical variation of C_{pk} with respect to C_p when the actual yield α is constant. It immediately follows from this curve that for a given pair (C_p, C_{pk}), the value of actual yield is fixed. But for a given actual yield value, there exist infinite such (C_p, C_{pk}) pairs.

3.3 A Supply Chain Perspective of the Process Capability Indices

In a typical manufacturing process, the size, shape, strength, and color of the product are usual quality characteristics and customers release specification for each one of them to the supplier. Similarly, for every business process which is a part of modern electronic or web enabled supply chain, delivery time of product is an important quality characteristic and end customers release specification for that too. Just like the variability in the dimension of the parts is an inherent feature of a machining process, the variability in the lead time is coherent to almost all the business processes. Therefore, it will be apt to apply the notion of process capability indices to measure the delivery capability or delivery quality of any business process [22]. To get a feel for delivery capability or delivery quality of a business process, let us consider an example of a logistics process. Posit in a supply chain goods are transferred from location P to location Q through trucks. The transportation time is random in nature which is assumed to be normally distributed with mean $\mu = 48$ hrs. and standard deviation 2 hrs. Let us assume that it is required that goods should not reach earlier than 36 hrs. due to limited storage space. Also, goods should reach within 60 hrs. after dispatching from A, otherwise potential customers will be lost at B. Ideal time suggested for arrival of goods at location B is 48 hrs. after loading the truck from location A. First we analyze this model and then change some of the parameters and see how the quality of the logistic process is going to be affected.

Case 1. Baseline Model
Following are the known parameters for the given logistics process.

$$\begin{aligned} \mu &= 48 \\ \sigma &= 2 \end{aligned} \qquad \begin{aligned} \tau = 48 \\ U = 60 \\ L = 36 \end{aligned}$$

For this model, process capability indices and yields are as follows.

$$\begin{aligned} C_p &= 2 \\ C_{pk} &= 2 \\ C_{pm} &= 2 \end{aligned} \qquad \begin{aligned} \text{Potential} &= 1 - 1.97317 \times 10^{-9} \\ \text{Actual Yield} &= 1 - 1.97317 \times 10^{-9} \end{aligned}$$

The above measurements of the process capability imply that the probability of shipping the goods within delivery window is approximately one which means that chances of defective deliveries are almost negligible. In the following model we change system parameters of the logistics process and show that index C_p alone is not sufficient to measure the quality of deliveries.

Case 2. C_p is not sufficient
Consider an alternative logistics service to satisfy above logistics requirement. The mean transportation time for this service is a bit higher, i.e. 51 hrs. but the standard deviation is the same i.e. 2 hrs. For this alternative logistics service, capability indices and yields take the following values.

$$\begin{aligned} C_p &= 2 \\ C_{pk} &= 1.5 \\ C_{pm} &= 1.1094 \end{aligned} \qquad \begin{aligned} \text{Potential} &= 1 - 1.97317 \times 10^{-9} \\ \text{Actual Yield} &= 1 - 3.39767 \times 10^{-6} \end{aligned}$$

It is easy to see that the probability of defective deliveries in this service is higher than in the previous case but C_p is the same for both. Therefore, C_p is not sufficient to measure the quality of deliveries. Index C_{pk} is also required along with this. Knowing both C_p and C_{pk}, we can compute the probability of defective delivery. However, the next model shows that even C_p and C_{pk} put together are also not sufficient. Index C_{pm} is needed to measure the quality of delivery process in a complete sense.

Case 3. The pair (C_p, C_{pk}) is also not sufficient
Let us compare two alternative logistics services A and B. The logistics requirement is the same as above but the mean and variance of two alternative services are as follows.

$$\mu_A = 48 \text{ hrs} \qquad \mu_B = 46.4541 \text{ hrs}$$
$$\sigma_A = 2.5833 \text{ hrs} \qquad \sigma_B = 2.3231 \text{ hrs}$$

For these two alternative systems process capability indices and yields come out to be as follows.

$$\begin{aligned} C_{p_A} &= 1.54835 & C_{p_B} &= 1.721814 \\ C_{pk_A} &= 1.54835 & C_{pk_B} &= 1.50001 \\ C_{pm_A} &= 1.54835 & C_{pm_B} &= 1.43346 \\ \text{Potential} &= 1 - 3.39994 \times 10^{-6} & \text{Potential} &= 1 - 2.39871 \times 10^{-7} \\ \text{Actual Yield} &= 1 - 3.39994 \times 10^{-6} & \text{Actual Yield} &= 1 - 3.39994 \times 10^{-6} \end{aligned}$$

In these two alternative systems, even though pair (C_p, C_{pk}) is different but chances of defective deliveries are still the same. Therefore, the pair (C_p, C_{pk}) which completely decides the probability of defective deliveries, is not sufficient in to completely quantify the quality of any process. Here C_{pm} is different for the two logistics service providers. Logistics service A is better than logistics service B because in service A most of the deliveries are close to target, i.e. 48 hrs.

The above example suggests that the 3-tuple (C_p, C_{pk}, C_{pm}) is sufficient to measure the delivery quality of any business process in a given supply chain. In next section, we use these three indices to define two metrics *delivery probability* and *delivery sharpness* for the purpose of measuring the delivery performance of the supply chain.

4 Delivery Probability and Delivery Sharpness

Note that a unique (C_p, C_{pk}) pair results in a unique actual yield, therefore, the 3-tuple (C_p, C_{pk}, C_{pm}) can be substituted by the pair (Actual yield, C_{pm}) to measure the delivery quality. Being an indicator for precision and accuracy of the deliveries, we prefer to call actual yield of the process as *Delivery Probability (DP)* and C_{pm} as *Delivery Sharpness (DS)*. In the present paper, we use these two indices to measure the quality of any delivery process in a given supply chain.

Motivated by the Motorola six sigma (MSS) program, we prefer to express DP in terms of $\theta\sigma$ levels, where $\theta \in \Re^+$, rather than expressing it in terms of numerical values. In the MSS program, each $\theta\sigma$ level corresponds to a unique number in the interval $[0, 1]$ and these numbers actually corresponds to upper bounds on the yield of the process. However, here we are assuming that these numbers correspond to the actual yield of the process. For example, according to the MSS program, in the presence of process mean shifts and drifts, if upper bound on yield of the process is equal to $1 - 3.4 \times 10^{-6}$ then its quality is 6σ quality. In the framework of DP and DS, we call a process DP is 6σ iff its actual yield is $1 - 3.4 \times 10^{-6}$. Moreover, in the framework of DP and DS no shifts and drifts are allowed in process mean, only bias is allowed between process mean and target value.

4.1 Motorola Six Sigma Quality: A Generalized View

The six sigma concept was proposed by Harry [7, 8] in 1987, at Motorola Inc. The core concept behind the MSS program is following: a unique σ level is attached with each value of *number of defects per million opportunities (npmo)*. The npmo is the probability, expressed on a scale of 10^{-6}, that a part is produced with quality characteristic X lying outside the specification limits. It is assumed in the MSS program that X is normally distributed and the target value τ is the midpoint of upper specification limit (U) and lower specification limit (L). It is common in all manufacturing processes that as the machine tool begins to wear and other independent variables such as room temperature, material hardness etc. come into play, μ begins to drift away from the nominal value of engineering specification. This shifting and drifting is captured in the Motorola six sigma model by assuming a one sided shift of 1.5σ in the process mean. For simplicity, the model also assumes that the variance in process mean is zero. Under these settings, the following scheme is recommends in MSS program for attaching the σ level with a given value of npmo. If U and L coincide with $(\tau + \sigma)$ and $(\tau - \sigma)$ respectively, which is different than $(\mu + \sigma)$ and $(\mu - \sigma)$ (see Figure 5), then corresponding *upper bound on yield* is assigned as 1σ level. Similar is the case with 2σ, 3σ, and others. Note that for the purpose of attaching the value with σ levels, MSS program uses upper bound on yield instead of actual yield. This is primarily because for normal distribution the tail area is too small and there is no appreciable difference between the actual yield and upper bound for 6σ and higher quality levels. However, when we say that our goal is to achieve $\theta\sigma$ quality, it seems more realistic and logical to have some target value for actual yield instead for upper bound. Therefore, we define $\theta\sigma$ quality as the actual yield equal to upper bound given by MSS program for the same quality level. For example, a process achieves 6σ quality iff actual yield of the process becomes $(1 - 3.4 \times 10^{-6})$ (not the upper bound as per Motorola 6σ program). We call DP of the process is 6σ iff actual yield of the process is $(1 - 3.4 \times 10^{-6})$ which is the upper bound for 6σ quality according to MSS program.

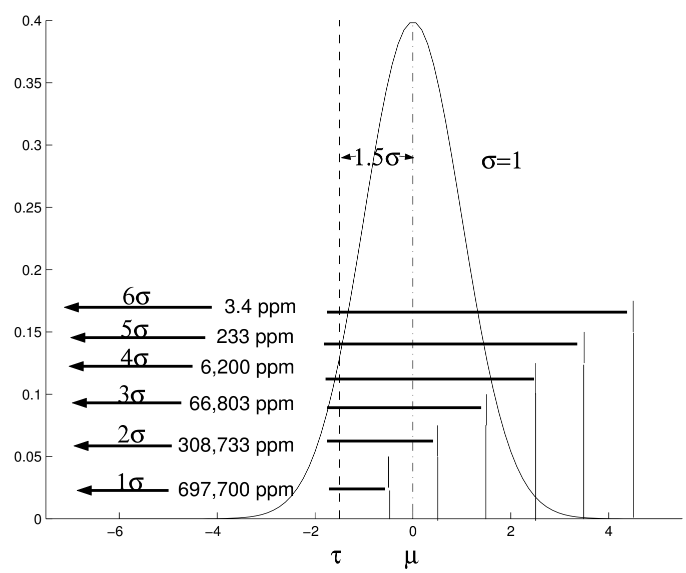

Figure 5: MSS quality in the presence of shifts and drifts in process mean

Recall that for a given (C_p, C_{pk}) pair, the value of actual yield is fixed. But for a given actual yield value, there exist infinite such (C_p, C_{pk}) pairs. Hence DP can be completely determined by knowing C_p and C_{pk}. However, there are numerous (in fact, infinitely many) ways in which we can choose the pair (C_p, C_{pk}) to achieve a given value of DP. This leads to a generalized view of six sigma quality. MSS is a special case of this in which bias is fixed i.e. 1.5σ. In order to explain this idea let us start with the equation:

$$\text{actual yield} = \Phi\left(3C_{pk}\right) + \Phi\left(6C_p - 3C_{pk}\right) - 1$$

If we fix the value of actual yield as α in the above equation then there will be two independent variables C_p, C_{pk}, hence the solution set will be unbounded. But we have earlier shown that for a given actual yield α, C_p and C_{pk} are bounded within certain range. Hence the solution is bounded by $\underline{C_{pk}^{\alpha}} \leq C_{pk} \leq \overline{C_{pk}^{\alpha}}$; $\underline{C_p^{\alpha}} \leq C_p \leq \infty$. If we substitute $\alpha = (1 - 3.4 \times 10^{-6})$ and plot a graph, then all points lying on the curve give those (C_p, C_{pk}) pairs that correspond to the 6σ quality level. This equation can be generalized for any $\theta\sigma$ level by expressing α in terms of θ. It is easy to see from Figure 5 that the upper bound in the MSS program for $\theta\sigma$ level is $\Phi(\theta - 1.5)$. Equating this to the actual yield of the process we get the following equation for $\theta\sigma$ quality curve on the $C_p - C_{pk}$ plane. Some of these curves are plotted in Figure 6.

$$\Phi(\theta - 1.5) = \Phi\left(3C_{pk}\right) + \Phi\left(6C_p - 3C_{pk}\right) - 1$$

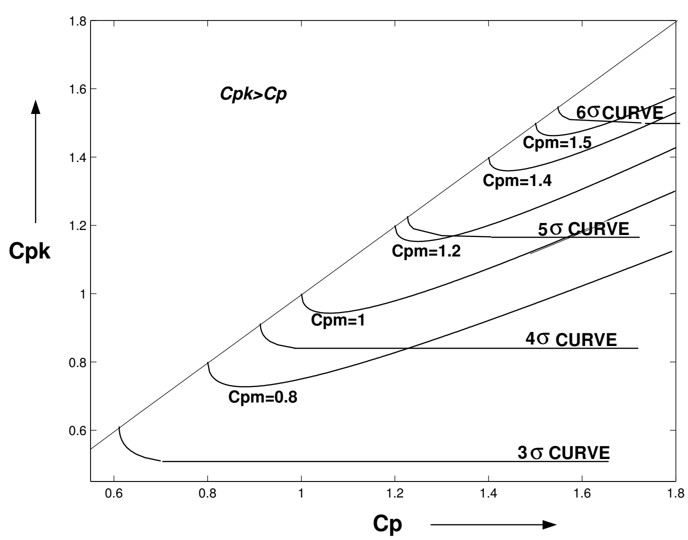

Figure 6: $\theta\sigma$ curves and C_{pm} curves on $C_p - C_{pk}$ plane

We can proceed one step further by looking at the connection between delivery probability and delivery sharpness in the light of our generalized notion of six sigma quality. For this, we consider the plots of σ quality levels on $C_{pk} - C_p$ plane and then see how C_{pm} behaves on the same plot. For this purpose we use the identity relation (6) among C_p, C_{pk}, and C_{pm} and plot this relation for a constant value of C_{pm} (say C_{pm}^*). The plot comes out to be a section of a hyperbola. From a process design point of view, it can be said that for a desired minimum level of DS (i.e. C_{pm}) and DP (i.e. C_p, C_{pk}), this curve provides a set of 3-tuples (C_p, C_{pk}, C_{pm}) which all satisfy these two requirements. The designer has to decide which one of the triples to choose depending upon the requirements. Figure 6 shows some C_{pm} curves on the $C_p - C_{pk}$ plane.

5 Variance Pool Allocation Problem

Using the paraphernalia developed in the previous section, we can formulate mathematical programming problems to describe design optimization and tactical decision making in supply chain networks. We now set out to define a particular design optimization problem, which we call the variance pool allocation problem. For the sake of simplicity, we consider make-to-order supply chains where the flow of materials is linear. The methodology can be easily extended (with extra computational requirements) to more general supply chains.

5.1 Description of the Problem

Consider a linear, make-to-order (MTO) supply chain with n stages as shown in Figure 7. This supply

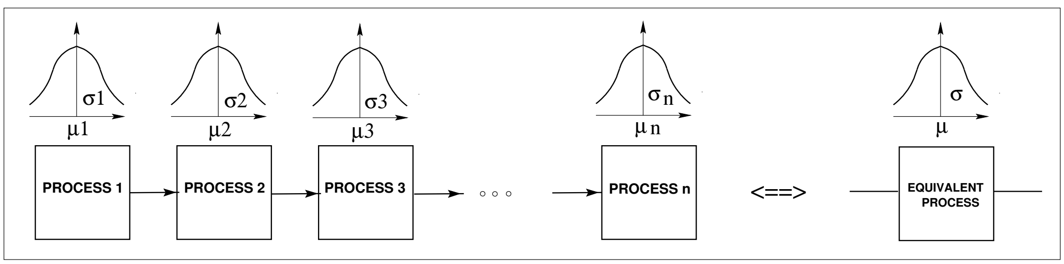

Figure 7: A linear supply chain model

chain is a single product supply chain. The product is delivered to the end customer from stage n. In the present model we are not concerned about how the orders are consolidated, how the production planning is done, and at which intermediate stages finished goods or semi-finished goods inventory will be maintained. Let us assume that as soon as any customer places an order for a unit of the product, the flow of material against the order starts from stage 1, undergoes processing at successive stages, and is finally delivered to the customer after processing at stage n. Let the lead time at each stage be considered as a continuous random variable (i.e. X_i, $i = 1, 2, \ldots, n$). As a consequence of this assumption, the end- to-end lead time is also a continuous random variable. Note that, in Figure 7, we have shown the probability density function of random variable X_i just above the corresponding process i ($i = 1, 2, \ldots, n$).

The first objective of the study here is to find out how the lead time variance of individual stages should be chosen, assuming that the mean lead time is given for each stage, such that the specified levels of DP and DS are attained for a given end-to-end lead time delivery window in a cost effective way. This we call as variance pool allocation (VPA) problem.

Depending on the nature of the objective function chosen, the solution of the VPA problem can be used in a wide variety of tactical decision making in supply chains. Typical such problems include: due date setting, choice of customers, inventory allocation, vendor selection, choice of logistics modes, choice of logistics providers, and choice of manufacturing control policies. The second objective of our study here is to explain one such compelling application of the VPA problem. As an example, we show

how the solution of the VPA can be used in choosing the best possible mix of alternatives for supply chain operation. This we call as supply chain partner selection problem. In what follows is a crisp idea behind how one can use the solution of the VPA problem for solving the supply chain partner selection problem.

Let us assume that each stage of the supply chain is a work process, e.g. transportation, machining, procurement, etc., then it is realistic to assume that in general there are number of service providers available for each individual stage. For example, there might be several logistics providers in the fray and each one of them can ship goods from one place to another. Typically, mean and variance of the shipment time vary from one candidate service provider to another and so is the freight charges. As a special case, assume that all candidate service providers promise for the same mean shipment time but quote the different variabilities. These variances may be available through past history also. If we thus know the pairs of cost and variance for each candidate service provider, the pairs can be used to fit a polynomial cost function in terms of the variance of shipping time. Such functions can be obtained for each stage of the supply chain. Knowing all these cost functions, the minimization of total cost of the supply chain will result in optimal values of variance for each stage. These optimal values can be used to pick up at most two service providers for each stage whose variance is closest to the corresponding optimal value. Thus the size of the problem of selecting optimal mix of service providers along the supply chain can be reduced to a case where we need to consider at most two candidates per stage. The best mix can now be easily obtained (for example by exhaustive enumeration). The best mix obtained will be optimal in the sense that it results in desired levels of DP and DS on the end-to-end delivery process with minimum possible total coat.

5.2 Assumptions

The model for VPA, proposed in this paper, is based upon the following assumptions about the nature of the business process and the customer delivery window:

1. Lead time X_i at stage i ($i = 1, \ldots, n$) is normally distributed, say, with mean μ_i and standard deviation σ_i. This assumption is quite standard and has received widespread justification in the literature through practical experimentation (see for example [7]). This assumption also has a theoretical basis through the central limit theorem.

2. Lead times X_i are mutually independent. This is a valid assumption since the lead times correspond to essentially independent subsystems or subprocesses.

3. There is no time elapsed between end of process i to commencement of process $i+1$ ($i = 1, \ldots, n-1$). This is a reasonable assumption since any interface time can be absorbed into the lead time of stage i or lead time of stage $i+1$. As an immediate consequence of this assumption, the end-to-end lead time, Y, is equal to the sum of lead times of the individual processes.

$$Y = \sum_{i=1}^{n} X_i \tag{9}$$

   Y can be easily seen to be normally distributed with $\mu = \sum_{i=1}^{n} \mu_i$ and $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$ since it is the sum of n independent normally distributed random variables.

4. Each customer specifies a customer delivery window (τ, T) where τ is the desirable amount of the time a customer is willing to wait after placing the order. The customer is prepared to wait

for a maximum period of $\tau + T$. Also, the customer does not want the delivery to occur before $\tau - T$. Therefore, τ is the target value for end-to-end delivery process and T is the tolerance.

5.3 Formulation of the VPA Problem

Essentially the VPA problem is a mathematical programming problem, hence it can be defined very well in the form of known parameters, decision variables, objective function, and constraints.

5.3.1 Known Parameters

The following parameters are known in a typical VPA problem.

1. End customer delivery window (τ, T).

2. Mean μ_i of random variable X_i, $i = 1, 2, \ldots, n$.

3. Delivery Probability and Delivery Sharpness for end-to-end lead time, Y.

4. Processing cost per unit product at each stage i, denoted by $\mathcal{K}_i$, of the supply chain. This cost is the part of the total processing cost that is associated with lead time. For example, in the case of a manufacturing process, it could be the opportunity cost of capital tied up with machinery. Similarly if it is the logistics process, it may represent the cost of transportation itself. As we showed earlier that for a given stage i, the mean processing time μ_i is almost same for all the potential service providers but their variance may differ and hence per unit processing cost may also vary. Assume that for each stage i, the processing time variances $\sigma_{iA}, \sigma_{iB}, \ldots$ and per unit processing costs $C_{iA}, C_{iB}, \ldots$ are known for all the service providers $A, B, \ldots$ of that stage. The pairs $(\sigma_{iA}, C_{iA}), (\sigma_{iB}, C_{iB}), \ldots$ can be used to get a polynomial function for per unit processing cost in terms of σ_i. For the sake of conceptual and computational simplicity we are motivated behind choosing a second order polynomial in the following manner:

$$\mathcal{K}_i = A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2 \qquad (10)$$

   Here A_{i0}, A_{i1}, A_{i2} are constants and not all are positive. These constants will be obtained by polynomial curve fitting for the pairs $(\sigma_{iA}, C_{iA}), (\sigma_{iB}, C_{iB}), \ldots$. Thus, at an abstract level we can safely assume that these constants are given to us for each stage of the supply chain.

   Here we would like to make an important remark. In most of the practical cases, getting the values for the constants A_{i0}, A_{i1}, A_{i2} is far more difficult. First of all shipment time of a given service provider may not be normal in nature and on top of that neither the shipper nor the carrier have record of past data of shipment time which makes it extremely hard to compute the variability in shipment time.

5.3.2 Decision Variables

The decision variables of the VPA problem are optimal standard deviations σ_i^* of each individual stage i ($i = 1, \ldots, n$). As we mentioned the scheme earlier, these optimal standard deviations σ_i^* can be used to compute the optimal partners (or service provider) $P_1^*, P_2^*, \ldots, P_n^*$ for each stage of the supply chain.

5.3.3 Objective Function and Constraints

As stated already, the objective in the VPA problem is to minimize the cost and the constraints are specified in terms of minimum expected levels of DP and DS on end-to-end lead time.

In the present model, we have confined our discussion only to lead time variability of the supply chain without considering other issues like demand variability, inventory levels, etc. Therefore, it seems to be reasonable to consider the total processing cost of a single unit of product, denoted by $\mathcal{K}$, as the objective function. This cost is simply the sum of processing costs of all the stages. Thus the problem formulation becomes:

Minimize

$$\mathcal{K} = \sum_{i=1}^{n} \mathcal{K}_i = \sum_{i=1}^{n} \left(A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2 \right) \tag{11}$$

subject to

$$\text{DS for end-to-end lead time} \quad \geq \quad C_{pm}^* \tag{12}$$
$$\text{DP for end-to-end lead time} \quad \geq \quad 6\sigma \tag{13}$$
$$\sigma_i \quad > \quad 0 \; \forall i \tag{14}$$

5.4 A 5-Step Solution Approach

In this section, we present a 5-Step procedure for solving the partner selection problem. First four steps constitute the solution of the VPA problem and fifth step explains how this solution can be used to solve the partner selection problem.

5.4.1 Step 1: Problem Formulation

The first step in solving the VPA problem is to collect all the known parameters specified in the problem and then formulate the problem in terms of a non-linear optimization problem as presented in Section 5.3.3. This includes:

- Extracting the value of μ_i, τ, and T.
- Extracting the desired level of DP and DS for end to end lead time
- Obtaining the pairs $(\sigma_{iA}, C_{iA}), (\sigma_{iB}, C_{iB}), \ldots$ for each stage i of the chain and then fitting it to get second order polynomial $\mathcal{K}_i$.

5.4.2 Step 2: Expressing the Constraints in terms of Decision Variables

Note that the first two constraints in the optimization problem, formulated in Step 1, are not being expressed in terms of decision variables. This step does the job of expressing the constraints in terms of decision variables.

Recall the following expression from Section 5.2:

$$\sigma^2 \quad = \quad \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2 \tag{15}$$

where σ is the standard deviation for end-to-end lead time. This can be expressed in terms of C_p^* and C_{pk}^* of end-to-end lead time Y in following manner:

$$\sigma^2 = \frac{T^2}{9C_p^{*2}} = \frac{d^2}{9{C_{pk}^*}^2} \tag{16}$$

where T, the tolerance of end customer delivery window, is a known parameter, and d, given by $\min(\tau + T - \mu, \mu - \tau + T)$, is also a known parameter. The only unknown quantities in Equation (16) are C_p^* and ${C_{pk}}^*$. Substituting Equation (16) in Equation (15),we get the following important relation.

$$\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2 = \frac{T^2}{9C_p^{*2}} = \frac{d^2}{9{C_{pk}^*}^2} \tag{17}$$

The following is an important observation derived out of the Equation (17).

"Once the pair (C_p^*, C_{pk}^*) is fixed for the end-to-end lead time Y, the feasible solution set gets automatically fixed as the set of all those n-tuples $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ which satisfy this equation for the chosen value of C_p^* and C_{pk}^*. "

The idea behind obtaining the pair (C_p^*, C_{pk}^*) is to choose such a pair which satisfies both Constraint (12) as well as Constraint (13). By using such a pair in Equation (17), we can get a single constraint, in terms of the decision variables, which captures both the constraints. In this way we express the constraints in terms of decision variables.

It is quite possible that multiple pairs satisfy the above requirement. In such a situation, selection of the best pair is an interesting exercise which we discuss in the next step. Moreover, there are situations when no pair satisfies the requirement. In such a situation, the VPA problem does not have any solution. Identifying such types of situations is also considered in the next section.

5.4.3 Step 3: Value Determination for C_p^* and C_{pk}^*

Note that the relation (17) forces the desired (C_p^*, C_{pk}^*) pair to lie on the line $C_{pk} = \frac{d}{T} C_p$ in the $C_p - C_{pk}$ plane. Also, it is easy to see that the Constraint (12) forces the desired pair to lie on or above the curve $C_{pm} = C_{pm}^*$ in the $C_p - C_{pk}$ plane. Similarly, the Constraint (13) forces it to lie on or above the 6σ curve in the same plane. All these result in a feasible region in the $C_p - C_{pk}$ plane. Depending on relative position of $C_{pm} = C_{pm}^*$ curve (call this C_{pm} curve for short) and 6σ curve (call this σ curve for short), the feasible region may take different shapes. Figure 8 shows the geometric shapes of such a feasible region. For the purpose of analysis, we classify these geometric shapes into 5 different cases where we discriminate them based on the number of points at which the two curves (σ curve and C_{pm} curve) intersect each other. It is clear from Figure 8 that the feasible region in each case is the part of the line $C_{pk} = \frac{d}{T} C_p$, denoted by OP, which intersects the shaded region. For the sake of clarity, we have shown the line OP only in Case 1. In all other cases it is understood. Each point of the feasible region satisfies both Constraints (12) and (13) and therefore can be used as a design point in Equation (17). The concern here is which point should be selected as the design point. Before we investigate further in this direction, let us consider a few interesting facts about such a (C_p^*, C_{pk}^*) pair. In what follows is two lemmas that describes a few facts about the pair (C_p^*, C_{pk}^*).

Lemma 5.4.1

For the given values of T and d, there is an upper bound on Delivery Sharpness (DS) which can be

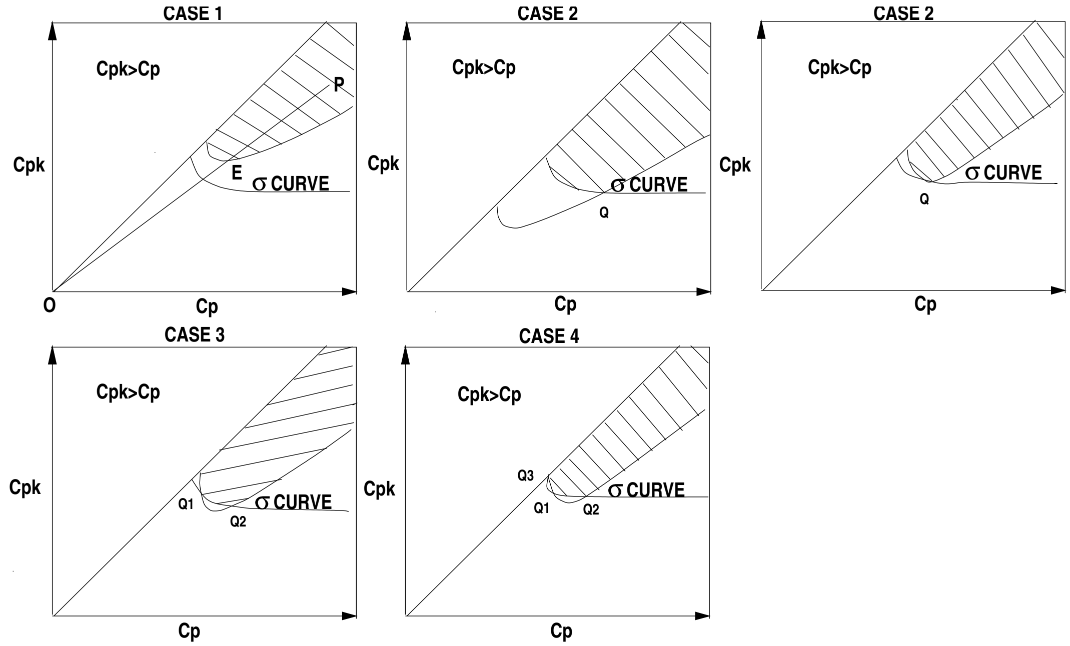

Figure 8: Possible geometric shapes of feasible region for C_p and C_{pk} of Y

achieved for Y. This is given by

$$\overline{C_{pm}} = \frac{T}{3\,(T-d\,)}$$

Proof: Observe from Equation (17) that, for a given values of T and d, C_p and C_{pk} of the process Y must satisfy the following relation which is a straight line when plotted on the $C_p - C_{pk}$ plane.

$$C_{pk} = \left(\frac{d}{T}\right) C_p \tag{18}$$

If we take any point on this line, it represents a unique combination of C_p, C_{pk}, and C_{pm}. Hence if we choose this point as design point then the DS for Y gets fixed. Now consider the following equation for a typical C_{pm} curve on the $C_p - C_{pk}$ plane.

$$\frac{1}{{C_{pm}}^2} = \frac{1}{C_p^2} + 9\left(1 - \frac{C_{pk}}{C_p}\right)^2 \tag{19}$$

It can be verified that this equation represents a hyperbola. It is quite possible that the line given by Equation (18) becomes an asymptote of such a hyperbola. Such a hyperbola is the plot of $\overline{C_{pm}}$ because it is clear from the geometry of the figure that this line cannot intersect any other C_{pm} curve which is more than $\overline{C_{pm}}$. Hence it is not possible to achieve the C_{pm} value (or DS) higher than $\overline{C_{pm}}$ for process Y.

It is easy to show that the slope of asymptotes of $\overline{C_{pm}}$ curve is $\left(1 \pm \frac{1}{(3\overline{C_{pm}})}\right)$. Equating these to the slope of the line (18) we get

$$\overline{C_{pm}} = \frac{T}{3(T-d)}$$

. ■

Lemma 5.4.2
For a fixed values of T and d, DP and DS have one-to-one correspondence with each other. Moreover, DP and DS have positive correlation.

Proof: Earlier we stated that (C_p^*, C_{pk}^*) pair is chosen for Y in such a way that apart from satisfying both the Constraints (12) and (13), the pair must lie on the line (18). It is easy to verify that for a given point on the line (18), a unique C_{pm} curve and a unique σ curve pass through the point. This σ values and C_{pm} values are the final DP and DS respectively which are achieved for Y if this particular point is chosen as design point. Hence, it can be concluded that once a value is chosen for DP of Y, it will automatically decide the corresponding value of DS and also vice versa. To prove the other statement of the lemma, observe that as we move from point $C_p = 0$ to point $C_p = \infty$ on the line (18), the values of both C_{pm} curve and σ curve which pass through that point increase. Therefore, DP increases (or decreases) as DS increases (or decreases) for given values of T and d. ■

The implication of Lemma 5.4.1 is following. If the desired C_{pm}^* is greater than $\overline{C_{pm}}$ for the given values of T and d then the problem is infeasible. In such a situation we need not proceed any further.

Lemma 5.4.2 also has a key implication on the problem of fixing the values of C_p^* and C_{pk}^* for Y. According to Lemma 5.4.2, DP and DS of Y get fixed immediately as soon as a feasible point from the line, given by Equation (18), is chosen as design point. It is easy to see that each point on the $C_p - C_{pk}$ plane is unique in its own because it has a unique combination of DP and DS. Therefore, it is quite possible the point which we have chosen results in either higher DP or higher DS than required for the end-to-end delivery process. Thus it is not always true that the DP and DS for Y obtained from the design will exactly be the same as specified in the Constraints (12) and (13).

In view of the above findings, the problem of fixing the values of C_p^* and C_{pk}^* can be addressed in following way. First step is to test the feasibility of the problem by means of Lemma 5.4.1. If the problem turns out to be feasible then each point in the feasible region is eligible to be selected as a design point. However, depending upon the point which is chosen as design point, the final cost $\mathcal{K}^*$ (which we get out of solving the optimization problem) may vary. At this point, we cannot say which feasible point will result in minimum cost. Hence, the problem is handled in an indirect manner. The proposed scheme is like this. First solve the optimization problem without any constraint and get the optimal variance σ^g for Y. It will result in global minimum cost. Now use this variance σ^g to get C_p^g and C_{pk}^g for Y which result in minimum cost. If the point (C_p^g, C_{pk}^g) falls in the feasible region then this point is used as a design point (C_p^*, C_{pk}^*), otherwise the point E where the line OP enters into the shaded region is taken as the desired (C_p^*, C_{pk}^*) pair. The reason behind choosing the point E as design point is following. The values DP and DS which result from point E are minimum possible values satisfying both the Constraints (12) and (13). If we choose any other feasible point then even though the resulting DP and DS for Y will satisfy the Constraints (12) and (13), yet their values will be a bit high and this will lead to higher cost. The point E can be determined by solving the corresponding equation of the plots. A detailed algorithm for determining the point E is discussed in [37]. In this way we obtain the constraints in terms of decision variables.

5.4.4 Step 4: Solving the Optimization Problem

The optimization problem which we presented in Step 1 can now be written as:

Minimize

$$\mathcal{K} = \sum_{i=1}^{n} \mathcal{K}_i = \sum_{i=1}^{n} \left(A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2 \right) \tag{20}$$

subject to

$$\sum_{i=1}^{n} \sigma_i^2 = \frac{T^2}{9{C_p^*}^2} = \frac{d^2}{9{C_{pk}^*}^2} \tag{21}$$

$$\sigma_i > 0 \; \forall i \tag{22}$$

This is a nonlinear optimization problem with an equality constraint. Therefore, we can use the Lagrange multiplier method to compute the stationary points. After getting the stationary points, we can apply the sufficiency condition to determine the points of minima. Here we just present the formulation for finding the stationary points and leave the remaining part to be covered in a detailed example.

Method of Lagrange Multipliers Applied to the VPA Problem

1. Lagrange Function

The Lagrange function $L(\sigma_1, \sigma_2, \ldots, \sigma_n, \lambda)$ is given by:

$$L(\sigma_1, \sigma_2, \ldots, \sigma_n, \lambda) = \mathcal{K} + \lambda \left(\sum_{i=1}^{n} {\sigma_i}^2 - {\sigma^*}^2 \right) \tag{23}$$

where ${\sigma^*}^2 = \frac{T^2}{9{C_p^*}^2} = \frac{d^2}{9{C_{pk}^*}^2}$.

2. Necessary Condition for Stationary Points

If the point $\mathcal{P}^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_n^*, \lambda^*)$ happens to be the optimal point, then this point must satisfy the following necessary conditions.

$$\left.\frac{\partial L}{\partial \sigma_1}\right|_{\mathcal{P}^*} = \left.\frac{\partial L}{\partial \sigma_2}\right|_{\mathcal{P}^*} = \ldots = \left.\frac{\partial L}{\partial \sigma_n}\right|_{\mathcal{P}^*} = \left.\frac{\partial L}{\partial \lambda}\right|_{\mathcal{P}^*} = 0$$

Substituting the value of Lagrange function L from Equation (23), we get the following necessary conditions.

$$\begin{aligned}
\left.\frac{\partial L}{\partial \sigma_1}\right|_{\mathcal{P}^*} &= A_{11} + 2A_{12}\sigma_1^* + 2\lambda^*\sigma_1^* = 0 \\
\left.\frac{\partial L}{\partial \sigma_2}\right|_{\mathcal{P}^*} &= A_{21} + 2A_{22}\sigma_2^* + 2\lambda^*\sigma_2^* = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\left.\frac{\partial L}{\partial \sigma_n}\right|_{\mathcal{P}^*} &= A_{n1} + 2A_{n2}\sigma_n^* + 2\lambda^*\sigma_n^* = 0 \\
\left.\frac{\partial L}{\lambda}\right|_{\mathcal{P}^*} &= \sum_{i=1}^{n} \left(\sigma_i^{*2} \right) - \sigma^{*2} = 0
\end{aligned}$$

Solving the above system of equations will give us all the stationary points. After discarding those stationary points that do not satisfy Constraint (22), we will be left with the stationary points that satisfy both the constraints. It is required now to apply the sufficiency condition in order to find out the points of minima. We omit this for the sake of brevity.

5.4.5 Step 5: Optimal Partner Selection Algorithm

Without loss of generality, let there be n stages in the supply chain and at each individual stage, there are several alternatives available (for example, if it is a logistics stage, we have several logistics providers). Each of these alternatives has a standard deviation associated with the promised lead time. Now the problem is to choose one service provider for each stage out of the given alternatives. It is not difficult to see that each combination of the partners will result in a unique DP, DS, and delivery cost for the end-to-end delivery process. We wish to choose the combination which meets the given standards for DP and DS in a minimum possible cost. Thus the problem is highly combinatorial in nature. The brute force technique of finding the solution involves computing the DP, DS, and end-to-end delivery cost for each combination and then picking up the optimal one. Note that complexity of such a technique would be of the order of $O(N_1 \times N_2 \times \ldots \times N_n \times \xi)$, where $N_i; i = 1, 2, \ldots, n$ is the number of alternative service providers for the stage i and ξ is the complexity involved in computing the DP, DS, and end-to-end delivery cost for one combination of the partners which can shown to be the constant. Thus, the exponential complexity of the brute force technique motivates us to investigate for a better algorithm. Here, we present an efficient algorithm for solving the same problem whose complexity turns out to be of the order of $O(2^n \xi + \psi)$, where ψ is the complexity involved in solving one instance of the VPA problem and that can be easily shown to be linear in terms of number of stages. The idea behind this algorithm is following.

First execute the Step 1 through 4 on the underlying VPA problem and get $\sigma_1^*, \sigma_2^*, \ldots, \sigma_n^*$. Now consider stage i. Out of all the available service providers for stage i, choose two service providers L_i and R_i such that the variance of processing time when the work is done by these two are σ_i^l, and σ_i^r respectively and these values are immediate below and immediately above σ_i^*. Note that it may happen that σ_i^* is less than all candidate standard deviations, in which case, we choose the one that is closest to σ_i^* (this will reduce the computational complexity than when there are two candidates).

After fixing two alternatives L_i and R_i for each stage i, we will be left with 2^n ways in which we can choose a mix of alternatives for the supply chain. For each possible mix, we can compute total cost $\mathcal{K}$ by using relation (11). The σ for Y can be computed by using relation (15) where σ_i will now be replaced by either σ_i^l or σ_i^r depending upon which service provider we have chosen. This σ can be used to compute C_p and C_{pk} for end-to-end lead time Y through the relation (16). After computing these values it is easy to decide the optimal combinations of service providers all along the supply chain. Note that the worst case number of combinations that need to be considered is 2^n, independent of the number of alternatives available at each individual stage.

The two remarks regarding the above algorithm are in order.

- Note that $O(2^n \xi + \psi)$ is a significant improvement over $O(N_1 \times N_2 \times \ldots \times N_n \times \xi)$. It ensures that the complexity of the partner selection algorithm is dependent only on the number of stages and is independent of the number of service providers at each stage of the supply chain.

- It is easy to see that in the event a supply chain partner drops out or a new supply chain partner enters the fray, the proposed algorithm handles the changes with ease. In the case of new entry, the all we have to do is just compute DP, DS, and delivery cost for all the $2^{(n-1)}$ new combinations of the supply chain partners that will emerge due to this new entry. Now check if any one of this new mix does better than the current optimal mix. Similarly, if any partner drops out then we need to do nothing if the leaving partner is not a member of the current set of optimal partners. Otherwise, we can just delete all those combinations which include this as one of the partner and find out the optimal one out of the remaining $2^n - 2^{(n-1)}$ combinations.

6 A Plastic Industry Case Study

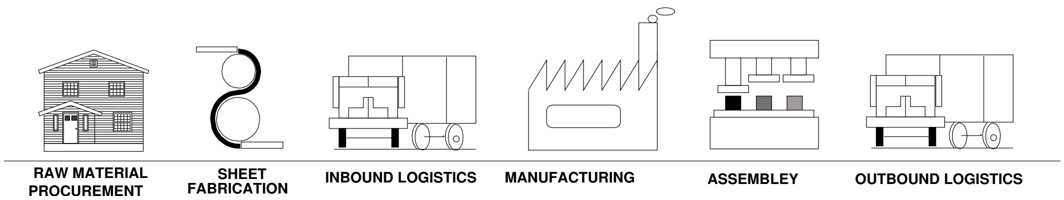

Figure 9: An example of a linear supply chain: A typical plastic industry supply chain

We now consider a supply chain for a plastics industry (a certain anonymous firm in the western state of Maharashtra, India) and apply the 5-Step approach for formulating and solving the partner selection problem. Figure 9 depicts the supply chain at an aggregate level. The supply chain has six business processes namely (1) Procurement, (2) Sheet Fabrication, (3) Inbound Logistics, (4) Manufacturing, (5) Assembly, and (6) Outbound Logistics. Let all the business processes in the supply chain satisfy the assumptions mentioned in Section 5.2. Assume for the sake of convenience that there are three alternatives (call them service providers) at each of the six stages.

The problem here is to determine the optimal mix of service providers for each stage such that the end -to end delivery probability is at least at 6σ level and delivery sharpness is at least, say, 1.4. Suppose for each stage, the mean lead time for all the three alternative service providers is same. Let the mean μ_i for $i = 1, 2, 3, 4, 5$ and 6 be 7 days, 30 days, 3 days, 30 days, 10 days, and 3 days respectively. Let the target value of supply chain lead time Y be 82 days and tolerance be 6.5 days. This implies:

$$\tau = 82 \text{ days}$$
$$T = 6.5 \text{ days}$$

The processing cost of one unit of product at each one of the six business processes, varies over the service providers as a function of variance of lead times. Table 1 gives the values of per unit processing cost and processing time variance for each service provider.

Stage	**Service Providers**					
i	A		B		C	
	σ_{Ai} (days)	C_{Ai} (\$/item)	σ_{Bi} (days)	C_{Bi} (\$/item)	σ_{Ci} (days)	C_{Ci} (\$/item)
1	0.50	256.18	0.75	161.44	1.00	125.69
2	0.40	436.40	0.50	413.03	0.60	390.37
3	0.10	077.79	0.20	049.33	0.30	034.81
4	0.40	436.40	0.50	413.03	0.60	390.37
5	0.50	185.79	1.00	080.34	1.50	065.27
6	0.10	077.79	0.20	049.33	0.30	034.81

Table 1: Standard deviation of lead times and cost for each service provider

This problem can be cast as a VPA problem which can be solved by using the 5-Step approach. The successive steps are discussed below.

6.1 Step 1

Some of the known parameters for VPA problem are provided explicitly in the given problem. The parameters which will be needed in further calculations and are implicit to the problem are μ, d, θ, C^*_{pm}, and A_{ij}. If we denote lead time distribution of Procurement, Sheet Fabrication, Inbound Logistics, Manufacturing, Assembly, and Outbound Logistics by X_1, X_2, X_3, X_4, X_5, and X_6 respectively, then it is easy to see that

$$\begin{aligned}
\mu &= \sum_{i=1}^{6} \mu_i = 83 \text{ days} \\
d &= \min(\tau + T - \mu, \mu - \tau + T) = 5.5 \text{ days} \\
\theta &= 6 \\
C^*_{pm} &= 1.4
\end{aligned}$$

The coefficients A_{ij} can be obtained by a second order polynomial curve fitting for the given three pairs (σ_{Ai} C_{Ai}), (σ_{Bi}, C_{Bi}), and (σ_{Ci}, C_{Ci}) for each stage i. The coefficients A_{ij} obtained by such an approximation are tabulated in Table 2.

Stage	A_{i_0} $\left(\frac{\$}{\text{item}}\right)$	A_{i_1} $\left(\frac{\$}{\text{item-day}}\right)$	A_{i_2} $\left(\frac{\$}{\text{item-day}^2}\right)$
Procurement	622.634	-968.872	471.928
Sheet Fabrication	537.011	-265.752	035.604
Inbound Logistics	120.186	-493.651	696.919
Manufacturing	537.011	-265.752	035.604
Assembly	381.625	-482.053	180.770
Outbound Logistics	120.186	-493.651	696.919

Table 2: Cost coefficients for plastic industry supply chain problem

Now the optimization problem can be formulated as follows:

Minimize

$$\mathcal{K} = \sum_{i=1}^{6} \mathcal{K}_i = \sum_{i=1}^{6} \left(A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2\right) \tag{24}$$

subject to

$$\begin{aligned}
\text{DS for end-to-end lead time} &\geq 1.4 \\
\text{DP for end-to-end lead time} &\geq 6\sigma \\
\sigma_i &> 0, \; i = 1, 2, \ldots, 6
\end{aligned}$$

6.2 Step 2

The constraints of the optimization problem presented in Step 1 can be expressed in terms of decision variables by invoking relation (17). This leads to:

$$\sum_{i=1}^{6} \sigma_i^2 = \frac{42.25}{9C_p^{*^2}} = \frac{30.25}{9C_{pk}^{*^2}} \quad (25)$$

$$\sigma_i > 0 \,\forall\, i = 1, 2, \ldots, 6 \quad (26)$$

6.3 Step 3

As discussed earlier, the first step towards fixing the values of C_p^* and C_{pk}^* is to test the feasibility of the problem. It is easy to see $\overline{C_{pm}}$ for this problem is 2.1667 which is greater than 1.4. Therefore, the problem is feasible. As a next step, we solve the corresponding unconstrained optimization problem and get the point (C_p^g, C_{pk}^g) which leads to global minimum cost and then test whether this point falls into feasible region or not.

For this, let $S = \{(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6) : \sigma_i \in \Re^+ \;\forall\, i = 1, 2, 3, 4, 5, 6\}$. It immediately follows from this definition of S that $\mathcal{K} : S \mapsto \Re$ where S is a nonempty open convex set. To test the convexity of function $\mathcal{K}$, we compute the gradient vector and Hessian matrix of the function. Note that the gradient vector $\nabla\mathcal{K}(\bar{\mathbf{X}})$ for function $\mathcal{K}$ at point $\bar{\mathbf{X}} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)^T$ can be given by

$$\nabla\mathcal{K}(\bar{\mathbf{X}}) = \begin{bmatrix} \frac{\partial\mathcal{K}(\bar{\mathbf{X}})}{\partial\sigma_1} \\ \frac{\partial\mathcal{K}(\bar{\mathbf{X}})}{\partial\sigma_2} \\ \frac{\partial\mathcal{K}(\bar{\mathbf{X}})}{\partial\sigma_3} \\ \frac{\partial\mathcal{K}(\bar{\mathbf{X}})}{\partial\sigma_4} \\ \frac{\partial\mathcal{K}(\bar{\mathbf{X}})}{\partial\sigma_5} \\ \frac{\partial\mathcal{K}(\bar{\mathbf{X}})}{\partial\sigma_6} \end{bmatrix} = \begin{bmatrix} A_{11} + 2A_{12}\sigma_1 \\ A_{21} + 2A_{22}\sigma_1 \\ A_{31} + 2A_{32}\sigma_1 \\ A_{41} + 2A_{42}\sigma_1 \\ A_{51} + 2A_{52}\sigma_1 \\ A_{61} + 2A_{62}\sigma_1 \end{bmatrix}$$

Also, the Hessian $H(\bar{\mathbf{X}})$ for function $\mathcal{K}$ at point $\bar{\mathbf{X}} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)^T$ turns out be

$$H(\bar{\mathbf{X}}) = \begin{bmatrix} 2A_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2A_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2A_{42} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2A_{42} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2A_{42} \end{bmatrix}$$

Observe that the gradient vector and Hessian exist for each $\bar{\mathbf{X}} \in S$. Hence function $\mathcal{K}$ is twice differentiable over open convex set S. Moreover, the Hessian is independent of $\bar{\mathbf{X}}$. Therefore, it is enough that we test the Positive Definiteness (PD) or Positive Semi Definiteness (PSD) of Hessian at any point of S instead of testing it over whole S.

It is easy to see that all the diagonal elements of Hessian are positive real numbers because A_{i2} are positive real numbers. Therefore, the Hessian is PD and the function $\mathcal{K}$ is strictly convex. It implies that a local minimum of function $\mathcal{K}$ is the unique global minimum. This can be obtained by equating $\nabla\mathcal{K}(\bar{\mathbf{X}})$ to 0. The optimal values of variance for the stages come out be $\sigma_1^g = 1.0265$ days, $\sigma_2^g = 3.732$ days, $\sigma_3^g = 0.3541$ days, $\sigma_4^g = 3.732$ days, $\sigma_5^g = 1.3333$ days and $\sigma_6^g = 0.3541$ days. The corresponding variance of end-to-end lead time Y comes out to be $\sigma^g = 5.5622$ days. Also $C_p^g = 0.3895$ and $C_{pk}^g = 0.3296$. In order to test the feasibility of this point (C_p^g, C_{pk}^g) first we compute the DP and DS values which will be obtained if this point is chosen as design point. According to Lemma 5.4.2, the C_{pm} curve and σ curve which pass through it are those desired DP and DS. These values comes out to be DP=2.17393σ and DS=0.383359.

Because these values are less than what are desired i.e. DP=6σ and DS=1.4, the point (C_p^g, C_{pk}^g) cannot be taken as design point and we will have to use point E (the point of intersection of the line OP and feasible region) as design point. The current problem falls in Case 2 (Subcase A) of Figure 8. Therefore point $E(C_p^*, C_{pk}^*)$ can be computed by solving the Equations (19) and (18). This point comes out to be:

$$
\begin{aligned}
C_p^* &= 1.834364282 \\
C_{pk}^* &= 1.552154393
\end{aligned}
$$

The DP and DS which are obtained for Y by using this point as design point are 6.15645σ and 1.4 respectively.

6.4 Step 4

Substituting the values of C_p^*, C_{pk}^* in Equation (25), we obtain the following constraint to work with while solving the optimization problem.

$$
\sum_{i=1}^{6} \sigma_i^2 = 1.389060165
$$

Now we will apply the Lagrange multiplier method to solve this optimization problem.

1. Lagrange Function

Lagrange function $L(\sigma_1, \sigma_2, \ldots, \sigma_6, \lambda)$ is given as:

$$
L(\sigma_1, \sigma_2, \ldots, \sigma_6, \lambda) = \mathcal{K} + \lambda \left(\sum_{i=1}^{6} \sigma_i^2 - 1.389060165 \right)
$$

2. Necessary Condition for Stationary Points

Let point $\mathcal{P}^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_6^*, \lambda^*)$ correspond to the optimal point, then this point must satisfy the following necessary conditions:

$$\begin{aligned}
-968.872 + 2(471.928)\sigma_1^* + 2\lambda^*\sigma_1^* &= 0 \\
-265.752 + 2(035.604)\sigma_2^* + 2\lambda^*\sigma_2^* &= 0 \\
-493.651 + 2(696.919)\sigma_3^* + 2\lambda^*\sigma_3^* &= 0 \\
-265.752 + 2(035.604)\sigma_4^* + 2\lambda^*\sigma_4^* &= 0 \\
-482.053 + 2(180.770)\sigma_5^* + 2\lambda^*\sigma_5^* &= 0 \\
-493.651 + 2(696.919)\sigma_6^* + 2\lambda^*\sigma_6^* &= 0 \\
\sigma_1^{*2} + \sigma_2^{*2} + \sigma_3^{*2} + \sigma_4^{*2} + \sigma_5^{*2} + \sigma_6^{*2} &= 1.389060165
\end{aligned}$$

Solving this system of equations by standard numerical methods we get the following solutions:

$$\begin{aligned}
\sigma_1^* &= 0.680498 \text{ days} \\
\sigma_2^* = \sigma_4^* &= 0.482201 \text{ days} \\
\sigma_3^* = \sigma_6^* &= 0.263456 \text{ days} \\
\sigma_5^* &= 0.572881 \text{ days}
\end{aligned}$$

Under this operating condition, the cost of delivery is:

$$\mathcal{K}^* = 802.299 \frac{\$}{\text{item}}$$

It can be verified easily by the sufficiency condition that this point indeed corresponds to the point of minima.

6.5 Step 5

By comparing the optimal standard deviations σ_i^* obtained in Step 4 with the given data in Table 1 we can compute, for each stage, the service providers whose variance is closest to the optimal. These are listed in Table 3. It is easy to see that we can construct 64 combinations out of these 12 service providers listed in Table 3, where each combination representing a particular mix of service providers. We have computed the end-to-end supply chain cost $\mathcal{K}$, process capability indices C_p and C_{pk}, DP, and DS for each of these 64 combination and the results are tabulated in Table 4. In this table, for each combination, rather than computing the exact sigma level for DP we have only specified whether or not DP is greater than 6σ level. If greater, we have indicated in the corresponding by 'Y', otherwise it is indicated by 'N' (for "no").

From Table 4, it can be easily seen that the combination which ensures the desired DP and DS level in minimum possible cost is the combination number 53 i.e. BBBBAB. Thus the optimal mix of service providers for stages 1,2,3,4,5, and 6 of the given problem are B,B,B,B,A, and B respectively.

Stage (i)	L_i	R_i
1	A	B
2	A	B
3	B	C
4	A	B
5	A	B
6	B	C

Table 3: Pairs of almost optimal service providers for each stage

Note that Table 4 can only have at most 64 entries. Often, if a service provider is already fixed for a particular stage, this number will be much less than 64. Also, note that whatever the number of candidate service providers at each stage, we will have to look at at most 64 combinations in this case because of the variance pool allocation that we have already done.

6.6 Some More Insights

A sensitivity analysis of the VPA problem is possible if we take different points on the line (18), which is $C_{pk} = \frac{6.5}{5.5}C_p$ in this case, and solve the optimization problem. We have solved this problem for a couple of points and tried to investigate the dependencies of optimal cost on DS. The plot is shown in Figure 10. Several inferences can be deduced from this plots.

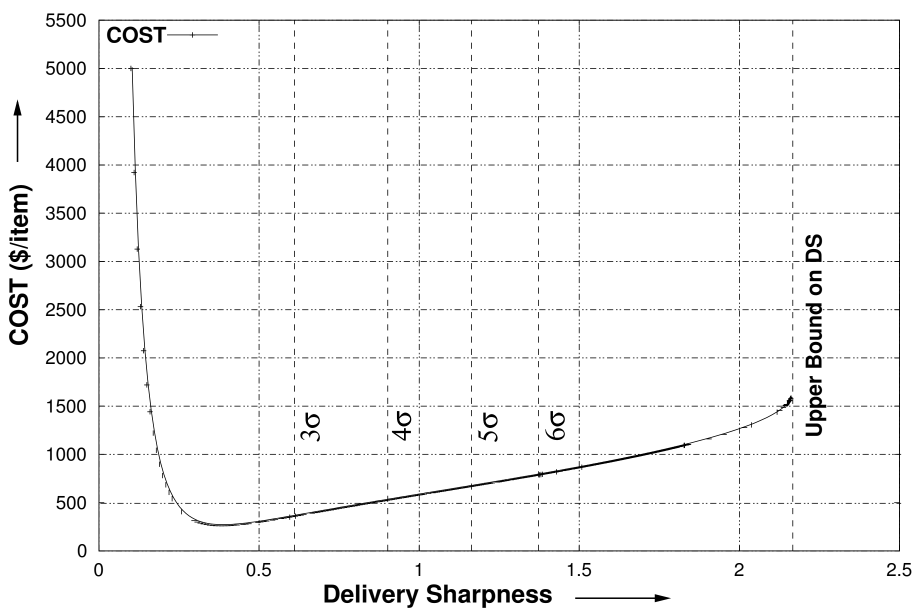

Figure 10: Effect of delivery quality on delivery cost

- A given DP results in a unique DS and also vice versa. Therefore, the DP and DS which are given as constraints in the problem may not be a valid pair and cannot be achieved as it is. However,

Combination		C_p	C_{pk}	DP (6σ level)	DS	$\mathcal{K}$ $/unit
1	AABAAB	2.283867	1.932503	N	1.571866	1413.429932
2	AABAAC	2.222953	1.880960	Y	1.551582	1398.910034
3	AABABB	1.686748	1.427248	Y	1.330972	1307.979980
4	AABABC	1.661757	1.406102	N	1.318591	1293.459961
5	AABBAB	2.177582	1.842569	N	1.535908	1390.059937
6	AABBAC	2.124591	1.797731	Y	1.516970	1375.540039
7	AABBBB	1.642546	1.389846	N	1.308930	1284.609985
8	AABBBC	1.619443	1.370298	Y	1.297150	1270.089966
9	AACAAB	2.222953	1.880960	Y	1.551582	1398.910034
10	AACAAC	2.166667	1.833333	N	1.532063	1384.390015
11	AACABB	1.661757	1.406102	N	1.318591	1293.459961
12	AACABC	1.637846	1.385870	N	1.306550	1278.939941
13	AACBAB	2.124591	1.797731	Y	1.516970	1375.540039
14	AACBAC	2.075290	1.756015	N	1.498715	1361.020020
15	AACBBB	1.619443	1.370298	Y	1.297150	1270.089966
16	AACBBC	1.597288	1.351551	Y	1.285680	1255.569946
17	ABBAAB	2.177582	1.842569	N	1.535908	1390.059937
18	ABBAAC	2.124591	1.797731	Y	1.516970	1375.540039
19	ABBABB	1.642546	1.389846	N	1.308930	1284.609985
20	ABBABC	1.619443	1.370298	Y	1.297150	1270.089966
21	ABBBAB	2.084876	1.764126	Y	1.502313	1366.689941
22	ABBBAC	2.038229	1.724655	Y	1.484575	1352.170044
23	ABBBBB	1.601646	1.355239	N	1.287950	1261.239990
24	ABBBBC	1.580204	1.337096	Y	1.276721	1246.719971
25	ABCAAB	2.124591	1.797731	Y	1.516970	1375.540039
26	ABCAAC	2.075290	1.756015	N	1.498715	1361.020020
27	ABCABB	1.619443	1.370298	Y	1.297150	1270.089966
28	ABCABC	1.597288	1.351551	Y	1.285680	1255.569946
29	ABCBAB	2.038229	1.724655	Y	1.484575	1352.170044
30	ABCBAC	1.994578	1.687720	Y	1.467452	1337.650024
31	ABCBBB	1.580204	1.337096	Y	1.276721	1246.719971
32	ABCBBC	1.559601	1.319662	N	1.265780	1232.199951

Combination		C_p	C_{pk}	DP (6σ level)	DS	$\mathcal{K}$ \$/unit
33	BABAAB	1.967665	1.664948	Y	1.456635	1318.689941
34	BABAAC	1.928308	1.631645	Y	1.440448	1304.170044
35	BABABB	1.546633	1.308689	N	1.258817	1213.239990
36	BABABC	1.527299	1.292330	N	1.248327	1198.719971
37	BABBAB	1.898468	1.606396	N	1.427882	1295.319946
38	BABBAC	1.863046	1.576423	N	1.412625	1280.800049
39	BABBBB	1.512344	1.279676	N	1.240122	1189.869995
40	BABBBC	1.494253	1.264368	Y	1.230088	1175.349976
41	BACAAB	1.928308	1.631645	Y	1.440448	1304.170044
42	BACAAC	1.891222	1.600264	Y	1.424790	1289.650024
43	BACABB	1.527299	1.292330	N	1.248327	1198.719971
44	BACABC	1.508673	1.276569	N	1.238094	1184.199951
45	BACBAB	1.863046	1.576423	N	1.412625	1280.800049
46	BACBAC	1.829535	1.548068	Y	1.397849	1266.280029
47	BACBBB	1.494253	1.264368	Y	1.230088	1175.349976
48	BACBBC	1.476796	1.249597	N	1.220295	1160.829956
49	BBBAAB	1.898468	1.606396	N	1.427882	1295.319946
50	BBBAAC	1.863046	1.576423	N	1.412625	1280.800049
51	BBBABB	1.512344	1.279676	N	1.240122	1189.869995
52	BBBABC	1.494253	1.264368	Y	1.230088	1175.349976
53	BBBBAB	1.836092	1.553616	Y	1.400767	1271.949951
54	BBBBAC	1.803990	1.526453	N	1.386356	1257.430054
55	BBBBBB	1.480238	1.252509	N	1.222234	1166.500000
56	BBBBBC	1.463263	1.238145	Y	1.212624	1151.979980
57	BBCAAB	1.863046	1.576423	N	1.412625	1280.800049
58	BBCAAC	1.829535	1.548068	Y	1.397849	1266.280029
59	BBCABB	1.494253	1.264368	Y	1.230088	1175.349976
60	BBCABC	1.476796	1.249597	N	1.220295	1160.829956
61	BBCBAB	1.803990	1.526453	N	1.386356	1257.430054
62	BBCBAC	1.773515	1.500667	Y	1.372381	1242.910034
63	BBCBBB	1.463263	1.238145	Y	1.212624	1151.979980
64	BBCBBC	1.446858	1.224264	Y	1.203239	1137.459961

Table 4: DP, DS, and delivery cost for combinations of near optimal service providers for each stage of plastic industry supply chain

VPA tries to choose a pair of DP and DS which suit the requirement in the best possible manner. For example, from the above plot it is clear that DS corresponding to DP=6σ is less than 1.4, therefore the point for which DS=1.4 is chosen as design point, even though the corresponding DP=6.15645σ is a little higher than 6σ. The reason is that this point suits the design requirements in the best possible manner.

- The curve can be divided into two parts
  - From $C_{pm} = 0$ to the $C^g_{pm} = 0.383359$ (point of global minima). In this part delivery cost decreases as the quality of delivery increases.
  - From $C^g_{pm} = 0.383359$ to $\overline{C_{pm}} = 2.1667$. In this part delivery cost increases as the quality of delivery increases.

  The behavior of second part of the curve is consistent with our intuition but the first part is counter-intuitive. A justification behind this is as follows. As described earlier, for a given actual yield α, there exist upper bounds and lower bounds for C_p and C_{pk}. In a similar way it is possible to get the lower bound for C_{pm} value also. It implies that in order to achieve a specified level of precision, a minimum level of accuracy is a must. If accuracy of the process is lower than that minimum level, then no matter how much effort one puts in, the precision can never reach the specified level. In one sentence it can be summarized as *"For being precise, one should be accurate also."* This is the fundamental cause behind the behavior of the curve in its first part. In the first part of the curve, accuracy is so low that even achieving a relatively low precision itself is very costly but in the second part of the curve accuracy is so high that achieving such high accuracy itself is very expensive.

- Table 5 lists some sample values of decision variables σ_i and optimal delivery costs at some representative DP values along with corresponding DS values.

DP	DS	σ_1 (days)	$\sigma_2 = \sigma_4$ (days)	$\sigma_3 = \sigma_6$ (days)	σ_5 (days)	Cost ($/item)
6σ	1.37214	0.694302	0.508311	0.267497	0.592825	785.952
5σ	1.16366	0.788656	0.746783	0.294104	0.745988	668.365
4σ	0.90195	0.885917	1.202550	0.319801	0.942760	525.029
3σ	0.61113	0.971198	2.126740	0.341016	1.160770	358.195

Table 5: Sample values for decision variables at some representative DP and DS pairs

The following is an interesting observation made through these sample values. As the quality level increases, variance of end-to-end lead time Y (i.e. σ) decreases. In order to accommodate such reduction in σ, variance σ_i of individual process(es) reduces. Observe that the processes which are expensive, for example Procurement, undergo a very little change in variance. However, the cheaper processes, such as sheet fabrication and manufacturing, are used as a vehicle to reduce the variance. The reason behind it is reducing the variance of cheaper processes is more cost effective than reducing the variance of expensive processes for achieving the same quality level of end-to-end lead time.

7 Summary and Future Work

In this paper, we have presented a novel approach to achieve variability reduction, synchronization, and therefore delivery performance improvement in supply chain networks. Our approach exploits connections between design tolerancing in mechanical assemblies and lead time compression in supply chain networks. The specific problem we solved here is the variance pool allocation problem. The VPA problem distributes a pool of variance across individual stages of a supply chain in a cost effective way, so as to achieve desired levels of delivery performance.

The contributions of this paper can be summarized as follows.

- Explaining the relevance of process capability indices C_p, C_{pk}, and C_{pm} in describing variability effects in the end-to-end supply chain delivery process.
- Defining two performance metrics, delivery probability and delivery sharpness, to describe the precision and accuracy aspects of supply chain deliveries
- Generalizing the notion of Motorola six sigma quality to include delivery sharpness
- Formulating the variance pool allocat ion problem, an important design optimization problem in supply chains
- Proposing a five step approach to solve the VPA problem for linear supply chains
- Illustrating the efficacy of the approach through a six stage plastics industry case study, by solving a specific problem, namely choosing an optimal mix of service providers in the supply chain stages.

The paper leaves plenty of room for further work in several directions. The VPA problem has been investigated only for linear supply chains. Apart from computational reasons, there is no major difficulty in solving the VPA problem for supply chains with non-linear flows. VPA is only one of a rich variety of design optimization problems that one can address in the framework developed in this paper. An immediate problem that would strike one here is allocation of a pool of nominals among individual business processes. This has implication for choosing resources such as logistics and suppliers in an optimal way. Choice of an optimal mix of customer orders is another problem that could be attempted in this framework. In a companion article [12, 13], we have looked at an inventory allocation problem in a multi-echelon supply chain, where we use the framework (variance pool allocation) developed in this paper to determine optimal inventory levels in different supply chain stages.

The supply chain example that we have looked at belongs to the MTO type. Here again, there is no reason why our approach cannot be applied for coordination types other than MTO, such as MTS and BTO (Build to Order). In fact, in the framework that we have developed in this paper, one can address almost any type of design optimization problem with variability reduction as the basic strategy. In a related paper [12] and a Master's thesis [37], we have defined the general notion of a *six sigma supply chain* and presented a general mathematical programming problem for supply chain design optimization.

Finally, variability is certainly not lead time alone. Variation is fundamental to any metric or process. In this paper, we have emphasized lead time (motivated by the importance of time based competition). End-to-end lead time is an encompassing metric that takes into account most aspects of system dynamics (for example, resource contention, queuing, inventories, etc.). The framework discussed in the paper will apply equally well to any metric other than lead time too. In general, let $X_1, \ldots, X_n$ represent n random variables that describe n phenomena in any system and let Y be a performance metric of interest, which is given as, $Y = f(X_1, \dots, X_n)$, where f is a deterministic function (analytic or computational). The framework developed here will apply to any Y, as long as f is known deterministically.

References

[1] Roshan Gaonkar, *Dynamic Configuration and Synchronization of Collaborative E-Supply Chain Networks*, Ph.D. thesis, Department of Mechanical Engineering, National University of Singapore, Singapore, 2002.

[2] N. Viswanadham and R.S. Gaonkar, "Strategic sourcing and collaborative planning in internet-enabled supply chain networks producing multi-generation products," Tech. Rep., Department of Mechanical Engineering, National University of Singapore, Singapore, 2003.

[3] M. J. Maloni and W.C. Benton, "Supply chain partnership: Opportunities for operations research," *European Journal of Operational Research*, vol. 101, pp. 419–29, 1997.

[4] W. H. Lp, Min Huang, K.L. Yung, and Dingwei Wang, "Genetic algorithm solution for a risk-based partner selection problem in a virtual enterprise," *Computers & Operations Research*, vol. 30, pp. 213–231, 2003.

[5] V. E. Kane, "Process capability indices.," *Journal of Quality Technology*, vol. 18, pp. 41–52, 1986.

[6] Samuel Kotz and Cynthia R. Lovelace, *Process Capability Indices in Theory and Practice*, Arnold, 1998.

[7] M. J. Harry, "The nature of six sigma quality," Tech. Rep., Motorola Inc., Motorola University Press, Schaumburg, IL 60196-1097, 1987.

[8] M. J. Harry and R. Stewart, "Six sigma mechanical design tolerancing," Tech. Rep., Motorola Inc., Motorola University Press, Schaumburg, IL 60196-1097, 1988.

[9] Hasiang TC and Taguchi G, "A tutorial on quality control and assurance - the taguchi methods," in *ASA Annual Meeting*, Las Vegas, Nevada, 1985.

[10] A. Mathur and K.R. Pattipati, "Single and multiobjective optimization problems in robust parameter design," *Sadhana*, vol. 22, no. 1, pp. 5–32, 1997.

[11] A. Song, A. Mathur, and K.R. Pattipati, "Design of process parameters using robust design techniques and multiple criteria optimization," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 25, no. 6, pp. 1437–1446, 1995.

[12] D. Garg, Y. Narahari, and N. Viswanadham, "Design of six sigma supply chains," *to appear in: IEEE Transactions on Automation Science and Engineering (first Issue)*, July 2004.

[13] D. Garg, Y. Narahari, and N. Viswanadham, "Design of six sigma supply chains," in *IEEE International Conference on Robotics and Automation, ICRA-2003*, Taipei, September 2003.

[14] W. J. Hopp, M. L. Spearman, and D. L. Woodruff, "Practical strategies for lead time reduction," *Manufact. Rev.*, vol. 3, no. 2, pp. 78–84, 1990.

[15] P.S. Adler, A. Mandelbaum, V. Nguyen, and Schwerer E., “From project to process management: An empirically-based framework for analyzing product development time,” *Manag. Sci.*, vol. 41, no. 3, pp. 458–484, 1995.

[16] Y. Narahari, R. Sudarsan, K.W. Lyons, M.R. Duffey, and R.D. Sriram, “Design for tolerancing of electro-mechanical assemblies: An integrated approach,” *IEEE Transactions on Robotics & Automation*, vol. 15, no. 6, pp. 1062–1079, 1999.

[17] Jackson S. Chao and Stephen C. Graves, “Reducing flow time in aircraft manufacturing,” *Production and Operations Management*, vol. 7, no. 1, pp. 38–52, 1998.

[18] W. J. Hopp and M. L. Spearman, *Factory Physics: Foundations of Manufacturing Management*, McGraw-Hill, New York, 1996.

[19] Y. Narahari, N. Viswanadham, and R. Bhattacharya, “Design of synchronized supply chains: A six sigma tolerancing approach,” in *IEEE International Conference on Robotics and Automation, ICRA-2000*, San Francisco, April 2000.

[20] D. Garg, Y. Narahari, and N. Viswanadham, “Achieving sharp deliveries in supply chains through variance pool allocation,” in *IEEE International Conference on Robotics and Automation, ICRA-2002*, Washington DC, May 2002.

[21] Russell A. Boyles, “The Taguchi capability index.,” *Journal of Quality Technology*, vol. 23, no. 1, pp. 17–26, 1991.

[22] D. Garg and Y. Narahari, “A process capability indices based approach for supply chain performance analysis,” in *International Conference on Energy, Automation and Information Technology EAIT-2001*, IIT Kharagpur, December 2001.

[23] David H. Evans, “Statistical tolerancing: The state of the art part I. Background,” *Journal of Quality Technology*, vol. 6, no. 4, pp. 188–95, 1974.

[24] David H. Evans, “Statistical tolerancing: The state of the art part II. Methods for estimating moments,” *Journal of Quality Technology*, vol. 7, no. 1, pp. 1–12, 1975.

[25] David H. Evans, “Statistical tolerancing: The state of the art part III. Shifts and drifts,” *Journal of Quality Technology*, vol. 7, no. 2, pp. 72–76, 1975.

[26] Madhav S. Phadke, *Quality Engineering Using Robust Design*, Prentice Hall PTR, Englewood Cliffs, New Jersey 07632, 1989.

[27] U. Roy, R. Sudarsan, Y. Narahari, R. D. Sriram, K. W. Lyons, and M. R. Duffey, “Information models for design tolerancing: From conceptual to the detail design,” Tech. Rep. NISTIR6524, National Institute of Standards and Technology, Gaithersburg, MD 20899, May 2000.

[28] Y Narahari, R. Sudarsan, K. W. Lyons, M. R. Duffey, and R. D. Sriram, “Design for tolerancing of electro-mechanical assemblies: An integrated approach,” Tech. Rep. NISTIR6223, National Institute of Standards and Technology, Gaithersburg, MD 20899, September 1998.

[29] J-S Song and D. Yao, “Performance analysis and optimization of assemble-to-order system with random lead times,” *Operations Research*, vol. 50, pp. 889–903, 2002.

[30] J-S Song, S. Xu, and B. Liu, "Order-fulfillment performance measures in an assemble-to-order system with stochastic leadtimes," *Operations Research*, vol. 47, pp. 131–149, 1999.

[31] Jing-Sheng Song, "The effect of leadtime uncertainty in a simple stochastic inventory model," *Management Science*, vol. 40, no. 5, pp. 603–13, 1994.

[32] Leroy B. Schwarz and Z. Kevin Weng, "The design of a JIT supply chain: The effect of leadtime uncertainty on safety stock," *Journal of Business Logistics*, vol. 21, no. 2, pp. 231–53, 2000.

[33] Y. Lu, J-S Song, and D. Yao, "Order fill rate, lead time variability, and advanced demand information in an assemble-to-order system," *Operations Research*, vol. 51, pp. 292–308, 2003.

[34] R. Hariharan and P. Zipkin, "Customer-order information, lead times, and inventories," *Management Science*, vol. 41, pp. 1599–1607, 1995.

[35] Sunil Chopra, Gilles Reinhardt, and Maqbool Dada, "The effect of lead time uncertainty on safety stocks," *Decision Sciences*, vol. 35, no. 1, 2004.

[36] DC Montgomery, *Introduction to Statistical Quality Control*, John Wiley and Sons, New York, 1996.

[37] D. Garg, "Design of six sigma supply chains," M.S. thesis, Department of Computer Science and Automation, Indian Institute of Science, Bangalore, India. URL: http://people.csa.iisc.ernet.in/∼dgarg, May 2002.